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Incentive Compatible Collusion and Investment *

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We consider a two-stage model in which two firms first invest in R&D to reduce their marginal production costs, and then either compete or collude in the output market. When they collude, they bargain over a cartel agreement to divide the collusive profit. If bargaining breaks down, they revert to duopolistic competition. For both a location model and a linear demand model, we show that firms invest more in R&D in the first stage under collusion than under competition. We demonstrate via example that social welfare may be greater under collusion than under competition in the location model. © 2005 Peking University Press

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1. INTRODUCTION

Whether monopoly or collusion among competing firms hinders innovation is a long standing question in economics. Arrow (1962) pointed out that monopoly weakens innovation incentives relative to competitive firms, because its marginal gain from cost-reducing innovation on top of its monopoly profit is smaller. This is called the "replacement effect" in Tirole

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(1988), which provides an excellent analysis and synthesis of the literature to that time.[1] In a recent important contribution, Fershtman and Pakes (2000) ("FP" hereafter) develop a dynamic model with heterogeneous firms and broad strategy choices (e.g., innovation investment, pricing, entry and exit). They present numerical solutions to their model, showing that when firms collude in the output market "more and higher-quality products" are offered to consumers, compared with the case when firms compete. Thus, collusion by firms results in higher consumer surplus. While their model is impressively general, analytical results are difficult to obtain, because of its complexity. In this paper, we show that similar results can arise in a very simple two-stage model. The intuition underlying our results is quite transparent, and so provides some insight into the numerical results of FP.

We consider situations in which firms first make cost-reduction investments, and then compete in the output market. We ask the following questions. What if the firms can collude in the output market, but not at the investment stage? Compared with the case in which they compete in the output market, will they reduce or increase their investment? How does social welfare change?

Using two standard competition models (a location model and a linear demand model), we show that firms in a duopoly actually invest more in cost-reducing innovations under collusion than under competition. The marginal cost of production is, therefore, lower under collusion. This effect is welfare-enhancing. Conversely, under collusion, the price offered to the consumers is higher, which can be welfare-reducing if it reduces consumer demand. While the latter effect is well-known, overall social welfare depends on the magnitude of these two effects. In the location model, we show that both social and consumer welfare can increase under collusion. While analytic results on welfare are difficult to obtain in the linear demand model, a numeric analysis suggests that welfare is lower under collusion.

The idea behind the results on investment is quite simple. The key lies in how competing firms reach collusive agreements (either explicit or implicit) to divide the benefits from collusion. Collusion outcomes must be incentive compatible, that is, each firm should get at least its competition profit (since any firm can reject a collusive outcome and revert to the competition mode). Thus, each firm's profit under collusion will be its competition profit plus a portion of the net gain from collusion, which is the difference between total collusion profit (i.e., the monopoly profit) minus

[1] In the presence of large spillover effects of innovation investments, the Schumpeterian view argues that rents in the output market are needed to provide incentives to innovate (for some recent contributions see D'Aspremont and Jacquemin 1988, Suzumura 1992, Cabral 2000). In this paper, we focus on situations without spillover effects, in order to present the main idea in the simplest way. The main idea of the paper can be extended to situations with spillover effects.

the sum of their competition profits. In any sensible competition model, if one firm lowers its production cost with all else equal, its opponent's competition profit must decrease. Therefore, under collusion, a firm has an additional incentive to invest in cost-reduction: at the margin, the net gain from collusion can be increased by lowering its opponent's competition profit. We call this the "strategic bargaining effect," meaning that firms can improve their bargaining positions in the collusive outcome by reducing their opponent's disagreement payoff.

Relative to the competition case, there is another effect of investments on collusive profits, which is the different marginal benefit of investment under monopoly, as compared to competition. This effect can be either positive (in the location model) or negative (in the linear demand model). In either case, the strategic bargaining effect dominates, and thus equilibrium investments are greater under collusion than under competition in both models. The existence of this positive effect on social and consumer welfare then offers a genuine tradeoff in comparing competition and collusion.

