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Income Taxation and Tax Evasion in a Finite Economy

Waly Wane*

Development Research Group, World Bank, 1818 H Street, NW
Washington D.C. 20433 USA
E-mail: wwane@worldbank.org

This paper introduces tax evasion in an optimal income taxation problem. It deals with finite economies. Two different problems are addressed. First, allowing the government to use generalized tax schedules (GTS, income distribution-contingent set of lump sum transfers) à la Piketty, we show that any first best Pareto optimum can be implemented, by proposing beside the GTS well defined audit strategy and fine function. Second, restricting the government to use classical tax schedules, we show that with the same type of audit strategy and fine function, (only) a subset of the first best Pareto allocations is implementable; moreover, all the agents except the more able evade some income and are not audited. © 2002 Peking University Press

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1. INTRODUCTION

The optimum income tax literature usually assumes that economies are composed by infinitely many agents whose income is perfectly and costlessly observable. The continuum assumption has been ruled out by Guesnerie and Seade (1982) who derive the properties of the optimal nonlinear tax schedule in a finite economy. Considering also a finite economy, we further assume in this paper that agents' income is private information obtainable only through a costly audit. As private information, income becomes a strategic variable at two different levels, the earning and reporting ones.

This paper lies at the intersection of three strands of literature, optimal income taxation, costly state verification and implementation theory.

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It has to do with optimal taxation since we consider a planner interested in redistributive Pareto efficient tax/transfer schemes in order to reduce inequality in a society where agents are not responsible of their exogenous income generating productivity which dispersion (variance) leads to "unfair" laissez-faire outcomes. However the planner does not observe the income earned by the agents and thus needs to announce an audit strategy to which it commits in addition to the tax schedule. The costly state verification therefore comes in since when announcing the tax schedule the planner can infer the distribution of income in the society, but will know any individual income only if it accepts to incur a cost for the acquisition of the information. The relation to the implementation theory stems from the fact that we are interested in determining the tax-cum-audit strategy that will implement a given feasible allocation.

Building on Townsend's (1979) classic paper, Border and Sobel (1987) analyze a tax evasion problem in which the risk neutral evader has private information on his realized income (out of n possibilities). The principal whose objective is to maximize expected revenue audits him with a probability depending on the reported income. They show that audit probability is a decreasing function of report and optimal audit is stochastic, while Townsend focused on deterministic audit. Mookherjee and Png (1989) generalize their model by allowing moral hazard, risk averse taxpayers and a social welfare maximizing principal. Their results are however similar to those of Border and Sobel. This is essentially due to the fact that moral hazard in their model applies to ex ante identical agents who therefore choose the same action. Cremer and Gahvari (1996) allow for different agents and endogenous labor supply as in the standard income tax literature and derive the optimal tax-cum-audit strategy of the government. However, they consider an economy with only two types of agents. All these papers consider only simple audit strategies i.e. audit probabilities depending solely on the reported income. On the contrary, considering optimal multilateral contracts in a costly state verification framework, Krasa and Villamil (1994) study audit probabilities that depend on the distribution of all announcements. There is however no effort variable in their setting in which individual incomes are completely exogenous and drawn by nature from a given distribution.

One of the main result in the optimal tax theory is the so–called taxation principle that delivers the basic message of equivalence between the sets of allocations achievable by a sophisticated planner using very general mechanisms met in the incentive theory, and by an unsophisticated planner using simple mechanisms such as tax schedules. This result shown by Hammond (1979) and Guesnerie (1981) in continuum economies has a counterpart as assessed by Dierker and Haller (1990) in large finite economies. It however relies on the assumptions of strong anonymity, i.e. the bundle allocated to an agent depends only on her own strategy, of the mechanisms and independence of the agents' characteristics drawn from the same common knowledge distribution. Arguing that strong anonymity is needlessly restrictive and independence not realistic when one deals with finite economies, Piketty (1993) shows how to implement any first best allocation via a generalized tax schedule that is a weakly anonymous mechanism, that is a mechanism that makes the tax liability of a given agent depend on her strategy and the whole distribution of income induced by the others' strategies. In fact, a GTS can be assimilated to a set of distribution–contingent lump-sum transfers.

The objective of this paper is to allow in a finite economy the possibility of tax evasion by assuming income observable only through a costly audit. It can be shown that the result of Piketty straightforwardly extends in this setting once the generalized tax schedule is supplemented with a suitably chosen audit strategy associated with a fine function. Afterwards, ruling out on the onset GTS and forcing the social planner to use classical tax schedules, we show that this same audit strategy implements a subset of the first best Pareto frontier.

2. THE MODEL

We consider an economy consisting of a set $\mathcal{H} = (1, \ldots, H)$ of agents; $h \in \mathcal{H}$ is the 'name' of the agent. Any agent h is characterized by a productivity parameter $n \in \mathcal{N}$, restricted to be a finite discrete set $\mathcal{N} = (n_1, \ldots, n_r)$ ranked in an increasing order without loss of generality. The consumers have the same preferences represented by a utility function over the only two commodities, x and y, present in the economy. These commodities are respectively consumption (net income, taken also to be the numeraire) and the amount of labor supplied. This utility function denoted $u(x, y)$ is assumed to be strictly concave on (x, y) and fulfills the conditions $u_x > 0$ and $u_y < 0$.

For any agent of productivity n who works y units of time, her gross income is given by $z = n \cdot y$. Let us define $U(x, z, n) \equiv u(x, z/n)$ and impose the condition of Agent Monotonicity of Seade (1982)

(AM) $s_n \equiv \partial s(x, z, n)/\partial n < 0$

Where $s(x, z, n) \equiv -\frac{U_z(x,z,n)}{U_x(x,z,n)} > 0$ is the marginal rate of substitution between gross income and consumption. This condition states that at any point (x, z) in the consumption-gross income space, the indifference curves are flatter the higher the productivity of the agent. This is also assumption B of Mirrlees (1971) and the usual single crossing condition of the screening literature. Moreover, we need the assumption of non-inferiority of leisure. A distribution of consumer characteristics is described by $\mu = (\mu(n_1), \ldots, \mu(n_r))$, where $\mu(n_s)$ is the number of agents whose productivity is n_s and $\sum_s \mu(n_s) = H$. A profile of characteristics p is a function that assignes to any agent $h \in \mathcal{H}$ a characteristic $n \in \mathcal{N}$. The set of possible characteristic profiles is

$$\begin{aligned} P =&\{(n(h), 1 \leq h \leq H) \text{ s.t. } \#(h \in \mathcal{H} \text{ s.t.} n(h) = n_s) = \mu(n_s) \\ &\forall s \in (1, \ldots, r)\} \end{aligned}$$

While we assume that the planner knows the actual distribution, he is not able to distinguish among agents, this is assignment uncertainty in the language of Roberts (1984). The government observes neither the productivity nor the level of labor supply but observes earned income through an audit. Auditing is however costly and audit costs are captured by the function $c(\cdot)$ which is increasing in the number of audited agents and strictly positive. Facing an economy $p \in P$, the planner to achieve his objective, to be defined later, adopts as a strategy the announcement of the following mechanism to the agents: $M = (\pi(\tilde{z}, \nu), \psi(\tilde{z}, \nu), f(\tilde{z}, z_a))$ [1], where $\tilde{z}$ is reported income audited with probability $\pi(\tilde{z}, \nu)$, z_a is earned income as revealed by the perfect audit, $f(\tilde{z}, z_a)$ the fine function that gives the penalty imposed on cheaters and $\nu = (\tilde{z}(h), h \in \mathcal{H}) \in I\!R_+^H$ is the distribution of announced incomes in the society as an outcome of the strategies adopted by the taxpayers. If an agent h reporting $\tilde{z}$ is not audited, she pays tax or receives transfer $\psi(\tilde{z}, \nu)$ but will have to pay or receive $f(\tilde{z}, z_a)$ if she is. These functions must satisfy the following limited liability restrictions: $\psi(z, \nu) \geq -z$, $\forall \nu$ and $f(z, z_a) \geq -z_a$. Note that the tax and audit functions may depend on the distribution inducing agents to take into account the strategies of others and thus use as an equilibrium concept that of Bayes–Nash. In contrast, the penalty function depends only on the announced and true incomes since there is no need to link it with the distribution. For instance, confiscating the entire income of the liers induces strong incentive for truthtelling.

