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Finite Horizon Negotiation Games*

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This paper studies the finite horizon version of the negotiation model of Busch and Wen (1995). Two players bargain over the division of a certain surplus in finitely many periods. In the absence of an agreement, players' payoffs in a period are determined by a disagreement game. The set of equilibrium payoffs is determined by backward induction. If at least one player has distinct Nash payoffs in the disagreement game, the set of subgame perfect equilibrium payoffs converges to that of the corresponding infinite horizon negotiation game as the game horizon increases to infinity. Otherwise, the finite horizon negotiation game will have a unique subgame perfect equilibrium outcome.

Key Words: Non-cooperative bargaining; Negotiation model; Finitely repeated games
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1. INTRODUCTION

There is a large literature on non-cooperative bargaining theory since Rubinstein's (1982) two-player infinite horizon bargaining model with per-

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fect information. Busch and Wen (1995) generalize Rubinstein's model by introducing a possible existing strategic relationship between the two players, called the (interim) disagreement game, into the model. This disagreement game is played in any period in which an offer is rejected, and hence determines the players' payoffs in that period. Such a negotiation game generally has multiple perfect equilibrium outcomes, including inefficient equilibria in which the two players do not reach an agreement immediately. The wage bargaining model of Haller and Holden (1990) and Fernandez and Glazer (1991) is a special case of this negotiation model. Recently, a number of issues in the negotiation model have been investigated. Houba (1997) considers the case when the bargaining frontier is the Pareto frontier of the disagreement game payoffs, Busch and Wen (2000) discuss the effects of observability of mixed disagreement game actions, and Furusawa and Wen (2000) consider the effects of inflexible disagreement actions where the players bargain for a number of rounds before they can change their disagreement game actions.

In this paper, we study the finite horizon negotiation model, and compare it to the infinite horizon negotiation model of Busch and Wen (1995). We show that, if at least one player has multiple Nash equilibrium payoffs in the disagreement game, then as the game horizon increases the set of average SPE (Subgame Perfect Equilibrium) payoffs in the finite horizon negotiation game will converge to that in the corresponding infinite horizon negotiation game when the players are sufficiently patient. The contribution of this paper is to bridge the finite and the infinite horizon negotiation games.

Our work in this paper is motivated by the similar studies in both the bargaining and repeated game literatures. It is well known that the unique SPE outcome in Ståhl's (1972) two-player finite horizon bargaining model converges to the unique SPE outcome of Rubinstein's (1982) discounting formulation as the game horizon goes to infinity. The relatively simple infinite horizon bargaining model is therefore a reasonable approximation of the finite horizon bargaining model. It is theoretically important to know if the same is true between the finite and the infinite horizon negotiation models.

The negotiation model of Busch and Wen (1995) is also related to repeated game models. As exemplified by Houba (1997), the negotiation model can be treated as an infinitely repeated game in which the two players may write a binding contract on their future actions. The study and comparison of finite and infinite horizon repeated games have received a lot of attention in the repeated game literature; see Aumann (1989) for a survey on this subject. One well-known result in repeated games is the Folk Theorem. It asserts that any feasible and individually rational payoff of a stage game can be supported or approximated by an equilibrium outcome in the corresponding repeated game, when the stage game satisfies certain conditions. For infinitely repeated games with discounting, Fudenberg and Maskin (1986) prove a Folk Theorem if the stage game has either only two players or satisfies the full dimensionality condition. Abreu, Dutta and Smith (1994) find a weaker dimensionality condition, called NEU (Non-Equivalent Utility), to replace the full dimensionality condition in Fudenberg and Maskin (1986). Wen (1994) introduces the notion of an effective minimax value to characterize the set of SPE payoffs when players are sufficiently patient. The effective minimax value will be equivalent to the standard minimax value if the stage game has only two players or satisfies the NEU condition.

For finitely repeated games, Benoît and Krishna (1985) establish a Folk Theorem if every player has distinct Nash payoffs in a stage game that satisfies the same conditions as in Fudenberg and Maskin's (1986) Folk Theorem. In a finitely repeated game, a feasible payoff vector can be approximated by an average SPE payoff when the game horizon is sufficiently long. Smith (1995) relaxes the condition that every player have distinct Nash payoffs, imposing the condition of recursively distinct Nash payoffs. For games with more than two players that do not satisfy the NEU condition, Wen's (1994) effective minimax values also characterize the limit (as the game horizon goes to infinity) of the average SPE payoffs in a finitely repeated game, as long as the stage game satisfies the condition of recursively distinct Nash payoffs. Therefore, with or without the NEU condition, as the game horizon goes to infinity, the set of average SPE payoffs of a finitely repeated game converges to that of the corresponding infinitely repeated game when players are sufficiently patient.

In this paper, we will establish a similar linkage between finite horizon and infinite horizon negotiation games. We show that if at least one player has distinct Nash payoffs in the disagreement game, then the limiting set of average SPE payoffs in a finite horizon negotiation game as the game horizon goes to infinity, and that in the corresponding infinite horizon negotiation game as players become sufficiently patient, are the same. The limiting set of average SPE payoffs is characterized by the players' minimax values and their highest effective disagreement payoffs in the disagreement game. A player's highest effective disagreement payoff, introduced by Busch and Wen (1995), is the player's highest disagreement payoff after compensating his opponent for a foregone one-shot gain from deviation.

While a player may have different effective and standard minimax values in the disagreement game, unlike in a repeated game, in the negotiation game a player's lowest SPE payoff is higher than his standard minimax value in the disagreement game. Indeed, his lowest SPE payoff in the negotiation game is constructed based on his standard minimax value and his opponent's highest effective disagreement payoff, where both values are determined by the disagreement game. In contrast to the Folk Theorem for finitely repeated games, the main result in this paper asserts that every feasible payoff vector where each player receives more than his lowest SPE payoff can be approximated by an average SPE payoff vector in the finite horizon negotiation game with a sufficiently long horizon.

On the other hand, when both players have a unique Nash payoff in the disagreement game, then any finite horizon negotiation game will have a unique SPE outcome by backward induction, while the corresponding infinite horizon negotiation game may still have multiple SPE outcomes [see Busch Wen (1995)]. As for finitely repeated games, if every player has a unique Nash payoff in the disagreement game, then there will be a unique SPE outcome in the last period of the negotiation game. There then also will be a uniquely determined SPE in the second last period, and so on. By backward induction, as long as the game horizon is finite, the negotiation game will have a unique SPE outcome.

