Equilibrium Cost Overruns[*]

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Cost overruns are endemic in military procurement projects and pervasive in other areas. This paper studies a model in which the apparent cost overruns arise not as systematic expectational errors but as equilibrium phenomena. The possibility of renegotiating payments when cost overruns occur results in firms bidding below their true estimate of expected project costs. This can cause the initial price for a project to be consistently lower than its expected cost, and hence the persistence of cost overruns in equilibrium. The trade-off between selecting the lowest cost source and inducing efficient investment effort is explored. © 2001 Peking University Press

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1. INTRODUCTION

Cost overruns are endemic on military projects and pervasive in certain other areas. Norman Augustine summarizes the evidence for defense projects as "there is only a 10% chance of meeting cost goals, there is a 15% chance of meeting schedule goals and a 70% chance of meeting performance goals" (Augustine, 1986 p341).[1] Whereas failure to meet performance goals

[1] Augustine is a defense engineer, the original version of his Laws was published by the American Institute of Aeronautics and Astronautics. He subsequently built Lockheed Martin into the largest US defense contractor.

can damage a firm's reputation and success in subsequent projects, there appears to be little reputational penalty for failing to meet cost goals. Augustine interprets the pervasive pattern of failing to meet goals in terms of psychologically biased expectations and quotes Thucydides: "Their judgement was based more on wishful thinking than on sound calculation of probabilities; for the usual thing among men is that when they want something they will, without any reflection, leave that to hope, while they will employ the full force of reason in rejecting what they find unpalatable." While such biased expectations are undoubtedly important, economists find it unsatisfactory to assume that agents repeatedly make the same mistakes, as they appear to with cost overruns. This paper investigates an alternative explanation in which the apparent cost overrun arises as an equilibrium of a procurement process where there is a trade-off between source selection and effort inducement and where the government agency is unable to commit not to renegotiate prices when cost overruns occur.

We consider a simple model where a government agency needs to select a firm to produce a product (a weapon system, for instance). Firms have private information about their parameters of cost efficiency. After firms submit cost estimates through some bidding process, a firm is selected for the project with a payment to be received when the project completes. The firm can then choose to make some investment (or effort) in cost reduction; after which but before production starts, the cost of production is realized and becomes publicly known. We can think of this investment as related to finalizing the product design and developing efficient production methods, see Lichtenberg (1995). If the cost realization is higher than the initially agreed payment, it is expected that there will be some renegotiation and a certain percentage of the cost overrun will be paid by the government.

Under the assumption that the competitive bidding is in the form of a second-price, sealed-bid auction, we show that firms always bid below their true estimates of the project cost. This happens because they expect to receive additional payments from the government if cost overruns occur, and thus the price that is necessary for a firm to receive zero expected profit from undertaking the project is lower than the expected cost of the project. We further show that, although in equilibrium the more efficient firm is always selected for the project, there is often underinvestment by the firm that wins the bidding. This comes about because a firm only receives positive profits when the cost realization is below the initially agreed price. When the realized cost is above this price, the firm does not bear the entire cost overrun. Hence, the firm does not fully internalize the social benefits of investment in cost reduction. Although our results are most conveniently illustrated when the competitive bidding is in the form of a second-price, sealed-bid auction, we shall also argue that they can hold in more general forms of procurement processes.

While our study is primarily motivated by the practice in military projects, where there is significant uncertainty about costs and renegotiation and cost sharing are the rule, our model can be applied to a much wider variety of projects. The same problems are characteristic of residential building and software development, for example. When you hold a competitive tender for work on your house you know that the builder's estimate is not going to cover the actual costs which will be revealed once the existing fittings are ripped out and it is discovered what work is actually required. Software development rivals military development for products which cost far more than estimated, are late and do not deliver the performance initially promised, but perhaps may eventually work in the end after large extra expenditures. On software McConnell (1996) provides a range of case studies and Brook (1975) is the classic statement of the problem.

There is a vast theoretical literature on these procurement problems[2]. The text book treatment is Laffont and Tirole (1993). One of the explanations they suggest for systematic cost overruns is that the initial price will only prevail in the good state of the world due to uncertainty of the costs. Our explanation is thus closely related to theirs since it also relies on cost uncertainty and the possibility of renegotiation. What we add to their argument, however, is the idea that because firms have private information about their cost efficiency, the need to choose the more efficient firm for the project requires some competitive process of source selection; and this, coupled with the possibility of renegotiation, results in the initial price being systematically lower than the expected cost of the project.