Competition in investment and collusion in the output market is easy to motivate.[2] Scherer (1980, ch. 6) presents some real world examples (e.g., the cigarette industry in the 1920's and 1930's). Firms often find it difficult to collude on innovative investments because these are difficult to verify and monitor.[3] On the other hand, market variables such as prices, market shares or quantities are fairly easy to verify and monitor, making collusion in the output market easier. Firms may collude in the output market either explicitly by signing a cartel agreement (in a loose anti-trust environment), or tacitly through repeated interactions. We use the Nash Bargaining Solution (NBS) to determine the division of collusive profits between the two firms. The NBS is sensible and standard in the explicit agreement case, and is also commonly used with repeated interaction. In particular, FP use the NBS in their dynamic model.[4] Therefore, the strategic bargaining effect identified in our simple model can also play a role in the more complex models such as FP.

There is an extensive literature that studies how collusion in output markets affects innovation incentives. In particular, it is shown that collusion in the output market can affect investment incentives and hence lead to greater consumer surplus (Matsui 1989), lower profits (Fershtman

[2]Similar assumptions have been made in Sutton (1991), Schmalensee (1987) and many others (see discussion of the related literature below). Note that our model fits the incomplete contract framework (Hart 1995): investments are not contractible, and market variables are not contractible ex ante but become contractible ex post.

[3]We assume that production costs are observable after investments are made. This provides a deterministic threat point at the bargaining stage. However, the qualitative results hold even if production costs are stochastic conditional on investments, or firms only observe noisy signals about each other's production cost.

[4]See equation (5) of FP.

and Gandal 1994), minimum product differentiation (Friedman and Thisse 1993), or excess capacity (Benoit and Krishna 1987, Osborne and Pitchik 1987, Davidson and Deneckere 1990). Some of these papers assume that colluding firms divide the monopoly profit according to some fixed rules; e.g., Matsui (1989) and Fershtman and Gandal (1994).[5] Other papers use repeated games to model tacit collusion in the output market; e.g., Benoit and Krishna (1987) and Davidson and Deneckere (1990).[6] Also related are Gans and Stern (2000) and Gans, Hsu and Stern (2002), which study the effects of license agreements on the incumbent firm's innovative activities. By using the standard NBS for different competition models, our model provides a simple framework to study the effects of collusion on investment incentives and social welfare. The idea of the strategic bargaining effect is robust: collusion in the output market provides additional investment incentives because reducing your opponent's disagreement payoff can improve your bargaining position.

The rest of the paper is organized as follows. The next section presents and analyzes the location model. Section 3 compares social welfare and consumer surplus between collusion and competition. In Section 4 we briefly consider a linear demand model to demonstrate the robustness of the main idea of the paper. Concluding remarks are in Section 5.

2. THE LOCATION MODEL AND ANALYSIS

Consider a standard horizontal differentiation model with two firms, indexed by $i = 1, 2$. Consumers are located uniformly on the line segment $[0, 1]$, each with a unit demand. The two firms are located on the two ends of the line segment, firm 1 at 0 and firm 2 at 1. Consumers incur transportation costs at the constant rate t to buy the good, and obtain a gross surplus of $R > 0$ from consuming the good. Hence, if a consumer at location x buys from firm 1, his net surplus is $u = R - xt - p_1$, where p_1 is firm 1's price.

We study the following two-stage complete information game. In the first stage, the two firms simultaneously and non-cooperatively make cost-reduction investments. Investments determine marginal costs, which be-

[5] Both papers use rules based on commonly used collusive technologies summarized by Schmalensee (1987): proportional reduction and market division. These rules may require quite strong enforcement (e.g., tying consumers to a particular firm regardless of the price differentials).

[6] Both papers (as well as Osborne and Pitchik 1987) follow Kreps and Scheinkman (1983), and study games in which firms make capacity choices first and then compete in prices. These papers show that firms use excess capacities in the punishment phases, to sustain collusion in the repeated game. In terms of welfare implications, excess capacity and collusion in the output market both reduce social welfare. In contrast, collusion in the output market in our paper can be welfare-enhancing, as firms become more efficient.

come publicly known at the end of the first stage. To simplify matters and focus on the more interesting cases, we make several assumptions. The first is that marginal production costs of both firms are constant in quantities. Firm i's marginal cost $c_i(I_i) \geq 0$ depends on its investment I_i, with $c_i' < 0$ (investment reduces marginal cost) and $c_i'' \geq 0$ (diminishing returns of investment). A second assumption is that firms have the same cost-reduction technology, so $c_1(I) = c_2(I)$ for the same I. By investing I_i, firm i incurs investment cost of $T(I_i)$, where $T' > 0$, and $T'' \geq 0$. Since $c(I)$ is strictly decreasing and hence invertible, we can think of firms as directly choosing marginal costs c_i with the associated investment costs $F(c_i) = T(c^{-1}(I))$, where $F' < 0$ and $F'' > 0$. We assume that F satisfies the Inada conditions: $F(\bar{c}) = 0$, $F'(0) = -\infty$ and $F'(\bar{c}) = 0$, for some $\bar{c} \in (0, R)$. To ensure concave profit functions, we further require F to be sufficiently convex, so that $tF''(c) > 1$ for all c.