Since an infinitely repeated game in which players can form binding (explicit) contracts on their future actions is an alternative interpretation of the negotiation model, the results in this paper, together with those in Busch and Wen (1995), imply that the set of average SPE payoffs in this case is different from that in the corresponding infinitely repeated game with implicit contracts only. Furthermore, a player's standard minimax value in the stage game alone is insufficient to characterize individual rationality when explicitly binding contracts are possible.

In the infinite horizon negotiation model, Busch and Wen (1995) derive the lower bound of a player's SPE payoffs following the arguments of Shaked and Sutton (1984). This lower bound is then supported by a SPE payoff when the players are sufficiently patient. Therefore, this lower bound is indeed the player's lowest SPE payoff in the infinite horizon negotiation game, and is used as the punishment to enforce this player's actions in other SPE outcomes. For finite horizon negotiation games, we find that a player's average SPE payoffs are also not less than his lowest SPE payoff in the corresponding infinite horizon negotiation game. We call a SPE as a player's optimal punishment if the player receives his lowest SPE payoff compared to any other SPE of the game. Directly calculating a player's optimal punishment is complicated, tedious and unnecessary. Instead, we construct a SPE in the finite horizon game where one player's payoff will converge to his lowest SPE payoff in the corresponding infinite horizon negotiation game, as the game horizon goes to infinity. This constructed SPE is then an approximation of the player's optimal punishment when the game horizon is sufficiently long.

The construction of this approximated optimal punishment in the finite horizon negotiation game is possible only when at least one player has distinct Nash payoffs in the disagreement game. In this case, each player will have distinct SPE payoffs in the last two periods of the negotiation game. When the game has a sufficiently long horizon, the difference between a player's SPE payoffs will be sufficient large to enforce almost all disagreement actions for finitely many periods. In particular, it is possible to support one player's minimax value and the other player's highest effective disagreement value alternately for every two periods. To approximate any feasible payoff vector where each player receives no less than his lowest SPE payoff, players' actions are enforced by the approximated optimal punishments in the finite horizon negotiation game.

The rest of this paper is organized as follows. We first introduce the finite horizon negotiation model in Section 2, and then formulate the backward induction to define SPE payoffs in Section 3. In Section 4, we investigate SPE outcomes in the negotiation game that are based on the Nash disagreement payoffs. In Section 5, we derive a SPE that approximates a player's optimal punishment in the finite horizon game and show that a player's average payoff in his approximated optimal punishment converges to his lowest SPE payoff in the corresponding infinite horizon game. Then we characterize the limit of the average SPE payoffs as the game horizon goes infinity in Section 6.

2. THE FINITE HORIZON NEGOTIATION MODEL

Two players, 1 and 2, bargain to share some surplus in T periods, where T is finite. The surplus to be shared in each period is normalized to 1. During each period before the players reach an agreement, one player makes a proposal and the other player may either accept or reject the proposal. A proposal is identified by the players' shares of the surplus, $(b, 1-b) \in \Delta$ for $b \in [0, 1]$, where Δ is the unit simplex in $\mathbf{R^2}$. Here b is player 1's proposed share and $1 - b$ is player 2's proposed share of the surplus, in each of the remaining periods. If the proposal is accepted, indicated by Y, then the proposal becomes an agreement, and the two players will share the surplus according to this agreement in each of the remaining periods of the negotiation game. If the proposal is rejected, indicated by N, then the two players play a one-shot game, called the disagreement game, before either a counter proposal will be made in the next period or the negotiation game ends. The two players alternate in making proposals as in Rubinstein's (1982) bargaining model. Without loss of generality, we assume that player 1 makes a proposal in the last period, period T. Since the game described here has a finite horizon T, we will apply backward induction to study Subgame Perfect Equilibria (SPE) of the negotiation model.

The disagreement game is a two-player game in normal form $G = \{A_1, A_2, u_1(\cdot), u_2(\cdot)\}$. A_i is the set of player i's disagreement actions and $u_i(\cdot) : A = A_1 \times A_2 \to \mathbf{R}$ is player i's payoff function in the disagreement game, where A is the set of disagreement outcomes. We assume that both players observe all past actions in the disagreement game. If A_i contains player i's mixed disagreement actions, this assumption implies that past mixed disagreement actions are observable.[1] The set of feasible payoff vectors of G is the convex hull of $u(A)$, denoted by $Co[u(A)]$. Without loss of generality, the two players may choose their disagreement actions contingent on the outcome of a public randomizing device. Hence, in terms of the expected value, every feasible payoff vector in $Co[u(A)]$ can be achieved in a one-shot play of the disagreement game G. The players' action sets and payoff functions satisfy the sufficient conditions under which the disagreement game G has at least one Nash equilibrium. If the players choose $a \in A$ after the rejection in a period, their payoffs for the period are given by $u(a)$. Therefore both players receive payoffs in every period, with or without an agreement. Players' payoffs from this finite horizon negotiation game are the sum of their payoffs in all T periods. In this paper, we focus on the case where players do not discount future payoffs. However, our results can easily be generalized to the case where players discount their payoffs. Let $NG(T)$ denote the negotiation game with finite T periods. According to this notation, after the first proposal is rejected in $NG(T)$, the players will play G and then $NG(T-1)$, which starts with a different player. Denote the subgame after the first rejection in $NG(T)$ by $GNG(T) = G \oplus NG(T-1)$.

Let m^i be a disagreement action profile that minimaximizes player i, for $i = 1, 2$, in the disagreement game G. That is, for $i \neq j$,

$$m^i \in \arg \min_{a_j \in A_j} \max_{a_i \in A_i} u_i(a_i, a_j). \tag{1}$$

Without loss of generality, we can normalize the disagreement game G so that every player has a minimax payoff of zero, $u_i(m^i) = 0$ for $i = 1, 2$. Let F denote the set of feasible and individually rational payoffs of the disagreement game G, where $F = Co[u(A)] \cap \mathbf{R}^{\mathbf{2}}_{+}$. The final assumption is that any disagreement outcome in F is (weakly) Pareto dominated by at least one agreement,

$$u_1(a) + u_2(a) \leq 1, \quad \text{for all } a \in A. \tag{2}$$

It is therefore in both players' interest to have an agreement sooner rather than later.

A strategy profile in $NG(T)$ consists of two functions which map from the sets of all appropriate histories into the sets of appropriate actions.

[1]For an infinite horizon negotiation game where only the realizations of past mixed disagreement actions, but not the mixed disagreement actions themselves, are observable to the two players see Busch and Wen (2000).