The literature on procurement and procurement auctions have taken two complementary approaches. One focuses on the analysis of equilibrium under a specific procurement process or auction format, such as Anton and Yao (1991), and Rogerson (1990). In these papers, as in ours, the emphasis is on a positive analysis of equilibrium in a particular institutional setting. The other approach is more normative in nature and designs optimal procurement mechanisms. Chapter 7 of Laffont and Tirole contains a careful analysis of optimal procurement auctions. Other studies on optimal procurement under different environments include Baron and Besanko (1987), Lewis (1986), McAfee and McMillan (1986), Riordan and Sappington (1987), and Tirole (1986).

The rest of the paper is organized as follows. The model is described in Section 2. Section 3 conducts the analysis of the model. Section 4 contains a discussion of the results and some concluding remarks.

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[2]Rogerson (1994, 1995) surveys the procurement issues as they apply to defense projects. McNaugher (1989) describes the institutional background.

2. THE MODEL

Two firms, 1 and 2, bid on a contract to produce a product in a second-price, sealed-bid auction.[3] Each firm's cost of production is c, which is a random draw from distribution $F(c\,|\,\theta,e)$ on support $[\underline{c},\bar{c}]$, where $0 \leq \underline{c} < \bar{c} < \infty$; θ is an efficiency parameter with $p.d.f.$ $h(\theta)$ on some subset of R^+, $[\underline{\theta},\bar{\theta}]$; and $e \in R^+$ is the firm's investment effort after the firm is selected for development but before production. The associated $p.d.f.$ for cost is $f(c\,|\,\theta,e)$, which is assumed to be differentiable. As we said earlier, the investment effort can be thought of as being devoted to refining the product design and finding the optimal method of production. Each firm knows privately her own θ at the time of bidding, but the production cost c is realized only after the investment is made and becomes publicly known at that time. A firm's investment e is not contractible. We assume that $\frac{\partial F(c|\theta,e)}{\partial\theta} > 0$ and $\frac{\partial F(c|\theta,e)}{\partial e} > 0$ for all $c \in (\underline{c},\bar{c})$. That is, c decreases in θ and in e in the sense of first-order stochastic dominance. The cost to the firm from investing e is normalized to be e. Denote the bids of the two firms by $b_i, i = 1, 2$. Then by the auction rule firm i wins the bidding if $b_i < b_j$, and i will be guaranteed a payment b_j for producing the product. We shall denote the winning firm by A, and the agreed payment to A by b, the base (initial) price. Each firm's objective is to maximize expected profits.

The buyer of the product is a government agency, who values the product at w, which is also assumed to be the social value of the product. We shall call the government agency G in what follows. For convenience, we assume $w \geq \bar{c}$ so that it is always socially desirable to produce the product. Cost overruns are said to occur when the cost realization is $c > b$. Before cost is realized, G is able to commit to pay only b for the project if $c \leq b$, but is unable to commit not to renegotiate price if $c > b$. We shall not explicitly model the renegotiation process since it is not crucial to our analysis, but will instead assume that the outcome of the renegotiation is such that when $c > b$, G is expected to make an extra payment $(1 - \beta)(c - b)$ to have the firm produce the product, where $\beta \in (0, 1)$.

One possible reason for cost sharing under cost overruns is that A is financially constrained and will not be able to continue production without an extra payment from G.[4] Alternatively, the initial contract between A and G may be incomplete because, say, there are features of the design that can be determined only at a later stage; and this will lead to rene-

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[3]The second-price, sealed bid auction is chosen for the convenience of equilibrium characterization. The results would be similar for a first-price auction, which is the characteristic defense practice. See Section 4 for more discussion about the source selection process.

