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The Economic Effects of Inflation Tax Instruments in an Overlapping-Generations Economy with Production

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Most inflation tax literature considers currency as the entire monetary base. In reality, however, many countries impose inflation tax on the required reserves of the banking system as well as on currency. Developing countries in particular usually augment the currency component of the monetary base by imposing high reserve requirements on bank deposits. This paper incorporates financial intermediaries into a general equilibrium setting in order to analyze the reserve component of the inflation tax. We present a Diamond-type overlapping-generations model in the context of a developing open economy, and analyze the economic consequences of changes in the reserve requirement and the rate of inflation. The model displays some results that are different from those of the existing literature.

Key Words: Inflation tax; Reserve requirement; Factor-price frontier.
JEL Classification Numbers: E44, E60, F41.

1. INTRODUCTION

In the monetary macroeconomics literature, the existence of a commercial banking sector is often neglected, and the whole banking system is assumed to be made up only of a central bank. For this reason, analyses of inflation tax, for example, are usually restricted only to those of that portion of government revenue that is attributable to its power to create central bank money, i.e., the ability to impose a tax on the currency holdings of the public. In practice, however, governments typically levy inflation tax on the required non-interest-bearing reserves of the banking

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system as well. In particular, developing countries usually augment the currency component of the monetary base by imposing high reserve requirements on bank deposits. From Brock (1984), we know that, during the 1970's, three large Latin American countries, Mexico, Colombia, and Brazil, obtained 72, 56, and 60 percent of their seigniorage from the required reserve components, respectively. A similar tendency was found in some of the Southern European countries during the 1980's (Bacchtta and Caminal 1992).

Therefore, in order to have a realistic view of inflation tax particularly in developing countries, we must explicitly consider the commercial banking sector of an economy so as to analyze its reserve component. In general, the failure to explicitly include the commercial banking sector would prevent us from analyzing many of the important features of a regulated financial system, such as reserve requirements, inside money, interest rate ceilings, and other features that are central to the interaction of monetary and real forces.

This fact is not unknown in the literature. Romer (1985), for example, has explicitly introduced financial intermediation into a general equilibrium model of overlapping generations[1], and analyzed the comparative static effects of changes in the reserve requirement and the rate of money growth. Romer's analysis mainly deals with the response of interest rates and generally does not support conventional conclusions. For example, under his pure banking economy model, a rise in the reserve ratio leads to an unambiguous rise in the loan interest rate but the response of the deposit interest rate is indeterminate, which is different from the conventional result that the loan interest rate is unaffected and the deposits interest rate falls (e.g., Fama 1980). Moreover, a rise in the rate of money growth leads to a rise in the real interest rate on loans and a fall in the real interest rate on deposits, which is also different from the conventional result that an increase in the rate of money growth reduces the real return on capital as long as the individuals have finite lives (e.g., Abel 1987, Drazen 1981, and Weiss 1980).

It should be emphasized, however, that these unconventional results of Romer (1985) were obtained in essentially an exchange economy model because individuals are endowed with goods, and not labor, thereby ruling out the important dynamic aspect of the economy working through the factor-price frontier. Thus one is tempted to question the robustness of his major conclusions in the light of production and other real world complica-

[1] After Romer (1985), several other general equilibrium models, in which financial intermediation is explicitly included, have been developed. See, for example, Bacchtta and Caminal (1992), Baltensperger and Jordan (1997), Bencivenga and Smith (1992), Bhattacharya, Guzman, Huybens and Smith (1997), Brock (1989), Davis and Toma (1995), Espinosa and Russell (1998), and Huybens and Smith (1998).

tions. For this reason, we will extend the analysis of Romer to a standard Diamond (1965) economy. It will turn out that Romer's conclusion regarding the effect of a change in the reserve requirement is robust even with production, but that his conclusion on the effect of a change in the rate of money growth is not.

The Tobin effect (i.e., the positive effect of higher inflation on capital intensity) or monetary superneutrality (i.e., invariance of the steady state capital-labor ratio with respect to the rate of monetary growth) has been a subject of heated discussion in monetary growth theory since the work of Tobin (1965) and Sidrauski (1967). Sidrauski (1967) has used a growth model based on explicit individual utility maximization to show that if individuals have infinite lives, money is superneutral. In contrast, several other writers have demonstrated that if individuals have finite lives, superneutrality does not hold, such that the Tobin effect emerges.

Our model shows that, even in an optimizing model with finite horizons, the Tobin effect (or the superneutrality of money) may well be ambiguous. This suggests that whereas the literatures suggest that finite lives are necessary to invalidate the superneutrality of money, they may not be a sufficient condition. In other words, whether the Tobin effect (or the superneutrality of money) holds in finite horizon utility-maximization models seems still an unsolved issue.

The remainder of the paper is organized as follows. Section 2 builds a Diamond-type overlapping-generations model with reserve requirements and capital control in the context of a small open developing economy. Section 3 describes the momentary equilibrium and the steady state equilibrium of the system, and derives the stability condition of the steady state. Section 4 and 5 use the model to discuss the effects of changes in the reserve requirement and the rate of inflation, respectively. Finally, section 6 presents some concluding remarks.

2. THE GENERAL SETUP

The model proposed in the paper incorporates the imposition of reserve requirements and capital control in the context of a small open developing economy. Individuals in this economy live two periods; they work in the first period of their lives, and retire in the second. In every time period ($t \geq 0$), a new generation is born, so that there are a young generation and an old generation overlapped in each time period ($t > 0$). In the first period of life, people are endowed with a unit of labor which is supplied inelastically to firms, earning a wage w , which they allocate to consumption and savings for retirement. In the second period of life, people receive both the principal and the interest on their savings and consume all their income; there is no bequest. Let the population of generation t (and consequently the size of the labor force in period t) be denoted by N_t , which grows at the constant rate n , so that $N_{t+1} = (1+n)N_t$. The term 'per capita' is defined with respect to the population of generation t . In this economy, there is only one good that can be traded with the rest of the world.