There are three types of histories in every period. The first type histories are those at the beginning of a period, $H_1 = \Delta^{t-1} \times A^{t-1}$ for $t = 1, \cdots, T$. They contain all past rejected proposals and disagreement action profiles. Type 2 histories are those after a proposal is made in a period, $H_2(t) = H_1(t) \times \Delta$. Type 3 histories are those after a proposal is rejected in a period, $H_3(t) = H_2(t) \times \{N\}$. For example, when T is odd, a strategy profile in $NG(T)$ is a pair of functions,

$$
\begin{array}{ll}
(f_1, f_2) : H_1(t) \to \Delta \times \{S\};\ H_2(t) \to \{S\} \times \{Y, N\};\ H_3(t) \to A & \text{for odd } t, \\
(f_1, f_2) : H_1(t) \to \{S\} \times \Delta;\ H_2(t) \to \{Y, N\} \times \{S\};\ H_3(t) \to A & \text{for even } t,
\end{array}
$$

where S denotes that the player keeps silent while waiting for either his opponent's proposal or his opponent's response to his proposal. Every strategy profile induces a unique outcome path and every outcome path yields a unique payoff to every player. Let $\pi(T) = (a^1, \cdots, a^{T_0-1}, b^{T_0}, \{Y\})$ denote a generic outcome path of $NG(T)$, where a^t is the disagreement outcome played in period t for $t < T_0$, and b^{T_0} is the agreement reached in period T_0 if there is an agreement. If the players do not have an agreement, then set $T_0 = T + 1$. The players' total payoffs from such an outcome path are the sum of their payoffs in all T periods,

$$
\sum_{t=1}^{T_0-1} u(a^t) + (T - T_0 + 1)(b^{T_0}, 1 - b^{T_0}). \tag{3}
$$

In this paper, we investigate SPE of the finite horizon negotiation game. A SPE is a strategy profile that induces a Nash equilibrium outcome in the continuation game after any history. In other words, no player may benefit from any unilateral deviation after any history in the negotiation game. We will focus on SPE payoffs in the negotiation game. The set of SPE payoffs can be found by backward induction. It is straightforward to recover SPE strategy profiles from the set of SPE payoffs.

3. BACKWARD INDUCTION AND SPE PAYOFFS

In this section we derive the set of SPE payoffs in $NG(T)$ by backward induction. In contrast to the finite horizon repeated games in Benoît and Krishna (1985, 1993), and Wen (1996), the backward induction for the finite negotiation model is based on the structure of the negotiation game as well as the subgame perfection criterion.

First, consider the last period of the negotiation game, which is denoted as $NG(1)$. After a rejection, any SPE must induce a Nash equilibrium of G, since the game ends after the players play G once and G is given in normal form. Let $P(1)$ be the set of SPE payoffs in game G. $P(1)$ is a compact subset of F. In general it contains only finitely many points.

Recall that player 1 makes the proposal in the last period. Player 1 should never offer player 2 more than player 2's best payoff in $P(1)$, since player 2 will certainly accept any offer in which player 2's share is not less than his best payoff in $P(1)$. On the other hand, player 1 should never offer player 2 less than player 2's worst payoff in $P(1)$, since such an offer will definitely be rejected. Denote player 2's best and worst payoffs in $P(1)$ as, respectively,

$$B_2(1) = \max_{y \in P(1)} y_2 \text{ and } W_2(1) = \min_{y \in P(1)} y_2. \tag{4}$$

Note that both $B_2(1)$ and $W_2(1)$ are well defined due to the compactness of $P(1)$ in $\mathbf{R^2}$.

In $NG(1)$, player 1's proposal $(b, 1-b)$ can be a SPE outcome if and only if $1 - b \in [W_2(1), B_2(1)]$. The following strategy profile constitutes a SPE that supports such an equilibrium outcome: Player 1 proposes $(b, 1 - b)$, and player 2 will accept. If player 1 demands more than b then player 2 will reject and in G the players will play the SPE in which player 2's payoff is $B_2(1)$, to punish player 1. If player 2 rejects player 1's offer $(b, 1 - b)$, then the two players will play the SPE from which player 2's payoff is $W_2(1)$, to punish player 2. Player 1 has no incentive to deviate by demanding more in his proposal since such a proposal will be rejected (as player 2's payoff from rejection is $B_2(1) \geq 1 - b$) and player 1's payoff will be less than $1 - B_2(1) \leq b$. Player 2 has no incentive to reject player 1's proposal $(b, 1 - b)$ either as player 2's payoff from rejection is $W_2(1)$ which is less than $1 - b$. Therefore, any convex combination of $(1 - B_2(1), B_2(1))$ and $(1 - W_2(1), W_2(1))$ is a SPE payoff vector in NG(1). Player 1's and player 2's lowest SPE payoffs in $NG(1)$ are, respectively,

$$L_1(1) = 1 - B_2(1) \text{ and } L_2(1) = W_2(1). \tag{5}$$

Other than the efficient SPE outcomes described above, $NG(1)$ may also have inefficient SPE outcomes. If there is a Nash equilibrium a^*, i.e., $u(a^*) \in P(1)$, such that $u_i(a^*) \geq L_i(1)$ for $i = 1,\ 2$, then $u(a^*)$ is a SPE outcome in $NG(1)$. Consider the following strategy profile in $NG(1)$: Player 1 offers player 2 less than $L_2(1)$, player 2 rejects, and then the two players will play a^* in G. If player 1 offers player 2 no less than $L_2(1)$, player 2 accepts only if his share is not less than $B_2(1)$, and then the two players will play the SPE of G that gives player 2 $B_2(1)$ after player 2's rejection. For subgame perfection of the strategy profile, consider that if player 1 offers player 2 more than $L_2(1)$, player 2's payoff from rejection will be $B_2(1)$ so player 2 will accept only if his share is not less than $B_2(1)$. This implies that player 1's payoff cannot be higher than $L_1(1)$ by offering player 2 more than $L_2(1)$, which certainly makes player 1 worse off than by following the prescribed strategy profile. On the other hand, if player 1 does not deviate, player 2's payoff from accepting player 1's proposal will be less than $L_2(1)$ which is less than $u_2(a^*)$. Therefore, neither player will deviate from the prescribed strategy profile. In this SPE, the two players do not reach an agreement and then play a^* in G.

