

## Decision for the Optimal Location — Waiting Timing Relationship in A Real Options Model

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The Cobb-Douglas production function with Abel's (1983) model is extended herein, and real options analysis for entry-exit decision making with Dixit's (1989) model under exchange rate uncertainty. A general form with the first order of degree homothetic production functions is also considered by the rule of decision-making in the proposed model. The firm is risk neutral and this study adopts the real options analysis for valuing the behavior of the transferable location. This investigation extends Lin and Wu (2002) from considering only threshold value to expected arrival time for exporter deciding to transfer the production location from domestic to foreign and Management's flexibility could be explained to Time's flexibility. Furthermore, a closed form solution of the difference of the expected arrival time for exporter deciding to transfer its location obtained by the real options analysis and NPV method, sensitivity analysis, and some characteristics of optimal production strategy are sought, providing for another way of thinking. © 2004 Peking University Press

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### 1. INTRODUCTION

Following the collapse of the Bretton Woods System in 1973, industrialized uncertainty significantly influences the cash flows of exporting manu-

facturers and influences exporters' choices concerning the location for their production base. Most industries use real exchange rates to determine expected remuneration. Blonigen (1997) and Tomlin (1998) have claimed that the threshold value of the real exchange rate is an important decision-making index. Management's flexibility to respond to altered future market conditions increase an investment opportunity's value by improving its upside potential while limiting the downside losses, in relation to initial expectations under the assumption passive management. The omit asymmetry due managerial adaptability requires an "expanded NPV" rule that reflects both value components: the traditional (static or passive) NPV of direct cash flows, and the option value of operating and strategic adaptability. This claim doesn't mean that traditional NPV should be ignored, but rather that it should be seen as a crucial and necessary input to an options-based, expanded NPV analysis, like that of Trigeorgis (1995). Accordingly, the complete NPV method, correctly stated, is as follows

$$\text{Expanded(Strategic) NPV} = \text{Static(Passive) NPV of expected cash flows} \\ + \text{value of options from active management.}$$

However, corporate managers and practitioners in their capital investment decision-making are not currently using real options model. The primary reasons are these models are not well known or understood by corporate managers and practitioners and many individuals do not have the required mathematical skills to use the models comfortably and knowledgeably and so on (Lander and Pinches, 1998). Furthermore, Abel (1983) assessed the value of competitive firm using the Cobb-Douglas production function. A competitive firm applied labor and capital to produce output according to a Cobb-Douglas production function. The firm hires labor at a fixed wage rate and undertakes gross investment, by incurring an increasing convex adjustment costs. Sales quantity is the function of labor and capital and the price of goods following the Geometric Brownian Motion, but the instantaneous conditional expected percentage change in price per unit time equals zero. Consequently, the production is basically planned production. Suppose that the firm is risk neutral and maximizes the expected present value of its cash flow subject to the capital accumulation equation. The value of the firm is then becomes the maximized expected present value of cash flow. Under Uncertainty of Exchange Rate, Dixit (1989) applies the real options method to model multiple industries entry to or exit from the U. S. market. Campa (1993) uses a Tobit specification to test the effects of real exchange rate fluctuations on foreign direct investment in the United State during the 1980s. Campa finds exchange rate volatility, the drift of the exchange rate, sunk costs and labor to be negatively correlated with the number of foreign investments occurring in these industries. Mean-

while, the exchange rate is positively correlated with the number of foreign investments occurring in these industries. Tomlin (2000) employs a count data specification to model counts of FDI occurrences and finds the results are similar to Campa (1993).

Industrial structural changes and batch production have slowly replaced planned production, making the method of determining the value to industry of adopting the batch process, extremely important. In managerial perspective, most managers don't have more sense about the threshold value. But, most people can catch the feeling of time sensibly. So we should transform the index from the threshold value of mathematics to expected exercise date kindly. Thus, Management's flexibility could be explained to Time's flexibility and the managers would get longer time in their decision-making. Then most people can gradually touch the essence of the real options. Therefore, few investigations have discussed about expected arrival time (Alvarez, 1999; Grenadier and Weiss, 1997) and investment decisions relating to the batch process and referring mainly to Lin et al. (2002a, 2002b), however, established a decision valuation model to select an optimal location, determined expected arrival time, and then explained its economic meaning. This investigation extends Lin and Wu (2002b) from considering only threshold value to expected arrival time for exporter deciding to transfer the production location from domestic to foreign. The transferable model is established in the CES batch process.

