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A Note on the Welfare Effects of Horizontal Mergers in Asymmetric Linear Oligopolies

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This paper extends Farrell and Shapiro (1990) and Levin (1990) by providing necessary and sufficient conditions for horizontal mergers to be both profitable and welfare-enhancing when market demand and firms' costs are linear. We show that profitable, welfare-enhancing mergers are likely to involve firms whose combined pre-merger market shares exceed 50%, and that mergers may be profitable and welfare-enhancing even when they do not generate any direct cost efficiencies. Our results suggest that any approach to evaluating the welfare effects of horizontal mergers which does not account for industry-wide strategic effects is seriously flawed. © 2006 Peking University Press

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1. INTRODUCTION

A central objective of antitrust policy is to balance the efficiency gains associated with horizontal mergers against any increases in market power.[1] Since the stakes for the firms involved are usually very large, uncertainty about the application of the policy can be very costly. The Department of Justice (DOJ) and Federal Trade Commission (FTC) have attempted to reduce the uncertainty by issuing joint horizontal merger guidelines (DOJ and FTC, 1992).[2] One of the ways in which the guidelines work is by establishing "safe harbors" — criteria that firms can use to assess whether a merger will be immune from antitrust scrutiny. The idea, of course, is to eliminate any uncertainty about the antitrust implications of mergers when the circumstances are sufficient to presume that they will have positive welfare effects. While the use of the safe harbors is clearly preferable to a more ad hoc policy, further research on the welfare effects of horizontal mergers may help to improve the way in which the safe harbors are defined. It may also help to improve the way that mergers which fall outside the safe harbors are analyzed for antitrust purposes.

Two recent papers which have provided major results are by Farrell and Shapiro (1990) and Levin (1990). In fact, in a "linear market" — one in which market demand and firms' costs are linear — Farrell and Shapiro's and Levin's most significant results are equivalent[3]: a merger increases welfare if it is profitable and if the pre-merger market share of the insider firms is less than 50%. We shall refer to this as the Farrell-Shapiro-Levin (FSL) condition. Although the FSL result provides a surprisingly simple way of identifying welfare-enhancing horizontal mergers, it is only a sufficient condition and may be grossly under-inclusive.

This paper extends Farrell and Shapiro's and Levin's results by providing a more precise characterization of the welfare effects of horizontal mergers in linear markets. We provide necessary and sufficient conditions for horizontal mergers to be both profitable and welfare-enhancing in linear markets with asymmetric firms. Indeed, we show that the set of profitable, welfare-enhancing mergers is generally much larger than the set defined by the FSL condition. Our results indicate that, in addition to the number of insiders and their market shares, the welfare effects of a merger will also de-

[1]See Williamson (1968) for the classic analysis of the tradeoff. For more recent perspectives, see the papers by Fisher (1987), Salop (1987), Schmalensee (1987), and White (1987).

[2]The joint 1992 Horizontal Merger Guidelines replaced the DOJ's 1984 Merger Guidelines and the FTC's 1982 Statement Concerning Horizontal Merger Guidelines. Since they were the first antitrust guidelines ever issued by the DOJ and FTC jointly, in spite of their overlapping responsibilities for antitrust enforcement, they achieved a significant advancement in antitrust policy coordination.

[3]In a linear market, Farrell and Shapiro's Proposition 5 (the Farrell-Shapiro condition) is equivalent to Levin's Theorem 4 (Levin's 50% rule). See the appendix for a proof.

pend on whether it generates any direct cost efficiencies as well as the cost structure of the industry. Interestingly, however, we show that mergers may be profitable and welfare-enhancing even when they do not generate any direct cost efficiencies. This implies that any approach to merger analysis which does not account for the industry-wide strategic effects which can generate indirect cost efficiencies is seriously flawed.

The paper is organized as follows: Section II describes the problem and presents the basic analytic framework. Section III provides necessary and sufficient conditions for horizontal mergers to be profitable, and Section IV provides necessary and sufficient conditions for horizontal mergers to be welfare-enhancing. Section V discusses the results and Section VI presents our conclusions. The appendix provides the proofs.

2. THE PROBLEM

An oligopoly market for a homogeneous good is defined by an inverse market demand function, $p(\sum x_j)$, and n cost functions, $C_i(x_i), i \in N = \{1, \ldots, n\}$, one for each of the n firms. Following Farrell and Shapiro (1990), we assume that the post-merger cost function for any subset of firms which merge, S, is given by

$$C_s(y) = \min\{\sum_{j \in S} C_j(x_j) | \sum_{j \in S} x_j = y, x_j \geq 0, j \in S\}, y \geq 0 \tag{1}$$

In a model with linear costs and without capacity constraints, this assumption in effect implies the exit of less efficient firms subsequent to their merger with more efficient firms. Our analysis is thus confined to the class of mergers which Farrell and Shapiro (1990) categorize as "generating no synergies." As they show, if a merger generates no synergies then it necessarily causes the market price to rise.

