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The Growth-Inequality Nexus without Borrowing Restrictions and Government Intervention: Some Implications from a Barebones Model *

Isaac Ehrlich†

Department of Economics,
State University of New York at Buffalo,
Buffalo, NY 14260-1520, USA.

and

Chi-Wa Yuen‡

School of Economics and Finance,
University of Hong Kong, Pokfulam, Hong Kong.
E-mail: cwyuen@hku.hk

Using a prototype human capital based growth model without borrowing restrictions and government intervention, we study the dynamic evolution of aggregate output and income inequality. We show how even barebones models can yield some testable implications about the growth-inequality relation that may square nicely with the empirical reality. We also provide some useful speculations about this relation over different stages of economic development.

Key Words: Endogenous growth; Income inequality; Human capital; Family economics; Barebones model
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† Leading Professor of Economics and Melvin H. Baker Professor of American Enterprise
‡ Corresponding author. Associate professor of economics and finance.

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1. INTRODUCTION

In the recent literature on growth and distribution[1], two questions have been placed at the center-stage: (1) How does (initial) income or wealth inequality affect growth in income per capita? (2) How does output growth affect earnings inequality?

Concerning the first question, most empirical studies based on cross-country regressions of average GDP growth on some measure of initial inequality have arrived at the same conclusion that inequality is harmful for growth.[2] By classifying countries into rich and poor categories, however, Barro (2000) has recently found that this conclusion applies only to poor countries—that inequality can actually be beneficial for growth in rich countries.

The second question is much harder to deal with. Theoretically, both income growth and income inequality are endogenous variables that evolve simultaneously along an economy's development path—especially in the context of endogenous growth models. Empirically, therefore, it is not straightforward to simply label output growth as an independent variable and earnings inequality as a dependent variable in a single regression equation without worrying about simultaneous equation bias and related estimation problems. In this regard, the literature has chosen to focus on how inequality changes as income in an economy grows. In particular, the focus is on whether Kuznets' (1955) inverted-U relation—whereby inequality first rises and ultimately falls during the development process—holds. Despite Barro's (2000) recent finding that the Kuznets curve is a clear empirical regularity, the overall evidence on such relation has been mixed.

Not all the papers in the theoretical branch of this literature produce results that are consistent with the empirical findings we have just described. In explaining why inequality may be detrimental to growth, however, most of them assume that growth is generated endogenously by human capital investment that may somehow be constrained by borrowing restrictions in the presence of imperfect capital markets. Such market imperfections are singled out as one important determinant of the inequality-growth relation. Another determinant that has also been highlighted in this line of research is government intervention in the form of income redistribution either in money terms or in kind (say, through some tax-transfer schemes).[3]

In this paper, we would like to take a step backward and ask what the fundamental relation between growth of per capita income and the

[1]See, for instance, the surveys by Aghion, Caroli, and García-Peñalosa (1999) and Bénabou (1996).

[2]See, e.g., Alesina and Rodrik (1994), and Persson and Tabellini (1994). See also Li and Zou (1999) for an exception.

[3]See Galor and Zeira (1993) for an example of the first determinant, and Persson and Tabellini (1994) for an example of the latter.

distribution of incomes across households would look like *theoretically* in an ideal world where both capital market imperfections and government redistribution are absent. This is not to deny the significance of these two factors in actually accounting for the *observed* relation between income growth and income distribution. Rather, our objective in carrying out this theoretical exercise is to better understand whether and where, if any, the dual assumptions of *laissez faire* and absence of borrowing constraints may fail to explain the growth-inequality relation *empirically*. For this reason, we shall resort only to a prototype growth model with a minimal set of economic features that is necessary to study the dynamic evolution of aggregate output and income inequality simultaneously—hence, the term "barebones model" in the title of the paper.

What exactly are the bare bones that we would like to incorporate into our model? They include the following five salient features:

(1) endogenous (rather than exogenous) growth—driven by

(2) investment in human capital—so we can talk about inequality in labor earnings (rather than wealth inequality)—by

(3) heterogeneous households—characterized by differences in initial endowments of human capital and/or innate abilities—which may interact with one another through work in

(4) a perfectly competitive labor market—and through

(5) knowledge spilllovers during the process of human capital formation.

Here, as in Ehrlich and Lui (1991), we identify families as providing the institutional support mechanism for aggregate growth in the economy at large. As we shall see, the last feature listed above facilitates the transmission of income growth from rich to poor families and thus stimulates growth for the macroeconomy over time.

The main point we would like to make in this paper is that even barebones models can yield (i) some testable implications about the growth-inequality relation that may square nicely with the empirical reality; and (ii) some rich implications that more complicated models in the literature have not tackled. In particular, we are able to provide some useful speculations about the relation between income growth and income inequality over different stages of economic development—something that has not been given too much attention by the recent literature.