## 2. CES BATCH PROCESS MODEL

This section states the assumptions and notation of the batch process. Using the CES production function. An export-oriented manufacturer with a constant capacity is considered to produce a fixed quantity of goods,  $\psi$ , which exactly meets market demand. Only the labor,  $L$ , the raw materials,  $\xi$ , and the fixed technology parameter,  $A$ , influence the productive function. The exporter sells overseas and the net profit is measured in local currency. Tariff,  $\tau$  are levied on the exporter overseas, and the price of the goods is  $P_s^*$ , in foreign currency, at time,  $s$ . Let  $R$  be the real exchange rate, namely the real price of the foreign currency in terms of the local currency. The real exchange rate is assumed to be follow geometric Brownian motion:

$$\frac{dR_t}{R_t} = \mu dt + \sigma dZ(t) \quad (1)$$

Here  $dz$  denotes an increment in the standard wiener process;  $\mu$  represents the drift of the real exchange rate, and  $\sigma$  is the volatility of the real exchange rate. The eq. (1) is a SDE (stochastic differential equation) and its solution is equal to  $R_t = R_0 \exp((\mu - 0.5\sigma^2)t - \sigma z(t))$ , where  $R_0$  is the initial value of the real exchange rate. As a first approximation, this ap-

proach has considerable empirical support from Frankel and Meese (1987). The batch process model produces output according to a constantly elastic substitution production function in domestic (foreign) market. The factors in the function include domestic (foreign) labor,  $L_s(L_s^*)$ , at time,  $s$ , domestic (foreign) raw materials,  $K_s(K_s^*)$ , at time,  $s$ , and the technological parameters, domestic (foreign),  $A_s(A_s^*)$ , at time,  $s$ . Other external factors include as real wages,  $\omega_s(\omega_s^*)$ , in domestic (foreign) market, at time,  $s$ , the prices of real raw materials,  $\xi_s(\xi_s^*)$ , in domestic (foreign) market, at time,  $s$ , and the quantity of sales,  $\psi_s(\psi_s^*)$ , in domestic (foreign) market, at time,  $s$ . Assume risk neutrality, so that all assets are priced to yield an expected rate of return equal to the risk-free interest rate,  $r$ . If a local export oriented manufacturer decides to transfer their production, then the cost of transferring production location  $E^*$  should be paid in foreign currency.

This section explores the batch process using the CES production function, satisfying the maximum value of the exporter's product both domestically and overseas. This study is to optimize Labor,  $L_s$ , and raw materials,  $\xi_s$ , and then to obtain special solutions to them. This solvable process uses the value matching and smooth pasting conditions from the real options method to determine the optimal solution to optimize the threshold value for exporter deciding to transfer their production location. The optimal transfer threshold value and expected arrival time for exporter deciding to transfer their locations using real options analysis is  $R_{ROA}^*$  and  $E_{R_0} [T (R_{ROA}^*)]$ , while that for the exporter deciding to transfer their location using the NPV method is  $R_{NPV}^*$  and  $E_{R_0} [T (R_{NPV}^*)]$ .

DEFINITION 2.1. The value of exporter's domestic production is defined as

$$V^d(R_t) = \max_{\{L_t, K_t\}} E_t \left[ \int_t^\infty e^{-r(s-t)} [(1-\tau) R_s \psi_s P_s^* - \omega_s L_s - \xi_s K_s] ds \right] \quad (2)$$

s.t.

$$\begin{aligned} \psi_s &= A_s [\alpha L_s^{-\rho} + (1-\alpha) K_s^{-\rho}]^{-\frac{1}{\rho}} \\ dR_s &= \mu R_s ds + \sigma R_s dZ(s) \end{aligned}$$

Equation (2) specifies a *Stochastic Optimization Control Problem*, in which  $E_t$  is the conditional expectation operator given that the information available at time  $t$  that includes  $R_t$ . The value of the exporter's domestic production is the maximum expected present value of the cash flows. In the CES built-to-order model, the cash flow at time  $s$  is discounted to time  $t$  using the risk-free interest rate  $r$ , and the decision-variables are labor,  $L_s$ , and raw materials,  $\xi_s$ . The exporter considers customer demand and

production output according to the CES production function. In Eq. (2), the revenue is  $(1 - \tau) R_s \psi_s P_s^*$ , where  $\psi_s$  represents sales volume and is a constant that depends on time,  $s$ . The revenue function equals the sale function, which does not depend on labor or raw materials. Herein, the sales volume is defined in batches, and an exporter produces output in response to orders for goods. An exporter thus determines customer demand and production output from the order levels, which determine optimal the quantity of labor,  $L_s$ , and raw materials,  $\xi_s$ .