[4]The nationalization of Rolls Royce in the UK after it went bankrupt on the fixed price RB211 (civil) project and the effective US government rescue of Lockheed after its losses on the C5A (military) contract are examples.

gotiating payment when $c > b$ but not when $c \leq b$ perhaps becuase there is a rule that forbids the government agency to pay more than b if $c \leq b$. [5] While renegotiations seem fairly likely in these situations, our assumption that the sharing for cost overruns takes the linear form is admittedly strong. However, this assumption makes our analysis tractable, and we believe that the insight of our analysis can still be valid if more complicated sharing rules are used. In practice, linear cost-sharing rules are common in defence programs. Interestingly, in a somewhat different context, Laffont and Tirole (1993) shows that linear cost sharing is in fact part of an optimal procurement mechanism. We emphasize that although G cannot commit not to renegotiate price upwards when $c > b$, it can commit not to reduce payment below b when the cost realization is low. This is important because without such a commitment A may have little incentive to invest in cost reduction.[6] We assume that $G's$ objective is to maximize the expected value of $w - c - e$, or the expected social surplus, subject to a budget constraint of paying no more than $\bar{c}$.[7] Clearly, in our model this objective is equivalent to minimizing the expected combined cost of production and investment, $c + e$[8].

For a bidder of type θ, the expected value from winning with a base price b is:

$$v(\theta, b) = \max_{e} \left\{ \int_{\underline{c}}^{b} [b - c] f(c \mid \theta, e) dc - \int_{b}^{\bar{c}} \beta(c - b) f(c \mid \theta, e) dc - e \right\}. \quad (1)$$

Without loss of generality, we can restrict the search for optimal e on interval $[0, \bar{c}]$. The objective function is obviously continuous in b, θ, and e. By the Maximum Theorem, $v(\theta, b)$ is a well-defined function and is continuous in θ and b. For any θ and b, define $e(\theta, b)$ as an e that achieves $v(\theta, b)$. Then, we have

$$v(\theta, b) = \int_{\underline{c}}^{b} [b - c] f(c \mid \theta, e(\theta, b)) dc - \int_{b}^{\bar{c}} \beta(c - b) f(c \mid \theta, e(\theta, b)) dc - e(\theta, b).$$
$$(2)$$

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[5] We thus expect β to be larger (or less cost sharing by G in the event of a cost overrun) if firms are better financially positioned and/or if there is less uncertainty about product design.

[6] Thus b can be viewed as a floor price that G commits to from which only price increases can be negotiated when $c > b$. Chen and Rosenthal (1996) develop a theory of the use of ceiling (or floor) price commitments by one party in a potential transaction when there can be renegotiations after asset-specific investment is made by another party.

[7] This implies that G will not have unlimited ability to make transfer payments.

[8] In practice, the Government may not be maximising welfare, but rather be trying to minimise the payment required to obtain the product. However, this assumption is not critical to our analysis.

To summarize the model: At stage 1 (the bidding stage), Firms 1 and 2 each chooses a bidding strategy $b_i(\theta)$, given each's private information about θ and given the auction rule. The firm submitting the lower cost bid wins the auction, call it A. A is entitled to receive a payment of b, equal to the higher bid, for completing the project. At stage 2, A chooses an optimal investment (effort), $e(\theta, b)$. The investment is then made. At stage 3, c is realized and becomes known publicly. If $c \leq b$, A delivers the product in exchange for a payment of b; and if $c > b$, A delivers the product in exchange for a payment $b + (1 - \beta)(c - b)$. Notice that our model captures two key institutional features of most government procurements, competitive source selection and some sharing of cost overruns. We study Nash equilibria of this game. A strategy of each firm in this game is a bidding rule $b_i(\theta)$. A Nash equilibrium of this game is a pair of the firms' strategies such that $b_i(\theta)$ is optimal to i given $b_j(\theta)$, $e(\theta, b)$, the auction rule, and the expected cost sharing when cost overruns occur.[9]

3. ANALYSIS

LEMMA 1. $\forall \theta$, $v(\theta, b)$ *strictly increases in b and in θ.*

Proof. We first show that for any $b' > b''$ and any θ, $v(\theta, b') > v(\theta, b'')$.