The rest of the paper is organized as follows. Section 2 lays out our barebones model and derives optimal rules for human capital investment as well as equilibrium in the labor market. Section 3 examines the dynamics of growth and inequality under both stagnant and persistent growth conditions. Section 4 concludes with an assessment of the empirical implications from our simple model. The growth and inequality issues can be addressed in the context of either an overlapping generations framework or a dynasty framework. We choose the former for its slightly richer demographic structure, but shall also spell out some similarity between the two in the Appendix.

2. THE BAREBONES OVERLAPPING GENERATIONS GROWTH MODEL WITH HETEROGENEOUS FAMILIES

Consider an overlapping generations economy populated by a continuum of heterogeneous families, indexed by i in the unit interval $[0,1]$. *Inter alia*, we focus on two possible sources of heterogeneity, viz., (i) differences in endowments of raw human capital at birth ($\overline{H}^i$); and (ii) differences in innate learning abilities (A^i).[4]

2.1. The demographic structure

Each agent lives three periods: childhood (0), working age (1), and old age (2). For simplicity, we abstract from marriage and fertility decisions within families. Each person is brought up by her single parent, gives birth to a single child after reaching adult age, and passes away after her child has grown up to become the income earner for the family. Hence, the size of each family equals the constant "three" in any given period (i.e., child + parent + grand-parent). As a child, one only eats, enjoys leisure (plays and sleeps), and receives education. As an adult, one has to work, feed her kid and mom (in addition to herself), and educate the kid. As an old-age person, one retires and enjoys life (consumes goods and leisure).[5]

2.2. Individual endowments and family division rule

We label an agent who is born into family i at date $t-1$ and becomes an adult at date t as the (i,t) agent. Young age is emphasized in such labels because all decisions within a family is made by the adult worker—who has to plan for both her child's future and her own retirement needs as well as to take care of her old-age mother. The (i,t) agent possesses a

[4] As explained in Ehrlich, Yuen, and Zhong (1998), the former may include such things as different life expectancies and different cost conditions associated with investments in children's education due to families' differential access to funds. Together with the latter, these sources of heterogeneity reflect differences arising from states of nature and market opportunities (i.e., family constraints) rather than from individual/family preferences.

[5] As Ehrlich and Lui (1991) make clear, the presence of a child-rearing cost (in addition to the child education cost) will induce a dominance of investment in child quality over investment in child quantity, thus yielding a corner solution in fertility at its minimum possible level, if the only motive linking the overlapping generations is mutual material benefits. This is because the cost of bringing an additional child to the world automatically entails the subsequent cost of educating that child, whereas the cost of the latter activity is technically independent of that of bearing the child. This argument may provide a justification for our assumption of a *single* child and time-invariant family size.

total productive capacity of $\overline{H}^i + H_t^i$ at date t—made up of a 'raw' stock of knowledge ($\overline{H}^i$) and a 'refined' asset of human capital (H_t^i). She is endowed with one unit of non-leisure time,[6] which she can split between child education (h_t^i) and work ($1 - h_t^i$). The amount of effective labor supplied by the (i,t) agent to the labor market is $(1-h_t^i)(\overline{H}^i + H_t^i)$. At the competitive market wage of w_t per effective labor unit, therefore, the household income for the i^{th} family in period t equals $w_t(1-h_t^i)(\overline{H}^i + H_t^i)$.

Suppose there is a social norm in this economy that dictates the following division rule within each family: The family income is shared among the three overlapping generations, with a fraction $\theta_j \in (0,1)$—assumed to be family-invariant and time-invariant—going to the age-j ($j = 0,1,2$) member such that $\theta_0 + \theta_1 + \theta_2 = 1$. This sharing rule can be interpreted either as a reflection of altruism or as a result of an implicit contract between the living generation and the yet-to-be born generation, "signed" at the point when the former is about to decide whether to give birth to the latter.[7] Under this rule and assuming that wages are the only source of family income,[8] the consumption flows of the (i,t) agent are determined as follows:

$$c_{0,t-1}^i = (1-\theta_1-\theta_2)w_{t-1}(1-h_{t-1}^i)(\overline{H}^i + H_{t-1}^i); \tag{0}$$

$$c_{1,t}^i = \theta_1 w_t(1-h_t^i)(\overline{H}^i + H_t^i); \tag{1}$$

$$c_{2,t+1}^i = \theta_2 w_{t+1}(1-h_{t+1}^i)(\overline{H}^i + H_{t+1}^i). \tag{2}$$

In principle, one can label $c_{0,t-1}^i$ as bequests from her (young age) mother, and $c_{2,t+1}^i$ as gifts from her (adult) daughter. For the sake of simplicity, however, we shall abstract from explicit modelling of altruistic bequest and gift motives in what follows.

[6] Implicitly, we assume inelastic leisure although the amount of leisure time for each agent can vary across ages. By leisure, we include also the time people spend with their family members, e.g., time devoted to child-rearing as well as to keeping their folks company.

[7] This idea of implicit inter-generational intra-familial contracts is explored in detail by Ehrlich and Lui (1991). See also Ehrlich, Yuen, and Zhong (1998), which extends its single agent framework to a heterogeneous agent setting.