In the second stage, we consider two possible cases. Under *competition*, firms compete in the output market as duopolists, by choosing prices simultaneously. Under *collusion*, they can collude by fixing prices to maximize their joint profit in the second stage. Collusion can be done by signing a cartel agreement (in a loose anti-trust environment), or through repeated interaction (FP sustain collusion in this manner). While the two firms may collude in the output market, we suppose that they cannot do so in the investment stage, either because investments are not easy to verify or monitor or because firms make one-time investments (e.g., building a plant) and then repeatedly make pricing and production decisions (e.g., monthly price adjustments). Our main focus is on comparing firms' incentives to make cost-reduction investments, and the resulting social welfare under these two different environments in the output market.

Before analyzing the competition and collusion cases, we first study the socially optimal solution as the benchmark case.

2.1. The Socially Optimal Solution

Let x_i be the measure of consumers purchasing from firm i, where $x_1 + x_2 \leq 1$. The total social welfare is given by

$$\begin{aligned} W &= x_1 R - \int_0^{x_1} zt dz - c_1 x_1 - F(c_1) + x_2 R \\ &- \int_{1-x_2}^{1} (1-z) t dz - c_2 x_2 - F(c_2) \\ &= x_1(R - 0.5 x_1 t - c_1) - F(c_1) + x_2(R - 0.5 x_2 t - c_2) - F(c_2). \quad (1) \end{aligned}$$

A social planner chooses (x_1, x_2, c_1, c_2) to maximize W subject to the constraints that $x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1$. Let λ be the Lagrangian multiplier for $x_1 + x_2 \leq 1$. The Kuhn-Tucker conditions are

$$\begin{aligned} R - x_i t - c_i &= \lambda \\ (1 - x_1 - x_2)\lambda &= 0 \\ -x_i - F'(c_i) &= 0. \end{aligned}$$

It can be verified that the second order condition is satisfied under our assumption $tF''(c) > 1$.

We will focus on the "full market coverage" case, i.e., $x_1 + x_2 = 1$, and the symmetric solution, in which $c_1 = c_2$ and $x_1 = x_2 = 0.5$.[7] Then $c_1 = c_2 = c^s$, where c^s ("s" stands for social optimum) is the solution to

$$-F'(c) = 0.5. \tag{2}$$

An additional condition is that $\lambda \geq 0$, or $R \geq 0.5t + c^s$. Under this condition, the market is fully covered, so the maximum social welfare is

$$W^s = R - 0.25t - c^s - 2F(c^s)$$

2.2. The Competition Case

Next, we consider the case in which the two firms choose investments non-cooperatively in the first stage, and compete in the output market in the second stage. We solve for the symmetric, subgame-perfect equilibrium of the game. Given marginal costs (c_1, c_2) (chosen at stage 1), firm i in the second stage chooses p_i to maximize $\pi_i = x_i(p_i - c_i) - F(c_i)$. Since no consumer will buy if the price is higher than R, without loss of generality, we restrict attention to $p_i \in [c_i, R]$. Consider the full market coverage equilibrium, with $x_1 + x_2 = 1$. Given (p_1, p_2), $x_i = 0.5 + 0.5(p_j - p_i)/t$. The first order condition is

$$p_i = 0.5(t + c_i + p_j). \tag{3}$$

This defines firm i's second stage reaction function. Solving these equations gives $p_i = t + (2c_i + c_j)/3$, leading to corresponding market shares $x_i = 0.5 + (c_j - c_i)/(6t)$. Hence, we have $p_i - c_i = 2tx_i$, so that the duopoly profit of firm i is $\pi_i(c_1, c_2) = 2tx_i^2 - F(c_i)$.

[7] In any regime in which the market is not fully covered (that is, the constraint $x_1 + x_2 \leq 1$ is not binding), the firms are local monopolists, and hence independent of each other. If full market coverage is not attained under competition or collusion, the comparison is trivial. There exists a set of parameters such that it is attained under competition, but not collusion, in which case there is an additional welfare-reducing effect of collusion.