$$
\begin{aligned}
v(\theta, b') &= \int_{\underline{c}}^{b'} [b' - c] f(c \,|\, \theta, e(\theta, b')) dc - \int_{b'}^{\bar{c}} \beta(c - b') f(c \,|\, \theta, e(\theta, b')) dc - e(\theta, b') \\
&\geq \int_{\underline{c}}^{b'} [b' - c] f(c \,|\, \theta, e(\theta, b'')) dc - \int_{b'}^{\bar{c}} \beta(c - b') f(c \,|\, \theta, e(\theta, b'')) dc - e(\theta, b'') \\
&> \int_{\underline{c}}^{b''} [b'' - c] f(c \,|\, \theta, e(\theta, b'')) dc - \int_{b''}^{\bar{c}} \beta(c - b'') f(c \,|\, \theta, e(\theta, b'')) dc - e(\theta, b'') \\
&= v(\theta, b''),
\end{aligned}
$$

---

where the first inequality above holds because $e(\theta, b')$ is a solution to (1) when $b = b'$, and the second inequality holds because

$$\frac{d}{db}\left\{\int_{\underline{c}}^{b}[b-c]f(c\,|\theta, e(\theta, b''))dc\right\} > 0$$

and $\frac{d}{db}\left\{\int_{b}^{\bar{c}}\beta(c-b)f(c\,|\theta, e(\theta, b''))dc\right\} < 0$.

We next show that $v(\theta', b) > v(\theta'', b)$ for any $\theta' > \theta''$ and any b.

$$
\begin{aligned}
v(\theta', b) &= \int_{\underline{c}}^{b}[b-c]f(c\,|\theta', e(\theta', b))dc \\
&\quad - \int_{b}^{\bar{c}}\beta(c-b)f(c\,|\theta', e(\theta', b))dc - e(\theta', b) \\
&\geq \int_{\underline{c}}^{b}[b-c]f(c\,|\theta', e(\theta'', b))dc \\
&\quad - \int_{b}^{\bar{c}}\beta(c-b)f(c\,|\theta', e(\theta'', b))dc - e(\theta'', b) \\
&> \int_{\underline{c}}^{b}[b-c]f(c\,|\theta'', e(\theta'', b))dc \\
&\quad - \int_{b}^{\bar{c}}\beta(c-b)f(c\,|\theta'', e(\theta'', b))dc - e(\theta'', b), \\
&= v(\theta'', b),
\end{aligned}
$$

where the first inequality above is due to the fact that $e(\theta', b)$ is optimal under θ', and the second inequality is due to $\int_{\underline{c}}^{b}[b-c]f_{\theta}(c\,|\theta, e(\theta'', b))dc > 0$ and $\int_{b}^{\bar{c}}(c-b)f_{\theta}(c\,|\theta, e(\theta'', b))dc < 0$ by the first-order stochastic dominance assumption. ∎

We note that if $e(\theta, b)$ is differentiable, then Lemma 1 would follow directly from the Envelope Theorem.

LEMMA 2. *For any given* θ, *there exists a unique* $b^*(\theta) \in (\underline{c}, \bar{c})$ *such that* $v(\theta, b^*(\theta)) = 0$.

Proof.
First, for any given θ,

$$v(\theta, \underline{c}) = \max_{e}\{-\int_{\underline{c}}^{\bar{c}}\beta(c-\underline{c})f(c\,|\theta, e)dc - e\} < 0.$$

Next, for any given θ,

$$
\begin{aligned}
v(\theta, \bar{c}) &= \int_{\underline{c}}^{\bar{c}} [\bar{c} - c] f(c \,|\, \theta, e(\theta, \bar{c})) dc - e(\theta, \bar{c}) \\
&> \int_{\underline{c}}^{\bar{c}} (\bar{c} - c) f(c \,|\, \theta, 0) dc - 0 \\
&> 0.
\end{aligned}
$$

The conclusion then follows from Lemma 1. ▮

We can think of $b^*(\theta)$ as the base price b that enables a firm of type θ undertaking the project to earn a zero expected profit.