A Cournot/Nash equilibrium of the model is a vector of outputs satisfying the best response property (i.e., each x_i is i's best response to $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$). For simplicity, we assume that there exists a unique pre-merger equilibrium and a unique post-merger equilibrium subsequent to the merger of any subset of firms, and that in any pre- or post-merger equilibrium all firms have positive outputs. The existence of a Cournot/Nash equilibrium is guaranteed by assuming the inverse market demand function is monotonic and the firms' profit functions are continuous and quasi-concave. The requirements for the uniqueness of a Cournot/Nash equilibrium are more complicated; these are surveyed in Zhang and Zhang (1996). Since we assume linear demand and costs throughout, we do not encounter any existence or uniqueness problems and shall not discuss them further.

Farrell and Shapiro (1990) consider a market with general nonlinear costs and nonlinear demand. Levin (1990) considers a market with linear costs and nonlinear demand. It is useful to note that when both demand and costs are linear the Farrell-Shapiro condition (their Proposition 5) is identical to Levin's 50% rule (his Theorem 4). In general, a linear market may be defined by an $n+1$ dimensional vector, $(a, c) \in R_{++}^{n+1}$, where $a > 0$ is the intercept of market demand and $\mathbf{c} = (c_1, \ldots, c_n) >> 0$ is the vector of all firms' (constant) marginal costs.[4] Assume $c_2, \ldots, c_n \geq c_1$. Then (1) becomes $C_s(y) = c_s y$, where $c_s = \min\{c_j | j \in S\}$, which is the cost structure assumed by Levin (1990). Farrell and Shapiro's Proposition 5 and Levin's Theorem 4 thus provide the FSL condition: a merger raises welfare if it is profitable and if the joint pre-merger market shares of the insider firms are less than 50%.

While this is an important result, it is only a sufficient condition and may be grossly under-inclusive. Indeed, we can obtain a more precise characterization of the welfare effects of horizontal mergers by focusing on markets with specific structures. In this paper we focus on linear markets with asymmetric firms.

3. PROFITABILITY CONDITIONS

We confine our attention throughout to linear markets with asymmetric firms and assume that any group of merged firms has a cost function of the form given by (1). This allows us to investigate the trade-off between cost efficiencies and market power without having to introduce any production synergies into our analysis. Of course, if there were additional costs efficiencies associated with production synergies a merger would have an even more positive (or less negative) welfare effect. But since claims about production synergies are difficult to verify, we are primarily interested in exploring the welfare effects of mergers in which they are not a significant motivation.

Consider a linear market as defined above by $(a, c) \in R_{++}^{n+1}$. The firms' Cournot/Nash equilibrium outputs and profits will be given by

$$x_i = (a - nc_i + \sum_{j \neq i} c_j)/(n+1); \pi_i = (x_i)^2, i = 1, \ldots, n \tag{2}$$

Suppose there is one efficient firm (firm 1) with marginal costs c_1 and there are $(n-1)$ identical inefficient firms with marginal costs $c \geq c_1$. Let

[4] The slope of market demand is assumed to be -1 without loss of generality.

$\triangle c = (c - c_1)$ represent the difference in marginal costs. Then (2) implies

$$x_1 = (a + n\triangle c - c)/(n+1); \pi_1 = (x_1)^2 \tag{3a}$$
$$x_i = (a - c - \triangle c)/(n+1); \pi_i = (x_i)^2, i = 2, \ldots, n \tag{3b}$$

and the market shares of the n firms will be given by

$$s_1 = (x_1/X) = (a + n\triangle c - c)/(na + \triangle c - nc) \tag{4a}$$
$$s_i = (x_i/X) = (a - \triangle c - c)/(na + \triangle c - nc), i = 2, \ldots, n \tag{4b}$$

Firms are motivated to merge only if doing so will increase their profits. Therefore, the post-merger profits of a combination of firms must exceed the sum of the firms' pre-merger profits. The increase in joint profits could result from the realization of cost efficiencies or a reduction in the industry's competitiveness or both.

Consider a merger of $k \leq n$ firms. We define a type I merger as one which excludes the efficient firm (firm 1) and a type II merger as one which includes the efficient firm. Thus, if $S = \{i_1, \ldots, i_k\}$ is the set of k merging firms, $1 \notin S$ in a type I merger and $1 \in S$ in a type II merger. In type I mergers there is no reduction in marginal costs, so $c_s = c$. In type II mergers the insiders' marginal costs fall to equal those of the efficient firm, so $c_s = c_1$. Type II mergers, therefore, generate direct cost efficiencies but type I mergers do not.

Let $\varepsilon = \triangle c/(a - c_1)$ denote a measure of the difference between the marginal costs of the efficient firm and those of the inefficient firms. It is, of course, also a measure of the potential marginal cost savings from a type II merger. Note that $\varepsilon \in [0, 1/2]$. To see why, recall that $c \geq c_1$, so ε is bounded from below by 0, and note that if $\varepsilon > 1/2$ firm 1 would be a pure monopolist prior to any merger and could therefore have no incentive to merge. We can use ε to rewrite 4a and 4b as follows:

$$s_1 = [1 + (n-1)\varepsilon)]/[n - (n-1)\varepsilon] \tag{5a}$$
$$s_i = (1 - 2\varepsilon)/[n - (n-1)\varepsilon], i = 2, \ldots, n. \tag{5b}$$

The combined pre-merger market shares, s_k, of the k merging firms for type I and II mergers will therefore be given by

$$s_k^I = k \cdot s_2 = k[1 + (n-1)\varepsilon)]/[n - (n-1)\varepsilon] \tag{5c}$$
$$s_k^{II} = s_1 + (k-1)s_2 = [k + (n - 2k + 1)\varepsilon)]/[n - (n-1)\varepsilon]. \tag{5d}$$