Now, at stage 1, firm i chooses c_i to maximize π_i. The first order condition is $4tx_i\frac{\partial x_i}{\partial c_i} - F'(c_i) = 0$, or $-2x_i/3 - F'(c_i) = 0$, or

$$\frac{3t + c_j - c_i}{9t} = -F'(c_i). \tag{4}$$

In the symmetric equilibrium, $c_1 = c_2$, so

$$-F'(c) = \frac{1}{3}. \tag{5}$$

Let c^d be the solution to the above equation ("d" stands for duopoly). In the symmetric equilibrium, $p^d = t + c^d$ and $\pi^d = 0.5t - F(c^d)$. Then, the social welfare is

$$W^d = R - 0.25t - c^d - 2F(c^d).$$

2.3. The Collusion Case

We now study the case in which the two firms choose investments non-cooperatively in the first stage, but collude in the output market in the second stage. Once investments are made and the cost structure is known, the monopoly profit is higher than the sum of the two firms' profits if they compete against each other. Hence the firms have incentives to collude on the monopoly outcome. Whether collusion is incentive compatible, however, will depend on how the firms divide the net surplus from collusion. If they do not reach a collusive agreement, they will compete against each other. Thus no firm should get less than its competition profit in the collusion outcome. We use the Nash Bargaining Solution as the outcome of the surplus division, with the competitive outcome as the disagreement point.[8] Without loss of generality, the two firms are assumed to have equal bargaining power in the bargaining process.[9] In a repeated pricing game one can get the same outcome as the static Nash Bargaining Solution when the discounting factor is sufficiently close to one and the static competition equilibrium is used in punishment phases (see FP).

Let (c_1, c_2) be the marginal costs after the investments at stage 1 have been made. As a perfect cartel, the two firms will choose prices to maximize

[8] Equivalently, one can adopt the non-cooperative alternating offer bargaining games of Rubinstein (1982) or Binmore, Rubinstein and Wolinsky (1986). To adopt the Rubinstein game, suppose the two firms have a common discount factor close to one, and compete against each other in each period if no agreement is reached. To adopt the Binmore *et al* game, suppose there is a small exogenous probability of bargaining breaking down upon each rejection of an offer, and the two firms compete against each other forever if bargaining breaks down. Both games have a unique equilibrium outcome that converges to the Nash Bargaining Solution as the discount factor goes to 1.

[9] It is straightforward to generalize the results of the model to unequal bargaining power.

their joint gross profit in the second stage (since the investment costs are sunk).[10] The maximization problem is

$$\max \quad \Pi^m = \Pi_1 + \Pi_2 = \sum_{i=1}^{2} x_i(p_i - c_i)$$

subject to the constraints $x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1$.

Consider full market coverage, so that $x_1 + x_2 = 1$. Then, $p_i = R - x_i t$, and the solution is $x_i = 0.5 + (c_j - c_i)/(4t)$ if $|c_j - c_i| < 2t$, $x_i = 0$ if $c_j - c_i < -2t$ and $x_i = 1$ if $c_j - c_i > 2t$. In addition, it is required that $c_1 + c_2 \leq 2(R - t)$ when x_i is interior, and $R - 2t \geq c_i$ when $x_i = 1$. We will assume that R is sufficiently large so that the x_i are interior. Then $\Pi_i^m = x_i(R - x_i t - c_i)$, and $\Pi^m = R - 0.5t - 0.5(c_1 + c_2) + (c_1 - c_2)^2/(8t)$.

If the two firms cannot agree on a collusive solution, they will compete against each other, leading to duopoly profits $(\pi_1^d(c_1, c_2), \pi_d^d(c_1, c_2))$ as derived in the preceding subsection. The net gain from collusion is $\Delta = \Pi^m - \Pi_1^d - \Pi_2^d$. The surplus-division outcome in the second stage, according to the Nash Bargaining Solution, is that firm i will get its disagreement profit Π_i^d plus half of the net gain from collusion. Hence, firm i's profit at the second stage is

$$\Pi_i^c = \Pi_i^d + \Delta/2 = (\Pi^m + \Pi_i^d - \Pi_j^d)/2, \tag{6}$$

where "c" stands for collusion or cartel.