PROPOSITION 1.   *Bidding $b_i(\theta) = b^*(\theta)$ is a weakly dominant strategy for each firm.*

Proof.
First notice that $b_i(\theta)$ only affects the probability that i will win the bidding, not how much payment i will receive if she wins. Next, since $v(\theta, b_j) < 0$ if $b_j < b^*(\theta)$, any strategy $b_i(\theta) < b^*(\theta)$ is as good as $b^*(\theta)$ to i if $b_j < b_i$ or if $b_j > b^*(\theta)$. But bidding $b_i(\theta) < b^*(\theta)$ is worse than bidding $b^*(\theta)$ to i if $b_j \in (b_i(\theta), b^*(\theta))$, since in that case i wins the bidding with $b_i(\theta) < b^*(\theta)$ but $v(\theta, b_j) < 0$, while bidding $b^*(\theta)$ avoids the loss. Hence any bidding strategy $b_i(\theta) < b^*(\theta)$ is weakly dominated by $b^*(\theta)$. Next, since $v(\theta, b_j) > 0$ if $b_j > b^*(\theta)$, any strategy $b_i(\theta) > b^*(\theta)$ is as good as $b^*(\theta)$ to i if $b_j < b^*(\theta)$ or if $b_j > b_i(\theta)$. But bidding $b_i(\theta) > b^*(\theta)$ is worse than bidding $b^*(\theta)$ to i if $b_j \in (b^*(\theta), b_i(\theta))$, since in that case i loses the bidding with $b_i(\theta) > b^*(\theta)$, while bidding $b^*(\theta)$ allows i to capture the expected gain because $v(\theta, b_j) > 0$. Hence any strategy $b_i(\theta) > b^*(\theta)$ is weakly dominated by $b^*(\theta)$. Therefore $b^*(\theta)$ is a weakly dominant strategy for $i = 1, 2$. ▮

The following result then follows immediately from the construction above.

PROPOSITION 2.   *The procurement game has a Nash equilibrium where each firm's bidding strategy at the auction stage is $b^*(\theta_i)$, where θ_i is firm $i's$ type, and the winning firm chooses $e = e(\theta_A, b)$, where θ_A is the type of the winning firm and $b = \max\{b^*(\theta_1), b^*(\theta_2)\}$. Furthermore, this is the unique Nash equilibrium surviving the refinement that players do not use weakly dominated strategies.*

One interesting issue here is whether the more efficient firm (the one with a higher θ) will always win the auction. From Lemma 1 and the definition of $b^*(\theta)$, we immediately have:

PROPOSITION 3. $b^*(\theta') < b^*(\theta'')$ *if and only if* $\theta' > \theta''$.

Although the more efficient firm is always chosen through the auction, the winning firm's investment is often inefficient. To see this, $\forall\theta$, define $e^o(\theta)$ as

$$e^o(\theta) = \arg\max_{e\in[0,\bar{c}]}\{-[\int_{\underline{c}}^{\bar{c}} cf(c\,|\theta,e)dc + e]\}. \tag{3}$$

Then $e^o(\theta)$ is the socially optimal level of investment. Notice that $e^o(\theta) < \bar{c}$ for any θ. We have:

PROPOSITION 4.   *Assume that* $\{-[\int_{\underline{c}}^{\bar{c}} cf(c\,|\theta,e)dc+e]\}$ *is strictly concave in* e *and* $e^o(\theta) > 0$, $\forall\theta$. *In equilibrium, the investment in cost reduction by the winning firm will be below the socially optimal level, unless* $\beta = 1$ *or* $b = \bar{c}$.

Proof.
Suppose that a firm of type θ wins the bidding. By definition, $e(\theta, b)$ solves

$$\max_{e\in[0,\bar{c}]}\left\{\int_{\underline{c}}^{b}(b-c)f(c\,|\theta,e)dc + \int_{b}^{\bar{c}}\beta(b-c)f(c\,|\theta,e)dc - e\right\}. \tag{4}$$

That is, $e(\theta, b)$ solves

$$\max_{e\in[0,\bar{c}]}\left\{b - [\int_{\underline{c}}^{\bar{c}} cf(c\,|\theta,e)dc + e] + (1-\beta)\int_{b}^{\bar{c}}(c-b)f(c\,|\theta,e)dc\right\}.$$

If $\beta = 1$ or if $b = \bar{c}$, then the problems defined by equations (3) and (4) are equivalent and produce the same e.