[8] In other words, we are assuming the absence of savings and capital income (or income from non-human assets for that matter).

2.3. Human capital formation

The law of motion of human capital in the i^{th} family is given by:

$$H_{t+1}^i = A^i h_t^i(\overline{H}^i + H_t^i)(E_t^i)^\xi, \tag{3}$$

where $\xi \in [0,1]$ is an externality parameter, and

$$E_t^i \equiv \frac{\overline{H}^1 + H_t^1}{\overline{H}^i + H_t^i} \geq 1 \quad \text{for } i \leq 1, \tag{4}$$

represents the effect of knowledge spillovers from family 1—the smartest or most human-capital-rich family, with 'raw' human capital of $\overline{H}^1$ and 'refined' human capital of H_t^1—onto the i^{th} family,[9] and serves as an inequality index in our model.

This human capital production function captures both intra-familial intergenerational and inter-familial intra-generational transmissions of knowledge and skills, the former being a result of conscious family decision and the latter a purely unintended, external effect. This latter effect is generated in the process of human capital accumulation when agents with poorer knowledge/skill mingle with those (in the same cohort) of superior knowledge/skill either at 'school' or at 'work'. *Ex ante*, agents do not know for sure whether they will turn out to be the top one, or are unable to take advantage of their (possibly superior) ultimate position *a priori*. In this sense, the spillover effect generated by the top family is 'external' to all other families. *Ex post*, the family that turns out to be at the top cannot benefit from knowledge spillovers from other families. For this particular family ($i = 1$), $E_t^1 = 1$ so that its human capital production function becomes effectively linear, i.e.,

$$H_{t+1}^1 = A^1 h_t^1(\overline{H}^1 + H_t^1). \tag{3'}$$

2.4. The utility maximization problem facing the family decision-maker

[9] Two points are worth noting about this spillover effect. First, implicit in the presence of this effect is the assumption that human capital is formed through formal schooling and/or on-the-job learning, instead of, or in addition to, family education. Second, for simplicity, we model it as a trickle-down merely from the topmost family without allowing for possible spillover from, say, the $i = 0.7$ family to the $i = 0.1$ family. The bigger the inequality between the 1^{st} family and the i^{th} family, the stronger is this trickle down effect.

The (i, t) agent is selfish and derives utility U_t^i merely from her own lifetime consumption (i.e., $c_{0,t-1}^i, c_{1,t}^i, c_{2,t+1}^i$):

$$U_t^i = \beta^{-1} \ln(c_{0,t-1}^i) + \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i), \tag{5}$$

where $\beta \in (0, 1)$ is the usual subjective discount factor. The optimization problem that the (i, t) agent faces involves choosing $\{c_{0,t-1}^i, c_{1,t}^i, c_{2,t+1}^i, h_t^i$ or $H_{t+1}^i\}$ to maximize (5) subject to the consumption division rules (0), (1), and (2) as well as the human capital accumulation equation (3), taking the spillover effect E_t^i and the wage rate w_t as given. But since $c_{0,t-1}^i$ is exogenously given to agent (i, t) when she is born at date $t - 1$ and is irrelevant to her decision making at date t, she can ignore equation (0) as a constraint and revise her objective function (5) to:

$$V_t^i = \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i). \tag{5$'$}$$

Selfish as she is, the (i, t) agent is still willing to invest in child education because her child's earning ability when grown up (H_{t+1}^i), which depends on h_t^i through (3), will directly affect her old age consumption through (2). Substituting constraints (1), (2), and (3) into (5$'$) and maximizing the resulting function with respect to h_t^i yields a first-order condition that can be used to solve for h_t^i as follows:

$$h_t^i = \frac{\beta A^i(\overline{H}^i + H_t^i)(E_t^i)^\xi - \overline{H}^i}{(1+\beta)A^i(\overline{H}^i + H_t^i)(E_t^i)^\xi}. \tag{6}$$

Plugging this into equation (3) and rearranging terms, we obtain a reduced form evolution equation for human capital:

$$H_{t+1}^i = a_t^i H_t^i + b_t^i, \tag{7}$$

where

$$a_t^i = \left(\frac{\beta}{1+\beta}\right) A^i (E_t^i)^\xi, \tag{7a}$$

and

$$b_t^i = \left(a_t^i - \frac{1}{1+\beta}\right) \overline{H}^i. \tag{7b}$$

Observe that a_t^i and b_t^i both depend on E_t^i, which in turn depends on H_t^i. Hence, H_{t+1}^i is actually a nonlinear function, say, $G_t^i(.)$, of H_t^i. Indeed,

a_t^i and b_t^i are increasing functions of E_t^i, so $G_t^i(.)$ is also time-varying. The pseudo-linear representation in (7) is chosen to highlight the growth-inequality relation at the family level. In particular, the 'slope' parameter, a_t^i, which has the interpretation of the marginal growth rate of human capital in the i^{th} family, rises over time with the degree of income inequality, E_t^i.[10] This implies that inequality is beneficial for the growth of relatively poor families. Note, on the other hand, that for the highest-skill family, $E_t^1 = 1$, implying that a^1 and b^1 no longer depend on H_t^1. The law of motion of human capital for such family is thus linear and stationary from the outset.