As explained in Cremer and Gahvari, the agents have widened opportunities to misrepresent. Two different ways of cheating labeled "reporting" and "earning" misrepresentation respectively, can be distinguished. In the first type, an agent may cheat by reporting an income that is different from that she actually earns, an audit will reveal this type of cheating. In the second type, the agent may report her effectively earned income but this income may differ from that the planner assigns to an agent of her productivity, this is undetectable by audit. Given the mechanism proposed by the planner, agent h of type n chooses her pure strategy $\sigma_h(n) = (\sigma_h^1(n), \sigma_h^2(n)) = (z, \tilde{z}) \in I\!R \times I\!R$, where $z = n \cdot y$ is income

[1]Unlike Cremer and Gahvari (1996) and most of the mechanism design literature, we do not consider a report of type, but one of income.

really earned, assessable by audit, by providing y units of labor. The agent thus chooses to earn z and to report $\tilde{z}$ on the basis of which she will be taxed and audited. A pure strategy profile is $\sigma = (\sigma_1, \ldots, \sigma_H) \in I\!R^{2H}$. The optimal strategy $\sigma_h^*(n)$ is the one that maximizes her expected utility given the reporting behavior adopted by the others:

$$I\!E_{\tilde{z}} U(\xi, z, n) = (1 - \pi(\tilde{z}, \nu))U(z + \psi(\tilde{z}, \nu), z, n) + \pi(\tilde{z}, \nu)U(z + f(\tilde{z}, z), z, n)$$

where ξ is the stochastic or lottery consumption $((\pi(\tilde{z}, \nu), z_b); (1-\pi(\tilde{z}, \nu), z_g))$ with $z_g = z + \psi(\tilde{z}, \nu)$ and $z_b = z + f(\tilde{z}, z)$.

In a world where no tax evasion possibilities exist because income is observable, the set of possible allocations can be defined as:

$$A = \{((x(h), z(h) = n(h)y(h), 1 \leq h \leq H) \text{ s.t. } \sum_{h \in \mathcal{H}} x(h) + R \leq \sum_{h \in \mathcal{H}} z(h)\}.$$

where R represents any amount the government needs for other policy purposes. For any characteristics profile $p \in P$, we note the set of Pareto optima of the economy also called the set of first best allocations $FB(p)$, a subset of A. By the second welfare theorem, any allocation $a \in FB(p)$ is achievable by means of suitable lump-sum transfers which decentralize the competitive allocation. However, the *optimal* redistributive tax scheme which is usually unique depends on the social welfare function. We assume here that the objective of the government, reflecting his social judgements and preferences, is to maximize a generalized utilitarian social welfare function:

$$W = \sum \lambda_s \mu(n_s) U(x_s, z_s, n_s) \tag{1}$$

where λ_s is the (non negative) weight put on the utility of n_s-agents, and $\lambda = \{\lambda_s\}_{s=1}^r$ denote the vector of social weights. Depending on this vector, the optimal first best allocation is noted $a^\lambda \in FB(p)$. Varying λ (which could be normalized) allows to visit alternatively all the optimal feasible allocations, depending on the constraints under which the planner is maximizing. Assuming that the government is mainly interested in redistribution for inequality reduction, we restrict the vector of social weights to belong to the following simplex:

$$\Lambda = \{\lambda \in I\!R^r \text{ such that } \sum_{s=1}^r \lambda_s = 1 \text{ and } \lambda_s \geq \lambda_t \text{ for } s \leq t\},$$

meaning that the less able an individual is, the more his well–being is valued by the social planner, a reasonable assumption. Denote by $\bar{\Lambda}$ the simplex without restriction. The optimal allocation is anonymous when it depends only on the characteristics of the agents not on their names. Any two profiles p and $p' \in P$ which are permutations must lead to the same optimal allocation, anonymity is thus equivalent to symmetry in this setting. An anonymous allocation is determined by a vector $a = ((x_s, z_s), 1 \leq s \leq r) \in A$, where (x_s, z_s) is the bundle chosen by n_s-agents. Let us note for later reference $a^{\lambda}_{|z}$ the restriction of a feasible allocation to the earned incomes components. Once we allow for tax evasion, the utility function must be replaced by the expected utility which captures the gambling behavior of the agents. For a planner to be able to implement $a^{\lambda} \in FB(p)$, optimal tax theory shows that he must not only know μ the distribution but also p the realized profile and imposes lump sum transfers that are non anonymous. Once p becomes unobservable because of parsimonious information while income is still observable, the planner has to content himself with incentive compatible i.e. information–constrained allocations. The set of attainable allocations is called second best and denoted $SB(p)$,[2] a subset of A. More formally, by the second welfare theorem a first best allocation, for the profile $p = (n(h), h \in \mathcal{H}) \in P$, is a sequence of bundles $((x(h), z(h), h \in \mathcal{H}) \in A$ that can be defined by a vector of transfers $(\psi(h), h \in \mathcal{H}) \in I\!R^H$ meeting (i) the budget constraint and (ii) the bundle allocated to an agent is the one that maximizes her utility given the transfer:

(i) $\sum_{h \in \mathcal{H}} \psi(h) + R \leq 0$
(ii) $\forall h \in \mathcal{H}, (x(h), z(h)) = \mathrm{Arg\,max}_{x,z}\, U(x, z, n(h))$ s.t. $x \leq z + \psi(h)$,

Whereas a second best allocation which depends on the distribution not the profile, is a sequence meeting (iii) the feasibility constraint and (iv) the incentive compatibility constraints:[3]

(iii) $\sum_{n_s \in \mathcal{N}} \mu(n_s)x(n_s) + R \leq \sum_{n_s \in \mathcal{N}} \mu(n_s)z(n_s)$
(iv) $U(x(n_s), z(n_s), n_s) \geq U(x(n_{s'}), z(n_{s'}), n_s), \quad \forall n_s, n_{s'} \in \mathcal{N}$.

3. THE IMPLEMENTATION PROBLEM

Assuming incomes to be perfectly and costlessly observable, the main focus of the optimal income tax literature has been to determine and precisely

[2]Throughout the paper, the allocations which are called first and second best are the usual ones, i.e allocations that would be implemented in a first and second best frameworks when income is observable and henceforth there is no tax evasion.

[3]A symmetric expression to that of the first best case involving the net transfers is obtained by writting $x(n_s) = z(n_s) + \psi(n_s)$ and therefore (iii) and (iv) are rewritten:

(iii') $\sum_{n_s \in \mathcal{N}} \mu(n_s)\psi(n_s) + R \leq 0$

(iv') $U(z(n_s) + \psi(n_s), z(n_s), n_s) \geq U(z(n_{s'}) + \psi(n_{s'}), z(n_{s'}), n_s), \ \forall n_s, n_{s'} \in \mathcal{N}$.

define the set of allocations a planner can implement when he is subjected to the resource and informational constraints. This set has been found to be the same whether the planner uses complex mechanisms that are direct, anonymous and truthful where agents are asked to report a given vector (their type, the profile) on which their allocation will depend, or simple mechanisms such taxation schemes where agents are only given a function (the tax schedule) to which they respond rationally by choosing the income to earn. This equivalence result does rely neither on the informational assumptions made i.e. what do the agents and the planner know, nor on the equilibrium concept used that reflects these informational assumptions. It remains however to determine whether the observability of incomes only through costly audits affects in some way the set of achievable allocations. One should in fact expect to enter in third best world due to this additional constraint.