Otherwise, suppose $\beta < 1$ and $b < \bar{c}$. If $e(\theta, b) = 0$, then clearly $e(\theta, b) < e^o(\theta)$. If $e(\theta, b) > 0$, and notice $e(\theta, b) < \bar{c}$, $e(\theta, b)$ must satisfy the first-order condition:

$$\frac{-\partial[\int_{\underline{c}}^{\bar{c}} cf(c\,|\theta,e)dc + e]}{\partial e} + (1-\beta)\frac{\partial\int_{b}^{\bar{c}}(c-b)f(c\,|\theta,e)dc}{\partial e} = 0.$$

Since $\frac{-\partial[\int_{\underline{c}}^{\bar{c}} cf(c|\theta,e)dc+e]}{\partial e} = 0$ uniquely at $e = e^o(\theta)$, and $\frac{\partial\int_{b}^{\bar{c}}(c-b)f(c|\theta,e)dc}{\partial e}$ < 0 by the assumption of first-order stochastic dominance, we must have $e(\theta, b) < e^o(\theta)$. ∎

The result is intuitive, and highlights the difficulties in achieving an efficient procurement outcome in the presence of both adverse selection and moral hazard. If a firm does not expect to bear all the extra costs of an overrun, the firm will not internalize all the benefits from investment in cost reduction. The only price that would induce efficient investment is $b = \bar{c}$. But if a winning firm will be paid $\bar{c}$ to undertake the project, it becomes impossible to ensure that only the efficient firm will be selected for the job.

Although there are problems of both adverse selection and moral hazard in our model, the key reason why the equilibrium is not socially efficient is the inability of G to commit not to share the costs of an overrun. In fact, from Proposition 4 above, if $\beta = 1$ so that A has to bear all the extra costs of an overrun, $A's$ investment level will be socially efficient. Furthermore, since the more efficient firm will always be selected for the project under our procurement process, we have

COROLLARY 1. *If $\beta = 1$, then the procurement method studied here is an optimal procurement mechanism for the government, in the sense that it achieves the highest possible social surplus among all possible incentive-compatible mechanisms.*

Since b is higher than the winning firm's cost bid in our model, one may think that on average the realized cost should be lower than b. This, however, need not be true, because each firm underbids her true cost in equilibrium. We can see this from the following:

The expected true cost of the project to a firm of type θ with a base price b is[10]

$$\int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e(\theta, b)\,)dc + e(\theta, b).$$

But each firm bids $b^*(\theta)$, where by equation (2) and the definition of $b^*(\theta)$, we have

$$
\begin{aligned}
b^*(\theta) &= \int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e(\theta, b^*(\theta)\,)dc \\
&\quad - \int_{b^*}^{\bar{c}} (1 - \beta)(c - b^*(\theta))f(c\,|\theta, e(\theta, b^*(\theta)))dc + e(\theta, b^*(\theta)) \\
&< \int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e(\theta, b^*(\theta))\,)dc + e(\theta, b^*(\theta)).
\end{aligned}
$$

---

[10]The expected cost of the project is endogenous, depending on b, and is lower with a higher b.

Therefore, in equilibrium, each firm's bid is always below its true cost of undertaking the project if it wins with the bidding price.

Furthermore, for any equilibrium initial price b, the expected production cost for any type θ can be higher than b. This is seen from the following:

PROPOSITION 5. *Assume again that* $\{-[\int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e)dc + e]\}$ *is strictly concave in* e *and* $e^o(\theta) > 0, \forall\theta$. *There exists some* $\bar{\beta} \in (0, 1)$ *such that when* $\beta \leq \bar{\beta}$, $b < \int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e(\theta, b))dc$ *for any equilibrium initial price* b *and any* θ.

Proof.

Since $v(\theta, b)$ increases in b by Lemma 1 and $v(\theta, b)$ decreases in β from equation (2), $b^*(\theta)$ must increase in β for all θ. Therefore, since $b^*(\theta) = 0$ for all θ if $\beta = 0$, and $\int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e(\theta, \bar{c}))dc > 0$, there must exist some $\bar{\beta} \in (0, 1)$ such that when $\beta \leq \bar{\beta}$, $b^*(\theta) < \int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e(\theta, \bar{c}))dc$ for any θ. But in equilibrium, $b \leq b^*(\underline{\theta})$. Thus, when $\beta \leq \bar{\beta}$, for any equilibrium initial price b, we must have

$$b < \int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e(\theta, \bar{c}))dc < \int_{\underline{c}}^{\bar{c}} cf(c\,|\theta, e(\theta, b))dc,$$

where the last inequality holds because $e(\theta, \bar{c}) = e^o(\theta)$, $b < \bar{c}$, and $e(\theta, b) < e^o(\theta)$ from Proposition 4. ∎