2.5. Production technology, the representative firm's profit maximization problem, and labor market equilibrium

Above, we have described the supply side of the labor market. On the other side, there is a representative firm run collectively by all the families in the economy, hiring a total of L_t^d units of effective labor at the competitive wage w_t in each period t and using them to produce an amount of output Y_t through a simple linear technology: $Y_t = KL_t^d$ (where $K > 0$ is a production efficiency parameter). Hence, the firm's date-t problem is to choose L_t^d to maximize profit $KL_t^d - w_t L_t^d$, implying an equilibrium time-invariant wage rate of $w_t = K$.

Since the total supply of effective labor L_t equals $\int_0^1 (1-h_t^i)(\overline{H}^i + H_t^i)di$, equilibrium in the labor market requires $L_t^d = \int_0^1 (1-h_t^i)(\overline{H}^i + H_t^i)di$. As a result, aggregate equilibrium output is given by:

$$Y_t = K \int_0^1 (1-h_t^i)(\overline{H}^i + H_t^i)di. \tag{8}$$

We can also obtain the economy-wide resource constraint, hence verify the Walras law, by aggregating the budget constraints (0), (1), and (2) across families at any given point in time (say, t), i.e.,

$$C_t \equiv \int_0^1 (c_{0,t}^i + c_{1,t}^i + c_{2,t}^i)di = w_t \int_0^1 (1-h_t^i)(\overline{H}^i + H_t^i)di = Y_t.$$

Note that, for simplicity, we have not introduced a credit market to allow for private borrowing and lending. Such omission does not imply the possibility of knowledge accumulation being limited by borrowing restrictions,

[10] The 'slope' parameter a_t^i is also increasing in the subjective discount factor β and the ability parameter A^i. So is the 'intercept' parameter b_t^i, which also increases with the raw human capital at birth $\bar{H}^i$. All this is pretty intuitive. What may seem weird, though, is that h_t^i as well as a_t^i and b_t^i do not depend on the parameters $(\theta_0, \theta_1, \theta_2)$ governing the family division rule. This special property is a result of the logarithmic utility function assumed in (5), and will not hold for more general preference specifications.

however, as explicit costs of human capital investment (such as tuition fees) are not modelled either. In addition, as emphasized by the title of the paper, we have abstracted from modelling any form of government intervention in the labor market.

3. STAGNATION VS. PERSISTENT GROWTH: THE DYNAMICS OF GROWTH AND INEQUALITY

Given the equilibrium laws of motion of human capital (7) for the various families, we are now ready to analyze the evolution of the distribution of skills in both the short and long runs. Following the convention of the growth literature, we define the 'long run' as the steady-state growth path and the 'short run' as the path along which the family/economy transits from an initial position to a steady-state equilibrium position. In the context of our model, a steady state requires that the fractions of productive capacity (or time) devoted to educating children (h_t^i) be asymptotically constant. From equation (6), this will be true only if the inequality index converges to a constant level over time. How does this convergence come about? In addressing this difficult issue of the dynamics of growth and inequality, we shall first treat two special cases before returning to the more general setup.

3.1. Special case 1: Homogeneous families

Let us begin by considering the case where the initial endowments and all basic parameters are the same across families. It is then trivial to see that there will not exist any skill or income inequality over time—the economy simply becomes a blown-up version of the single family. The homogeneity of agents also eliminates the inter-familial spillover of knowledge. Given $E_t^i \equiv 1$, H_{t+1}^i in equation (7) becomes a linear and stationary function of H_t^i, i.e., $H_{t+1} = aH_t + b$, where $a = \left(\frac{\beta}{1+\beta}\right) A$ and $b = (a - \frac{1}{1+\beta})\overline{H}$. Regarding the economy's growth prospects, there are two possible steady states depending on the value of the marginal (gross) growth rate a.

(i) Stagnant equilibrium. If $a < 1$ in equation (7), then the level of human capital in all families will converge to a stagnant level of $H_\infty \equiv \lim_{t\to\infty} H_t = b/(1-a)$; and

(ii) Persistent growth equilibrium. If $a \geq 1$ ($\Rightarrow b > 0$), all families will grow simultaneously at the same rate in both the short and long runs, with $H_{t+1} = aH_t + b > H_t$ for all t.[11]

Whether the economy is in a stagnant or growth equilibrium depends on the magnitudes of the basic parameters of the model. In particular, higher

[11] The human capital growth rate $g_{H_t} \equiv H_t/H_{t-1} - 1 = (a-1) + b/H_t$ will converge to an asymptotic level of $a-1$ (≥ 0 for $a \geq 1$) as $H_t \to \infty$.

values of β and A will raise a, hence the likelihood of a persistent growth equilibrium.[12]