Now, at the first stage, firm i chooses c_i to maximize $\pi_i^c = \Pi_i^c - F(c_i)$ where Π_i^c is given by Equation 6. The first order condition of firm i's profit maximization problem is

$$0.5\left\{\frac{2t + c_j - c_i}{4t} + \frac{3t + c_j - c_i}{9t} + \frac{3t + c_i - c_j}{9t}\right\} = -F'(c_i). \tag{7}$$

Under symmetry, $c_i = c_j$, so we have

$$-F'(c) = \frac{7}{12}. \tag{8}$$

Let the solution to this equation be c^c. In symmetric equilibrium, $x_i = 0.5$ and the social welfare is

$$W^c = R - 0.25t - c^c - 2F(c^c)$$

[10]Since we consider a static model, to achieve joint profit maximization, some sort of side payments may be needed. In the case of repeated interactions in the output market, joint profit maximization can arise as an equilibrium outcome supported by future actions, as in FP. Thisse and Vives (1992) make a similar point about repeated interaction and collusion on basing point pricing.

It is now immediate that, compared to the socially optimal solution, firms over-invest at stage 1 under collusion, and under-invest under competition.

PROPOSITION 1. *Suppose R is sufficiently large so that there is full market coverage. Then $c^c < c^s < c^d$.*

Proof. Follows directly from equations (2), (5), and (8), given that $F'(\cdot) < 0$ and $F''(\cdot) > 0$. ∎

Proposition 1 shows that firms invest more in cost-reduction when they collude, rather than compete, in the output market. To understand this result, first consider the observation that competing firms invest too little relative to the social optimum (that is, $c^s < c^d$). The intuition is simple. The social marginal benefit of investment by a firm, $\partial W/\partial(-c_i)$, is 0.5, since the equilibrium market share of each firm is 0.5. Under competition, a firm's cost reduction will lower its price, and therefore its opponent's price, since p_1 and p_2 are strategic complements in the spatial competition model (Equation 3). In other words, cost reduction by firm i will lead to both firms competing more fiercely, and hence reducing the profit for firm i. By comparison of Equations 2 and 5, this strategic competition effect reduces the marginal benefit of investment by 1/6. Thus, compared to the first best, firms invest less in cost reduction when they compete in the output market.

Next, consider the result that $c^c < c^d$; that is, firms invest more under collusion than competition. Under collusion, if a firm reduces its marginal cost, it has two effects. First, it raises the monopoly profit. Second, by shifting its own disagreement point, it captures a larger share of the monopoly profit. This leads to an over-investment in cost reduction.

Formally, from equation (6) the marginal benefit of investment differs in the two cases by

$$\frac{\partial \Pi_i^c}{\partial(-c_i)} - \frac{\partial \Pi_i^d}{\partial(-c_i)} = 0.5\frac{\partial \Delta}{\partial(-c_i)} = 0.5[\frac{\partial \Pi^m}{\partial(-c_i)} - \frac{\partial \Pi_i^d}{\partial(-c_i)}] - 0.5\frac{\partial \Pi_j^d}{\partial(-c_i)}. \quad (9)$$

In the location model we have $\partial\Pi^m/\partial(-c_i) = 0.5 + (c_j - c_i)/(4t)$, $\partial\Pi_i^d/\partial(-c_i) = 2x_i^d/3 = 1/3 + (c_j - c_i)/(9t)$, and $\partial\Pi_j^d/\partial(-c_i) = -2x_j/3 = -1/3+(c_j-c_i)/(9t)$. Under symmetry, $\partial\Pi^m/\partial(-c_i) = 0.5 > \partial\Pi_i^d/\partial(-c_i) = 1/3$ and $\partial\Pi_j^d/\partial(-c_i) = -1/3 < 0$, therefore the marginal benefit of investment under collusion, $\partial\Pi_i^c/\partial(-c_i)$, is greater than that under competition, $\partial\Pi_i^d/\partial(-c_i)$.

Since, in any sensible competition model, one firm's profit decreases as its competitor becomes more efficient, the result that $\partial\Pi_j^d/\partial(-c_i) < 0$ is quite robust. Under collusion, firm i benefits from reducing firm j's competition profit because it reduces firm j's disagreement payoff under collusion, and thus increases the collusion rent to i. This gives firms more incentives to invest in cost-reduction under collusion than under competition. We call this the "strategic bargaining effect." It arises because, in trying to reach a collusive agreement, firms can always fall back to competition, so their competition profits affect their relative bargaining positions. Clearly firms gain from weakening the bargainin position of their opponent: this is achieved by investing more in cost-reduction to make themselves more efficient. Because the strategic bargaining effect is quite strong in this model,firms actually invest more in cost-reduction under collusion than in the first best.