The individuals in this economy do not have direct access to the production technology as it requires a minimum level of investment, so that the savings must be channeled through financial intermediaries. The modeling strategy in this paper places the banking sector as the only link between savers and investors in view of the ability of developing country governments to impose very high reserve requirements on the banking system without causing banks to lose their central position as financial intermediaries. Thus, all physical capital is financed through loans of the banking system. The government regulates the banking sector by imposing reserve requirements; it also prohibits the private sector from international borrowing and leading. Only the government can borrow or lend abroad.

2.1. Production

There are two factors of production, namely, capital and labor. Because the economy has only one sector, capital is simply non-consumed output. Output in the t th period (Y_t), is produced according to a neo-classical constant-returns-to-scale production technology $Y_t = F(K_t, N_t)$, where N_t is the labor force and K_t is the stock of capital carried over from period $t-1$. For simplicity, we assume that capital will depreciate completely after one period of use. In per capita terms, output can thus be expressed by,

$$y_t = f(k_t) \tag{1}$$

where y_t is per capita output and k_t is the capital-labor ratio, and $f(k) > 0$, $f'(k) > 0$, $f^{''}(k) < 0$, for $k > 0$.

Because labor is supplied inelastically, the only decision for the profit-maximizing firms concerns the amount of investment in period t , or the gross amount of capital held over to period $t+1$, which satisfies,

$$f'(k_{t+1}) = r^l_{t+1} \tag{2}$$

where r^l_{t+1} is the real interest rate on bank loans. We assume that the firms can not borrow directly from the individuals; instead, they must rely on banking loans to finance their investment. Given constant returns to scale, the per capita real wage in terms of t -period goods is

$$w_t = f(k_t) - k_t f'(k_t) \tag{3}$$

Equation (2) and (3) imply the following factor-price frontier (FPF),

$$w_t = \psi(r^l_t) \tag{4}$$

The economy's initial stock of physical capital $k(1)$ is given.

2.2. Consumption and Saving

Individuals are identical both within and across generations. The amount of first-period consumption of a member of generation t is denoted by c_t^1 , and second-period consumption (also of a member of generation t) is denoted by c_{t+1}^2 . The two assets that individuals can hold are domestic currency and bank deposits. Let M_{t+1} and D_{t+1} denote the per capita stocks of domestic currency and bank deposits, respectively, at the beginning of period $t+1$. The corresponding real values in terms of t -period goods are M_{t+1}/p_t and D_{t+1}/p_t , which in turn can be expressed as $(1+\pi)m_{t+1}$ and $(1+\pi)d_{t+1}$, where p_t is the money price of t -period goods; $1+\pi = p_{t+1}/p_t$ is the constant inflation factor from time t to $t+1$; $m_{t+1} = M_{t+1}/p_{t+1}$; and $d_{t+1} = D_{t+1}/p_{t+1}$.

The decision problem for young people born at t can be stated as follows:

$$\max U(c_t^1, c_{t+1}^2, (1+n)(1+\pi)m_{t+1})$$ [2]

subject to:

$$w_t - c_t^1 = (1+n)[(1+\pi)m_{t+1} + (1+\pi)d_{t+1}] \tag{5}$$

$$c_{t+1}^2 = (1+n)[m_{t+1} + (1+r_{t+1}^d)(1+\pi)d_{t+1}] \tag{6}$$

where r_{t+1}^d is the real rate of interest on domestic bank deposits. This means that, for every unit of t -period goods deposited in domestic banks in period t , individuals will receive $1+r_{t+1}^d$ units of $t+1$ -period goods in period $t+1$. The factor $(1+n)$ exists in equations (5) and (6) because the stock of assets at the beginning of period $t+1$ is purchased with the savings carried over from period t .

Let $\tilde{m}_{t+1}$ and $\tilde{d}_{t+1}$ denote $(1+n)(1+\pi)m_{t+1}$ and $(1+n)(1+\pi)d_{t+1}$ respectively. From equation (5) and (6), we can derive the lifetime budget constraint as follows,

$$w_t - c_t^1 - \frac{1}{1+r_{t+1}^d} c_{t+1}^2 = \left[1 - \frac{1}{(1+\pi)(1+r_{t+1}^d)}\right] \tilde{m}_{t+1} \tag{7}$$

Then, we can easily obtain the additional first-order conditions for the individual's optimization problem as follows.

$$U_1 = (1+r_{t+1}^d)U_2 \tag{8}$$

$$U_m = \left[(1+r_{t+1}^d) - \frac{1}{1+\pi}\right] U_2 \tag{9}$$

[2]We choose to put money in the utility function because this is a simple way to model the medium of exchange function of money. Abel (1987) has used the same utility function in analyzing the issue of optimal monetary growth.

where $U_1 \equiv \partial U() / \partial c_t^1$, $U_2 \equiv \partial U() / \partial c_{t+1}^2$, and $U_m \equiv \partial U() / \partial \tilde{m}_{t+1}$.