3.2. Special case 2: Absence of raw human capital at birth

Suppose there is no such thing as 'raw' human capital at birth (i.e., $\overline{H}^i \equiv 0$). In this case, the only shoulder on which a new-born child can build her human capital (H^i_{t+1}) is that of her parent's (H^i_t). Upon eliminating $\bar{H}^i$ as a source of heterogeneity, equation (6) simplifies to:

$$h^i_t = \frac{\beta}{1+\beta} \equiv h, \tag{6$'$}$$

which is both family-invariant and time-invariant. Substituting this into equation (3) yields

$$H^i_{t+1} = A^i h (E^i_t)^\xi H^i_t. \tag{7$'$}$$

Dividing this into $H^1_{t+1} = A^1 h H^1_t$ and recalling the definition of our inequality index from equation (4) with $\overline{H}^i \equiv 0$, we obtain a first-order difference equation in E^i_t as follows:

$$E^i_{t+1} = \left(\frac{A^1}{A^i}\right)(E^i_t)^{1-\xi}. \tag{9}$$

Solution of (9) takes the form

$$E^i_t = \left(\frac{A^1}{A^i}\right)^{\frac{1-(1-\xi)^t}{\xi}} (E^i_0)^{(1-\xi)^t}, \ t \geq 0, \tag{9$'$}$$

given E^i_0 (or H^i_0 and H^1_0), implying a long-run inequality level of

$$E^i_\infty \equiv \lim_{t\to\infty} E^i_t = (A^1/A^i)^{\frac{1}{\xi}} \geq 1 \quad \text{as} \quad A^1 \geq A^i. \tag{9$''$}$$

[See Figure 1 for a diagrammatic representation of equation (9) as well as E^i_∞.] In other words, differences in innate abilities will generate persistent

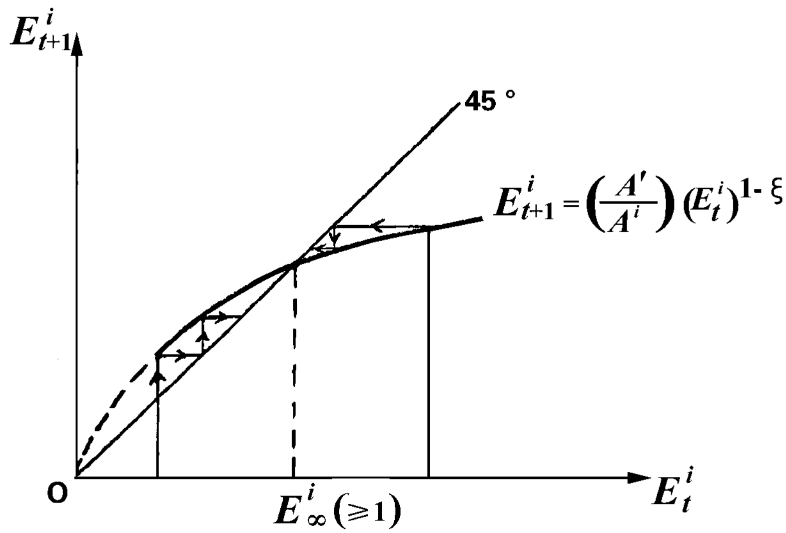
[12] Here, we are describing growth in terms of increase in the stock of 'refined' human capital. Given the production function (8), growth in aggregate output requires:

$$\frac{Y_{t+1}}{Y_t} = \frac{K(1-h_{t+1})(\overline{H}+H_{t+1})}{K(1-h_t)(\overline{H}+H_t)} > 1.$$

Since $\lim_{t\to\infty} h_t$ is constant—so that $h_{t+1} = h_t$—in a steady state equilibrium (whether stagnant or growth), the condition above implies that asymptotic growth in 'refined' human capital is equivalent to asymptotic growth in output.

inequality whereas differences in (initial) parental human capital will not. From this, we can infer that a more 'able' but poorer family will ultimately overtake a less 'able' but richer one. In addition, equation (9) implies monotone convergence and stability; i.e., income inequality will be rising or falling over time depending on whether the initial level (E_t^i) is lower or higher than its long-run counterpart $((A^1/A^i)^{\frac{1}{\xi}})$.

FIG. 1. Growth and inequality in special case 2



In the absence of knowledge spillovers (i.e., $\xi = 0$), however, equation (9) becomes $E_{t+1}^i = \left(A^1/A^i\right) E_t^i$. Hence, $E_t^i = (A^1/A^i)^t E_0^i$ $(t \geq 0)$, implying constant (in)equality if $A^1 = A^i$ and ever-increasing inequality if $A^1 > A^i$. This means that initial conditions in terms of parental human capital matter just as much for persistent inequality as innate abilities. Put differently, the spillover effect is an important equalizing force (absent ability differentials).[13]

The results spelled out in the preceding two paragraphs hold whether the economy is in a stagnant or sustained growth equilibrium. As regards growth possibilities for the economy as a whole, equations (8), (3), and (6′)