3.1. Implementation with Generalized Tax Schedules

In a recent paper, Piketty (1993) successfully challenges the equivalence result and shows that in a finite economy almost every first best allocation is implementable. The planner can thus implement many more allocations than stated by the taxation principle by using slightly more complicated tax schemes. This result however heavily relies on the finite economy and common knowledge fixed distribution of types assumptions. We adopt here these assumptions but rule out the costless observability of incomes. What can the planner then implement if he must incur a cost in order to know "true" incomes? Obviously the first best is unattainable as soon as a proportion $\epsilon > 0$ of the population is audited since it entails a loss of scarce resources. In this special setting, we can however show that using a generalized tax schedule (GTS) as defined by Piketty, supplemented with well defined audit strategy and fine function, the planner can implement any first best allocation he desires. Let us first define and give the properties of a GTS.

A generalized tax schedule is a mapping ψ which for every pre-tax income and distribution of incomes associates a transfer, hence a consumption level (post-tax income) satisfying:

(a) $\forall \nu \in I\!R^H$, $\forall n \in \mathcal{N}$, $\forall h \in \mathcal{H}$, such that $n(h) = n$, $\exists!\ (x(n), z(n)) = \operatorname{Arg\,max} U(x, z, n)$ s.t $x \leq z + \psi(z, \nu)$,

(b) $\sum_{s=1}^{r} x(n_s)\mu(n_s) + R \leq \sum_{s=1}^{r} z(n_s)\mu(n_s)$.

Where ν is the distribution of income in the economy induced by the strategies adopted by the agents. Condition (a) states that the pre-tax income chosen by an agent of type n is the only that maximizes her utility given the distribution of earned incomes and the consumption that the GTS allocates her, while condition (b) is merely a feasibility constraint that must hold at the equilibrium. It is also worth noting at this point that the arguments of the GTS as defined are reported and distribution of reported incomes, not necessarily true incomes. This matters since there exist strategies such that the distribution of reported incomes is the "expected" one while the true distribution is completely different. Since the reports are such that the planner gets the desired distribution, he therefore has no incentive to carry out audits which are costly and the society ends up with a different outcome than the one expected. We will however be able to show that this situation cannot be sustained as an equilibrium. The information structure assumed is the following one: each agent knows only her type and the statistical distribution of types while the planner knows only the latter. The agents can thus rationally behave in a Bayesian Nash way. We are now in the position to state the following proposition:

PROPOSITION 1. *Let $p \in P$ be the profile. Suppose the social judgements of the planner are given by $\lambda \in \Lambda$ and his first best optimal allocation is $a^\lambda \in FB(p)$. Then, there exists a mechanism $M = (\pi(\tilde{z}, \nu), \psi(\tilde{z}, \nu), f(\tilde{z}, z_a))$, where ν is the distribution of announced incomes, $\pi(\cdot, \cdot)$ is the audit strategy and $\psi(\cdot, \cdot)$ a GTS, that implements a^λ in Bayesian Nash equilibrium.*

Proof. The proof is divided into steps. Let $a^\lambda = ((x_s, z_s), 1 \leq s \leq r)$ be the first best Pareto optimal allocation.

Step 1: a^λ is such that $z_s > z_t$ and $z_s - x_s > z_t - x_t$ for $s > t$. The optimal allocation is such that more able agent have greater earned incomes. This is obvious because of the social weights and the proof is left to the reader.

Step 2: From the proposition 1 of Piketty [1993, pp. 36–37], there exists a sequence of lump sum transfers $(\psi_{st}, 1 \leq s, t \leq r)$ such that:

$$\forall s, t, u \text{ such that } 1 \leq s \leq r, \quad s \leq u \leq r, \quad 1 \leq t \leq r, \quad t \neq s, \quad \text{then}$$

$$\psi_{ss} = \psi_s \text{ and } U(z_s + \psi_{su}, z_s, n_s) > U(z_t + \psi_{tu}, z_t, n_s) \qquad (D)$$

Note that the observabilty of incomes is quite important in this sequence. Indeed, the transfers are defined with respect to incomes. Once unobservability enters the scene, it becomes necessary to take into account the maximizing behavior of the agents. For instance if those who declare z_s are not audited, it is not clear whether it is their real income. Given the GTS they face, they could have chosen to earn something else. This adjustment process is not taken into account and needs not to be if pre-tax income is known. In this context, the sequence defined above may not work well. One needs a more restrictive property to hold. Let us then assume for the time being that there exists a sequence of transfers $(\psi_{st}, 1 \leq s, t \leq r)$

satisfying the following condition:

$$\forall s, t, u \text{ such that } 1 \leq s \leq r, \quad s \leq u \leq r, \quad 1 \leq t \leq r, \quad t \neq s, \quad \text{then}$$

$$\psi_{ss} = \psi_s \text{ and } U(z_s + \psi_{su}, z_s, n_s) > U(z^*_{st} + \psi_{tu}, z^*_{st}, n_s) \qquad (D')$$

where $z^*_{st} \equiv Arg\max_z U(z + \psi_t, z, n_s)$ is the income that maximizes the utility of an n_s-agent who reports z_t and receives the transfer ψ_t assigned to n_t-agents. Futhermore, $z^*_{st} \neq z_t$ for any $s \neq t$. Let us define the different functions of the mechanism. The GTS ψ is defined by:

(a) $\psi(\tilde{z}, \nu) = \psi_{st}$ if $\tilde{z} = z_s \in a^\lambda_{|z}$ and
$t = Min\{t' \text{ s.t. } 1 \leq t' \leq r \text{ and } \nu(z_{t'}) < \mu(n_{t'})\}$,
$t = s$ if $\nu = \mu$, for any $1 \leq s, t \leq r$.
(b) $\psi(\tilde{z}, \nu) = -\tilde{z}$ if $\tilde{z} \notin a^\lambda_{|z}$.

The audit strategy is given by:

$$\pi(\tilde{z}, \nu) = \begin{cases} 1 & \text{if } \tilde{z} \notin a^\lambda_{|z} \\ 1 & \text{if } \tilde{z} = z_s \in a^\lambda_{|z} \text{ and } \nu(z_s) > \mu(n_s) \\ 0 & \text{otherwise.} \end{cases}$$

And finally the fine function is simply:

$$f(\tilde{z}, z_a) = \begin{cases} -z_a & \text{if } \tilde{z} \neq z_a \\ \psi(z_a, \nu) & \text{if } \tilde{z} = z_a. \end{cases}$$

Step 3: $(z, \tilde{z})$ with $\tilde{z} \notin a^\lambda_{|z}$ is a strictly dominated strategy for every agent and $\forall z \in I\!R$.

3i) $z \neq \tilde{z}$. The agent is misrepresenting at both reporting and earning levels. Given the audit strategy, she will be audited for sure and fined since the true and reported incomes do not coincide and her income is entirely confiscated as suggested by the fine function. It is never in the agent's interest to adopt this strategy.