That $\partial\Pi^m/\partial(-c_i) > \partial\Pi_i^d/\partial(-c_i)$ is specific to the location model. Under full market coverage and symmetry, the marginal benefit of investment under monopoly $\partial\Pi^m/\partial(-c_i)$ is identical to that under the social optimal solution $\partial W/\partial(-c_i)$, and as argued above the latter is greater than that under competition. This is a rather special feature of the location model, as the monopoly has the same market size as the social optimum under full market coverage. In competition models other than the location model, it is usually the case that $\partial\Pi^m/\partial(-c_i) < \partial\Pi_i^d/\partial(-c_i)$. This will tend to reduce firms' investment incentives under collusion. The net effect on investment will then depend on whether this effect dominates or is dominated by the strategic bargaining effect.

3. WELFARE IN THE LOCATION MODEL

In this model, for the parameters considered, all consumers receive the good under either collusion or competition because of full market coverage. Hence, consumption utility is identical across the two regimes. Since payments from consumers to firms are transfers that do not affect social welfare, the latter depends only on the cost of production, c, and the investment cost, $F(c)$. In this model, since firms over-invest under collusion and under-invest under competition, neither collusion nor competition can provide the same welfare level as the social optimum. A comparison of welfare under collusion and competition shows that

$$W^c - W^d = (c^d - c^c) + 2(F(c^d) - F(c^c)).$$

Hence, $W^c > W^d$ if and only if

$$-\frac{F(c^d) - F(c^c)}{c^d - c^c} < \frac{1}{2}. \tag{10}$$

By convexity of F, we know that $F'(c^c) < \frac{F(c^c)-F(c^d)}{c^d-c^c} < F'(c^d)$, so that $-\frac{F(c^d)-F(c^c)}{c^d-c^c} \in (\frac{1}{3}, \frac{7}{12})$. Hence if $-\frac{F(c^d)-F(c^c)}{c^d-c^c} \in (\frac{1}{3}, \frac{1}{2})$, then $W^c > W^d$; otherwise, $W^c \leq W^d$.

We consider two examples which demonstrate that welfare can be higher or lower under collusion, as compared to competition.

Example 1

Suppose $F(c) = Ae^{-\beta c}$, where $A, \beta > 0$. Then, $W^c > W^d$.

Here, $F'(c) = -\beta F(c)$. Hence, the solution to the equation $-F'(c) = k$ is $c = \frac{\ln \beta A - \ln k}{\beta}$, and, at this solution, $F(c) = \frac{k}{\beta}$. Therefore, for this function,

$$\begin{aligned} W^c - W^d &= \frac{\ln \beta A - \ln(\frac{1}{3})}{\beta} - \frac{\ln \beta A - \ln(\frac{7}{12})}{\beta} + 2(\frac{1}{3\beta} - \frac{7}{12\beta}) \\ &= \frac{\ln 7 - \ln 4}{\beta} - \frac{1}{2\beta} > 0. \end{aligned}$$

That is, in this example, the welfare under collusion exceeds that under competition.

Modifying the above investment cost function, we can show that welfare under collusion may be lower than welfare under competition. Let c^s be the solution to the equation $\beta Ae^{-\beta c^s} = \frac{1}{2}$.

Example 2

Define $\phi(c)$ as follows:

$$\phi(c) = \begin{cases} A_1 e^{-\gamma c} & \text{if} \quad c \leq c^s \\ A_2 e^{-\beta c} & \text{if} \quad c > c^s, \end{cases}$$

where $\gamma > 0$, and A_1 is given by $A_1 e^{-\gamma c^s} = A_2 e^{-\beta c^s}$, to ensure continuity at the point c^s.

Suppose the investment cost function is $\phi(\cdot)$, as defined above. Fix A_2, β. Then, there exists a $\hat{\gamma} > \beta$ such that, for $\beta \leq \gamma < \hat{\gamma}$, $W^c > W^d$, whereas for $\gamma > \hat{\gamma}$, $W^c < W^d$.

