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Asset Pricing Simultaneities: Phases and Patterns

Robert D. Coleman

3715 Bryn Mawr Drive, Dallas, Texas 75225
E-mail: rcoleman @ mba1971.hbs.edu

We show that asset pricing models of return with risk factors that entail either shares or dividends are logically circular simultaneities and thus are fallacious, meaningless, non-interpretable, indeterminate and not valid when tested and estimated by scientific statistical methods. This extends the findings for such models with risk factors that entail price. We also show that stock-split events are not a counter-example. Further we demonstrate that shares-, dividends- and price-entailing asset pricing simultaneities conform to three phases: events, individual risk factors and multifactor return models, and these simultaneities reflect patterns that have a common source which suggests a grand design.

Key Words: Capital asset pricing; Portfolio; Factor model; Price; Dividends; Shares; Stock splits; Simultaneity; Fallacy of circular reasoning; Logical validity; Scientific validity.
JEL Classification Numbers: G12, C12, C14.

1. INTRODUCTION

A single-equation simultaneity (SES) is distinguished from a simultaneous-equations system in introductory econometrics. In a SES both sides of the model equation simultaneously contain a variable that influences a variable on the other side. In a simultaneous-equations system two or more equations are determined simultaneously, and none of the equations in the system is a SES.

As Davis (1985, p.10) says:

> Social research aims to develop causal propositions supported by data and logic. The principles of statistics and probability provide part of the logic, but not all of it. In addition, research workers draw on a more general "logic of causal systems" that applies to diverse statistical schemes. The core of this logic is the notion of "causal order."

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Two types of SES can be distinguished based on the logic of casual order: isolated and circular. The isolated type of SES is stochastic, is symmetrical two-way ($X \leftrightarrows Y$) or reciprocal influence and is not the fallacy of circular reasoning. The criterion to identify an isolated SES is no identical variable appears on both sides of the model equation, either as a factor or embedded in a factor, and thus the left-hand side is isolated from the right-hand side. Isolated SES is usually unavoidable bias but can be corrected by the use of instrumental variables if available.

In contrast to the isolated type of SES, the circular type of SES is non-stochastic, is definitional ($X \equiv Y$) and is the fallacy of circular reasoning. The criterion to identify a circular SES is the same identical variable appears on both sides of the model equation, either direct or embedded, and thus there is a fallacy of circular reasoning or begging the question. Circular SES in principle is a topic of beginning algebra and basic logic. In econometrics it is always an avoidable fallacy and is the result of incompetence, carelessness or reckless disregard for truth.

Examples of isolated SES include supply and demand, crime and punishment, and education and income. Examples of circular SES are found in financial asset pricing, for example, return and size (market equity), return and value (book-to-market equity ratio), the Three-Factor Model, and more generally, any econometric model of expected return that specifies explanatory variables that are related to price, shares or dividends, or any such model that is tested and estimated with data sorted on such variables. Circular SES, not isolated SES, is at issue here. Any model equation that is specified or estimated as a circular SES is fallacious, meaningless, non-interpretable, indeterminate and unscientific. The circular SES fallacy is usually fatal, i.e., irremediable, irrefutable and terminal because it ends an argument [Coleman, R.D. (2005, pp. 91-93)].

Asset pricing is estimated by return models. Total return can not be observed naturally; rather it must be calculated. The three component variables of return are share price, cash dividends and number of shares of common stock issued and outstanding in the total capitalization of a firm. Thus the definition of return entails price, dividends and shares. A model of return is a direct simultaneity if it specifies a risk factor that entails price, dividends or shares. A return model is an indirect simultaneity if it is estimated using data that are sorted by a variable that entails price, dividends or shares. Sorted data are not random drawings of observations.

In general a direct circular SES occurs when both the dependent variable and a specified explanatory variable either are identical or entail an identical component, and an indirect circular SES occurs when such an explanatory variable is used to sort the data. A circular SES can be avoided by using scientific estimation methodology and not sorting the data. Such methods require that the model equation isolate the left-hand-side (LHS) variable and its components. A single-equation asset pricing model of price-, dividends- and shares-entailing return that either specifies or sorts data on a price-, dividends- or shares-entailing risk factor is not scientifically valid because it is logically circular due to circular SES.