Equations (8) and (9) have standard interpretations. Equation (8) indicates the optimal intertemporal consumption decision of individuals: an individual who chooses to hold an additional unit of deposits by giving up a unit of consumption in the first period suffers a utility loss U_1 , but obtains an increase of $(1 + r_{t+1}^d)$ units of consumption in the second period which raises his utility by $(1+r_{t+1}^d)U_2$. Equation (8) thus shows that an optimizing individual will invest up to the point where the utility loss equals the utility gain. On the other hand, equation (9) expresses the portfolio strategy of an optimizing individual: an individual who rearranges his portfolio in the first period by increasing his holding of deposits by one unit and reducing his holding of real balances by one unit gains $[(1+r_{t+1}^d)-1/(1+\pi)]$ units of consumption in the second period. This means that he obtains a utility gain of $[(1 + r_{t+1}^d) - 1/(1 + \pi)]U_2$ in exchange for a utility loss of U_m. Then, equation (9) shows that he will rearrange his portfolio until the utility gain from holding an additional unit of deposits equals the utility loss from holding a less unit of real balances.

From equation (7), (8), and (9), we can derive the consumption functions $c_t^1 = c^1(w_t, r_{t+1}^d, \pi)$ and $c_{t+1}^2 = c^2(w_t, r_{t+1}^d, \pi)$, as well as the currency demand function $\tilde{m}_{t+1} = \tilde{m}(w_t, r_{t+1}^d, \pi)$. Substituting them into equation (5) and the utility function, we obtain the following functions for deposit demand and indirect utility, respectively, $\tilde{d}_{t+1} = \tilde{d}(w_t, r_{t+1}^d, \pi)$ and $V_t = V(w_t, r_{t+1}^d, \pi)$.

For our later use, it may be useful to specify the signs of the partial derivatives of the deposits demand function with respect to its arguments, by assuming that c_t^1 and $\tilde{m}_{t+1}$ are normal goods. Hence, by implication, we have $0 < \partial\tilde{d}_{t+1}/\partial w_t < 1$, $\partial\tilde{d}_{t+1}/\partial r_{t+1}^d > 0$ and $\partial\tilde{d}_{t+1}/\partial\pi > 0$ (see, for example, Brock, 1984 and Drazen, 1981). On the other hand, the signs of the partial derivatives of total savings $s_t = \tilde{m}_{t+1} + \tilde{d}_{t+1}$ with respect to r_{t+1}^d and π are both ambiguous.

2.3. Financial Intermediation

The developing economy setting allows us to focus our attention on a relatively simple structure of financial intermediation, made up only of the banking sector. The banking sector is assumed to be perfectly competitive and banks have access to a costless intermediation technology. They accept deposits from domestic individuals, with a portion ϕ deposited at the central bank without remuneration as a compulsory reserve. The remaining portion is lent out as loans to firms. The banks are assumed to pay for domestic deposits at the real interest rate r_{t+1}^d and receive interest from the loans at the real interest rate r_{t+1}^l . We restrict the stock of loans to be greater than zero in equilibrium, in order to be sure that, in equilibrium, the real rate of return on loans $(1+r^l)$ be at least as high as the real return on currency $1/(1+\pi)$. This follows from the fact that a bank can always attain the real rate of return $1/(1+\pi)$ by holding currency. This requires that banks not hold excess reserves in their portfolios. In real terms, the per capita banking profit is,

$$(1+r^l_{t+1})(1-\phi)\tilde{d}_{t+1} + \frac{1}{1+\pi}\phi\tilde{d}_{t+1} - (1+r^d_{t+1})\tilde{d}_{t+1} \tag{10}$$

In equilibrium, banks will make zero profits, such that

$$(1+r^l_{t+1})(1-\phi) + \frac{\phi}{1+\pi} = 1+r^d_{t+1}. \tag{11}$$

2.4. The Government

Here, we consider the government as a consolidated public sector consisting of the nonfinancial public sector and the central bank. The government finances public spending either through seigniorage (based on the level of inflation and required reserves) or through the depleting of foreign reserves. Let $E_t B^*_{t+1}$ denote the per capita stock of foreign reserves at the beginning of period $t+1$, where E_t is the nominal exchange rates (defined as the domestic currency price of foreign currency). Because there is only one good in the economy, perfect price flexibility assures that the law of one-price holds. Normalizing the foreign price level as unity, p_t becomes the nominal exchange rate E_t . Therefore, in real terms of t -period goods, the foreign reserves become $b^*_{t+1} = B^*_{t+1}/p^*_{t+1} = B^*_{t+1}$. Then, the government budget constraint in per capita real terms can be expressed as

$$g_t - r^*_t b^*_t = [\tilde{h}_{t+1} - \frac{1}{(1+n)(1+\pi)}\tilde{h}_t] - [(1+n)b^*_{t+1} - b^*_t]. \tag{12}$$

Where g_t is real per capita government spending in terms of t -period goods in period t , r^*_t is the real world rate of interest which is exogenously given to this small economy, and $\tilde{h}_{t+1} = \tilde{m}_{t+1} + \phi\tilde{d}_{t+1}$ is the real per capita stock of high-powered money at the beginning of period $t+1$. Equation (12) means that the deficit is financed either by seigniorage revenue or by depleting the foreign reserves of the central bank.

2.5. The Balance of Payments Identity

The balance of payments identity of this economy in real per capita terms is

$$(1+n)b^*_{t+1} - b^*_t = s_t - g_t - (1+n)k_{t+1} + r^*_t b^*_t \tag{13}$$

The left-hand side of equation (13) corresponds to reserve accumulation by the central bank (the overall balance of payments), and the right-hand side is the current account. Because there are no private capital flows, the capital account is zero. Because private net intertemproal trade in goods is matched by a secular change in the government's net foreign reserves, the government thus acts as a "financial intermediary" for the private sector in this capital control system.