PROPOSITION 1. *Consider a type I merger of $k < n$ firms. Then the following four claims are equivalent:*

(i) the merger is profitable;
(ii) $[(n+1)^2 - k(n-k+2)^2] > 0$;[5]
(iii) $k > k_1$; *and*
(iv) $s_k^I > \theta_1$;

where k_1 *and* θ_1 *are given by*

$$k_1 = k_1(n) = (3 + 2n - \sqrt{5+4n})/2 \tag{6}$$

and

$$\theta_1 = \theta_1(\varepsilon, n) = \frac{(1-2\varepsilon)}{[n-(n-1)\varepsilon]} \frac{(3+2n-\sqrt{5+4n})}{2} \tag{7}$$

The values of k_1 and θ_1 define the number of insiders and the insiders' combined pre-merger market share necessary for a type I merger to be profitable. A type I merger is profitable if and only if the number of insiders and their combined pre-merger market share exceed these critical values.

Remark 1: The FSL condition can apply in type I mergers if and only if $\varepsilon > \sigma_1$ where

$$\sigma_1 = \sigma_1(n) = \frac{(3+n-\sqrt{5+4n})}{(7+3n-2\sqrt{5+4n})}. \tag{8}$$

The value of σ_1 defines the critical size of the cost differential between the efficient and inefficient firms necessary for the FSL condition to apply in type I mergers. If the cost differential is any smaller than σ_1, a type I merger in which the insiders' combined pre-merger market share is less than 50% cannot be profitable. In a type I merger, therefore, the FSL condition can apply if and only if the cost differential exceeds this critical value. It is easy to show that σ_1 is strictly positive for all positive values of n. The following is therefore a corollary:

Remark 2: In symmetric linear markets, where $\varepsilon = \triangle c/(a - c_1) = 0$, the FSL condition cannot apply.

It is interesting to note that the critical number of insiders, k_1, depends only on the number of firms in the industry and not on the cost differential, ε. The critical size of the insiders' combined pre-merger market share, θ_1, on the other hand, does depend on ε. Indeed, it is easy to show that θ_1 is

[5]The inequality in (ii) is a generalization of a condition stated by Salant et al (1983), who consider a symmetric linear market ($\triangle c = 0$) with n firms and $(m+1)$ insiders. Salant et al's condition (i.e., their equation (3') on page 191) states that a merger of $(m+1) = k$ firms is profitable if and only if $[(n+1)^2 - (m+1)(n-m+1)^2] > 0$, which becomes $[(n+1)^2 - k(n-k+2)^2] > 0$ by substituting k for $m+1$.

strictly decreasing in ε. In other words, as the cost differential between the efficient and inefficient firms increases, the combined pre-merger market share that a group of merging inefficient firms must have in order for their merger to be profitable actually falls. This is a surprising result. Indeed, it may not be immediately apparent how a merger of inefficient firms with less than a 50% market share in an asymmetric Cournot model can be profitable when a merger of efficient firms with a 50% market share in a symmetric Cournot model cannot.

In an asymmetric Cournot model the inefficient firms compete at a disadvantage. The greater the cost differential, the greater their disadvantage. If the cost differential becomes sufficiently great, the efficient firm becomes dominant and the inefficient firms are relegated to the fringe of the market. The greater the cost differential, and the smaller the fringe, the less the dominant, efficient firm needs to respond to the inefficient, fringe firms' strategic actions. If the cost differential is large, for instance, and the number of inefficient firms increases (say, through new entry), the efficient firm reduces its output in response, but by less than it would if the cost differential was small. And if the number of inefficient firms decreases (say, through a merger), the efficient firm increases its output by less if the cost differential is large than if it is small. The larger the cost differential, therefore, the more the market price rises in response to any merger of inefficient firms, and the more profitable the merger is for the participants. Although a type I merger of firms with less than a 50% market share can never be profitable if there is no cost differential (as in a symmetric Cournot model), such a merger can be profitable if the cost differential is sufficiently large.

Proposition 2. *Consider a type II merger of $k \leq n$ firms. Then the following four claims are equivalent:*

(i) *the merger is profitable;*
(ii) $\varepsilon > \frac{(n+1)^2 - k(2n-k+3)}{(4n-2k+6)(n-k+1)}$.
(iii) $k > k_2$; *and*
(iv) $s_k^{II} > \theta_2$;

where

$$k_2 = \frac{2n+3-\varepsilon(6n+8)-\sqrt{4n+5-[12n+16]\varepsilon+[4n^2+16n+16]\varepsilon^2}}{2(1-2\varepsilon)} \tag{9}$$

and

$$\theta_2 = \frac{2n+3-\varepsilon(4n+6)-\sqrt{4n+5-[12n+16]\varepsilon+[4n^2+16n+16]\varepsilon^2}}{2(n-(n-1)\varepsilon)} \tag{10}$$

define the critical number of insiders and the insiders' combined pre-merger market share necessary for a type II merger to be profitable. A type II merger is profitable if and only if the number of insiders and their combined pre-merger market share exceed these critical values.