3ii) $z = \tilde{z}$. The agent is now misrepresenting only at the earning level since she earns an income different from those recommended by the planner. An audit will be carried out and reveal that the agent is not cheating. However, from condition (b) of the GTS, her consumption level will be set at zero and this is a strictly dominated strategy.

Step 4: From step 3, the only strategies that remain are $(z, \tilde{z})$ with $\tilde{z} \in a^\lambda_{|z}$. It is always in the interest of the agent to report an income in $a^\lambda_{|z}$. It remains now to show that each agent must earn and report the income assigned to her type. Let us first consider an agent h such that $n(h) = n_1$ and show that (z_1, z_1) is a strictly dominant strategy. Indeed, given the tax schedule, it is never in the interest of agent h to report $z_s > z_1$ since her tax liability will increase and her utility decrease. Reporting z_1 being a dominant strategy, since given (D') we have

$$\forall z \geq 0, \quad z \neq z_1, \quad \forall \nu, \quad U(z_1 + \psi(z_1, \nu), z_1, n_1) > U(z + \psi(z, \nu), z, n_1).$$

$\sigma_h^*(n_1) = (z_1, z_1)$ is a strictly dominant strategy for all $h \in \mathcal{H}$ such that $n(h) = n_1$. What is then the strategy of an agent k such that $n(k) = n_2$? Knowing the distribution, the agent rationally anticipates that she will never face a distribution of reports such that $\nu(z_1) < \mu(z_1)$ then if she reports to earn z_1, an audit will be performed and truly earning that income is the best she could do. However, given (D'), the strategy (z_2, z_2) dominates (z_1, z_1) and all other reports that could only increase tax liability therefore cost without providing any benefit. It is thus the unique iteratively non-strongly dominated strategy for any n_2-agent. Repeating this argument for all the agents until $n(l) = n_r$ shows that (z_s, z_s) is the only strategy that survives the iterative removal of strongly dominated strategies for any n_s-agent, for $s = 1, \ldots, r$.

Step 5: Given the reporting strategies, the reported profile appeals for no audit and thus no cost. Moreover, all the agents earn the income assigned to their type and get the optimal transfer. The first-best is thus implemented in Bayesian Nash equilibrium. ▌

A last remark is that no evasion occurs in the economy. Indeed, despite the fact that no audit takes place, the agents are not attracted by the possibility of evasion. As a matter of fact, they could earn more by working more without paying additional tax. However, evasion is not rewarding in this setup since agents are offered undistorted bundles, i.e. given the transfer they receive, the pre-tax utility maximizing income is nothing else than the one they are required to report, hence the one they earn and report.

3.2. Implementation without Generalized Tax Schedules

An important feature of any GTS is that it assumes too much power for the planner. Indeed, this scheme resembles to the “shoot them all” type mechanisms such as those encountered in the auction theory, see Crémer and McLean (1985, 1988). In these mechanisms, when the distribution of announced characteristics is different from the exact distribution, the principal punishes all the players. In the context of the preceding subsection, the GTS proposing distribution-contingent transfers punishes all the agents whenever the distribution of incomes (reports confirmed by audit) is different from the desired one induced by the true distribution of characteristics. GTS are as such hardly defendable on ethical grounds and politically unsustainable since all the agents except the less skilled disapprove it. Futhermore, no-envy allocations tend to be the main focus of the modern equity theory and GTS implements first best allocations which are not envy free. Suppose now that the government gets his hands tied and is bound to use distribution-independent tax schedules. This could be the case for several reasons. For instance, we suppose that the agents have the right or power to destroy all or part of their endowment (here labor supply), i.e. earn the income they want without fearing any confiscation of income, as long as they are honest in their report. This assumption is a realistic one in modern economies where the sovereignty of the individual is an unalienable right but dishonesty is considered as a condemnable behavior. Governments generally have a popular mandate to punish cheaters.

Being bound to use distribution-independent tax schedules or simple mechanisms in the language of Dierker and Haller, the government will propose a tax function, $\psi(z)$, that takes income to tax liability. In order to be implementable, this tax schedule must be incentive compatible. We know from Guesnerie and Seade (1982) that in the case of finite economies, there is no loss of generality by restricting the tax schedules to be in the class of increasing step-functions. Denote the optimal second best allocation by $b^\lambda = ((x_s, z_s), 1 \leq s \leq r)$, given the social welfare function, when income is observable. It is then a sequence of bundles satisfying requirements (iii) and (iv). If income becomes unobservable, we will only use the tax schedule and associate to any reported income $\tilde{z}$ a transfer $\psi(\tilde{z})$. It is well known that this function is non-increasing. For a report $\tilde{z}$, the consumption of an n-agent is $x = z + \psi(\tilde{z})$ where z is the truly earned income and her corresponding utility $U(z + \psi(\tilde{z}), z, n)$, whenever she is not audited. The objective of this section is to identify the equilibria allocations, if any, that will result if the planner proposes the mechanism $M = (\pi(\tilde{z}, \nu), \psi(\tilde{z}), f(\tilde{z}, z_a))$, the same as in section 3.1 except that the GTS is replaced by a simple tax function. Even if we are not directly interested here in the structure of the optimal second best allocation, it is useful to remind its basic properties. Contrarily to the continuum case where there is no distortion at the endpoints, i.e. at the bottom and top, in the finite case all the agents face a strictly positive distortion except the most able (top). As shown by Guesnerie and Seade (1982), the optimal schedule is a downward adjacent incentive compatible chain, i.e. the n_s-agents are only attracted by the bundle proposed to their immediate less able n_{s-1} neighbors. It is assumed here that there is no bunching at the optimum. Formally then, the optimal second best allocation exhibits the following feature:

$$U(x_s, z_s, n_s) \geq U(x_t, z_t, n_s) \quad \forall \quad 1 \leq s, t \leq r$$

with equality only for $t = s - 1$. Let us also assume that:

(A1) $U(x_s, z_s, n_s) > U(x_{s+1}+e, z_{s+1}+e, n_s)\ \forall e \geq 0,\ \forall s = 1, \ldots, r-1.$

Assumption (A1) means that there is no gain for a given agent to claim being more productive than she is, even in the more favorable case which entails no audit. This is quite a natural assumption and it always holds for $s = r-1$ or if $r = 2$. Mimicking more skilled agents is never rewarding.[4]

Suppose the planner announces the mechanism $M = (\pi(\tilde{z}, \nu), \psi(\tilde{z}), f(\tilde{z}, z_a))$ such that:

$$\pi(\tilde{z}, \nu) = \begin{cases} 1 & \text{if } \tilde{z} \notin b^{\lambda}_{|z} \\ 1 & \text{if } \tilde{z} = z_s \in b^{\lambda}_{|z} \text{ and } \nu(z_s) > \mu(n_s) \\ 0 & \text{otherwise.} \end{cases}$$

and

$$f(\tilde{z}, z_a) = \begin{cases} -z_a & \text{if } \tilde{z} \neq z_a \\ \psi(z_a) & \text{if } \tilde{z} = z_a. \end{cases}$$

$\psi(\tilde{z})$ is the tax schedule that would generate the second best allocation b^{λ} if income were observable. Let us determine the equilibrium allocation associated with this mechanism:

Step 1: (Reporting strategies) Let us consider the reaction of an agent h such that $n(h) = n_1$ that faces this mechanism. What will she report? We will show that the strategy (z_1, z_1) is strictly preferred to any strategy $(z, \tilde{z})$ with $z \in I\!R$ and $\tilde{z} \neq z_1$. Two configurations are then possible. First, suppose $\tilde{z} \notin b^{\lambda}_{|z}$, given the audit strategy, the agent will be audited and the most favorable case for her is the one in which $\tilde{z} = z$ but the tax schedule being incentive compatible, we have $U(z_1 + \psi(z_1), z_1, n_1) > U(\tilde{z} + \psi(\tilde{z}), \tilde{z}, n_1)$ thus this report is strictly dominated. If her report is $z_s \in b^{\lambda}_{|z}$ with $z_s > z_1$ then whether audit occurs or not, using (A1), the agent is worse off than if she announced and earned z_1, i.e. $U(z_1 + \psi(z_1), z_1, n_1) > U(z^* + \psi(z_s), z^*, n_1), \forall s = 2, \ldots, r$ where z^* is the income that maximizes her utility if she receives the transfer $\psi(z_s)$. Reporting z_1 is thus a strictly dominant strategy for all n_1-agents.