Since $c^c < c^s$ and $c^d > c^s$, we have

$$W^c - W^d = \frac{\ln \gamma A_1 + \ln 3}{\gamma} + \frac{2}{3\gamma} - \frac{\ln \beta A_2 - \ln 7 + \ln 12}{\beta} - \frac{7}{6\beta}.$$

Since by definition, $\ln A_1 = \ln A_2 + (\gamma - \beta)c^s$, and $\ln \beta + \ln A_2 - \beta c^s = -\ln 2$, the above expression is equal to

$$\begin{aligned} W^c - W^d &= \frac{\ln \gamma + \ln A - \beta c^s + \ln 3 + 2/3}{\gamma} + c^s - \frac{\ln \beta A + \ln 7 - \ln 12}{\beta} - \frac{7}{6\beta} \\ &= \frac{\ln \gamma - \ln \beta + \ln 1.5 + 2/3}{\gamma} + \frac{\ln 7/6 - 7/6}{\beta} \end{aligned}$$

As shown above, if $\gamma = \beta$ (which ensures $A_1 = A$), $W^c > W^d$. The first term is declining in γ for $\gamma \geq \beta$, and go to zero as $\gamma \to \infty$. Hence, there exists a $\hat{\gamma}$ with the required properties.

Overall welfare in this model, therefore, may be higher or lower under collusion, as compared to competition. For completeness, we consider consumer welfare as well. Consumer surplus under competition is $S^d = R - \frac{t}{4} - p^d = R - \frac{5t}{4} - c^d$. Under collusion (as in monopoly), consumer surplus is $S^c = R - \frac{t}{4} - p^c = R - \frac{t}{4} - (R - 0.5t) = \frac{t}{4}$. Therefore, $S^c > S^d$ if and only if $R - c^d < 1.5t$. Hence, there exists an open set of parameters $R \in (0.5t + c^d, 1.5t + c^d)$ such that $S^c > S^d$, whereas if R is large enough, $S^c < S^d$.

4. LINEAR DEMAND MODEL

The results of the location model extend to other market competition situations. We briefly present an example. Suppose the inverse demand function for firm $i = 1, 2$ is given by

$$A - q_i - \alpha q_j = p_i$$

where $\alpha \in [0, 1]$ measures the degree of substitution between the two products. This demand function can be derived from the following consumer utility function:

$$U = Aq_1 + Aq_2 - 0.5q_1^2 - 0.5q_2^2 - \alpha q_1 q_2 + y,$$

where y is the amount of money the consumer has available.

The social welfare function then is

$$W = Aq_1 + Aq_2 - 0.5q_1^2 - 0.5q_2^2 - \alpha q_1 q_2 - c_1 q_1 - c_2 q_2.$$

We focus on symmetric solutions throughout. We assume that $\alpha > 0$ (so that there is some substitutability between the products), but α is low enough so that both firms are always operating.[11] One can show that the first best investment level and hence marginal cost, c^s, is defined by

$$q^s = (A - c)/(1 + \alpha) = -F'(c). \tag{11}$$

Consider a monopolist which chooses investments and outputs for the two firms to maximize joint profit. The monopoly solution, c^m, is then

[11] If α is close to 1, both the social optimum and monopoly solutions have only one firm active.

given by

$$q^m = (A - c)/[2(1 + \alpha)] = -F'(c). \tag{12}$$

In the competition case, the two firms compete first in investment, and then in Cournot fashion in the output market. For a given cost structure (c_1, c_2), solving their reaction functions yields equilibrium quantities $q_i^d = [(1 - 0.5\alpha)A + 0.5\alpha c_j - c_i]/(2 - 0.5\alpha^2)$, and gross profits $\Pi_i^d = (q_i^d)^2$. Further, at the investment stage, the symmetric equilibrium outcome, c^d, is the solution to

$$4(A - c)/[(4 - \alpha^2)(2 + \alpha)] = -F'(c). \tag{13}$$

Finally, when the two firms compete in investment but collude in the output market, the second period profits are identical to equation (6). At the investment stage, the symmetric equilibrium outcome c^c can be found as the solution to

$$(A - c)[0.25/(1 + \alpha) + 1/(4 - \alpha^2)] = -F'(c). \tag{14}$$

In this model as well, collusion leads to a greater investment in cost reduction than competition.