2.6. The Exchange Rate Regime

Finally, we need assumptions about the exchange rate regime. Under flexible exchange rates, the central bank chooses the path of the money stock and the reserve level while allowing the nominal exchange rate E_t to be determined by market forces. Under pegged exchange rates, the path of E_t , hence p_t , is chosen by the central bank, and the nominal money stock and the foreign reserve stock passively accommodate the money demand equation and the balance of payments identity, respectively. Here, we assume a crawling peg, which is often found in developing countries, as a special case of the latter regime in which the policy variables are the reserve requirement (ϕ) and the rate of crawling, hence, the rate of inflation (π).

3. THE EQUILIBRIUM

3.1. The Momentary Equilibrium

In equilibrium, it is necessary that individuals maximize utility, firms and banks maximize profits and all markets clear. In addition, the government budget constraint and the balance of payments identity should also be satisfied. The equations describing a momentary equilibrium in this small open economy are,

$$f'(k_{t+1}) = r^l_{t+1} \tag{2}$$

$$w_t = f(k_t) - k_t f'(k_t) \tag{3}$$

$$\tilde{m}_{t+1} = \tilde{m}(w_t, r^d_{t+1}, \pi) \tag{14}$$

$$s_t = \tilde{m}_{t+1} + \tilde{d}_{t+1} \tag{15}$$

$$(1 + r^l_{t+1})(1 - \phi) + \frac{\phi}{1+\pi} = 1 + r^d_{t+1} \tag{11}$$

$$(1 - \phi)\tilde{d}_{t+1} = (1 + n)k_{t+1} \tag{16}$$

$$g_t - r^*_t b^*_t = [\tilde{h}_{t+1} - \frac{1}{(1+n)(1+\pi)}\tilde{h}_t] - [(1+n)b^*_{t+1} - b^*_t] \tag{12}$$

$$(1 + n)b^*_{t+1} - b^*_t = s_t - g_t - (1 + n)k_{t+1} + r^*_t b^*_t \tag{13}$$

where equations (13), (14) and (16) are the market clearing conditions in goods, the currency and the bank loan markets, respectively. By Walras' law, the market clearing condition for deposits can be omitted.

In each period, a momentary equilibrium is established through the above system of equations. Last period's investment and the labor endowment in the present period have already determined the economy's capital-labor ratio, so that the real wage is determined by equation (3). Given the zero profit condition of the banking sector (given by equation (11)), the market clearing condition for loans (given by equation (16)), the deposits demand function $\tilde{d}_{t+1}$ and equation (2), the real deposit and loan interest rates are determined. Likewise, the firms' investment plan as well as the individual's saving and portfolio plans are determined. Then, the balance of payments identity (given by equation (13)) and the government budget constraint (given by equation (12)) determine the changes in the stock of foreign reserves and high-powered money. The capital stock carried over to the next period gives rise to a new momentary equilibrium and so the process continues.

3.2. The Steady States and Their Stability Conditions

By substituting equation (16) into equation (2), we obtain

$$r_{t+1}^{l} = f'\left(\frac{1-\phi}{1+n}\tilde{d}(w_t, r_{t+1}^{d}, \pi)\right) \tag{17}$$

Given equations (3) and (11), and with ϕ and π exogenous, equation (17) is a nonlinear difference equation in r_t^l . Equation (17), along with the other two difference equations in b_t^* and $\tilde{h}_t$ (equations (12) and (13)), constitute the dynamic system of this economy. Obviously, equation (17), (12) and (13) constitute a recursive system, so that we can focus our attention on equation (17).

A steady state is a situation where r_t^l , b_t^* , and $\tilde{h}_t$ are unchanging over time (hence, the other endogenous variables are also unchanging over time). This kind of equilibrium is described by equation (17), (12) and (13).

$$r^{l} = f'\left(\frac{1-\phi}{1+n}\tilde{d}(w, r^{d}, \pi)\right) \tag{17'}$$

$$g - r^*b^* = [1 - \frac{1}{(1+n)(1+\pi)}]\tilde{h} - nb^* \tag{12'}$$

$$nb^* = s - g - (1+n)k + r^*b^* \tag{13'}$$

where time subscripts are no longer added.

To proceed further, we follow the analysis of national debt in Diamond (1965), and assume that equation (17') has a unique solution and that the associated steady state is stable[3]. Calculating the derivative of equation (17), we obtain,

$$0 < \frac{dr^l_{t+1}}{dr^l_t} = \frac{-k_t f'' \frac{1-\phi}{1+n} \frac{\partial \tilde{d}_{t+1}}{\partial w_t}}{1 - f'' \frac{(1-\phi)^2}{1+n} \frac{\partial \tilde{d}_{t+1}}{\partial r^d_{t+1}}} \tag{18}$$

The assumption of stability, given by $dr^l_{t+1}/dr^l_t < 1$, implies that, in the steady state, we have,

$$1 + kf'' \frac{(1-\phi)}{1+n} \frac{\partial \tilde{d}}{\partial w} - f'' \frac{(1-\phi)^2}{1+n} \frac{\partial \tilde{d}}{\partial r^d} > 0 \tag{19}$$

This stability condition will be used to derive the direction of change in steady state values in the subsequent sections.

4. CHANGES IN THE RESERVE REQUIREMENT

Let us first consider the economic effects of changing the reserve requirement.

4.1. The Effects on the Equilibrium Interest Rates

Given equation (3) and (11), we can derive the change in the equilibrium loan interest rate arising from a change in the reserve rate by differentiating equation (17') with respect to ϕ ,

$$\frac{dr^l}{d\phi} = \frac{-f'' \frac{1}{1+n} \tilde{d} + f'' \frac{1-\phi}{1+n} \frac{\partial \tilde{d}}{\partial r^d} [\frac{1}{1+\pi} - (1 + r^l)]}{1 + kf'' \frac{1-\phi}{1+n} \frac{\partial \tilde{d}}{\partial w} - f'' \frac{(1-\phi)^2}{1+n} \frac{\partial \tilde{d}}{\partial r^d}} \tag{20}$$

From the stability condition (19), we know that the denominator of the right hand side (RHS) is positive. The first term of the numerator of RHS is positive because, given $\tilde{d}$, a rise in ϕ reduces the fraction of deposits available for loans, raising the loan interest rate. The second term of the numerator of RHS also has a positive effect on r^l because a rise in ϕ raises the inflation tax on banks. This means that the banks will reduce the deposit interest rate so as to preserve the zero profit condition, thereby reducing the demand for deposits and hence loanable funds, pushing up the loan interest rate[4]. Because these two effects work in the same direction, the equilibrium loan interest rate rises unambiguously.