The problem of finding the optimal control  $\{L_t, K_t\}$  in a short time interval from  $t$  to  $t + dt$ , is considered: the exporter's maximum domestic production is,

$$\begin{aligned} V^d(R_t) &= \max_{\{L_t, K_t\}} E_t [(1 - \tau) R_t \psi_t P_t^* - \omega_t L_t - \xi_t K_t] dt + e^{-rdt} V^d(R_{t+dt}) \\ &= \max_{\{L_t, K_t\}} [(1 - \tau) R_t \psi_t P_t^* - \omega_t L_t - \xi_t K_t] dt \\ &\quad + E_t(1 - rdt) [V^d(R_t) + dV^d(R_t)], \end{aligned} \tag{3}$$

s.t.

$$\begin{aligned} \psi_t &= A_t [\alpha L_t^{-\rho} + (1 - \alpha) K_t^{-\rho}]^{\frac{-1}{\rho}} \\ dR_t &= \mu R_t dt + \sigma R_t dZ(t) \end{aligned}$$

implying

$$rV^d(R_t) = \max_{\{L_t, K_t\}} \left\{ [(1 - \tau) R_t \psi_t P_t^* - \omega_t L_t - \xi_t K_t] + \frac{1}{dt} E_t dV^d(R_t) \right\}$$

s.t.

$$\begin{aligned} \psi_t &= A_t [\alpha L_t^{-\rho} + (1 - \alpha) K_t^{-\rho}]^{\frac{-1}{\rho}} \\ dR_t &= \mu R_t dt + \sigma R_t dZ(t) \end{aligned}$$

In which  $dV^d(R_t)$  is defined as  $V^d(R_{t+dt}) - V^d(R_t)$  and can be evaluated by It'o Lemma given the stochastic differential equation (1) for  $dR_t$

$$dV^d(R_t) = \left[ \mu V_R^d + \frac{1}{2} \sigma^2 V_{RR}^d \right] dt + \sigma V_R^d dz. \tag{4}$$

By using (4) to evaluate  $E_t dV^d(R_t)$  and substituting into (??), one obtains the Bellman equation

$$rV^d(R_t) = \max_{\{L_t, K_t\}} \left[ [(1 - \tau) R_t \psi_t P_t^* - \omega_t L_t - \xi_t K_t] + \left( \mu R_t V_R^d(R_t) + \frac{1}{2} \sigma^2 (R_t)^2 V_{RR}^d(R_t) \right) \right]. \tag{5}$$

s. t.

$$\psi_t = A_t [\alpha L_t^{-\rho} + (1 - \alpha) K_t^{-\rho}]^{-\frac{1}{\rho}}$$

A standard approach to solving this optimal control problem by dynamic programming consists of two steps. First, the value function  $V^d(R_t)$  is assumed to be known; then, the optimum  $\{L_t, K_t\}$  is found by solving the maximization problem of Eq. (5). For differentiable functions, Lagrange Multipliers obtained by differentiation with respect to  $\{L_t, K_t\}$  may be used. Since  $\frac{\omega_s}{\xi_s} = \frac{\alpha L_s^{\rho-1}}{(1-\alpha)K_s^{\rho-1}}$ ,

$$\begin{aligned} & \max_{\{L_t, K_t\}} [(1 - \tau) R_t \psi_t P_t^* - \omega_t L_t - \xi_t K_t] \\ &= \left[ (1 - \tau) R_t \psi_t P_t^* - (A_t)^{-1} \times \right. \\ & \quad \left. \left[ (\alpha)^{\frac{1}{\rho+1}} (\omega_t)^{\frac{\rho}{\rho+1}} + (1 - \alpha)^{\frac{1}{\rho+1}} (\xi_t)^{\frac{\rho}{\rho+1}} \right]^{\frac{1+\rho}{\rho}} \right]. \end{aligned} \quad (6)$$