[13] Tamura (1991) has also shown how a spillover effect in the human capital formation technology may provide below-average human capital agents with a higher rate of return on investment than above-average human capital agents, so that the former will have a bigger incentive to invest in human capital, and eventually catch up with the latter. In a similar vein, Razin and Yuen (1997) have proved that in an open economy with the extent of knowledge spillovers confined within national boundaries, cross-border labor mobility can facilitate international transmission of knowledge and skills to bring about income convergence across countries.

imply that the gross growth rate of aggregate output equals:

$$\frac{Y_{t+1}}{Y_t} = \frac{\int_0^1 H_{t+1}^i di}{\int_0^1 H_t^i di} = \frac{\left(\frac{\beta}{1+\beta}\right)\int_0^1 A^i H_t^i (E_t^i)^\xi di}{\int_0^1 H_t^i di}.$$

Given $E_t^i \geq 1$ by definition, a sufficient growth condition at the family level at *any* point in time is that $a^i \equiv \left(\frac{\beta}{1+\beta}\right) A^i > 1$ for all i. As the argument in subsection 3.3.2 below makes clear, due to the trickle-down-type spillover effect, it is sufficient for the above condition to apply just to the most able family—i.e., $a^1 \equiv \left(\frac{\beta}{1+\beta}\right) A^1 > 1$—in order to guarantee sustained growth at both the family and macroeconomy levels in the *long run*. Note that, in form, these growth conditions are similar to the one specified in subsection 3.1 above.

3.3. The more general case: Heterogeneity in innate abilities, raw human capital at birth, and parental heritage of human capital; and the crucial role of the leading family

Having discussed two special cases, let us get back to the original setup where heterogeneity can arise in innate abilities (A^i), 'raw' human capital at birth $(\overline{H}^i)$, as well as parental stocks of human capital (H_t^i) across families. In this more general setup, two possible sub-cases can be distinguished.

3.3.1. Case 1: All families are in stagnant equilibria

The case of 100% stagnant families will emerge if

$$a^1 \equiv \left(\frac{\beta}{1+\beta}\right) A^1 < 1, \tag{10}$$

i.e., if the growth condition does not hold even for the top family. Obviously, the economy as a whole will converge to a stationary state in the long run with contant levels of output and human capital. Defining $x_\infty^i \equiv \lim_{t\to\infty} x_t$ for any variable x, equation (7) applied to this stagnant case implies that

$$H_\infty^i = \frac{b_\infty^i}{1 - a_\infty^i} = \left[\frac{\left(\frac{\beta}{1+\beta}\right) A^i (E_\infty^i)^\xi - \left(\frac{1}{1+\beta}\right)}{1 - \left(\frac{\beta}{1+\beta}\right) A^i (E_\infty^i)^\xi}\right] \overline{H}^i. \tag{11}$$

Substituting this expression into the definition of our inequality index (4) and noting that $E_\infty^1 \equiv 1$, we get a nonlinear equation that can be used to solve for the steady state level of inequality E^i_∞ in terms of the underlying parameters $(\beta, \xi, A^1, A^i, \overline{H}^1, \overline{H}^i)$ for $i \neq 1$.[14] A sufficient condition for complete equality (i.e., $E^i_\infty = 1$) is that $A^i = A^1$ and $\overline{H}^i = \overline{H}^1$. In other words, equality in parental human capital (or initial endowments) is not necessary for income equality in the long run—unless there do not exist ability differentials either.

3.3.2. *Case 2: All families are in growth equilibria*

In the case of 100% growth families, the marginal human capital growth rate a^i_t as defined in equation (7a) exceeds unity for all $i \in [0, 1]$. For there to exist steady-state growth in the long run, we require a^i_t to converge to some time-invariant value. From (7a), this requirement in turn restricts E^i_t to be time-invariant in the long run. But E^i_t will be asymptotically constant only if H^i_t and H^1_t both grow at the same asymptotic rate. Such *balanced* growth restriction[15] can be stated as $a^i_t = a^1_t$, which can be simplified to

$$E^i_\infty = (A^1/A^i)^{1/\xi}. \tag{9''}$$

Since this relation (9″) is the same as the one in special case 2 above, the same conclusion that only differences in innate abilities matter-that initial conditions in terms of raw human capital at birth and parental heritage of human capital have no role to play in persistent inequality-carries over to this more general case.

One may wonder why, starting from some initial position where $a^i_t \neq a^1_t$, all families will ultimately converge to the same growth rate. In essence, this is a direct consequence of the knowledge spillovers from human-capital-rich to human-capital-poor families. Recall from the discussion following equations (7a) and (7b) that both the slope (a^i_t) and intercept (b^i_t) terms in the equilibrium law of motion of human capital (7) are increasing in the inequality index (E^i_t), a proxy for the spillovers. So long as some families grow at slower rates than the top family, this trickle-down effect will operate to pull them up. [See Figure 2 for the evolution of H^1_t in panel 1 and of H^i_t $(i \neq 1)$ in panel 2.] The same effect works when some families are initially in stagnant states and others in growth states. In the final analysis, one