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[3]Galor and Ryder (1989) provide a sufficient condition for the existence of a unique and globally stable (non-trivial) steady state equilibrium.

[4]It should be noted that $\frac{1}{1+\pi} - (1 + r^l) < 0$ because, in equilibrium, the real loan rate cannot be less than the real rate of return on currency.

Next, we turn to the effect of the same policy on the equilibrium deposit interest rate. From the zero profit condition (11), we know that, in equilibrium,

$$\frac{dr^d}{d\phi} = (1-\phi)\frac{dr^l}{d\phi} + [\frac{1}{1+\pi} - (1+r^l)] \tag{21}$$

The first term of RHS is positive from equation (20), while the second term is negative, so that the sign of $dr^d/d\phi$ is generally ambiguous. Therefore, although an increase in the reserve requirement forces the banks to hold a larger fraction of their portfolios as currency and hence imposes a larger inflation tax on them, the equilibrium deposit interest rate does not necessarily fall. This result is also obtained in Romer (1985), in contrast to the conclusion of the conventional partial equilibrium literature, where a rise in reserve requirements decreases the equilibrium deposit interest rate (e.g., Fama 1980). From equation (21), we know that $\frac{dr^d}{d\phi} > 0$ iff

$$(1-\phi)\frac{dr^l}{d\phi} > (1+r^l) - \frac{1}{1+\pi} \tag{22}$$

As the term $(1+r^l) - 1/(1+\pi)$ is the opportunity cost of holding one unit of reserves, if a rise in r^l is large enough to compensate for the opportunity cost, the deposit interest rate needs not fall in order for the banks to maintain zero profits. Therefore, whether the equilibrium deposit interest rate falls or rises depends on the elasticity of the loan demand. The deposit interest rate rises if the loan demand is inelastic and falls if it is elastic.

The above results regarding the effect of a change in the reserve requirement is essentially the same as those of Romer. We have thus shown that Romer's result regarding the effect of a change in the reserve ratio is robust with respect to the inclusion of production $a'\ la$ Diamond (1965).

4.2. The Effects on the Welfare of Individuals

We next turn our attention to the welfare effect of a change in the reserve requirement. The effect of a change in the reserve requirement on the welfare of individuals arises from the change in the wage income associated with the change in the loan interest rate (through the factor-price frontier) as well as from the change in the asset income caused by the change in the deposit interest rate. In an exchange economy, utility V depends only on asset income, so that we would expect the utility of an individual living in the steady state to fall in the conventional case of $dr^d/d\phi < 0$, but we expect it to rise in the non-conventional case of $dr^d/d\phi > 0$. In contrast, in a production economy, utility V depends not only on asset income r^d but also on labor income, which in turn depends on the loan interest rate r^l through the factor-price frontier.

As shown in the appendix, a change in the utility of an individual living in the steady state is given by,

$$\frac{dV}{d\phi} = U_1 \left[-k \frac{dr^l}{d\phi} + \frac{(1+n)k}{(1+r^d)(1-\phi)} \frac{dr^d}{d\phi} \right] \tag{23}$$

There are two terms in RHS of equation (23). The first term represents the welfare loss arising from the distortions on the production side of the economy, which is induced by the change in the reserve requirement; a rise in ϕ raises the loan interest rate, causing the wage income to fall through the factor-price frontier. The second term measures the change in welfare arising from the misallocation in the individual's asset portfolio resulting from the change in r^d . Combining the two effects, we can conclude that as long as the conventional case of $dr^d/d\phi < 0$ holds, the steady state level of welfare declines with an increase in the reserve requirement. However, if the non-conventional case of $dr^d/d\phi > 0$ holds, the welfare effect is indeterminate as the two effects in the bracket work in opposite directions.

4.3. The Effect on Government Seigniorage

Now, consider government seigniorage, $g^s \equiv (1 - \frac{1}{(1+n)(1+\pi)})\tilde{h}$.
From equations (12') and (13'), we have,

$$g^s = s - (1+n)k \tag{24}$$

Given equation (2), the change in government seigniorage that occurs in response to a change in the reserve requirement is calculated as,

$$\frac{dg^s}{d\phi} = \frac{\partial s}{\partial r^d} \frac{dr^d}{d\phi} - \frac{1+n}{f''} \frac{dr^l}{d\phi} \tag{25}$$

The change in the steady state level of seigniorage that occurs in response to a change in the reserve requirement has two components, as shown by the two terms on the RHS of equation (25). As typically assumed in the literature, we assume that $\partial s/\partial r^d > 0$, i.e., the substitution effect on savings is greater than the income effect when there is a change in r^d . Then, in the non-conventional case of $dr^d/d\phi > 0$, a rise in the reserve ratio raises the amount of government seigniorage. This results from the following two effects. First, a rise in ϕ induces an increase in the deposit interest rate, which in turn increases the individual's demand for deposits, thereby extending the inflation tax base. On the other hand, the rise in ϕ raises the loan interest rate, which in turn crowds out the private capital accumulation. Given the saving of individuals, this increases the amount of government revenue (expressed by the second term in RHS). In the conventional case of $dr^d/d\phi < 0$, however, the effect of a change in the reserve requirement on government seigniorage is indeterminate because the two terms in equation (25) work in opposite directions.