It is interesting to note that, in contrast to a type I merger, the critical number of insiders in a type II merger does depend on the cost differential, ε, as well as the number of firms. As with type I mergers, however, the critical combined pre-merger market share of the merging firms is also strictly decreasing in ε. The intuition in this case is more obvious: other things being equal, the greater the cost differential, the greater the direct cost efficiencies and thus the greater the profits realized as a result of a type II merger. In fact, it is not difficult to show that, for any given cost differential, ε, and for any given number of firms, n, the combined pre-merger market share necessary for a profitable type II merger is always less than the combined pre-merger market share necessary for a profitable type I merger. This is because a type II merger generates direct cost efficiencies and a type I merger does not.

Remark 3: The FSL condition will apply in type II mergers if and only if $\varepsilon \geq \sigma_2$, where

$$\sigma_2 = \frac{3n^2 + 10n + 13 - (n+1)\sqrt{4n^2 + 16n + 37}}{(n+3)(5n+11)} \tag{11}$$

The value of σ_2 defines the critical size of the cost differential, ε, necessary for the FSL condition to apply in type II mergers. If the cost differential is any smaller than σ_2 a type II merger involving firms with a combined pre-merger market share of less than 50% can never be profitable and the FSL condition cannot apply. A comparison of σ_1 and σ_2 shows that, for any given number of firms, ε must always be greater for the FSL condition to apply in type I mergers than in type II mergers. This is somewhat surprising upon first impression because type I mergers do not generate any direct cost efficiencies. But as Proposition 1 and Remarks 1 and 2 show, the profitability of type I mergers is nonetheless positively related to the size of the inefficient firms' cost disadvantage. In fact, it is because type I mergers do not generate any direct cost efficiencies that the cost differential must be larger in order for them to be profitable.

Taken as a whole, Propositions 1 and 2 and Remarks 1 - 3 characterize the profitability conditions for type I and II mergers. Intuitively, the results imply that either type of merger is more likely to be profitable the greater the cost differential, ε, and the greater the number (and market shares) of participants. Alternatively, the number (and market shares) of insiders necessary for a merger to be profitable is likely to be greater the smaller the cost differential. It is reasonable to infer, therefore, that the smaller the cost differential, and the smaller the number (and market shares) of insiders, the more likely the merger is motivated by production synergies or the desire to improve the prospects for sustaining industry-wide collusion.

4. WELFARE EFFECTS

This section characterizes the welfare effects of mergers in linear markets. Even though we assume the merging firms do not realize any production synergies, social welfare may still increase if enough production is transferred from the inefficient firms to the efficient firm. Intuitively, the net welfare effects of a merger should depend on the number of merging firms, since this will affect the extent of the decrease in competition, and the difference in marginal costs between the efficient and inefficient firms, since this will affect the amount of cost savings.

PROPOSITION 3. *Consider a type I merger of $k < n$ firms. The following three claims are equivalent:*

(i) the merger raises welfare;
(ii) $k < k_3$; and
(iii) $s_k^I < \theta_3$;

where

$$k_3 = \frac{\varepsilon(2n^2 + 10n + 10) - (2n + 3)}{\varepsilon(2n + 4) - 1} \tag{12}$$

and

$$\theta_3 = \frac{(1 - 2\varepsilon)(\varepsilon(2n^2 + 10n + 10) - (2n + 3))}{(n - (n - 1)\varepsilon)(\varepsilon(2n + 4) - 1)} \tag{13}$$

define the critical number of insiders and the insiders' combined pre-merger market share necessary for a type I merger to have a positive welfare effect. A type I merger has a positive welfare effect if and only if the number of insiders and their combined pre-merger market share satisfy these critical values.

Proposition 3 is counter-intuitive because a type I merger does not generate any direct cost efficiencies. It is not difficult to understand how a type I merger can be profitable, since the merging firms reduce their combined output and the market price rises, but since higher prices mean less consumer surplus, and since there are no direct cost efficiencies, it is not immediately apparent how a type I merger can raise welfare. To understand the welfare effects of a type I merger fully, however, one must broaden one's perspective to encompass not just consumers and the merging firms, but the industry as a whole. A type I merger does not generate any direct cost efficiencies, but it does generate indirect cost efficiencies by reducing the outputs of the inefficient firms and raising the output of the efficient firm. Thus, even though the market price will rise, and even though the merged firms themselves do not realize any cost savings, the average marginal costs of the industry as a whole may fall by enough to raise welfare overall.

Other things being equal, a type I merger is more likely to be both profitable and welfare-enhancing the greater the cost differential between the efficient and inefficient firms. Indeed, the minimum cost differential, ε, necessary for a profitable, welfare-enhancing type I merger is significantly less than the minimum cost differential necessary for the FSL condition to apply. Alternatively, if the cost differential is large enough for the FSL condition to apply, a type I merger with strictly more than a 50% market share can be both profitable and welfare enhancing. Thus, even without production synergies, and even if they do not generate any direct cost efficiencies, the set of potential mergers which can be both profitable and welfare-enhancing mergers is much larger than the set defined by the FSL condition.