Substituting Eq. (5) into Eq. (5) yields

$$\begin{aligned} & \frac{1}{2} \sigma^2 (R_t t)^2 V_{RR}^d(R_t) + \mu R_t V_R^d(R_t) - r V^d(R_t) \\ & + \left[ (1 - \tau) R_t \psi_t P_t^* - (A_t)^{-1} \times \right. \\ & \quad \left. \left[ (\alpha)^{\frac{1}{\rho+1}} (\omega_t)^{\frac{\rho}{\rho+1}} + (1 - \alpha)^{\frac{1}{\rho+1}} (\xi_t)^{\frac{\rho}{\rho+1}} \right]^{\frac{1+\rho}{\rho}} \right] = 0, \end{aligned} \quad (7)$$

where the productive value of exporter produces in domestic is  $V^d(R_t)$

$$\begin{aligned} V^d(R_t) &= \frac{R_t}{(r - \mu)} (1 - \tau) \psi_t P_t^* \\ & - \frac{\psi_t}{r A_t} \left[ (\alpha)^{\frac{1}{\rho+1}} (\omega_t)^{\frac{\rho}{\rho+1}} + (1 - \alpha)^{\frac{1}{\rho+1}} (\xi_t)^{\frac{\rho}{\rho+1}} \right]^{\frac{1+\rho}{\rho}}. \end{aligned} \quad (8)$$

The above result resembles that for a static economy. Financially,  $r - \mu > 0$ . Otherwise, investing in the asset depends on risk aversion, since money could be borrowed at  $(r - \mu)$ , and then invested without risk at  $\mu$ , yielding unlimited profits. Moreover, from a traditional economic perspective, the value of an exporter's domestic production equals the market value of the exporter's domestic production. However, if the real exchange rate continues to rise, then the exporter will transfer the location of its domestic production to reduce the cost of production. Otherwise, opportunity costs will apply, meaning that, exporter's domestic production is likely to hold an American call options. The dividend rate is assumed to be zero. Thus, the value of the option to transfer the location of production (i.e., the productive value of the exporter produces in domestic prior to acceptance),

$F^d(R_t)$ , satisfies an ordinary differential equation of the form specified in Dixit and Pindyck (1994).

$$\frac{1}{2}\sigma^2 F_{RR}R_t^2 + \mu F_R R_t - rF_R = 0. \tag{9}$$

The characteristic equation is  $Q \equiv \frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$  and two roots are defined as  $\beta_1 = \frac{1}{\sigma^2} \left\{ -(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2} \right\} > 1$ ,  $\beta_2 = \frac{1}{\sigma^2} \left\{ -(\mu - \frac{1}{2}\sigma^2) - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2} \right\} < 0$ .

Imposing the appropriate boundary conditions (value matching and smooth pasting), the solution is given by,

$$F(R_t) = A_1R_t^{\beta_1} + A_2R_t^{\beta_2} \tag{10}$$

(Proof available from the author on request),  $A_1R_t^{\beta_1}$  diverges if the real exchange rate,  $R_t$ , approaches infinity. The exporter won't transfer production location perpetually and the waiting-time value approaches zero. Therefore,  $A_1 = 0$  must be set and Eq. (10) must be corrected to  $F(R_t) = A_2R_t^{\beta_2}$ . The market value of the exporter's domestic production is then defined as

$$\begin{aligned} V^D(R_t) &= V^d(R_t) + A_2R_t^{\beta_2} \\ &= \frac{R_t}{(r - \mu)}(1 - \tau)\psi_t P_t^* \\ &\quad - \frac{\psi_t}{rA_t} \left[ \alpha^{\frac{1}{1+\rho}} \omega_t^{\frac{\rho}{1+\rho}} + (1 - \alpha)^{\frac{1}{1+\rho}} \xi_t^{\frac{\rho}{1+\rho}} \right]^{\frac{\rho+1}{\rho}} + A_2R_t^{\beta_2}. \end{aligned} \tag{11}$$

Notably, the market value of the exporter's domestic production is a nonlinear function of the real exchange rate.