4.4. The Effects on the Trade Balance and the Stock of Foreign Reserves

We now turn our attention to the effect of a change in the reserve requirement on the trade balance and the position of foreign reserves. From equations (12') and (13'), the trade balance is given by,

$$q \equiv (n - r^*)b^* = s - g - (1+n)k \tag{26}$$

Consider first the non-conventional case of $dr^d/d\phi > 0$. Because there is more saving, some private capital has been crowded out, and government spending is unchanged, the trade balance surplus must increase in response to a rise in ϕ. As to the effect on the position of foreign reserves, because $db^*/d\phi = 1/(n - r^*)dq/d\phi$ and $dq/d\phi > 0$, the balance of foreign reserves will rise if $n > r^*$ and fall if $n < r^*$. In the conventional case of $dr^d/d\phi < 0$, the effects of a change in the reserve requirement on the trade balance and the foreign reserve position are indeterminate. For convenience, all of the preceding results are summarized in Table 1.

TABLE 1.

The Steady-State Effects of an Increase in the Reserve Requirement

	Welfare	Seigniorage	Trade surplus	Foreign exchange reserves	Loan interest rate
Conventional case	$-$	?	?	?	$+$
Non-conventional case	?	$+$	$+$	$+$ if $n > r^*$ $-$ if $n < r^*$	$+$

Notes: 1) The conventional case means $dr^d/d\phi < 0$.
2) The non-conventional case means $dr^d/d\phi > 0$.

5. CHANGES IN THE RATE OF INFLATION

Now, we will consider the economic effects of changing the rate of crawling, hence the rate of inflation. The change in the loan interest rate that results in response to a change in the inflation rate can be obtained by differentiating equation (17') with respect to π , given equations (3) and

(11).

$$\frac{dr^l}{d\pi} = \frac{f'' \frac{1-\phi}{1+n}[-\frac{\partial \tilde{d}}{\partial r^d} \frac{\phi}{(1+\pi)^2} + \frac{\partial \tilde{d}}{\partial \pi}]}{1 + kf'' \frac{1-\phi}{1+n} \frac{\partial \tilde{d}}{\partial w} - f'' \frac{(1-\phi)^2}{1+n} \frac{\partial \tilde{d}}{\partial r^d}} \tag{27}$$

By equation (19), the denominator is positive, so that the sign of $dr^l/d\pi$ depends on the sign of the numerator. The second term in the bracket is the individual portfolio effect of an increase in the inflation rate on the loan interest rate, hence on the steady state capital-labor ratio. This is usually called the Tobin effect, which works negatively on r^l . The first term in the bracket is the *financial intermediary effect* of an increase in the inflation rate, which is often neglected in the conventional literature on the Tobin effect. A rise in the rate of inflation increases the amount of inflation tax on banks, forces the banks to reduce the deposit interest rate, thus reducing the individual's demand for deposits. As a result, the loanable fund of banks will decline, thereby raising the loan interest rate. Because the two effects work in different directions, how a change in the rate of inflation affects the equilibrium loan interest rate is thus indeterminate. This result differs from the result of the conventional literature on the Tobin effect (e.g., Drazen 1981 and Weiss 1980) as well as that of Romer (1985).

The Tobin effect (i.e., the positive effect of higher inflation on capital intensity) or monetary superneutrality (i.e., invariance of the steady state capital-labor ratio with respect to the rate of monetary growth) has been a subject of heated discussion in monetary growth theory since Tobin (1965) and Sidrauski (1967). Sidrauski (1967) has used a growth model based on explicit individual utility maximization to show that if individuals have infinite lives, money is superneutral. In contrast, Drazen (1981) and Weiss (1980) have demonstrated that if individuals have finite lives, superneutrality does not hold, such that the Tobin effect emerges.

Now, what our analysis has shown in this paper is that, even in an optimizing model with finite horizons, the Tobin effect (or the superneutrality of money) may well be ambiguous. This suggests that whereas Drazen (1981) and Weiss (1980) suggest that finite lives are necessary to invalidate the superneutrality of money, they may not be a sufficient condition. In other words, whether the Tobin effect (or the superneutrality of money) holds in finite horizon utility-maximization models is still an unsolved issue[5].

Now, let us discuss why the result of our analysis concerning the effect of a change in the rate of inflation differs from the celebrated result of Romer (1985). Romer based his argument on a diagrammatic analysis, in

[5]Although the policy variable in our model is the rate of inflation, the same argument holds because the steady state rate of inflation is equal to the rate of monetary growth less the rate of population growth.

which two loci (i.e., the zero-profit condition of banks and the loan market equilibrium condition) determined the equilibrium loan and deposit interest rates, as follows

$$r^d = (1-\phi)r^l - \phi\frac{h-n}{1+h} \tag{28}$$

$$L(r^l) = (1-\phi)(D(r^d) - B) \tag{29}$$

where h is the growth rate of high-powered money; B is the stock of government bonds; $L(r^l)$ is the demand for loans, and $D(r^d)$ is the demand for deposits. Because his model is essentially an exchange economy model, the dynamic mechanism that works through the factor-price frontier is ruled out. As a result, the locus satisfying equation (29) is necessarily downward sloping,

$$\frac{dr^d}{dr^l} = \frac{\frac{dL}{dr^l}}{(1-\phi)\frac{dD}{dr^d}} < 0 \tag{30}$$

An increase in the rate of inflation does not affect the combinations of interest rates that equate the supply of and the demand for loans. However, an increase in the rate of inflation does reduce the real rate of return on the bank holding of currency, thereby reducing the real deposit interest rate that banks can afford to pay for a given real loan interest rate: the locus of real interest rate combinations that lead to zero profits for banks thus shifts down. It is for this reason that, in the Romer model, the real loan interest rate rises and the real deposit interest rate falls when there is an increase in the rate of inflation. This effect is shown in Figure 1.