Since the safe harbors established under the 1992 Horizontal Merger Guidelines are even less inclusive than the FSL condition's 50% pre-merger market share criterion (see Farrell and Shapiro (1990)), Proposition 3 implies that some profitable, welfare-enhancing mergers which do not provide production synergies or direct cost efficiencies may be subject to antitrust scrutiny. It also implies that any analysis of the welfare effects of a merger which ignores the cost structure of the industry and the effect of the merger on the distribution of industry output will be seriously flawed. Since the Guidelines omit any consideration of indirect cost efficiencies, such as those generated by type I mergers, this is one dimension along which they could clearly be improved.

Proposition 4. *Consider a type II merger of $k \leq n$ firms. Then the following three claims are equivalent:*

(i) the merger raises welfare;
(ii) $k < k_4 = k_3$;
(iii) $s_k^{II} < \theta_4$,

where

$$\theta_4 = \frac{(2n^2+14n+16)\varepsilon^2 - (2n^2+13n+15)\varepsilon + (2n+3)}{((n-1)\varepsilon - n)(\varepsilon(4n+2)-1)} \tag{14}$$

and k_4 and θ_4 define the critical number of insiders and the insiders' combined pre-merger market share necessary for a type II merger to have a *positive welfare effect. A type II merger has a positive welfare effect if and only if the number of insiders and their combined pre-merger market share satisfy these critical values.*

As one would expect, Proposition 4 implies that the subset of type II mergers that increase welfare becomes larger as the cost differential, ε, increases. Indeed, the subset of type II mergers that are both profitable and welfare-enhancing is generally much larger than the subset of mergers that satisfy the FSL condition. Nonetheless, there is often some critical size beyond which type II mergers will have negative welfare effects. If the number of insiders (and their market shares) become too large the reduction in output may dominate the cost efficiencies and the merger may have a negative welfare effect overall. In this regard, consider Remark 4.

Remark 4: A consolidation of the entire industry into a monopoly will increase (but not necessarily maximize) welfare if and only if $\varepsilon > (n+3)/(6n+10)$.

Remark 4 provides necessary and sufficient conditions for the cost efficiencies associated with a type II merger to dominate the impact of any consequent increase in market power. It is interesting to note that the cost differential between the efficient and inefficient firms does not have to be great enough to confer a pure monopoly on the efficient firm in order for a merger to monopoly to have a positive welfare effect. If the cost differential is sufficiently great, a merger to monopoly could reduce industry-wide marginal costs by enough to more than offset the deadweight loss generated by the reduction in output, even though the efficient firm could not rationally have chosen to price its competitors out of the market prior to the merger.

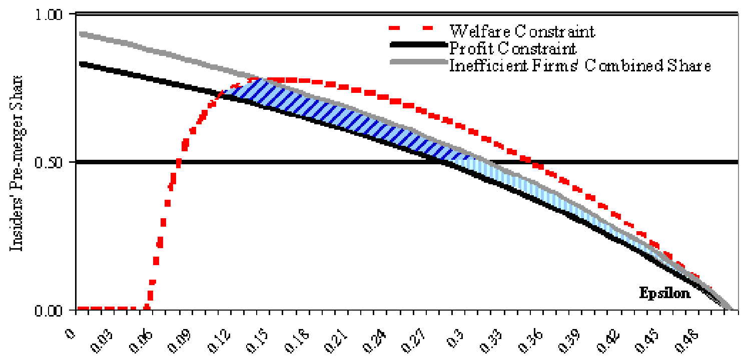

FIG. 1. Type I Merger with $N = 15$. The entire shaded area is the set of profitable and welfare-enhancing mergers, the light-shaded area is the set of mergers satisfying the FSL condition.

Figure 1 illustrates the results for type I mergers with $n = 15$.[6] The bold, downward-sloping line shows the insiders' pre-merger market share necessary for a profitable merger at different values of ε. The dotted line shows the insiders' pre-merger market share necessary for a welfare-enhancing merger and the light, downward-sloping line shows the inefficient firms' aggregate pre-merger market share. The feasible set of mergers which satisfy the FSL condition lies in the area bounded from above by the 50% line and the light line showing the inefficient firms' aggregate pre-merger market share and from below by the bold profit line. The feasible set of profitable, welfare-enhancing mergers, on the other hand, lies in the area bounded from above by the dotted welfare line and the light line showing the inefficient firms' aggregate pre-merger market share and from below by the bold profit line. It is striking that the subset of feasible, profitable, welfare-enhancing type I mergers in which the insiders have aggregate pre-merger market shares greater than 50% is larger than the subset in which the insiders have aggregate shares smaller than 50%.

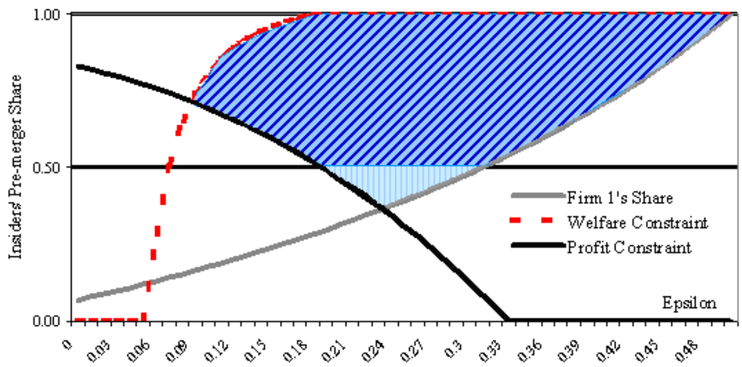

FIG. 2. Type II Merger with $N = 15$. The entire shaded area is the set of profitable and welfare enhancing mergers, the light-shaded area is the set of mergers satisfying FSL condition.