Hence, given $P(1)$, the set of SPE payoffs in $NG(1)$ is

$$\begin{aligned} Q(1) \;=\; & Co[\,(L_1(1), 1-L_1(1)),\ (1-L_2(1), L_2(1))\,] \bigcup P(1) \bigcap \\ & Co[\,(L_1(1), 1-L_1(1)),\ (1-L_2(1), L_2(1)),\ (L_1(1), L_2(1))\,]. \end{aligned} \tag{6}$$

Note that $Q(1) = Co[\,(L_1(1), 1-L_1(1)),\ (1-L_2(1), L_2(1)),\ (L_1(1), L_2(1))\,]$ if players are allowed to make inefficient proposals, i.e., (b_1, b_2) with $b_1 + b_2 < 1$.

For $T > 1$, the negotiation game $NG(T)$ can be treated as a one period negotiation game with the disagreement game $GNG(T)$ and a total surplus of T. Let $Q(T)$ and $P(T)$ be the sets of SPE payoffs in $NG(T)$ and $GNG(T)$, respectively. $Q(T)$ and $P(T)$ are defined by backward induction based on $Q(T-1)$, the set of SPE payoffs in the last $T-1$ periods of $NG(T)$, which is $NG(T-1)$.

Given $Q(T-1)$, for $i = 1, 2$, denote the players' lowest SPE payoffs in $NG(T-1)$ as, respectively,

$$L_1(T-1) = \min_{(y_1,y_2)\in Q(T-1)} y_1 \quad\text{and}\quad L_2(T-1) = \min_{(y_1,y_2)\in Q(T-1)} y_2 \tag{7}$$

In $GNG(T)$, a SPE payoff vector must have the form of $u(a) + (y_1, y_2)$ for some $(y_1, y_2) \in Q(T-1)$. The necessary and sufficient condition for the two players to play $a \in A$ and (y_1, y_2) to be a SPE in $GNG(T)$ is

$$\max_{a_i' \in A_i} u_i(a_i', a_j) + L_i(T-1) \le u_i(a) + y_i, \ \text{for } i, j = 1, 2, \ \text{and } i \ne j \tag{8}$$

Inequality (8) states that player i's payoff by deviating from a_i, the sum of player i's best one-shot payoff in G and his lowest equilibrium payoff in the last $T-1$ periods as the punishment, is not more than player i's payoff if player i plays a_i followed by a payoff of y_i. Hence the set of SPE payoffs in $GNG(T)$ is

$$\begin{aligned} P(T) \;=\; \Big\{\, & u(a) + (y_1, y_2) \mid (y_1, y_2) \in Q(T-1) \ \text{and for } i, j = 1, 2, i \ne j, \\ & \max_{a_i' \in A_i} u_i(a_i', a_j) + L_i(T-1) \le u_i(a) + y_i \,\Big\}. \end{aligned} \tag{9}$$

Given $P(T)$, player i's worst and best SPE payoffs in $GNG(T)$ are given by, respectively

$$W_i(T) = \min_{y \in P(T)} y_i \text{ and } B_i(T) = \max_{y \in P(T)} y_i.$$

To find $Q(T)$, we need to identify the player who makes a proposal in the first period of $NG(T)$. Recall that player 1 makes a proposal in the last period. In the first period of $NG(T)$, player 1 makes the proposal when T is odd, and player 2 makes the proposal when T is even.

First consider the case that T is odd. As in the last period of the negotiation game, player 2's SPE payoffs in $GNG(T)$ determine the set SPE payoffs in $NG(T)$. Any proposal $(b, 1-b)$ where player 2's total share in the T periods $T(1-b)$ is between $W_2(T)$ and $B_2(T)$ can be supported as a SPE outcome in $NG(T)$. Suppose player 1 makes such a proposal and player 2 accepts. If player 1 deviates then player 2 will accept only if his share is not less than $B_2(T)$ and the players will play a SPE in $GNG(T)$ in which player 2's payoff is $B_2(T)$. If player 2 deviates (rejects) then the players will play a SPE where player 2's payoff in $GNG(T)$ is $W_2(T)$. Therefore, any payoff vector in the convex hull of $(T - B_2(T), B_2(T))$ and $(T - W_2(T), W_2(T))$ can be supported as a SPE outcome. In addition, $NG(T)$ may have inefficient SPE outcomes. Any outcomes in $P(T)$ in which player 1 receives more than $T - B_2(T)$ and player 2 receives more than $B_2(T)$ can be supported by a SPE. The set of SPE payoffs in $NG(T)$ is therefore

$$\begin{aligned}
&Q(T) = \\
&Co[\ (L_1(T), 1 - L_1(T)),\ (T - L_2(T), L_2(T))\ ] \bigcup P(T) \bigcap \\
&Co[\ (L_1(T), 1 - L_1(T)),\ (T - L_2(T), L_2(T)),\ (L_1(T), L_2(T))\ ], \quad (10)
\end{aligned}$$

where $L_i(T)$ is player i's lowest SPE payoff in $NG(T)$,

$$L_1(T) = T - B_2(T) \text{ and } L_2(T) = W_2(T). \tag{11}$$

By symmetry, when T is even, player 2 makes the offer in the first period of $NG(T)$. Then $Q(T)$ is given by (10) with

$$L_1(T) = W_1(T) \text{ and } L_2(T) = 1 - B_1(T). \tag{12}$$

Equations (9)—(12) give the set of SPE payoffs in $NG(T)$ by backward induction. Note that both $P(T)$ and $Q(T)$ are formulated in terms of players' total payoffs. This backward induction argument is different from those in Benoît and Krishna (1985, 1992), and Wen (1996), due to the bargaining structure in the negotiation model. In order to characterize the limiting behavior of SPE payoffs in a finite horizon negotiation game as the game horizon goes to infinity, we will study the limit of the average SPE payoffs, $Q(T)/T$ or $P(T)/T$, under the Hausdorff metric as the game horizon T goes to infinity.

From (9)—(12), it is straightforward to show the following facts:

1. Both $P(T)$ and $Q(T)$ are compact and

$$\frac{Q(T)}{T} \subset Co[\ (0,1),\ (0,0),\ (1,0)\ ] \text{ and } \frac{P(T)}{T} \subset Co[\ (0,1),\ (0,0),\ (1,0)\ ].$$

2. For any even T' and any T, we have

$$P(T) + Q(T') \subset P(T+T') \text{ and } Q(T) + Q(T') \subset Q(T+T'),$$

and hence for $i = 1, 2$,

$$W_i(T) + L_i(T') \geq W_i(T+T') \text{ and } L_i(T) + L_i(T') \geq L_i(T+T'). \quad (13)$$

3. If G has a unique Nash equilibrium payoff vector, then $P(T)$ and $Q(T)$ are singletons for all T. In fact, $Q(T)/T = P(1)$.