In contrast, in our production economy, the demand for deposits depends not only on the deposit interest rate but also on wage income which in turn depends on the loan interest rate through the factor-price frontier. By differentiating our loan market equilibrium condition (given by equation (16)), we have,

$$\frac{dr^d}{dr^l} = \frac{\frac{1}{f''} + (1-\phi)k\frac{\partial\tilde{d}}{\partial w}}{(1-\phi)\frac{\partial\tilde{d}}{\partial r^d}} \tag{31}$$

The numerator of RHS has two terms. The first term measures the negative effect of a rise in the loan interest rate on the equilibrium deposit interest rate: a rise in the loan interest rate reduces the demand for loans, which in turn reduces the bank's demand for deposits. This acts to reduce the deposit interest rate. Note that the first term in the numerator of RHS ($1/f^{''}$) is identical to dL/dr^l in Romer's equation (30).

The second term measures the positive effect of an increase in the loan interest rate on the equilibrium deposit interest rate which does not exist

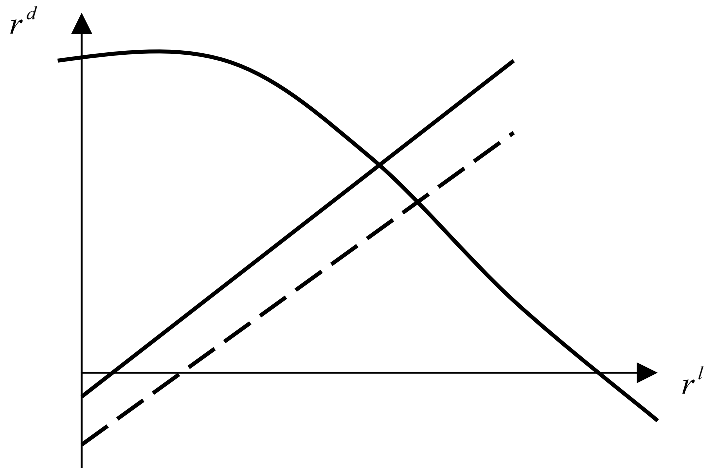

FIG. 1. The Possible Effect of an Increase in the Inflation Rate in the Exchange Economy of Romer (1985)

in the Romer model: an increase in r^l reduces the wage income of individuals through the factor-price frontier, which in turn reduces the supply of deposits to the banks. This effect acts to raise the deposit interest rate. Because these two forces work in different directions and the net effect is thus ambiguous, the slope of the loan market equilibrium locus has an ambiguous sign. If the second effect dominates the first, the result will become completely opposite of Romer's. This possibility is shown in Figure 2.

Thus, we have demonstrated that Romer's conclusions regarding the effect of a change in the inflation rate do not necessarily extend to a Diamond-type production economy[6], where the effect of a change in the inflation rate is ambiguous. Because the effects on the loan and deposit interest rates are both ambiguous, the effects of a change in the inflation rate are also indeterminate with respect to the welfare of individuals living in the steady

[6]In addition to a pure banking economy, Romer (1985) also examined a multi-intermediary economy to show that an increase in the inflation rate reduces the loan interest rate (i.e. the Tobin effect). Davis and Toma (1995) argue that this result is attributable to Romer's assumption that there is no first period consumption and hence no intertemporal substitution of consumption. When this assumption is relaxed, whether or not an increase in the inflation rate raises the loan rate depends on whether or not the intertemporal consumption effect dominates the intermediary switching effect. It should be noted, however, that the model of Davis and Toma is also essentially an exchange economy because the individual's endowment is given in goods, and not in labor, so that it has the same problem as Romer (1985).

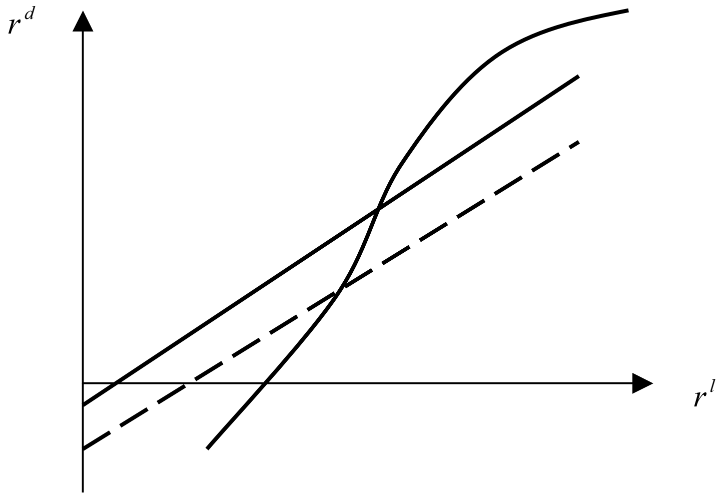

FIG. 2. The Possible Effect of an Increase in the Inflation Rate in a Production Economy.

state, government seigniorage, the trade balance, and the stock of foreign exchange reserves[7].

6. CONCLUDING REMARKS

In this paper, we have constructed a Diamond-type overlapping-generations model in the context of a small open developing economy, and examined the economic effects of changes in the reserve requirement and the inflation rate. As with Romer (1985), we find that responses to changes in parameter values are generally complex and ambiguous even in this simple model. However, this production economy model does display some results that are different from those of Romer's exchange economy model as well as those of conventional monetary growth literature.