Figure 2 illustrates the results for type II mergers with $n = 15$. The bold, downward-sloping line shows the insiders' pre-merger market share necessary for a profitable merger at different values of ε. The dotted line shows the insiders' pre-merger market share necessary for a welfare-enhancing merger and the light, upward-sloping line shows the efficient firm's pre-merger market share. Note that the feasible set of mergers which satisfy the FSL condition lies in the area bounded from above by the 50% line and from below by the profit line and the efficient firm's pre-merger market share. The feasible set of profitable, welfare-enhancing mergers, on the

[6]The results are not significantly different for other relevant n values.

other hand, lies in the area bounded from above by the dotted welfare line and from below by the bold profit line and the line showing the efficient firm's pre-merger market share. The size of the latter is striking in contrast to that of the former.

It is perhaps even more striking to observe that, for $n = 15$, the dotted welfare line reaches a market share of 100% before the bold profit line reaches a market share of 50%.[7] This means that by the time the cost differential, ε, becomes large enough to enable a type II merger of insiders with only a 50% pre-merger market share to become profitable, it will already be large enough to ensure that even a merger to monopoly would be welfare-enhancing. Although the FSL result is only a sufficient condition, this illustrates how grossly under-inclusive a 50% pre-merger market share criterion would be as a safe harbor. And since the FSL condition is more inclusive than the safe harbors established under the 1992 Horizontal Merger Guidelines, it may also heighten concerns about the scope of the discretion employed by antitrust enforcers in conducting merger reviews.

5. DISCUSSION

In contrast to Farrell and Shapiro (1990) and Levin (1990), which mainly provide sufficient conditions, Propositions 1 - 4 provide both necessary and sufficient conditions for horizontal mergers of different types to be profitable and welfare-enhancing. Our results show that relatively small mergers tend to be welfare enhancing, and relatively large mergers tend to be profitable. There is, nonetheless, significant overlap between the subset of mergers that are profitable and the subset of mergers that are welfare-enhancing even when the mergers do not generate any direct cost efficiencies. Moreover, the subset of mergers which are both profitable and welfare-enhancing is much larger than the subsets of mergers defined by either the FSL condition or the 1992 Horizontal Merger Guidelines.

We have assumed linear demand and costs throughout. While this may limit our results, it also allows us to characterize the welfare effects of horizontal mergers more precisely than previous authors have done. Indeed, because we are able to provide both necessary and sufficient conditions, our results are very amenable to simulations, which are proving to be increasingly useful in antitrust analysis (see Dalkir et al (2000)). We hope to use our results to simulate the welfare effects of mergers in future work.

Our analysis is squarely in the equilibrium tradition advocated by Farrell and Shapiro (1990). We have therefore assumed that both pre-merger and post-merger market outcomes can be approximated by Cournot/Nash equilibria. Indeed, by assuming an immediate transition from a pre-merger

[7] This is not true for smaller values of n.

Cournot/Nash equilibrium to a post-merger Cournot/Nash equilibrium, we have assumed the absence of any capacity constraints. This is ironic, since it is well known that the Cournot model best applies to industries in which there are, in general, short-run capacity constraints (see Kreps and Scheinkman (1983)). Our results are best interpreted, therefore, as characterizing the welfare effects of mergers over a horizon long enough for the firms in an industry to make any strategic adjustments in their capacities. This has both its advantages and disadvantages.

One disadvantage is that the analysis obviously provides no insights into the transition from one equilibrium to another. Another is that it mischaracterizes the welfare effects of mergers that occur in industries which are not initially in equilibrium. At the same time, however, we believe that equilibrium models provide a more rigorous approach to merger analysis than the one described in the 1992 Horizontal Merger Guidelines. The Guidelines assume that any group of merging firms will have a post-merger market share equal to their combined pre-merger market share. As Farrell and Shapiro (1990) note, this assumes the absence of any kind of strategic response to the merger by either the merged firm itself or the non-merging firms. It thus completely ignores the possibility that a merger will generate indirect cost efficiencies as we have defined them above. In light of the overwhelming importance accorded to strategic behavior in modern industrial organization economics, we believe that merger analysis should incorporate strategic effects and that this is a major advantage of the equilibrium approach.

Finally, we have assumed no entry. In a more general framework, any increase in price would tend to stimulate new entry. The threat of entry would tend to curb price increases and diminish firms' incentives to merge. In addition to making mergers less likely overall, this would increase the likelihood that any mergers which did occur were welfare-enhancing. In that respect, our results are more conservative than they would be if entry were possible.

6. CONCLUSION

We have provided necessary and sufficient conditions for horizontal mergers to be profitable and welfare-enhancing when market demand and firms' cost are linear. In general, the set of profitable, welfare-enhancing mergers is significantly larger than the set defined by either the FSL condition or the safe harbors established under the 1992 Horizontal Merger Guidelines. Moreover, mergers may be profitable and welfare-enhancing even when they do not generate any direct cost efficiencies. Indeed, any attempt to evaluate the welfare effects of a merger that does not account for indirect cost efficiencies will be seriously flawed. Since indirect cost efficiencies result from industry-wide strategic effects, a satisfactory analysis of horizontal mergers requires an equilibrium approach.