Consider now the agent k such that $n(k) = n_2$. Since she knows the distribution and anticipates that all n_1-agents will report z_1, she knows that she will never face a distribution such that $\nu(z_1) < \mu(n_1)$. First of all, note that this agent is indifferent between (z_2, z_2) and (z_1, z_1); then following the whole implementation literature let us assume that whenever an agent is indifferent between two strategies she chooses the one her principal wants her to choose.[5] Given the strategy of n_1-agents, (z_2, z_2) is strictly preferred

[4]Cremer and Gahvari (1996) implicitly make this assumption by ignoring in their program the upward incentive constraint (p. 239).

[5]This assumption is made for the sake of simplicity and allows to have an already defined departure point, namely the second best optimum. It is in fact inessential

to (z, z_1). Suppose the agent adopts $(z, \tilde{z})$ with $\tilde{z} \notin \{z_1, z_2\}$. As above, there are two possible configurations. First, suppose $\tilde{z} \notin b^{\lambda}_{|z}$, audit will occur and the agent's utility is higher if she effectively earns $\tilde{z}$, but given the tax schedule $U(z_2 + \psi(z_2), z_2, n_2) > U(\tilde{z} + \psi(\tilde{z}), \tilde{z}, n_2)$, thus this strategy is dominated. Second, the report is $z_s \in b^{\lambda}_{|z}$ with $z_s > z_2$ then whether audit occurs or not and using (A1) we have $U(z_2 + \psi(z_2), z_2, n_2) > U(z^* + \psi(z_s), z^*, n_2), \forall s = 3, \ldots, r$ where z^* is the income that maximizes her utility if she receives the transfer $\psi(z_s)$. Therefore reporting z_2 is the unique iteratively non-strongly dominated strategy for any n_2-agent. As in the preceding section, repeating these arguments until $n(l) = n_r$ leads to the fact that z_s is the surviving reporting strategy for any n_s-agent for any $s = 1, \ldots, r$. However, we only analyzed here the reporting strategies, what about the earning ones?

Step 2: Given the reporting strategies from step 1, the distribution of announced incomes is such that $\nu(z_s) = \mu(n_s)\ \forall\ s = 1, \ldots, r$. Therefore, given the audit strategy to which the government is committed, no audit will be carried out.

Step 3: (Earning strategies) Anticipating step 2, the agents could have incentives to earn incomes different from those they announce. Indeed, they will choose to earn the income that maximizes their utility given the transfer they receive corresponding to the report they made. The optimal earned income is thus denoted by z^*_s for n_s-agents (for $s = 1, \ldots, r$), defined by

$$z^*_s \equiv \operatorname{Arg}\max_z U(z + \psi(z_s), z, n_s)$$

it can easily be shown that $z^*_s = z_s + e_s$ meaning that if the optimal second best bundle of a type n_s agent is $b^{\lambda}_s = (x_s, z_s)$, then under unobservability and the mechanism proposed her "effective" bundle is $(x_s + e_s, z_s + e_s)$ lying on the 45 degree line passing through b^{λ}_s, e_s is the amount evaded by the agent. One can directly remark that all the agents except the most able evade some amount. Indeed, these latter are the only whose bundle is undistorted, hence given the transfer they receive the income they are required to announce is the one that maximizes their utility level.

Step 4: Moving from the strategies as defined in the three steps entails a strictly positive loss for either the government or the agents except the most able ones. Indeed, these latter can switch from (z_r, z_r) to (z_{r-1}, z_{r-1}) and get the same utility, but given our assumption, they will stick with the first strategy. For any other n_s-agent, moving from (z^*_s, z_s) to $(z, \tilde{z})$

in this model, as it will be shown in the appendix. Indeed, the same equilibrium will obtain if the planner proposes a tax schedule that would implement an allocation slightly different to the second best that is budget balancing and has the feature that no incentive constraint binds, eliminating by the way this indifference and rendering the report of the assigned income a strictly dominant strategy.

given the others' strategies can only be harming, irrespective of whether she changed the reporting or earning strategy or both. Let us consider all three cases:

(i) Changing only the reporting strategy, i.e. switch from (z_s^*, z_s) to $(z_s^*, \tilde{z})$. The agent will be audited anyway and fined unless $\tilde{z} = z_s^*$ in which case her utility is $U(z_s^* + \psi(z_s^*), z_s^*, n_s) < U(z_s^* + \psi(z_s), z_s^*, n_s)$ since the tax schedule is incentive compatible. It is thus never optimal to change the reporting strategy.

(ii) Changing only the earning strategy, i.e. switch from (z_s^*, z_s) to (z, z_s). In this case, the agent is not audited since the announced profile of earned income is the one expected by the planner. Earning z is then dominated since the agent can do better by earning the income that will maximize her utility, namely z_s^*.

(iii) Changing both strategies, i.e. switch from (z_s^*, z_s) to $(z, \tilde{z})$. Audit will occur and the agent can by no means do better than with the original strategy.

For the government, auditing will be harming at two levels:

- it entails loss of scarce resources;
- agents will be punished and social welfare will decrease.

There is thus no incentive to audit even if agents cheated. Finally let us denote the equilibrium allocation by b^λ, and summarize our discussion in the following proposition:

PROPOSITION 2. *Let $p \in P$ be the profile. Suppose the social judgements of the planner are given by $\lambda \in \Lambda$ and his second best optimal allocation is $b^\lambda = ((z_s + \psi(z_s), z_s), 1 \leq s \leq r) \in SB(p)$. Then, by using a mechanism $M = (\pi(\tilde{z}, \nu), \psi(\tilde{z}), f(\tilde{z}, z_a))$ where ν is the distribution of announced incomes, $\pi(\cdot, \cdot)$ is the audit strategy and $\psi(\cdot)$ the tax function, the allocation $d^\lambda = b^\lambda + e$ where $e = ((e_s, e_s), 1 \leq s \leq r)$, is implemented in Bayesian Nash equilibrium. Moreover, $d^\lambda \in FB(p)$ and $e \in I\!R^r$ is the vector of evaded revenues.*

One immediately remarks that this result is in sharp contrast with those found in the literature on tax evasion and auditing. First of all, at the equilibrium almost everyone evades (see step 3) and there is no audit. The tax evasion literature using the principal agent framework usually finds that at the equilibrium everybody except the most able is audited but no one cheats, hence the commitment issue which does not arise in this setup. On the contrary, the government's commitment to the audit strategy is strengthened. Furthermore, in this paper the revelation principle does not *fully* hold in the sense that agents go to earn incomes that are different from that defined by the mechanism even though there are those they report.