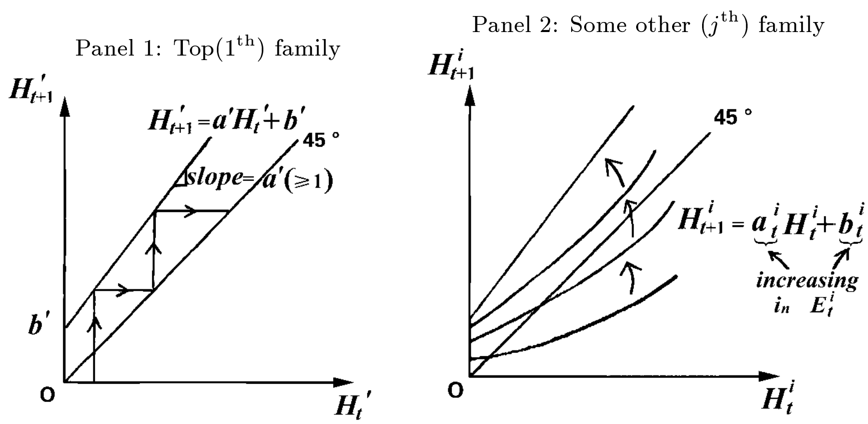
[14] Plugging E^i_∞ into equation (11) yields a solution for H^i_∞ in terms of the same set of parameters. The steady state value of aggregate output is then given by $Y_\infty = K\int_0^1 (1 - h^i_\infty)(\overline{H}^i + H^i_\infty)di$.

[15] In the absence of knowledge spillovers (i.e., $\xi = 0$), however, *balanced* growth will not exist as long as $A^i \neq A^1$. In such cases, inequality will rise without bounds over time.

leading growth family is sufficient to ultimately generate growth in the whole economy.[16] In other words, a sufficient growth condition is:

$$a^1 \equiv \left(\frac{\beta}{1+\beta}\right) A^1 > 1. \tag{10'}$$

FIG. 2. Growth and inequality in the more general case



So far, we have discussed how the long-run value of the inequality index is determined in the stagnant equilibrium and in the growth equilibrium. There is no reason to expect that these two values are equal to one another—especially since the former depends in general on $(\beta, \overline{H}^1, \overline{H}^i)$ whereas the latter does not. In addition, we have not examined how an economy can transit from a stagnant equilibrium to a growth equilibrium and how the inequality level will change during the transition process of economic development. In Ehrlich, Yuen, and Zhong (1998), we provide a more careful analysis of these issues at both the theoretical and empirical levels. In a nutshell, we find there that the growth-inequality relation can be dramatically different across stages of development. Most probably, inequality will first increase when the economy starts taking off on a growth path and will end up converging to some constant level (possibly lower than the initial level) in the long run. In other words, a Kuznets-type inverted-U relation between growth and inequality is likely, though not absolutely necessary.

[16]This result may provide a justification for a "picking the winner" or "letting a select group first get rich" (rather than a more balanced) type growth-enhancing policy.

4. CONCLUSION

In this paper, we have attempted to put down a simple theoretical model that would enable us to organize our thinking about the relation between income growth and income inequality. In carrying out this exercise, we have managed to come up with some intriguing inferences, if not testable propositions.

Regarding the first major issue raised in the introduction (i.e., the effect of inequality on growth), we have shown that inequality is beneficial to the growth of relatively poor families. This is basically a consequence of our assumed production function of human capital with its spillover effects. From this, one may be tempted to jump to the conclusion that inequality would enhance growth in poor countries—given the likelihood that there will be a higher proportion of poor families in poor countries. As such, it may seem that our theoretical result contradicts Barro's (2000) recent finding that initial inequality is harmful for growth in poor countries. Does it therefore imply that we should follow the literature and bring into our otherwise barebones model such complications as capital market imperfections and political economy considerations in government redistribution in order to generate harmful effects of inequality on growth?

The answer is not clear. Our specific prediction about the role of inequality in enhancing the growth rate of relatively poor families is not really about growth effects of *initial* inequality and, in any event, can only be tested against intergenerational family data. Besides, depending on how one interprets it, our prediction is not necessarily inconsistent with Barro's finding. Recall that the inequality level can in principle vary across stages of development. If inequality turns out to be higher in a stagnant state than in a growth state, then it would appear (in the data) that higher inequality is detrimental to growth even though it is also a stimulus that kicks the economy off from the stagnant equilibrium onto a positive growth path.

Turning to the second major issue raised in the introduction (i.e., the effect of growth on inequality), we have seen—in the special case 3.2—how inequality may either rise or fall over time as the economy grows, and—in the more general case 3.3—how inequality may assume a Kuznets-type inverted-U curve as the economy transits from a stagnant equilibrium to a growth equilibrium during the process of development. Roughly, these theoretical results are in agreement with the mixed evidence on the Kuznets curve. More rigorously, what our analysis suggests is the necessity of a more serious test of these hypotheses based on classification of countries into different income categories according to their stages of development. This is taken up in our companion paper (Ehrlich, Yuen, and Zhong (1998)).