There is a sense in which the inefficient firms in an industry provide competition at the expense of the social resources they waste on inefficient production. The social optimum inevitably lies somewhere along the trade-off. Our analysis suggests that a merger policy which fails to account for industry-wide strategic effects will tend to bias the trade-off in favor of competition and result in an excessive amount of inefficient production.

APPENDIX

Proof of the equivalence between the Farrell-Shapiro and Levin Results: Levin states the 50% condition outright (hence the title). Farrell and Shapiro (1990) state that the insiders' pre-merger share must be less than the weighted sum of the outsider's share. In linear markets, all weights are one, and the condition becomes: $\sum_{j\in S} s_i \leq \sum_{j\notin S} s_i$, or equivalently, $2\sum_{j\in S} s_i \leq \sum s_j$, which leads to $\sum_{j\in S} s_i \leq 1/2$. ∎

Proof of Proposition 1: Consider each part in turn:

(i) $\Longleftrightarrow$ (ii): Let π_S denote the profits of the new merged firm, and $\sum_{i\in S} \pi_i$ denote the sum of its members' pre-merger profits. In a type I merger,

$$\begin{aligned}\sum_{i\in S} \pi_i &= \sum_{i\in S} (a - nc_j + \sum_{k\neq j} c_k)^2/(n+1)^2 \\ &= \sum_{i\in S} (a - c_i - 2\triangle c)^2/(n+1)^2 = k(a - c_i - 2\triangle c)^2/(n+1)^2; \\ \pi_S &= (a - (n-k+1)c_S + \sum_{k\notin S} c_k)^2/(n-k+2)^2 \\ &= (a - c_1 - 2\triangle c)^2/(n-k+2)^2.\end{aligned}$$

Thus,

$$\begin{aligned}\pi_S - \sum_{i\in S} \pi_i &= (a - c_1 - 2\triangle c)^2/(n-k+2)^2 - k(a - c_1 - 2\triangle c)^2/(n+1)^2 \\ &= [(n+1)^2 - k(n-k+2)^2](a - c_1 - 2\triangle c)^2/[(n+1)^2(n-k+2)^2].\end{aligned}$$

Let $\lambda(k,n) = [(n+1)^2 - k(n-k+2)^2]$, then we have

$$\pi_S - \sum_{i\in S} \pi_i > 0 \iff \lambda(k,n) > 0,$$

which proves the equivalence between (i) and (ii).

(ii) $\Longleftrightarrow$ (iii): $[(n+1)^2 - k(n-k+2)^2] = 0$ has three roots:
$k = 1$, and $(3/2) + n \pm (\sqrt{5+4n})/2$.

The only root in $[2, n]$, the potential range of k, is k_1. The slope of $[(n+1)^2 - k(n-k+2)^2]$ is $[2k(n-k+2) - (n-k+2)^2] = (n-k+2)(3k-n+2)$. The first term is always positive, so the sign of $(3k-n+2)$ determines the sign of the slope. The second term is positive for $k_1 \leq k \leq n$. Therefore, any k in the interval $[k_1, n]$ will satisfy part (ii).

(iii) $\Longleftrightarrow$ (iv): Since all the firms are the same, each has the same market share s_2 given in (5c). The critical market share is equal to the critical number of firms multiplied by an individual firm's market share, or $\theta_1 = k_1 s_2$. ▌

Proof of Remark 1: Set $\theta_1 = .5$ and solve for ε in terms of n to get σ_1. ▌

Proof of Remark 2: It follows from Remark 1 by $\varepsilon = 0$ and $\sigma_1 > 0$. ▌

Proof of Proposition 2: Consider each part in turn:

(i) $\Longleftrightarrow$ (ii): For a type II merger, S, post-merger joint profits, π_S, and the sum of members' pre-merger profits, $\sum_{i \in S} \pi_i$, are given by

$$\begin{aligned}
\sum_{i \in S} \pi_i &= \pi_1 + (k-1)(a - c_i - 2\triangle c)^2/(n+1)^2 \\
&= [(a - c_1 + (n-1)\triangle c)^2 + (k-1)(a - c_1 - 2\triangle c)^2]/(n+1)^2; \\
\pi_S &= (a - (n-k+1)c_1 + \sum_{k \notin S} c_k)^2/(n-k+2)^2 \\
&= [a - c_1 + (n-k)\triangle c]^2/(n-k+2)^2.
\end{aligned}$$

Let $\theta = \triangle c$, and

$$h(\theta) = (n-k+2)^2(n+1)^2(\pi_S - \sum_{i \in S} \pi_i).$$

Thus, we have

$$
\begin{aligned}
h(\theta) &= (n+1)^2[a-c_1+(n-k)\triangle c]^2 \\
&\quad -(n-k+2)^2[(a-c_1+(n-1)\triangle c)^2+(k-1)(a-c_1-2\triangle c)^2] \\
&= [(n+1)^2-k(n-k+2)^2](a-c_1)^2 \\
&\quad -2[(n-k)(n+1)^2-(n-k+2)^2(n-1-2(k-1))](a-c_1)\triangle c \\
&\quad +[(n-k)^2(n+1)^2-(n-k+2)^2((n+1)^2+4(k-1))]\triangle c^2 \\
&= (k-1)[k(2n-k+3)-(n+1)^2](a-c_1)^2 \\
&\quad +2(k-1)[(n+1)^2+(2n-k+3)(n-2k+1)](a-c_1)\theta \\
&\quad -4(k-1)(2n-k+3)(n-k+1)]\theta^2 \\
&= (k-1)[(a-c_1)-2\theta]\{[k(2n-k+3)-(n+1)^2](a-c_1) \\
&\quad +2(2n-k+3)(n-k+1)\theta\}.
\end{aligned}
$$