The fact that both $P(T)$ and $Q(T)$ are compact implies that players worst or lowest SPE payoffs are well defined. (13) states that $L_i(T)$ is sub additive hence $L_1(T)/T$ has a limit as T goes to infinity. Lastly, if the player who responds to the proposal in the last period has a unique Nash equilibrium payoff in G then the negotiation game will have a unique SPE outcome in the last period. If the player who responds in the second last period also has a unique Nash payoff in G will $NG(T)$ have a unique SPE outcome for all finite T. In the rest of this paper, we will consider the case when at least one player has distinct Nash payoffs in the disagreement game G. In other words, G has recursive distinct Nash payoffs.

4. SPE WITH NASH DISAGREEMENT OUTCOMES

Before analyzing all SPE outcomes in $NG(T)$, we first consider a simple class of SPE in which players only play Nash disagreement outcomes. Players therefore have no incentive to deviate in the disagreement game if their continuation payoffs do not depend on their current disagreement actions. Their proposal and response strategies are uniquely determined by the future disagreement outcomes. First, Proposition 1 provides the unique acceptable proposal when disagreement outcomes are exogenously given. The results of Proposition 1 will subsequently be used to study SPE with Nash disagreement outcomes.

PROPOSITION 1. *Let a^t be the disagreement outcome in period t for $t = 1, \cdots, T$. Then $Q(T) = \{(V_1(T), V_2(T))\}$ where, $V_1(0) = V_2(0) = 0$ and*

$$V_2(t) = u_2(a^{T-t+1}) + \delta V_2(t-1),\ V_1(t) = t - V_1(t) \text{ for odd } t \le T, \quad (14)$$
$$V_1(t) = u_1(a^{T-t+1}) + \delta V_1(t-1),\ V_2(t) = t - V_2(t) \text{ for even } t \le T. \quad (15)$$

Proof. Consider the following strategy profile: In period $T - t + 1$ the proposing player proposes $(V_1(t), V_2(t))$. This leaves the responding player indifferent between rejection and acceptance, while the proposing player will claim all the residual surplus, as shown by the equations in the proposition. ∎

Proposition 1 implies that the player who makes the last proposal (player 1) has a bargaining advantage in the finite horizon negotiation game. When the disagreement outcome is fixed at $u(a)$ for all periods, then as the game horizon goes to infinity the two players will equally share the residual value $1 - u_1(a) - u_2(a)$ in addition to their disagreement payoff $u(a)$ per period. So player i's average payoff converges to $u_i(a) + [1 - u_1(a) - u_2(a)]/2$. Proposition 1 corresponds to the stationary SPE outcome in the infinite horizon negotiation model as players are sufficiently patient. Also as in the infinite horizon model, the SPE proposals depend on only the responding players' disagreement payoffs. Since the proposing player proposes the responding player's continuation payoff after rejection, the proposing player's disagreement payoff is not relevant. Finally note that the existence of Nash equilibrium in G guarantees the existence of SPE in the negotiation game $NG(T)$.

We now study SPE with Nash disagreement outcomes. By Proposition 1, we have

PROPOSITION 2. *Let a^t be a Nash equilibrium of G for all $t = 1, \cdots, T$. Then $NG(T)$ has a SPE with payoff vector $(V_1(T), V_2(T))$, where $V_1(T)$ and $V_2(T)$ are given by (14) and (15).*

Proof. Consider the following strategy profile: The proposing player in period t proposes $(V_1(t), V_2(T))$, and the responding player will accept a proposal only if his share is not less than that in $(V_1(t), V_2(T))$. After a rejection in period t (which will never happen in the SPE), the two players will play the Nash equilibrium a^t during period t for $t = 1, \cdots, T$.

During the disagreement game in any period, a player's continuation payoff after the disagreement game does not depend on the disagreement outcome in the current period. Therefore, neither player will deviate in the disagreement outcome, since a^t itself is a Nash equilibrium outcome in G. Since the two players do play the Nash outcome a^t of G in period t, neither player will deviate from their proposing or responding strategies, as Proposition 1 shows. ▌

Proposition 2 describes all the SPE payoffs with Nash disagreement outcomes. Recall that $B_i(1)$ and $W_i(1)$ are player i's best and worst Nash payoffs in G, respectively. The Nash-based punishment outcomes are obtained if the two players play the punisher's best Nash outcome when the punishee proposes, and the punishee's worst Nash equilibrium outcome when the punisher proposes. Because of the two-period cyclic structure of the disagreement outcomes, we are able to provide the closed form solutions of the SPE outcomes in terms of the players' disagreement payoffs.

Proposition 3. *In $NG(T)$, player i's worst SPE with Nash disagreement outcomes is $(V_1^i(T), V_2^i(T))$, where $V_1^i(T) + V_2^i(T) = T$ and*

$$V_1^1(T) = \begin{cases} (K+1)[1 - B_2(1) + W_1(1)] - W_1(1) & \text{for odd } T = 2K+1, \\ K[1 - B_2(1) + W_1(1)] & \text{for even } T = 2K, \end{cases} \tag{16}$$

$$V_1^2(T) = \begin{cases} (K+1)[1 - W_2(1) + B_1(1)] - B_1(1) & \text{for odd } T = 2K+1, \\ K[1 - W_2(1) + B_1(1)] & \text{for even } T = 2K. \end{cases} \tag{17}$$

Proposition 3 implies that player 2 payoffs are

$$V_2^1(T) = \begin{cases} K[1 + B_2(1) - W_1(1)] + B_2(1) & \text{for odd } T = 2K+1, \\ K[1 + B_2(1) - W_1(1)] & \text{for even } T = 2K, \end{cases} \tag{18}$$

$$V_2^2(T) = \begin{cases} K[1 + W_2(1) - B_1(1)] + W_2(1) & \text{for odd } T = 2K+1, \\ K[1 + W_2(1) - B_1(1)] & \text{for even } T = 2K. \end{cases} \tag{19}$$

Now note that in the disagreement game G every player's gain from a one shot deviation is finite. Denote the maximum one-shot gain from a deviation in G by any player as D, where D is finite. When the disagreement game G has multiple Nash equilibrium payoffs for at least one player, we know that $B_1(1) + B_2(1) - W_1(1) - W_2(1) > 0$. Therefore by Proposition 3, we have