The question that immediately jumps at mind then is can the planner propose another mechanism that will lead to the same allocation without inducing agents to cheat? The answer is no. Indeed, it is straightforward to see that d^{λ} is on the first best Pareto frontier since all the agents are on a 45 degree line thus face no distortion. d^{λ} is not an incentive compatible allocation. If the planner wants to implement it, he would compulsorily rely on a GTS which is excluded. Effectively, if the planner directly proposes d^{λ}, he will be obliged to threaten the agents of strong punishment if too many people claim for the envied bundles, thus to propose a distribution–contingent tax schedule.

Proposition 2 merely identifies the equilibrium allocation that would result if the government announces the mechanism M. The tax schedule proposed is the one that would maximize the planner's objective if income were observable and lead to the second best allocation b^{λ} for social judgements embodied by the vector of welfare weights λ. The planner can anticipate the strategies of the agents. Note however, that the proposition assigns him quite a myopic behavior in the choice of the tax schedule. In fact, anticipating the agents' reaction and the final allocation, the planner can and will effectively play strategically by proposing the mechanism that will implement the allocation the closest to his favorite first best. Let us first introduce some notations for later reference. Define

$$\Psi^i(p) = \{\psi^i_{\lambda} \in I\!R^r, p \in P, \lambda \in \bar{\Lambda}, \text{ such that } \psi^i_{\lambda} \text{ implements } b^{\lambda} \in i(p)\}$$

for $i = FB, SB$ the set of transfers that implement first and second best allocations for a given profile. A planner that assigns the social weights λ is called a λ-planner, denote by ψ^{FB}_{λ} the vector of lump sum transfers that implements his optimal first best allocation and ψ^{SB}_{λ} the vector of distortionary transfers that generates the second best, and let us assume that these vectors are unique for any $\lambda \in \bar{\Lambda}$. In fact, the latter vector represents the vector of individual revenue requirements (IRR) that is implemented by the incentive compatible tax function[6] $\psi^{\lambda}(z) : I\!R^+ \longmapsto I\!R$. The level of social welfare achieved at the equilibrium allocation when all the behavioral adjustments already took place, i.e. the agents choose their optimal labor supply given the vector of IRR ψ, is defined by:

$$W^*(\psi) = \sum \lambda_s \mu(n_s) U(z^*_s + \psi_s, z^*_s, n_s) \tag{2}$$

where z^*_s depends on the transfer and maximizes the utility of n_s-agent.

It is obvious that $\Psi^{SB} \subset \Psi^{FB}$ (where p is dropped without risk of confusion). Indeed, all the transfers that can take place in an asymmetric

[6] See Berliant and Gouveia (1994) and Berliant and Page (1996) for more on individual revenue requirements.

information framework can also be done if information is perfect. The objective of any λ-planner is to implement the allocation that will achieve the social welfare level the closest to his first best, and will then behave strategically by proposing in his mechanism the incentive compatible tax function of the λ'-planner solution to the following program:

$$\lambda' \in Arg\min_{\lambda''}\left(W^*(\psi_\lambda^{FB}) - W^*(\psi_{\lambda''}^{SB})\right) \qquad (ML).$$

This is a loss function and the planner minimizes this loss in social welfare. Note that there is no existence problem but there could exist many solutions for this program. When it is the case, the planner randomly chooses one of them. The following proposition captures his strategic behavior:

PROPOSITION 3. *Any λ-planner to achieve his objective behaves strategically by proposing the mechanism $M = (\pi(\tilde{z},\nu), \psi^{\lambda'}(\tilde{z}), f(\tilde{z}, z_a))$, mimicking a λ'-planner where λ' is a solution of (ML). Moreover, his first best is attained if and only if the vector of optimal lump sum transfers of the λ-planner is in the set of incentive compatible individual revenue requirements, i.e. iff $\psi_\lambda^{FB} \in \Psi^{SB}$.*

Proof. If $\psi_\lambda^{FB} \in \Psi^{SB}$, then $\exists\ \lambda' \in \Lambda$ such that $\psi_\lambda^{FB} = \psi_{\lambda'}^{SB}$. Thus, using proposition 2, the equilibrium allocation will coincide with the first best if the planner proposes the tax schedule $\psi^{\lambda'}(z)$. If, on the contrary $\psi_\lambda^{FB} \notin \Psi^{SB}$ because the planner wants to make too much redistribution, it is impossible to implement *his* first best. He is constrained to choose on the Pareto frontier his next best feasible allocation. ▮

It could be interesting to translate program (ML) in an alternative program that involves only the vectors of transfers. This would indeed lead to the complete characterization of the set of λ'-planners that corresponds to (are mimicked by) a given λ-planner, for all λ. This is however not an obvious task. For instance, one could think that the vector of IRR that will satisfy (ML) is the closest to the vector of optimal lump sum transfers with respect to a given norm. However, this reasoning can be misleading since many other dimensions such as the number of agents of a given productivity, the maximizing behavior of the agents also intervene. There is also the fact that we are on an r-dimensional space. To get more insight on all this, an illustration with a two-group model is given in the next section.

As with the GTS, this section shows that the equilibrium allocation belongs to the first best Pareto frontier. It is then legitimate to inquire about the strength of the restriction to classical tax schedules. Banning the use of GTS in fact narrows the set of implementable allocations. Indeed, too redistributive planners will not be able to attain their optimal point because it implies large transfers from rich to poor that are infeasible when agents enjoy private information. The restrictiveness of classical tax schedules is then well captured by $\Psi^{SB} \subset \Psi^{FB}$, meaning that there exists a set of allocations implementable via GTS but not with simple tax schemes. The analysis could also be done directly on the utility possibilities sets under first and second best. Unfortunately, the thinness of the literature on this topic when there are many different individuals implying multidimensional frontiers does not allow such an analysis. In next section, we give an illustration where there are only two types of agents.

3.3. A two-group illustration

To build up the intuition and enlighten the way the different mechanisms work, this section is devoted to an illustration in economies consisting of only two types of agents. Suppose that $r = 2$, $n_2 > n_1$ and $\mu(n_i) = \mu_i$ for $i = 1, 2$.

GTS allowed: Let us consider the first best anonymous allocation $a^\lambda = ((z_1, x_1); (z_2, x_2))$ a λ-planner, $\lambda \in \Lambda$, prefers. This allocation implies a large per capita redistribution from n_2- to n_1-agents, the former paying ψ_2 and the latter receiving ψ_1. How can the planner implement this allocation knowing that the bundle of type 1 agents is envied by the more able and moreover income is not observable?

Denote by $\tilde{\psi}$ the tax/transfer satisfying

$$U(z_1 + \tilde{\psi}, z_1, n_2) = U(z_2 + \psi_2, z_2, n_2),$$

clearly $\psi_1 > \tilde{\psi} > \psi_2$, (see fig 1). The government, who has a coercive power, imposes the following social contract to the agents:

- if you announce an income $\tilde{z} = z_2$ your transfer is ψ_2 and you will not be audited;
- if you announce $\tilde{z} = z_1$ and $\nu_1 \leq \mu_1$ your transfer is ψ_1 and you will not be audited;
- if you announce $\tilde{z} = z_1$ and $\nu_1 > \mu_1$ then you will be audited and
  1. if $z_a = z_1$ your transfer is ψ^* with $\psi_2 < \psi^* < \tilde{\psi}$;
  2. if $z_a \neq z_1$ your transfer is $-z_a$.
- if you announce $\tilde{z} \neq z_i$ for $i = 1, 2$ then you will be audited and assigned the transfer $-z_a$.

With this contract, any n_1-agent has as a strictly dominant strategy the report of z_1. Anticipating the report of the less skilled agents, any n_2-agent unique best report is z_2. The government will then collect the taxes on n_2-agents and distribute the proceeds to n_1-agents. All the agents will choose to earn the income they reported since it maximizes their utility given the transfer. The government thus implements a^λ by proposing the above contract which is a simplification of that proposed to r types of agents examined in section 3.1. Any point on the first best Pareto frontier can be attained by such a mechanism.