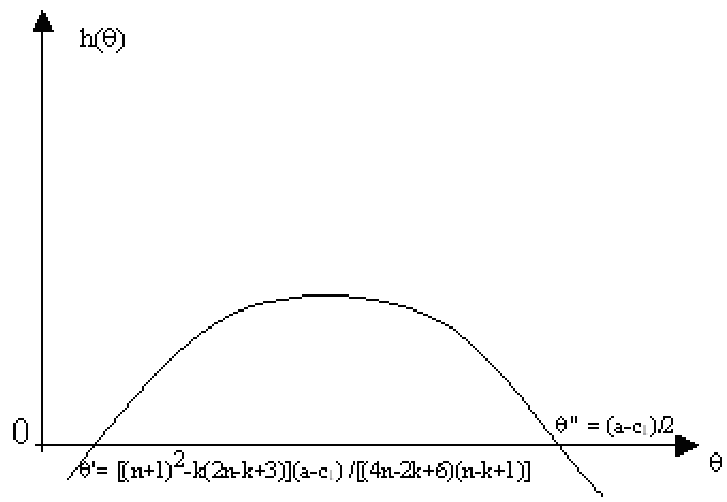

FIG. 3.

As illustrated in Figure 3, the two roots of $h(\theta)=0$ are respectively

$$\theta' = [(n+1)^2-k(2n-k+3)](a-c_1)/[(4n-2k+6)(n-k+1)] < \theta'' = (a-c_1)/2.$$

Since h has $\cap$-shape, and its two roots are θ' and θ'', we have

$$h(\theta) > 0 \iff \theta > \theta'.$$

Define

$$\mu(k,n) = [(n+1)^2 - k(2n-k+3)]/[(4n-2k+6)(n-k+1)] = \theta'/(a-c_1),$$

we have

$$\pi_S - \sum_{i\in S} \pi_i > 0 \iff \triangle c/(a-c_1) > \mu(k,n),$$

which proves the equivalence between (i) and (ii).

(ii) $\iff$ (iii): k_2 is found by solving for k in $\varepsilon = \mu(k,n)$. Given the above $\mu(k,n)$, the sign of its derivative with respect to k is determined by

$$[(n+1)^2 - k(2n-k+3)](3n-2k+4) - (2n-k+3)(n-k+1)(2n-2k+3).$$

This reduces to

$$-(2n-2k+3)(2n-k+3) + (n-k+1)(3n-2k+4) - \frac{(3n-2k+4)k}{n-k+1}.$$

The last term will be negative for all $k \le n$, and the first term is more negative than the second term, because

$$(n-k+3/2)(4n-2k+6) > (n-k+1)(3n-2k+4)$$

since $(n-k+3/2) > (n-k+1) > 0$ and $(4n-2k+6) > (3n-2k+4) > 0$. Therefore, $\mu(k,n)$ is strictly decreasing in k, and $\varepsilon > \mu(k,n)$ iff $k > k_2$.

(iii) $\iff$ (iv): By adding up the individual market shares of the insider firms, one gets the critical share $s_1 + (k_2-1)s_2 = \theta_2$. ▌

Proof of Remark 3: Set $\theta_2 = .5$ and solve for ε in terms of n to get σ_2. ▌

Proof of Proposition 3: Consider each part in turn:

(i) $\iff$ (ii): Zhao (1999, Proposition 4) shows that welfare increases if and only if

$$\varepsilon > \frac{2n-k+3}{2n^2 - 2n(k-5) - 2(2k-5) - (2n-k+3)} = \lambda(k,n).$$

The sign of the derivative of $\lambda(k,n)$ with respect to k is determined by

$$(-1)(2n^2 - 2n(k-4) - 3k + 7) - (2n-k+3)(-2n-4) = n^2 + 2n + 1,$$

so $\lambda(k,n)$ is strictly increasing in k and there exists a maximum value on k that will allow the inequality to be satisfied. Solving for k in $\varepsilon = \lambda(k,n)$, one gets

$$k_3 = \frac{\varepsilon(2n^2 + 10n + 10) - (2n + 3)}{\varepsilon(2n + 4) - 1}.$$

(ii) $\Longleftrightarrow$ (iii): The critical market share is $\theta_3 = k_3 s_2$. ▮

Proof of Proposition 4: Consider each part in turn:
(i) $\Longleftrightarrow$ (ii): The result is the same as in Proposition 3, since $(k-1)$ of the inefficient firms are being removed from the market.
(ii) $\Longleftrightarrow$ (iii): The critical share is given by $\theta_4 = s_2 \cdot (k_4 - 1) + s_1$. ▮

Proof of Remark 4: Set $\theta_4 = 1$ and solve for ε. ▮

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