PROPOSITION 2. *In the linear demand model, $c^s < c^c < c^d < c^m$. In particular, investments are higher when the firms collude in the output market than when they compete.*

Proof. By Equations 11, 12, 13 and 14, the comparison of investments hinges on the comparison of the following coefficients: $\nu^s = 1/(1 + \alpha)$, $\nu^m = 0.5/(1+\alpha)$, $\nu^d = 4/[(4-\alpha^2)(2+\alpha)]$, and $\nu^c = 0.25/(1+\alpha)+1/(4-\alpha^2)$. A larger ν implies higher marginal benefit of investment, leading to greater investments (and hence lower production costs) in equilibrium. It is easy to verify that $\nu^s > \nu^d \geq \nu^m = 0.5\nu^s$. To see that $\nu^c > \nu^d$, we have

$$\begin{aligned} & 4(4 - \alpha^2)(2 + \alpha)(1 + \alpha)(\nu^c - \nu^d) \\ = \; & (4 - \alpha^2)(2 + \alpha) + 4(2 + \alpha)(1 + \alpha) - 16(1 + \alpha) \\ = \; & 2\alpha^2 - \alpha^3 > 0. \end{aligned}$$

To see $\nu^c < \nu^s$, we have

$$4(4 - \alpha^2)(1 + \alpha)(\nu^s - \nu^c) = 3(4 - \alpha^2) - 4(1 + \alpha) = 8 - 4\alpha - 3\alpha^2 > 0.$$

Hence, $\nu^s > \nu^c > \nu^d > \nu^m$, so that $c^s < c^c < c^d < c^m$. ∎

In this model, we expect that $c^s < c^d < c^m$. A monopolist facing a downward-sloping demand curve under-invests in cost-reduction relative to the socially optimal level, because part of the benefit from its investment is captured by the consumers. In fact, by the Envelope Theorem, marginal returns to cost-reducing investments for both the social planner and the monopolist are simply the production levels in the second stage. With a downward-sloping demand curve, the social planner will set a higher production level than the monopoly, hence cost-reducing investments have greater marginal returns to the social planner than the monopoly.

Under Cournot competition, the production level in symmetric equilibrium is higher than the monopoly level but lower than the socially optimal level. Thus, firms invest more under competition than under monopoly, but less than the social optimum.[12]

The intuition behind the result $c^c < c^d$ is the same as before. Since $\partial\Pi_i^c/\partial(-c_i) = 0.5[\partial\Pi^m/\partial(-c_i) + \partial\Pi_i^d/\partial(-c_i)] - 0.5\partial\Pi_j^d/\partial(-c_i)$, the marginal benefit of investment (and hence cost-reducing investments) under collusion is somewhere between that in the monopoly case and the competition case, excluding the last term. The last term, $-0.5\partial\Pi_j^d/\partial(-c_i)$, represents the "strategic bargaining effect" discussed above, and is equal to $(A-c)/[(2+\alpha)(4-\alpha^2)]$ under symmetry. Since larger investment by firm i (hence a smaller marginal cost) reduces firm j's profit in the case of competition, which benefits firm i under collusion, this provides firm i with an additional incentive to make cost-reducing investments when anticipating the collusion outcome in the second stage. This "strategic bargaining effect" is strong enough that the overall marginal returns to cost-reducing investments under collusion are greater than those under competition, leading to the result that $c^c < c^d$.

Welfare in this model is difficult to determine analytically. There are two opposite effects on welfare. Under collusion, the firms produce the monopoly quantity (given their costs at stage 2). Hence, market coverage under collusion is significantly smaller than under competition, which reduces social welfare. On the other hand, firms invest more in cost reduction, and hence are more efficient under collusion than under competition, which increases social welfare. Numerical computations suggest that, for a variety of cost functions, including those considered in the previous section, welfare is higher under competition than collusion. This seems to suggest that the market effect tends to dominate the investment effect in this model.

[12] An additional effect is that lower marginal cost c_i will lower firm j's equilibrium production level, and therefore increases firm i's unit profit. This effect also increases firms' investments under competition.

5. CONCLUSION

In this paper we show that, compared with oligopolistic competition, collusion in the output market can provide additional incentives (via the "strategic bargaining effect") for firms to invest in cost-reduction. When this effect is strong, it is possible that overall social welfare is greater under collusion. This analysis provides another reason that one needs to be cautious in applying the conventional wisdom of competition policies to certain industries that may have strong strategic bargaining effects. We conjecture that such industries are likely to have the following features: (i) innovation investments are important and hard to monitor; (ii) marginal investment costs do not increase very fast; (iii) demand is not very elastic.

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