Proposition 4. *$\forall\, T^* > 0$, $\exists\, T_0$ such that $\forall\, T \geq T_0$*

$$V_1^2(T) - V_1^1(T) > T^* \cdot D \quad \text{and} \quad V_2^1(T) - V_2^2(T) > T^* \cdot D. \tag{20}$$

Proof. For either an odd $T = 2K + 1$ or an even $T = 2K$, due to the fact that $B_1(1) + B_2(1) - W_1(1) - W_2(1) > 0$, (16) and (17) imply that

$$V_1^2(T) - V_1^1(T) \geq K[B_1(1) + B_2(1) - W_1(1) - W_2(1)].$$

Choose $T_0 = 2K$ so that $K[B_1(1) + B_2(1) - W_1(1) - W_2(1)] \geq T^* \cdot D$. Then the first inequality in (20) holds. Similarly, (18) and (19) imply that

$$V_2^1(T) - V_2^2(T) \geq K[B_1(1) + B_2(1) - W_1(1) - W_2(1)].$$

Therefore, the second inequality of (20) holds as well for the same T_0. ▮

Proposition 4 implies that SPE with Nash disagreement outcomes are sufficient to enforce any disagreement outcome in G for a given number of T^* periods, as the two inequalities in (20) show. Since the set of SPE with Nash disagreement outcomes is a subset of the set of SPE outcomes in the negotiation game $NG(T)$, we must have $L_i(T) \leq V_i^i(T)$.

5. OPTIMAL PUNISHMENTS

In this section we investigate players' optimal punishments in $NG(T)$ and compare them to their lowest SPE payoffs in the corresponding negotiation game with infinite horizon. In the corresponding infinite horizon negotiation game, a player' lowest possible disagreement payoff is his minimax value in the disagreement game, which is 0 by assumption. A player's highest effective disagreement payoff is his highest disagreement payoff after compensating the other player with his best one-shot gain from deviation. From Busch and Wen (1995), player i's highest effective disagreement payoff w_i is defined as, for $i, j = 1, 2$ and $i \neq j$,

$$\begin{aligned} w_i &= \max_{a \in A} \left[u_i(a) - \left(\max_{a'_j \in A_j} u_j(a'_j, a_i) - u_j(a) \right) \right] \\ &= \max_{a \in A} \left[u_1(a) + u_2(a) - \max_{a'_j \in A_j} u_j(a'_j, a_i) \right]. \end{aligned} \tag{21}$$

Note that $W_i(1) \geq 0$ (player i's minimax value in G) and $B_i(1) \leq w_i$ for $i = 1, 2$. In the finite horizon negotiation game, player i's optimal punishment is also determined by w_j and player i's minimax value of 0.

PROPOSITION 5. *In $NG(T)$,*

$$L_1(T) \geq 1 - w_2 + L_1(T-1) \text{ and } L_2(T) = L_2(T-1) \text{ if } T \text{ is odd,} \tag{22}$$

$$L_2(T) \geq 1 - w_1 + L_2(T-1) \text{ and } L_1(T) = L_1(T-1) \text{ if } T \text{ is even.} \tag{23}$$

Proof. Due the similarity of (22) and (23), we will prove (22) only. When T is odd, player 1 makes the first offer in $NG(T)$. Recall that $L_1(T) = T - B_2(T)$ from (11).

From the definition of $B_2(T)$ and (9), we have

$$\begin{aligned} B_2(T) &= \max_{a\in A}[u_2(a) + y_2] \text{ s.t.} u(a) + y \in P(T) &(24)\\ &\le \max_{a\in A}[u_2(a) + y_2] \text{ s.t.} u_1(a) + y_1 \ge \max_{a_1'\in A_1} u_1(a_1', a_2) + L_1(T-1). &(25)\end{aligned}$$

Since $(y_1, y_2) \in Q(T-1)$, $y_1 + y_2 \le T-1$ and the constraint in (25) imply that

$$y_2 \le T - 1 - y_1 \le T - 1 - L_1(T-1) + u_1(a) - \max_{a_1'\in A_1} u_1(a_1', a_2). \quad (26)$$

Substituting (26) into (25), we have (note that $T - 1 - L_1(T-1)$ is a constant),

$$\begin{aligned} B_2(T) &\le \max_{a\in A}\left\{u_2(a) + u_1(a) - \max_{a_1'\in A_1} u_1(a_1', a_2)\right\} + T - 1 - L_1(T-1)\\ &= w_2 + T - 1 - L_1(T-1). &(27)\end{aligned}$$

Then (24) and (27) yield the first part of (22).

On the other hand, $W_2(T) = L_2(T-1)$ of (11) since the two players can play the minimax action profile against player 2 after player 2's first rejection. This concludes the proof. ∎

Proposition 5 corresponds to Proposition 2 in Busch and Wen (1995). It shows that any player will receive at least his minimax disagreement payoff after rejecting the other player's proposal, and at most his highest effective disagreement payoff after his proposal is rejected. Together with Proposition 5, Proposition 6 below asserts that a player's average payoff in his optimal punishment in $NG(T)$ is not less than that in the negotiation game with an infinite horizon when the two players are sufficiently patient [$(1 - w_2)/2$ for player 1 and $(1 - w_1)/2$ for player 2, see Busch and Wen (1995)].

Proposition 6. *For all K,*

$$\begin{aligned} L_1(2K+2) &= L_1(2k+1) \ge K(1 - w_2) + L_1(1), &(28)\\ L_2(2K+3) &= L_2(2k+2) \ge K(1 - w_1) + L_2(2). &(29)\end{aligned}$$

Player 1's and 2's average SPE payoffs in $NG(T)$ are not less than $(1 - w_2)/2$ and $(1 - w_1)/2$ in the limit as T goes to ∞, respectively.

Proof. For player 1 and an odd $T = 2K+1$ (so $T+1 = 2K+2$), by (22) and (23) from Proposition 5, we have,

$$\begin{aligned} L_1(2K+2) &= L_1(2K+1) \geq (1-w_2) + L_1(T-2) \\ &\geq 2(1-w_2) + L_1(T-4) \geq \cdots \geq K(1-w_2) + L_1(1), \end{aligned}$$

which is (28). (29) is proven similarly.