GTS prohibited: Suppose now that the government is bound to propose a simple tax function that relates income to tax liability which cannot be made contingent to the distribution of announced incomes. What can then the λ-planner whose first best allocation is a^λ implement? There are two cases to consider depending on whether $\hat{z}$ verifying

$$U(\hat{z} + \psi_1, \hat{z}, n_2) = U(z_2 + \psi_2, z_2, n_2),$$

is ≥ 0 (fig 2a) or < 0, (fig 2b). These two cases correspond to $\psi_\lambda^{FB} \in \Psi^{SB}$ (resp. $\notin$). In the first case, the planner is able to implement a^λ by proposing

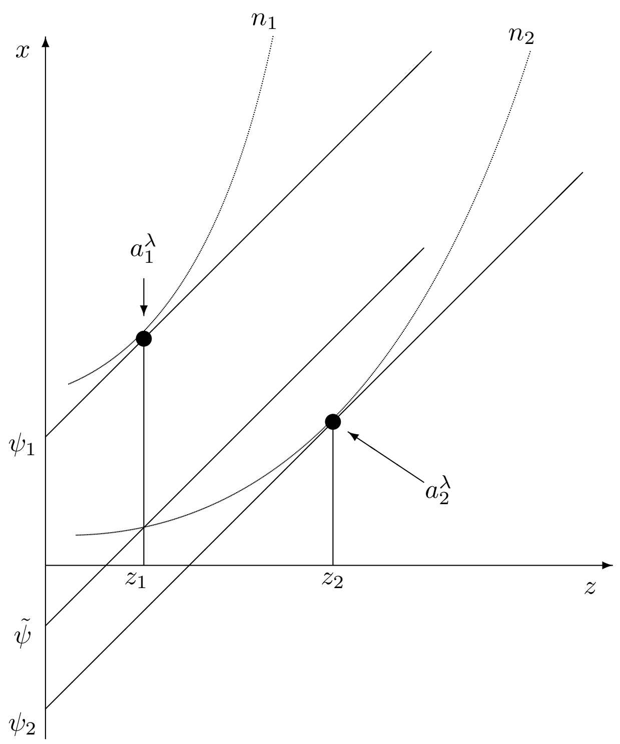

FIG. 1. First-Best allocation implemented with GTS

in his mechanism the incentive compatible tax function represented by the common budget set. The following mechanism is announced:

- for any announced income $\tilde{z}$ your transfer if you are not audited or audited and truthful is $\psi(\tilde{z}) = \tau(\tilde{z}) - \tilde{z}$;
- if you announce $\tilde{z} \notin \{\hat{z}, z_2\}$ you will be audited;
- if you announce $\tilde{z} = z_2$ or $\tilde{z} = \hat{z}$ and $\nu_1 \leq \mu_1$ you will not be audited;
- if you announce $\tilde{z} = \hat{z}$ and $\nu_1 > \mu_1$ then you will be audited
- when audit occurs, the fine function $f(\tilde{z}, z_a)$ is applied.

Again what are the strategies of the agents? Any n_1-agent clearly has a strictly dominant strategy in the announcement of $\hat{z}$. Anticipating this report, an n_2-agent can either report and earn $\hat{z}$ or announce and earn z_2 since audit will then occur and she is indifferent between these two bundles which strictly dominate all other possible bundles. Given our assumption, the agent will choose to report z_2. Knowing this, all the less skilled agents anticipating that they will not be audited, go to earn z_1 that maximizes their utility given the transfer ψ_1 they receive. They are thus evaders with the tacit agreement of the planner who by the way attains indirectly his first best outcome.

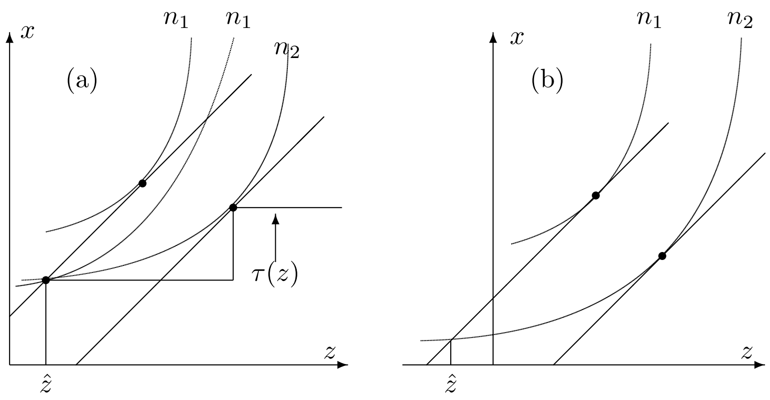

FIG. 2. (a) Indirectly Implementable (b) Not Implementable

In the second case, pictured in figure 2b, there are no means to implement a^λ since there does not exist an incentive compatible tax schedule which will tax ψ_2 to the more able to redistribute ψ_1 to the less skilled. To approach his ideal point, the planner must then mimic a Rawlsian planner by proposing this latter's tax schedule that redistributes the maximum amount to the poor. It is obvious that all the planners whose optimal first best is unattainable will mimic the Rawlsian one (including himself). This result relies on the fact that the planners attach a higher weight on the welfare of the less productive agents. Therefore they seek to give them the highest (efficient) transfer which in turn allows the agents to reach their maximum utility on the first best Pareto frontier. Thus, the less skilled agents will again evade in equilibrium.

Figure 3 summarizes in the utility possibilities space the different cases. C is the laissez faire equilibrium. The first best utility possibilities schedule is represented by AB, while the second best is given by CD (for $\lambda \in \Lambda$, the case we are interested in). R is the Rawlsian optimal point and D the equilibrium corresponding to a mechanism with the Rawlsian tax schedule. By using a GTS, all the points in AB (in fact CB) can be implemented. The restriction to simple mechanisms makes the set of implementable allocations shrink from CB to CD. All the planners whose ideal point lies on the arc DB will propose R then the high skilled agents will get their second best Rawlsian utility (U_2^R) while the less skilled agents will cheat thereby increasing their utility level. The equilibrium allocation is the point on the first best frontier with $U_2 = U_2^R$. The planners who are not very redistributive having their ideal point on CD will be able to attain that point by choosing the second best tax schedule that would give to the more able individuals their corresponding first best utility.

4. CONCLUSION

There are many interesting extensions to the above model. The first that comes at mind would be to relax the fundamental assumption stating that the distribution of characteristics is fixed, finite and the planner using sophisticated mechanisms has a perfect knowledge of it. As stated by Dierker and Haller (1990) such a planner is too powerful and can implement almost every allocation as shown in subsection 3.1. What happens if one introduces “slight” uncertainties about the distribution? Imagine for instance the distribution is μ_1 with probability α and μ_2 with probability $1 - \alpha$ where these distributions are quite close, a generalized tax schedule which uses a wrong distribution (for instance the average one) may lead to large a loss of social welfare because many agents will be punished even though they are right. The only problem that can arise with the restriction to simple tax schedules is infeasibilities, since agents are not punished if their reported and audited incomes coincide. These infeasibilities could be sustained if we are in a dynamic framework and allow some budget deficits at the early stages of the game. This planner would be able to extract the true distribution while the sophisticated planner will probably encounter a ratchet effect problem. In fact, the agents anticipate that the information will be used at their expense and will thus try to manipulate it.