We have demonstrated that Romer's result regarding the effect of a change in the reserve requirement on interest rates is robust, but that his result regarding the effect of a change in the inflation rate on interest rates is not robust when the analysis is extended to a production economy. We must thus use care in generalizing the conclusions obtained from an exchange economy or partial equilibrium analysis about the long-run effect of monetary policy changes. As an important by-product of our analysis,

[7]In contrast, the conventional conclusion of monetary growth theory is that the steady state level of welfare increases with money growth. See, for example, Weiss (1980).

the Tobin effect (or monetary superneutrality) has been shown to be ambiguous even in a finite horizon utility-maximization model, refuting the conjecture of Drazen (1981) and Weiss (1980) based on production economy models that finite lives necessarily invalidate the superneutrality of money.

APPENDIX

In this appendix, we derive equation (23) in the text. Because we are concerned only with the steady state value, the time subscripts will be dropped.

Using the optimality conditions (8) and (9), we see that,

$$\begin{aligned}\frac{dV}{d\phi} &= U_1\frac{dc^1}{d\phi} + U_2\frac{dc^2}{d\phi} + U_m\frac{d\tilde{m}}{d\phi} \\ &= U_1\left\{\frac{dc^1}{d\phi} + \frac{1}{1+r^d}\frac{dc^2}{d\phi} + [1 - \frac{1}{(1+\pi)(1+r^d)}]\frac{d\tilde{m}}{d\phi}\right\} \qquad \text{(A.1)}\end{aligned}$$

By differentiating the lifetime budget constraint (7) with respect to ϕ, we obtain,

$$\begin{aligned}&\frac{dw}{d\phi} - \frac{dc^1}{d\phi} - [-\frac{1}{(1+r^d)^2}\frac{dr^d}{d\phi}c^2 + \frac{1}{1+r^d}\frac{dc^2}{d\phi}] \\ =\ &\frac{1}{(1+\pi)(1+r^d)^2}\frac{dr^d}{d\phi}\tilde{m} + [1 - \frac{1}{(1+\pi)(1+r^d)}]\frac{d\tilde{m}}{d\phi} \qquad \text{(A.2)}\end{aligned}$$

By substituting this equation into equation (A.1), we have,

$$\frac{dV}{d\phi} = U_1\left\{\frac{dw}{d\phi} + \frac{1}{(1+r^d)^2}\frac{dr^d}{d\phi}[c^2 - \frac{1}{1+\pi}\tilde{m}]\right\} \qquad \text{(A.3)}$$

From the factor-price frontier (4), we have,

$$\frac{dw}{d\phi} = -k\frac{dr^l}{d\phi} \qquad \text{(A.4)}$$

Substituting equation (A.4), equation (6) and (16) into equation (A.3), we obtain,

$$\frac{dV}{d\phi} = U_1\left[-k\frac{dr^l}{d\phi} + \frac{(1+n)k}{(1+r^d)(1-\phi)}\frac{dr^d}{d\phi}\right] \qquad \text{(23)}$$

This is equation (23) in the text.

REFERENCES

Abel, A. B., 1987, Optimal monetary growth. *Journal of Monetary Economics* **19**, 437-450.

Bacchetta, P. and R. Caminal, 1992, Optimal seigniorage and financial liberalization. *Journal of International Money and Finance* **11**, 518-538.

Baltensperger, E. and T. J. Jordan, 1997, Seigniorage, banking, and the optimal quantity of money. *Journal of Banking and Finance* **21**, 781-796.

Bencivenga, V. R. and B. D. Smith, 1992, Deficits, inflation, and the banking system in developing countries: The optimal degree of financial repression. *Oxford Economic Papers* **44**, 767-790.

Bhattacharya, J., M. G. Guzman, E. Huybens, and B. D. Smith,1997, Monetary, fiscal, and reserve requirement policy in a simple monetary growth model. *International Economic Review* **38**, 321-350.

Brock, P. L., 1984, Inflationary finance in an open economy. *Journal of Monetary Economics* **14**, 37-53.

Brock, P. L., 1989, Reserve requirements and the inflation tax. *Journal of Money, Credit, and Banking* **21**, 106-121.

Davis, G. and M. Toma, 1995, Inflation, reserve requirements, and real interest rates with direct and indirect loan markets. *Journal of Macroeconomics* **17**, 515-531.

Diamond, P. A., 1965, National debt in a neoclassical growth model. *American Economic Review* **55**, 1126-1150.

Drazen, A., 1981, Inflation and capital accumulation under a finite horizon. *Journal of Monetary Economics* **8**, 247-260.

Espinosa, V. and S. Russell, 1998, Can higher inflation reduce real interest rates in the long run? *Canadian Journal of Economics* **31**, 92-103.

Fama, E. F., 1980, Banking in the theory of finance. *Journal of Monetary Economics* **6**, 39-57.

Galor, O. and H. E. Ryder,1989, Existence, uniqueness, and stability of equilibrium in an overlapping-generations model with productive capital. *Journal of Economic Theory* **49**, 360-375.

Huybens, E. and B. D. Smith, 1998, Financial market frictions, monetary policy, and capital accumulation in a small open economy. *Journal of Economic Theory* **81**, 353-400.

Romer, D., 1985, Financial intermediation, reserve requirements and inside money: A general equilibrium analysis. *Journal of Monetary Economics* **16**, 175-194.

Sidrauski, M., 1967, Rational choice and patterns of growth in a monetary economy. *American Economic Review* **57**, 534-544.

Tobin, J., 1965, Money and economic growth. *Econometrica* **33**, 671-684.

Weiss, L., 1980, The effects of money supply on economic welfare in the steady state. *Econometrica* **49**, 565-576.