As by-products, there are two other natural implications from our barebones model that we have not stressed, but are somehow consistent with two major stylized facts about inequality emphasized recently by the World Bank (based on a comprehensive data set on income distribution)—viz., intertemporal stability and cross-country variability.[17] First, our model predicts a constant level of inequality in the long run, under both the stagnant as well as growth equilibria. This prediction squares well with the relative constancy of inequality measures over time. Second, if we stretch our imagination and think of different countries as possessing different distributions of innate abilities and human capital endowments, then it is easy to see why there may exist some variations in inequality measures across countries.

In a highly stylized barebones model such as the one we have introduced in this paper, there are obviously a lot of omitted but important elements that may help explain the growth-inequality relation. Examples include the child quantity-quality tradeoff, altruism, intergenerational transfers in the form of bequests and gifts, intragenerational transfers in the form of charitable givings, credit market arrangements, and government redistribution. Even though our barebones model is based on *laissez faire*, the assumed presence of intra-generational knowledge spillovers may imply a normative role for government intervention (say, in the form of subsidization of tertiary education), which may in turn accentuate the growth-inequality tradeoff.

In fact, our prototype model can be viewed as a "reduced form" representation of the more elaborate models in Ehrlich, Yuen, and Zhong (1998)—which contains, among other things, an analysis of old age insurance as a basis for implicit intergenerational contracts. Some of the results are also quite similar to those we obtain in that paper. It is only natural that we relegate these extensions to this work-in-progress. (See also Yuen (1999).)

[17]See Li, Squire, and Zou (1998) for a discussion of the two stylized facts and Deininger and Squire (1996) for a description of the data set.

APPENDIX: GROWTH AND INEQUALITY IN THE DYNASTY MODEL

In this appendix, we examine the same issues about growth and inequality in the context of the dynasty framework. The purpose is to show the insensitivity of our key results to this alternative framework. The main advantage of the OLG framerwork from our point of view will be more apparent as soon as we make the intergenerational contracts endogenous, as in Ehrlich, Yuen, and Zhong (1998).

Consider an infinitely lived agent/family $i \in [0,1]$ born at time 0 with preferences given by:

$$U = \sum_{t=0}^{\infty} \beta^t \ln(c_t^i), \tag{A1}$$

where c_t^i is her consumption at date t, and $\beta \in (0,1)$ her subjective discount factor. She faces budget constraints (A2) and human capital accumulation equations (A3) as follows:

$$c_t^i = w_t(1 - h_t^i)H_t^i, \tag{A2}$$

$$H_{t+1}^i = A^i h_t^i H_t^i (E_t^i)^{\xi}, \tag{A3}$$

given an initial (also interpretable as "raw") stock of human capital at birth of H_0^i and the inequality index

$$E_t^i \equiv H_t^1 / H_t^i \geq 1 \quad \text{for} \quad i \leq 1. \tag{A4}$$

As in the text, $i = 1$ is the index applied to the family with the highest human capital. Here, H_t^i is her stock of human capital (or earning capacity) at time t, h_t^i the amount of time she devotes to accumulation of knowledge and skills, and $1 - h_t^i$ the residual time supplied to the labor market to earn a market wage per labor efficiency unit of w_t.

The utility-maximization problem she faces involves choosing $\{c_t^i, h_t^i, H_{t+1}^i\}_{t=0}^{\infty}$ to maximize (A1) subject to (A2) and (A3), given $\{w_t, E_t^i\}_{t=0}^{\infty}$ and H_0^i. Solution to this problem implies $h_t^i = \beta$ for all $t \geq 0$ and $i \in [0,1]$ so that, from (A3), $H_{t+1}^i = \beta A^i H_t^i (E_t^i)^{\xi}$. Dividing it into $H_{t+1}^1 = \beta A^1 H_t^1$ yields a first order difference equation in E_t^i:

$$E_{t+1}^i = \left(\frac{A^1}{A^i}\right)(E_t^i)^{1-\xi}, \tag{A5}$$

which is exactly identical to, and thus shares the same solution with, equation (9) in the main text. In particular, $\lim_{t\to\infty} E_t^i = (A^1/A^i)^{1/\xi} \geq 1$ as $A^1 \geq A^i$.

Under a linear production technology, $Y_t = K\int_0^1 (1-h_t^i)H_t^i di = (1-\beta)KH_t$, where $H_t \equiv \int_0^1 H_t^i di$. The familiar marginal productivity condition (i.e., $w_t = K$) clearly holds. So also does the economy-wide resource constraint (i.e., $C_t \equiv \int_0^1 c_t^i di = Y_t$). Growth in aggregate output requires $Y_{t+1} > Y_t$, which in turn requires growth in the aggregate stock of human capital, i.e., $H_{t+1} > H_t$ or $\beta\int_0^1 A^i H_t^i (E_t^i)^\xi di > \int_0^1 H_t^i di$. A sufficient (but not necessary) growth condition is $A^i > 1/\beta$ for all i, given $E_t^i \geq 1$ by definition. When this condition applies, inequality is beneficial for growth.

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