DEFINITION 2.2. The value of exporter's foreign production is defined as

$$V^f(R_t) = \max_{\{L_s^*, K_s^*\}} E_t \left[ \int_t^\infty e^{-r(s-t)} R_s [\psi_s^* P_s^* - \omega_s^* L_s^* - \xi_s^* K_s^*] ds \right] \tag{12}$$

s.t.

$$\begin{aligned} \psi_s^* &= A_s^* [\alpha^* L_s^{*\rho} + (1 - \alpha^*) K_s^{*\rho}]^{\frac{-1}{\rho}} \\ dR_s &= \mu R_s ds + \sigma R_s dZ(s). \end{aligned}$$

The production value of the exporter's overseas production is the maximized expected present value of cash flows, and the economic mean resembles Definition 1: By the same definition, the value of the exporter's foreign production should satisfy Eq. (12).

$$\begin{aligned} & \frac{1}{2}\sigma^2 R_t^2 V_{RR}^f + \mu R_t V_R^f - rV^f \\ & + R_t \left[ \psi_t^* P_t^* - A_t^{*-1} (\alpha^{*\frac{1}{\rho+1}} \omega_t^{*\frac{\rho}{\rho+1}} + (1-\alpha^*)^{\frac{1}{\rho+1}} \xi_t^{*\frac{\rho}{\rho+1}})^{\frac{1+\rho}{\rho}} \right] = 0, \end{aligned} \quad (13)$$

where the value of exporter's foreign production is  $V^f(R_t)$

$$V^f(R_t) = R_t \left\{ \frac{1}{(r-\mu)} \left[ \psi_t^* P_t^* - \frac{\psi_t^*}{A_t^*} \times \left[ \alpha^{*\frac{1}{1+\rho}} \omega_t^{*\frac{\rho}{1+\rho}} + (1-\alpha^*)^{\frac{1}{1+\rho}} \xi_t^{*\frac{\rho}{1+\rho}} \right]^{\frac{\rho+1}{\rho}} \right] \right\}. \quad (14)$$

Several results follow immediately from Eq. (14). First (a), the productive value of the exporter's overseas production in foreign is a linear function of the real exchange rate, and the second term on the right-hand side of (14), measures cost expenses.

$$\frac{R_t}{(r-\mu)} \frac{\psi_t^*}{A_t^*} \left[ \alpha^{*\frac{1}{1+\rho}} \omega_t^{*\frac{\rho}{1+\rho}} + (1-\alpha^*)^{\frac{1}{1+\rho}} \xi_t^{*\frac{\rho}{1+\rho}} \right]^{\frac{\rho+1}{\rho}}$$

That is similar to the result of static economy. In fixed CES productive function and production  $\psi_s$ , we can get the minimal cost expense is the  $(R_t/r-\mu)$  times of the second term in the right-hand side of (14) by Principle of Duality. (b) and (c) resemble results (b) and (c) of theorem 1. (d) Herein, from a traditional economic and the real option perspective are the same, the value of exporters domestic production will be equal to the market price of the exporters' foreign production.

If the exporter with local production facilities decides to transfer their production location, then it should pay off the cost of transfer productive location  $R_t E^*$ . The market value of exporter's foreign production is thus defined as

$$\begin{aligned} & V^F(R_t) \\ & = V^f(R_t) - R_t E^* \\ & = R_t \left\{ \frac{1}{(r-\mu)} \left[ \psi_t^* P_t^* - \frac{\psi_t^*}{A_t^*} * \left[ \alpha^{*\frac{1}{1+\rho}} \omega_t^{*\frac{\rho}{1+\rho}} + (1-\alpha^*)^{\frac{1}{1+\rho}} \xi_t^{*\frac{\rho}{1+\rho}} \right]^{\frac{\rho+1}{\rho}} \right] - E^* \right\} \end{aligned} \quad (15)$$



Notably, the market value of exporter’s foreign production is a linear function of the real exchange rate.