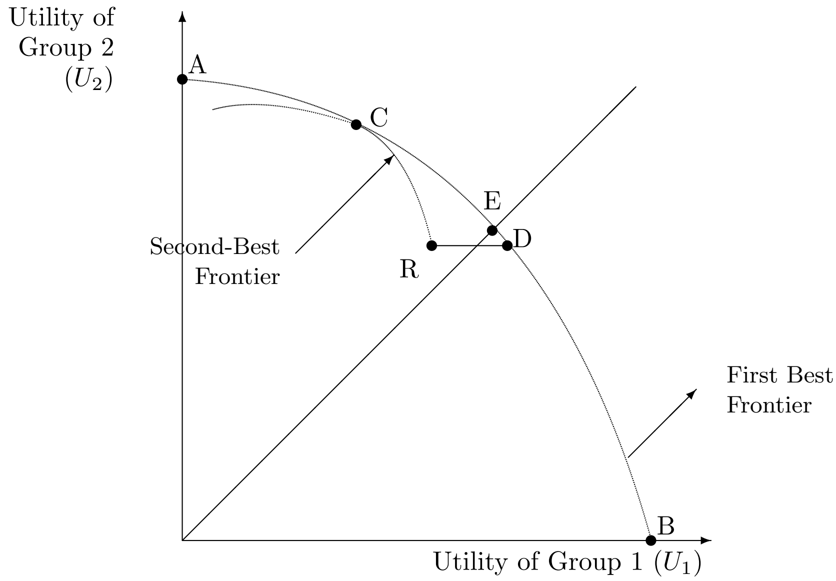

FIG. 3. First- and Second-Best Utility Possibilities Sets

As a first step, we have restricted the planner to use only simple mechanisms but he was still able to use a generalized audit strategy that links the likelihood of audit of a given income, to the distribution of announced incomes. What, if the planner is also restricted to use simple audit strategies? This means that planner is not allowed to use the information at its disposal and it is then as if he faces a continuum economy. This would be a generalization of the Cremer and Gahvari (1996) model to several types of agents and of the Mookherjee and Png (1989) model to ex ante different individuals.

In any case, this paper shows that in a finite economy where tax evasion possibilities exist because of costly observability of endogenous incomes, a sophisticated planner can implement any first best allocation by a combination of generalized tax schedule and audit strategy. Once generalized tax schedule is discarded for whatever reason, the use of a generalized audit strategy still allows the implementation of a subset of the first best Pareto frontier. All the agents, except the most able, evade some amount and nobody is audited.

APPENDIX

In propositions 2 and 3 the equilibrium relies on the assumption that any agent who is indifferent between two allocations would choose the one the planner assigns her. This equilibrium seems thus to be not robust to the introduction of randomization among equivalent allocations. This appendix shows that the assumption that sustains the equilibrium is not as restrictive as one would be inclined to think, in the sense that the same equilibrium would be implemented without it at the cost of replacing in the mechanism M the tax schedule $\psi(z)$ by another function $\psi'(z)$ that is slightly different. Recall that these functions can simply be increasing step functions linking the different bundles (see Guesnerie and Seade (1982)). The following lemma is helpful:

LEMMA 1. *If* $a = ((x_s, z_s),\ \ 1 \leq s \leq r)$ *is an optimal incentive compatible allocation implemented by a tax function* $\psi(z)$*, then there exists* $\boldsymbol{\eta} = ((\eta_s, \eta_s),\ 1 \leq s \leq r)$ *such that* $a' = a - \boldsymbol{\eta}$ *is* ***strictly*** *incentive compatible, budget balancing and implemented by a tax function* $\psi'(z)$.

Proof. First of all, it is known from Guesnerie and Seade (1982) that a is such that only the downward adjacent incentive constraints are binding, i.e. $U(x_s, z_s, n_s) \geq U(x_t, z_t, n_s)\ \forall t$ with equality for $t = s - 1$ only, and $(x_1, z_1) \ll \ldots \ll (x_s, z_s) \ll \ldots \ll (x_r, z_r)$. Since a is budget balanced, it is obvious that a' inherits the same feature. It is also obvious that $\boldsymbol{\eta} \in I\!R_+^{2r}$, since otherwise a would not be optimal. Suppose $\eta_s = \eta\ \forall\ s = 1, \ldots, r - 1$

and $\eta_r = 0$. Let us show that this vector satisfies the lemma. We have

$$U(x_s, z_s, n_s) = U(x_{s-1}, z_{s-1}, n_s),$$

it remains to demonstrate that

$$U(x_s - \eta, z_s - \eta, n_s) > U(x_{s-1} - \eta, z_{s-1} - \eta, n_s).$$

If this is the case, then the n_s-agents will no longer be attracted by the bundle proposed to the n_{s-1}-agents. Our strategy of proof is to show that

$$U(x_s, z_s, n_s) - U(x_s - \eta, z_s - \eta, n_s) < U(x_{s-1}, z_{s-1}, n_s) - U(x_{s-1} - \eta, z_{s-1} - \eta, n_s).$$

Both the lhs and the rhs can be rewritten $U(x_i, z_i, n_s)(1 - s(x_i, z_i, n_s))\eta$ after some straightforward manipulations, meaning that the inequality is satisfied whenever $U_x(x_s, z_s, n_s) < U_x(x_{s-1}, z_{s-1}, n_s)$. This condition holds since the utility function is concave in consumption. Hence a' is strictly incentive compatible since

$$U(x_r, z_r, n_r) > U(x_{r-1} - \eta, z_{r-1} - \eta, n_r)$$

and

$$U(x_s - \eta, z_s - \eta, n_s) > U(x_{s-1} - \eta, z_{s-1} - \eta, n_s) \ \forall \ s = 1, \ldots, r-1.$$

It remains to show that the transfers do not make attractive the bundle of n_s-agents to n_{s-1}-agents, a' must still satisfy

$$U(x_s - \eta, z_s - \eta, n_s) > U(x_{s+1} - \eta, z_{s+1} - \eta, n_s).$$

By a continuity argument, one can show that this holds. Indeed we have

$$U(x_s, z_s, n_s) > U(x_{s+1}, z_{s+1}, n_s) \ \forall \ s = 1, \ldots, r-1$$

thus $\forall s \ \exists \ \epsilon_s > 0$ s.t.

$$U(x_s - \epsilon_s, z_s - \epsilon_s, n_s) = U(x_{s+1}, z_{s+1}, n_s) \ \forall \ s = 1, \ldots, r-1$$

and a fortiori

$$U(x_s - \epsilon_s, z_s - \epsilon_s, n_s) > U(x_{s+1} - \epsilon_s, z_{s+1} - \epsilon_s, n_s).$$

The issue that can arise is when $z_1 = 0$, then ones considers ϵ_1 such that $U(x_1 - \epsilon_1, 0, n_1) = U(x_2, z_2, n_1)$. Taking $0 < \eta < Min(\epsilon_s)$ completes the proof. ∎

Lemma 1 and proposition 2 together establish that the equilibrium obtained by the mechanisms $M = (\pi(\tilde{z},\nu), \psi(\tilde{z}), f(\tilde{z}, z_a))$ and $M' = (\pi(\tilde{z},\nu), \psi'(\tilde{z}), f(\tilde{z}, z_a))$ is the same. With the former mechanism randomization is ruled out from the outset while with the latter there is no gain from randomization. We have thus shown that assumption (A2) is just for convenience and does not entail any loss of generality. However considering M is useful because one does not need to define another tax schedule than the already defined second best ones. Another difference between the two mechanisms concerns the amounts evaded. Indeed the agents evade e_s with M and $e_s + \eta$ with M'.

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