The second part of the proposition can be shown by backward induction. Alternatively, from the definition of w_2, $B_2(1) < w_2$ implies that $L_1(1) = 1 - B_2(1) \geq 1 - w_2$. Together with (28), we have

$$\begin{aligned} \frac{L_1(2K+1)}{2K+1} &\geq \frac{K+1}{2K+1}(1-w_2) \;\to\; \frac{1-w_2}{2}, \\ \frac{L_1(2K+2)}{2K+2} &\geq \frac{K+1}{2K+2}(1-w_2) \;\to\; \frac{1-w_2}{2}. \end{aligned}$$

Since $L_2(0) = 0$ and so $L_2(2) \geq 1 - w_1$, by (29), we have

$$\begin{aligned} \frac{L_2(2K+2)}{2K+2} &\geq \frac{K+1}{2K+2}(1-w_1) \;\to\; \frac{1-w_1}{2}, \\ \frac{L_2(2K+3)}{2K+3} &\geq \frac{K+1}{2K+3}(1-w_1) \;\to\; \frac{1-w_1}{2}. \end{aligned}$$

Note that $L_i(T)/T$ is player i's lowest average SPE payoff in $NG(T)$. ∎

The results of Proposition 6 suggest that the set of average SPE payoffs in the game with a finite horizon must be a subset of that with the infinite horizon when players are sufficiently patient. Proposition 7 shows that the limits in Proposition 6 can be approximated arbitrarily closely by SPE outcomes when the game has a sufficiently long horizon. Due to the similarity of the argument, we will only investigate a SPE to approximate player 1's optimal punishment in the limit as T goes to the infinity.

PROPOSITION 7. *$\forall\, \varepsilon > 0$, $\exists\, T_1$ such that $\forall\, T \geq T_1$, $NG(T)$ has a SPE in which player 1's average payoff is ε close to $(1-w_2)/2$.*

Proof. Let a^1 be an outcome that solves for w_2 and m^1 be a minimax action profile against player 1 in the disagreement game G. By Propositions 5 and 6, without loss of generality, there is an even T_0 such that $V^1(T_0) \in Q(T_0)$ and $V^2(T_0) \in Q(T_0)$, where both $V^1(T_0)$ and $V^2(T_0)$ are on the bargaining frontier, and $V_1^1(T_0) < V_1^2(T_0)$. For $i = 1$ and 2, $V^i(T_0)$ will be used as player i's punishment in the last T_0 periods in our construction.

Then every convex combination of $V^1(T_0)$ and $V^2(T_0)$ is also in $Q(T_0)$. In particular, $V(T_0) \in Q(T_0)$ where

$$V_1(T_0) = V_1^1(T_0) + \max_{a_1' \in A_1} u_1(a_1', a_2^1) - u_1(a^1) \text{ and } V_2(T_0) = T_0 - V_1(T_0). \quad (30)$$

In $GNG(T_0+1)$, $U(T_0+1) = u(a^1) + V(T_0) \in P(T_0+1)$ since (30) yields

$$U_1(T_0+1) = \max_{a_1' \in A_1} u_1(a_1', a_2^1) + V_1^1(T_0) \geq \max_{a_1' \in A_1} u_1(a_1', a_2^1) + L_1(T_0),$$

and for the sufficiently large T_0, player 2 will not deviate either when the difference between $V_2(T_0)$ and $V_2^2(T_0)$ is large enough. Recall that T_0+1 is odd. Then $V(T_0+1) \in Q(T_0+1)$ where,

$$V_1(T_0+1) = T_0 + 1 - V_2(T_0+1) \text{ and } V_2(T_0+1) = U_2(T_0+1).$$

Since $B_2(1) \leq w_2 = u_1(a^1) + u_2(a^1) - \max_{a_1' \in A_1} u_1(a_1', a_2^1)$, we have

$$\begin{aligned} V_1(T_0+1) &= T_0 + 1 - V_2(T_0+1) \\ &= T_0 + 1 - u_2(a^1) - V_2(T_0) \\ &= T_0 + 1 - u_2(a^1) - T_0 + V_1(T_0) \\ &= V_1^1(T_0) + 1 - w_2 \\ &\leq V_1^1(T_0+1). \end{aligned}$$

Thus $V_2(T_0+1) \geq V_2^1(T_0+1)$.

Now, suppose that for an even T we have $V(T_0+T) \in Q(T_0+T)$ such that $V_2(T_0+T) \geq V_2^1(T_0+T)$ and $U(T_0+T+1) = u(a^1)+V(T_0+1) \in P(T_0+1)$. Then $V(T_0+T+1) \in Q(T_0+T+1)$ where,

$$\begin{aligned} V_2(T_0+T+1) &= U_2(T_0+T+1) \text{ and} \\ V_1(T_0+T+1) &= T_0 + T + 1 - V_2(T_0+T+1). \end{aligned}$$

Since $B_2(1) \leq w_2 = u_1(a^1) + u_2(a^1) - \max_{a_1' \in A_1} u_1(a_1', a_2^1)$, we have

$$\begin{aligned} V_1(T_0+T+1) &= T_0 + T + 1 - V_2(T_0+T+1) \\ &= T_0 + T + 1 - u_2(a^1) - V_2(T_0+T) \\ &= T_0 + T + 1 - u_2(a^1) - T_0 - T + V_1(T_0+T) \\ &= V_1^1(T_0+T) + 1 - w_2 \\ &\leq V_1^1(T_0+T+1). \end{aligned}$$

Thus $V_2(T_0+T+1) \geq V_2^1(T_0+T+1)$.

Now, suppose that for an odd T we have $V(T_0+T) \in Q(T_0+T)$ such that $V_2(T_0+T) \geq V_2^1(T_0+T)$. Hence $U(T_0+T+1) = u(m^1)+V(T_0+T) \in P(T_0+1)$. Therefore, $V^*(T_0+T+1) \in Q(T_0+T+1)$ where

$$\begin{aligned} V_1^*(T_0+T+1) &= U_1(T_0+T+1) = V_1(T_0+T), \\ V_2^*(T_0+T+1) &= T_0+T+1-V_1^*(T_0+T+1). \end{aligned}$$

Since $w_1 \geq u_1(m^1) = 0$

$$V_1^*(T_0+T+1) = V_1(T_0+T) \leq V_1^1(T_0+T+1).$$

Thus $V_2^*(T_0+T+1) \geq V_2^1(T_0+T+1)$. Every convex combination of $V^*(T_0+T+1)$ and $V^2(T_0+t+1)$ is in $Q(T_0+T+1)$. In particular $V(T_0+T+1) \in Q(T_0+T+1)$ where

$$\begin{aligned} V_1(T_0+T+1) &= V_1^*(T_0) + \max_{a_1'} u_1(a_1', a_2^1) - u_1(a^1), \\ V_2(T_0+T+1) &= T_0+T+1-V_1(T_0). \end{aligned}$$