The next step is to consider the method of deciding production location. Dixit and Pindyck (1994) outline these conditions as follows:

$$V^D(R_t) = V^F(R_t) \text{ (Value-matching condition)}$$

$$V_R^D(R_t) = V_R^F(R_t) \text{ (Smooth-pasting condition)}$$

The solution of the optimal transfer threshold value can be written as

$$R_{ROA}^* = \frac{\beta_2}{(\beta_2 - 1)} \frac{H}{[G + H^* + E^*]} \tag{16}$$

where

$$H \equiv \frac{\psi_t}{rA_t} \left[ \alpha^{\frac{1}{1+\rho}} \omega_t^{\frac{\rho}{1+\rho}} + (1 - \alpha)^{\frac{1}{1+\rho}} \xi_t^{\frac{\rho}{1+\rho}} \right]^{\frac{\rho+1}{\rho}}$$

$$H^* \equiv \frac{\psi_t^*}{(r - \mu)A_t^*} \left[ \alpha^{*\frac{1}{1+\rho}} \omega_t^{*\frac{\rho}{1+\rho}} + (1 - \alpha^*)^{\frac{1}{1+\rho}} \xi_t^{*\frac{\rho}{1+\rho}} \right]^{\frac{\rho+1}{\rho}}$$

$$G \equiv \frac{1}{(r - \mu)} P_t^* [(1 - \tau)\psi_t - \psi_t^*]$$

Several results follow immediately from Eq. (16). First,  $H$  is the second term on the right of Eq. (2) and is the expense costs. Second,  $R_t H^*$  is the second term on the right of Eq. (14) and is the expense costs. The following section examines the expected arrival time in the real exchange rate between the real option analysis and the NPV method. Finally, the parameters are considered in relation to the expected arrival time.

### 3. COMPARING THE DECISION METHOD AND SENSITIVE ANALYSIS

From the real options analysis and NPV method, we can get the optimal transfer threshold value for exporter deciding to transfer their locations is  $R_{ROA}^*$  and  $R_{NPV}^*$ . Furthermore, we can get  $R_{ROA}^* = (\beta_2/\beta_2 - 1) R_{NPV}^*$  (Lin et al., 2002b). On the other hand, the expected arrival time is finite only if  $\mu < 0.5\sigma^2$ , that is, only if the drift of the real exchange rate is negative. In this case, if  $R_0 > R_{ROA}^* (R_{NPV}^*)$ , then the expected arrival time for exporter deciding to transfer the production locations is

$$E_{R_0} [T(R_{ROA}^*)] = (0.5\sigma^2 - \mu)^{-1} \ln(R_0/R_{ROA}^*)$$

and

$$E_{R_0} [T(R_{NPV}^*)] = (0.5\sigma^2 - \mu)^{-1} \ln(R_0/R_{NPV}^*)$$

**THEOREM 1.** *The difference of the expected arrival time for exporter deciding to transfer the production location obtained by the real options analysis,  $E_{R_0} [T (R_{ROA}^*)]$  and the expected arrival time for exporter deciding to transfer the production location using the NPV method  $E_{R_0} [T (R_{NPV}^*)]$  is  $(-1) (0.5\sigma^2 - \mu)^{-1} \ln (\beta_2/\beta_2 - 1)$ .*

*Proof.* We can get immediately

$$\begin{aligned}
 & E_{R_0} [T (R_{ROA}^*)] - E_{R_0} [T (R_{NPV}^*)] \\
 &= (0.5\sigma^2 - \mu)^{-1} \ln (R_0/R_{ROA}^*) - (0.5\sigma^2 - \mu)^{-1} \ln (R_0/R_{NPV}^*) \\
 &= (0.5\sigma^2 - \mu)^{-1} [-\ln (\beta_2/\beta_2 - 1) + \ln (R_0/R_{NPV}^*)] \\
 &\quad - (0.5\sigma^2 - \mu)^{-1} \ln (R_0/R_{NPV}^*) \\
 &= (-1) (0.5\sigma^2 - \mu)^{-1} \ln (\beta_2/\beta_2 - 1)
 \end{aligned} \tag{17}$$

■

**Remarks** Several results follow immediately from Eq. (16). (a) Since the parameter  $\beta_2 < 0$ , then  $0 < (\beta_2/\beta_2 - 1) < 1$ , which infers that  $E_{R_0} [T (R_{ROA}^*)] > E_{R_0} [T (R_{NPV}^*)]$ . (b) There are only the parameters  $\mu, \sigma, r$  can influence the difference. (c) The difference of the expected arrival time for exporter deciding to transfer its location obtained by the real options analysis and NPV method is **adding value of the wait time**.