Thus in our model the initial price can be always below the expected cost of production, and thus cost overruns occur in equilibrium not only because there can be bad realizations of the Nature, but also because there is a systematic bias in setting the initial price in equilibrium. The latter occurs because each firm expects that there will be some cost sharing if cost overruns occur, and thus the expected cost of the project to the firm is lower than the true cost. We emphasize that the cost overrun problem is inherently related to the need to select the more efficient firm under asymmetric information. In the absence of the asymmetric information about firms' cost efficiency, G could simply set $b = \bar{c}$ that not only avoids cost overruns but also induce the first-best investment.

Obviously, when $\beta \to 0$, the bids by both firms must approach $\underline{c}$. We thus have the following:

COROLLARY 2. *The probability that cost overruns occur approaches 1 when* $\beta \to 0$.

4. DISCUSSION AND CONCLUSION

We have studied a simple model of cost overruns in which the expectation of cost sharing in the event of an overrun results in firms' bidding below their true estimate of project costs. This can cause the initial price to be systematically below the expected cost of production and hence the persistence of cost overruns in equilibrium. A welfare-maximizing government agency under budget constraint faces the difficult problem of both selecting the efficient source and providing investment incentives for the producer in the procurement process. If the government cannot commit not to share the cost of the overrun, the socially efficient level of investment will occur only if the firm is guaranteed a price that is equal to the highest possible cost realization (so that all benefits from investment in cost reduction is internalized by the firm and there will be no cost overruns), but then the government cannot ensure that the most efficient firm will be selected.

Although we have considered a particular source-selection (bidding) process, the basic results of our analysis can hold in a more general setting. Consider any incentive-compatible procurement mechanism that G may use and that has the feature of sharing cost overruns as in our model. Without loss of generality, we can restrict attention to direct mechanisms where firm i is asked to report her type θ_i and truthfully reporting by each firm is an equilibrium. In many, if not all, situations we would expect that an optimal mechanism has the property that the relatively more efficient firm will be selected to undertake the project; and assume that this is the case here. Let $p(\theta)$ be the guaranteed payment to the firm who is selected for the project and who reports θ. Type $\underline{\theta}$, the least efficient type, must have an expected payoff of zero at an optimal mechanism since she will be selected with probability zero (assuming $\underline{\theta}$ is not a mass point). Since type $\underline{\theta}$ can always choose to report any θ that has a positive probability of being selected at the optimal mechanism, for any such θ, the truthfully-reporting constraint for $\underline{\theta}$ requires

$$
\begin{aligned}
& \int_{\underline{c}}^{p(\theta)} [p(\theta) - c] f(c \,|\, \underline{\theta}, e(\underline{\theta}, p(\theta))) dc \\
& - \int_{p(\theta)}^{\bar{c}} \beta(c - p(\theta)) f(c \,|\, \underline{\theta}, e(\underline{\theta}, p(\theta))) dc - e(\underline{\theta}, p(\theta)) \\
& \leq \; 0.
\end{aligned}
$$

By the definition of $e(\underline{\theta}, p(\theta))$, the inequality above implies

$$
\int_{\underline{c}}^{\bar{c}} [p(\theta) - c] f(c \,|\, \underline{\theta}, 0) dc + \int_{p(\theta)}^{\bar{c}} (1 - \beta)(c - p(\theta)) f(c \,|\, \underline{\theta}, 0) dc \leq 0.
$$

Notice that the left-hand side of the last inequality above increases in $p(\theta)$, decreases in β, and is zero when $p(\theta) = \underline{c}$ and $\beta = 0$. Therefore in order for the inequality to hold, $p(\theta)$ will have to be close to $\underline{c}$ and will thus be less than $\int_{\underline{c}}^{\bar{c}} c f(c \,|\, \theta, e(\theta, p(\theta))) dc$ when β is small enough.