By induction, $\forall\ T$, we have $V(T_0+T) \in Q(T_0+T)$. That is, $V(T_0+T)$ is a SPE payoff vector, such that for an odd $T=2K+1$ and even $T=2K$, we have

$$\begin{aligned} \frac{V_1(T_0+2K+1)}{T_0+2K+1} &= \frac{K(1-w_2)+V_1(T_0+1)}{T_0+2K+1}, \\ \frac{V_1(T_0+2K)}{T_0+2K} &= \frac{K(1-w_2)+V_1(T_0)}{T_0+2K}. \end{aligned}$$

It is then straightforward from the last two equations that $V_1(T)/T$ converges to $(1-w_2)/2$ as $T \to \infty$, or equivalently $K \to \infty$, which concludes the proof of Proposition 7. ▌

6. THE MAIN PROPOSITION

Now we are ready to present our main result, Proposition 8, which characterizes the limiting set of average SPE payoffs as the horizon goes to infinity. Any SPE payoff vector of the negotiation game with infinite horizon when players are sufficiently patient can be approximated by a SPE in the corresponding finite horizon negotiation game with a sufficiently long horizon.

Propositions 6 and 7 imply that $Q(T)/T$ is a closed subset of F, and its limit is a subset of set Q, where

$$Q = Co\left[\left(\frac{1-w_2}{2}, \frac{1+w_2}{2}\right), \left(\frac{1+w_1}{2}, \frac{1-w_1}{2}\right), \left(\frac{1-w_2}{2}, \frac{1-w_1}{2}\right)\right]. \quad (31)$$

Our main proposition states that every payoff vector in Q can be approximated by an average SPE payoff in $NG(T)$ for a large enough T. Therefore, Q is the limit of $Q(T)/T$ in the Hausdorff metric as T goes to infinity.

PROPOSITION 8. *Every payoff vector in Q can be approximated by the average payoff vector of a SPE in $NG(T)$ when T is large enough.*

Proof. We will prove Proposition 8 by contradiction. Suppose there exists a payoff vector $z^* \in Q$ that cannot be approximated by a SPE in $NG(T)$ even for a large enough T. Then $\exists\, \varepsilon^* > 0$ such that $\forall\, T$, $\exists\, T_0 \geq T$ such that $NG(T_0)$ has no SPE whose payoff is within $3\varepsilon^*$ of z^*. Choose another feasible payoff vector $z \in Q$ within 2ε of z^* such that for $\varepsilon \leq \varepsilon^*$,

$$\frac{1-w_j}{2} + 3\varepsilon \leq z_i \quad \text{for and } i \neq j. \quad (32)$$

By construction, $NG(T_0)$ has no SPE whose average payoff vector is within ε of z. (32) states that player i's payoff in z is at least 3ε higher than the limit of his optimal punishment SPE payoff.

Let z be a convex combination of $u(a)$ and $(b, 1-b)$ such that $u(a) < (b, 1-b)$, where $(b, 1-b)$ can be supported as a SPE in $NG(T)$ for $T \geq T_0$ and

$$L_i(T) - \frac{1-w_i}{2} < \varepsilon \ \text{ and } \ T \cdot \varepsilon \geq D, \quad (33)$$

where D is a player's highest one-shot gain by deviating in G,

$$D = \max_i \left\{ \max_{a_i' \in A_i} [u_i(a_i', a_j) - u_i(a)] \right\}.$$

By our assumptions on the disagreement game G, D is finite. The first condition in (33) requires that player i's optimal punishment SPE payoff is within ε of its limit, as guaranteed by Proposition 7. The second condition of (33) requires that a player's loss of ε per period from being punished for T periods out-weighs any possible one-shot deviation gain.

Since z is a convex combination of $u(a)$ and $(b, 1-b)$, for $\varepsilon > 0$, $\exists\, T^*$ such that $\forall\, T \geq T^*$, we have $T = T_1 + T_2$ such that $T_1 > T_0$ and

$$\frac{T_2 u(a) + T_1(b, 1-b)}{T} \quad (34)$$

is within ε of z. In the rest of this proof, we will show that $T_2u(a)+T_1(b,1-b) \in Q(T)$ by induction. It is easy to see that $u(a)+T_1(b,1-b) \in Q(T_1+1)$.

Now suppose that $tu(a)+T_1(b,1-b) \in Q(T_1+t)$ for $t < T_2$.

$$\frac{tu_1(a)+T_1(b,1-b)}{T_1+t} \geq z-\varepsilon \tag{35}$$

$$\geq \left(\frac{1-w_2}{2}, \frac{1-w_2}{2}\right)+2\varepsilon \tag{36}$$

$$\geq \frac{(L_1(T_1+t), L_2(T_1+t))}{T_1+t}+\varepsilon. \tag{37}$$

(35) is due to the fact that $u(a) \ll (b,1-b)$, (36) is due to (32) and (37) is due to (33). Therefore, (33) implies that neither player will deviate in G from a, when $tu(a)+T_0(b,1-b)$ will be followed. Hence $(t+1)u(a)+T_1(b,1-b) \in P(T_1+t+1)$. Then

$$(t+1)u(a)+T_1(b,1-b) > [L_1(T_1+t+1), L_2(T_1+t+1)]$$

implies that $(t+1)u(a)+T_1(b,1-b) \in Q(T_1+t+1)$. By induction, (34) is in $Q(T)/T$. However, this contradicts the supposition that there is no SPE whose average payoff vector is within ε of z, which concludes the proof. ∎

As the game horizon goes to infinity, the set of average SPE payoffs in the negotiation game converges to that in the corresponding negotiation game with an infinite horizon as long as at least one player has distinct Nash equilibrium payoffs in the disagreement game. For a two-player game, this requirement is equivalent to requiring that the game satisfy the recursive distinct Nash equilibrium payoffs condition of Smith (1995). Therefore, the necessary condition for the Folk Theorem in finitely repeated games is also necessary for our finite horizon negotiation game to have multiple SPE outcomes. If every player has a unique Nash equilibrium payoff in the disagreement game, then the finite horizon negotiation game will have a unique SPE. However, the corresponding infinitely repeated negotiation game will only have a unique SPE if and only if $w_i = 0$ for $i = 1, 2$, which is a stronger condition. The condition that at least one player has distinct Nash disagreement payoffs implies that at least one $w_i > 0$ for either $i = 1$ or 2.

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