**THEOREM 2.** *If real exchange rate volatility,  $\sigma$ , rises, then the difference of the expected arrival time for exporter deciding to transfer the production location obtained by the real options analysis and using the NPV method,  $(-1) (0.5\sigma^2 - \mu)^{-1} \ln (\beta_2/\beta_2 - 1)$ , declines.*

*Proof.* Because  $\frac{\partial}{\partial \sigma} \left[ \frac{\beta_2}{(\beta_2 - 1)} \right] < 0$  (Lin an Wu, 2002), we can get immediately

$$\begin{aligned}
 & \frac{\partial}{\partial \sigma} [E_{R_0} [T (R_{ROA}^*)] - E_{R_0} [T (R_{NPV}^*)]] \\
 &= (-1) \times \frac{\partial}{\partial \sigma} \left[ (0.5\sigma^2 - \mu)^{-1} \ln (\beta_2/\beta_2 - 1) \right] \\
 &= (-1) \left[ \ln (\beta_2/\beta_2 - 1) \times \frac{\partial}{\partial \sigma} (0.5\sigma^2 - \mu)^{-1} + (0.5\sigma^2 - \mu)^{-1} \times \frac{\partial}{\partial \sigma} \ln (\beta_2/\beta_2 - 1) \right] \\
 &= \sigma (0.5\sigma^2 - \mu)^{-2} \times \ln (\beta_2/\beta_2 - 1) - (0.5\sigma^2 - \mu)^{-1} \times (\beta_2 - 1/\beta_2) \frac{\partial}{\partial \sigma} (\beta_2/\beta_2 - 1) \\
 &< 0
 \end{aligned}$$

■

**THEOREM 3.** *If risk-free interest rate,  $r$ , increases, then the difference of the expected arrival time for exporter deciding to transfer the production location obtained by the real options analysis and using the NPV method,  $(-1) (0.5\sigma^2 - \mu)^{-1} \ln(\beta_2/\beta_2 - 1)$ , rises.*

*Proof.*

Because  $\frac{\partial \beta_2}{\partial r} < 0$  and  $\frac{\partial}{\partial \beta_2} \left( \frac{\beta_2}{\beta_2 - 1} \right) < 0$ ,  $\forall \beta_2 < 0$

Thus, we have

$$\begin{aligned} & \frac{\partial}{\partial r} [E_{R_0} [T(R_{ROA}^*)] - E_{R_0} [T(R_{NPV}^*)]] \\ &= \frac{\partial}{\partial r} \left[ (0.5\sigma^2 - \mu)^{-1} \ln(\beta_2/\beta_2 - 1) \right] \\ &= (0.5\sigma^2 - \mu)^{-1} \times \frac{\partial}{\partial r} [\ln(\beta_2/\beta_2 - 1)] > 0 \end{aligned}$$

■

#### 4. CONCLUSIONS

This investigation examines the manufacturing exporter's production in the batch process using the CES production function given exchange rate uncertainty. The model is established using one exporter who produces domestically, and one that produces at the overseas point of sale (foreign). Either the domestic and foreign production locations can be selected, and exporter can then transfer local production overseas and find an optimal productive location where production can be achieved for less than the market value of the existing production location. To calculate the expected arrival time of the exporter decides to transferable location. This investigation yields the expected arrival time of the exporter's decisions to transfer production location, and performs a sensitivity analysis between the internal and external factor with the expected arrival time of the exporters who decide to transfer their production location. The above-mentioned result can provide exporters with a reference in their decision-making.

This investigation establishes the decision model and performs a sensitivity analysis. The following conclusions are reached

The difference of the expected arrival time for exporter deciding to transfer the production location obtained by the real options analysis and using the NPV method is  $(-1) (0.5\sigma^2 - \mu)^{-1} \ln(\beta_2/\beta_2 - 1)$  and  $E_{R_0} [T(R_{ROA}^*)] > E_{R_0} [T(R_{NPV}^*)]$ .

If real exchange rate volatility rises, then the difference of the expected arrival time for exporter deciding to transfer the production location obtained by the real options analysis and using the NPV method declines.

If risk-free interest rate increases, then the difference of the expected arrival time for exporter deciding to transfer the production location obtained by the real options analysis and using the NPV method rises.

The authors hope that this model represent a useful beginning of the important examination of the effects of exchange rate fluctuations on an industry in which sunk costs are important. At various points, I possible extensions of the model in future research are indicated. Building a Syscom On-Line EERP (Extended Enterprise Resource Planning) system can help to choose the optimal production locations (more than one) anywhere in the world. This basic batch production model can help domestic industry become international industry.

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