Return is capital gains plus dividend income. The operational definition of return for a common stock includes five variables, four of which are indexed for time: share price (beginning and ending), cash dividends and number of shares outstanding (beginning and ending). The three generic non-indexed variables are price, dividends and shares. In models of return if one of these three generic variables appears on the right-hand side (RHS) of the model equation it is a circular SES. These three variables can appear either directly in the specification or indirectly in the sorting of the data, and they can either appear explicitly or be embedded in or entailed by another variable.

The fallacy of circular reasoning for shares-, dividends- and price-entailing risk factors in asset pricing models of return was a serendipitous discovery presented by R.D. Coleman (1995b; 1996, pp. 137-179). A convenience sample of published models is presented by R.D. Coleman (1996, pp. 178-179) in Table 25 that illustrates return models with specified direct explanatory variables and unspecified indirect portfolio formation variables, which entail logically circular variables that are simple or compound, positive or inverse.

2. SHARES

Stock splits change the capitalization of a firm by changing the total number of shares of common stock that are issued and outstanding, but they do not change the total capital of the firm. Shares is directly specified much less frequently than is price or dividends as a risk factor in a model of return. The most prominent example of shares as an explanatory variable in return models is found in event studies of stock splits. Event studies have been done on many other topics and provide evidence that the capital market incorporates new information very quickly and in that sense is efficient. The event is question is investigated in so-called event-time which allows many similar events to be looked at concurrently. This allows the event impact to be isolated from the impacts of market-wide events that also influence stock prices.

Event studies are used to analyze stock splits and any associated cash dividends. Although the event-study return model does not specify either shares or dividends on the RHS, the data are typically sorted and partitioned into four groups: stocks that split, stocks that did not split, stocks that paid a dividend and stocks that did not pay a dividend. Thus shares and dividends are indirectly included in the model testing and estimation, and this is equivalent to the specification of a dummy variable for stocks that split or did not split and a dummy variable for stocks that paid a cash dividend or did not pay a cash dividend.

Shares and thus data sorted and grouped on shares are logically circular in a return model and form a circular SES. Likewise, dividends and thus data sorted and grouped on dividends are logically circular in a return model and form a circular SES. Similarly, the joint specification of shares and dividends and thus the joint sorting and grouping of data on shares and dividends are logically circular in a return model and form a circular SES.

The original event study is Fama, Fisher, Jensen and Roll (1969). A variation on the conventional Capital Asset Pricing Model (CAPM) is used as the estimating equation. Return, the left-hand side (LHS) variable to be explained, entails price, shares and dividends. The only explanatory risk factor on the RHS is a market factor, an index of the New York Stock Exchange (NYSE) stocks that serves as a proxy for general market conditions or price level. There is no direct specification of either shares or dividends, so these two variables must be indirectly included in the estimation and testing of the model by partitioning the sample into groups after sorting the data on variables that are directly related to shares and dividends. The original event-study model equation, therefore, is an indirect circular SES.

On this very important point about circular SES the authors are silent, a serious inculpatory nondisclosure covered by misplaced emphasis on the diagnostics of Stable Paretian distributions, yet the circular SES is an undisclosed embedded fatal fallacy. This indirect circular SES violates the crucial independence assumption of classical statistical inference, the classical linear regression model (CLRM) behind the Gauss-Markov Theorem for best linear unbiased estimators, and is a serious deviation from standard scientific methodological practice as discussed in Gujarati (1988), Maddala (1992) and Pindyck and Rubinfeld (1991).

The variables of interest in an empirical study must be included somehow in the statistical hypothesis. Their influence cannot be transmitted by telekinesis from the error term of a model or from a sample that excludes observations of them. There are four equivalent ways to include a variable in the testing and estimating of an econometrics model. Two of these ways require direct specification in the model equation: (1) specify the risk factor for full-sample estimation, and (2) specify a dummy variable of the risk factor for full-sample estimation. The other two of these ways require indirect data manipulation: (3) partition the sample into subsamples according to one or more risk factors used to sort the data, and then estimate each sub-sample separately, and (4) sort the data by one or more risk factors, partition the sample into groups, and then estimate the full sample by group. Since no variable entailing either shares or dividends is directly specified in the single-equation model of the original event study, we must impute that these two variables were introduced indirectly and surreptitiously by undisclosed data manipulation.