Therefore, for any optimal direct mechanism where the more efficient firm is selected for the project and where there is the sharing of cost overruns as in our model, the expected production cost will be higher than the initial price, at least when β is small. Thus cost overruns will be expected. It is also clear that since at any such mechanism $p(\theta) < \bar{c}$ for any $\theta > \underline{\theta}$, there will often be underinvestment.

We now turn to some other assumptions of our model. The assumption that the number of bidding firms is 2 is not crucial to our analysis. In fact, if there are any n bidding firms, bidding $b_i(\theta) = b^*(\theta)$ will remain to be a weakly dominant strategy for each firm, and our results about equilibrium cost overruns and inefficient investment will continue to hold.[11]

In real procurement situations, it is possible that the cost realization is higher than w, in which case the project is cancelled. Projects with high cost overruns that are not cancelled are usually procured in smaller quantities (Lichtenberg, 1989; and Rogerson, 1990, 1991). Our model has abstracted from these more realistic considerations in order to make the model as transparent as possible. It is also likely that firms are risk averse. But risk aversion by firms would provide an efficiency reason why there will be cost sharing when cost overruns occur, and would thus strengthen our argument that cost overruns occur in equilibrium due to the expected cost sharing.

From the public policy perspective, cost overruns do not merely create transfer payments, but have real social costs. When cost overruns are expected in equilibrium, the firm undertaking the project tends to choose inefficiently low levels of investment in cost reductions. It would be desirable to study the institutional arrangements that can reduce cost overruns and encourage more efficient investment. In a more realistic model with risk averse firms, the need to provide optimal risk sharing will also have to be considered. These are interesting issues for future research.

REFERENCES

Anton, James and Dennis Yao, 1992, Coordination in split award auctions. *Quarterly Journal of Economics* **107**, 681-707.

Augustine, Norman R., 1986, *Augustine's Laws*, Viking Penguin.

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[11]Note that our model precludes splitting production between two firms. For models where such procurement methods are considered and may be preferred, see, for instance, Anton and Yao (1992).

Baron, D. and D. Besanko, 1987, Monitoring, moral hazard, asymmetric information, and risk sharing in procurement contracts. *Rand Journal of Economics* **18**, 509-532.

Brook, Frederick P, Jr, 1975, *The Mythical Man-Month*. Addison Wesley, Reading Mass.

Chen, Y. and R. W. Rosenthal, 1996, On the use of ceiling price commitments by monopolists. *Rand Journal of Economics* **27,2**, 207-220.

Laffont, J-J and J. Tirole, 1993, *A Theory of Incentives in Procurement and Regulation*. MIT Press, Cambridge MA.

Lewis, Tracy, 1986, Reputation and contractual performance in long-term projects. *Rand Journal of Economics* **17**, 141-157.

Lichtenberg, F. R., 1989, How elastic is the government's demand for weapons? *Journal of Public Economics* **40**, 57-78.

Lichtenberg, F. R., 1995, The economics of defense R&D, ch 15, p431-457. In: Hartley, K. and T. Sandler (eds) *Handbook of Defense Economics*, North Holland.

McAfee, R. P. and J. McMillian, 1986, Bidding for contracts: A principal-agent analysis. *Rand Journal of Economics* **17**, 326-338.

McConnell, 1996, *Rapid Development: Training Wild Software Schedules*. Microsoft Press.

McNaugher, Thomas L, 1989, *New Weapons Old Politics*. Brookings.

Riordan, M. and D. Sappington, 1987, Awarding monopoly franchises. *American Economic Review* **80**, 83-92.

Rogerson, William P, 1990, Quality vs quantity in military procurement. *American Economic Review* **77**, 375-387.

Rogerson, William P, 1991, Incentives, the budgetary process, and inefficiently low production rates in defense procurement. *Defense Economics* **3**, 1-18.

Rogerson, William P, 1994, Economic incentives and the defense procurement process. *Journal of Economic Perspectives* **8**, 65-90.

Rogerson, William P, 1995, Incentive models of the defense procurement process, Ch 12, p309-346. In: Hartley, K. and T. Sandler (eds) *Handbook of Defense Economics*, North Holland.

Tirole, Jean, 1986, Procurement and renegotiation. *Journal of Political Economy* **94**, 235-259.