In Fama, Fisher, Jensen and Roll (1969, p. 3) the stocks are classified by the stock-split ratio. A stock-split ratio of 5:4 is arbitrarily chosen as the breakpoint, and a stock split is defined as an exchange of shares in which five or more shares are distributed for every four shares formerly outstanding. Thus a stock split includes all stock dividends of at least 25 percent.

In Fama, Fisher, Jensen and Roll (1969, p. 9) the cash dividends are classified by the dividend change ratio. The dividend change ratio is defined as the total dividends (per equivalent non-split share) paid in the twelve months after the split, divided by total dividends paid during the twelve months before the split. Relative dividend increases are then defined as cases where the dividend change ratio of the split stock is greater than the ratio for the NYSE as a whole, while relative dividend decreases include cases of relative dividend decline.

Fama, Fisher, Jensen and Roll (1969, p. 5) says: “Moreover, the estimates of equation (1) for the different securities conform fairly well to the assumptions of the linear regression model”, but that is not true. The indirect inclusion in the model testing and estimation of the two classifications of the stocks by stock-split ratio and dividend change ratio creates a circular SES, and thus the testing and estimation violate the crucial assumptions of the CLRM.

Fama, Fisher, Jensen and Roll (1969) investigates stock splits, cash dividends and new information. The 33-year sample of monthly data includes for each stock its price, capitalization-adjusted price, cash dividends, total return and a complicated average return of NYSE stocks as a measure of general market conditions. The estimated univariate model implicitly includes shares and dividends through sorting and grouping of the data. The groups include (1) all stocks, (2) stocks with splits, (3) stocks without splits, (4) splits followed by relative dividend increases, and (5) splits followed by relative dividend decreases. The event studies of stock splits and dividends based on a model of return are neither scientifically interesting nor important because they are circular SES models.

Fama, Fisher, Jensen and Roll (1969) violates true method, which is to proceed from the better known to the lesser known. Circular reasoning is a fatal fallacy. If rational and scientific analysis is required, then there is no exception to the rules of scientific logic. The proper disposition of a logically circular variable is to either isolate it or reject it.

Fama (1991, p. 1599) says the original event study in Fama, Fisher, Jensen and Roll (1969) had a definite motivation: “The purpose was to have a piece of work that made extensive use of the newly developed CRSP monthly NYSE file, to illustrate the usefulness of the file, to justify continued funding." This is quite surprising and may be a precedent for an academic researcher to openly acknowledge that he was motivated by commercial interests and values rather than by scientific interests and values. Fama has been the head of the Center for Research in Securities Prices (CRSP), Graduate School of Business, University of Chicago, from 1985 to the present except for a term by French from 1989 to 1994. CRSP was originally funded by and is annually sponsored by Merrill Lynch, Pierce, Fenner & Smith Inc.

Fama (1991, pp. 1600, 1601) says: "In short, on all counts, the event-study literature passes the test of scientific usefulness." ... "The cross-breeding between finance and other areas of economics has resulted in a healthy burst of scientific growth." Those two statements in Fama (1991) are not true as can be seen in Fama, Fisher, Jensen and Roll (1969) and in other articles with an undisclosed embedded circular SES by Fama and his co-authors. Fama and his co-authors appear eager to claim the esteemed mantle of science for their unscientific research and to imply the imprimatur of their university that is renowned for the scientific achievements of its faculty including several Nobel Laureates in Economic Sciences.

Econometrics is not a separate area of economics, but rather it is an integral part of empirical economics. Yet introductory econometrics has been neglected to the detriment of valid scientific research in articles by Fama and his co-authors. This fatal fallacy of scientifically non-valid asset pricing circular SES ironically was either overlooked or approved by the 34 persons named in the Fama (1991) acknowledgements for their comments on the article. Even more surprising, the unscientific research reported in Fama, Fisher, Jensen and Roll (1969) was supported by funds made available to CRSP by the National Science Foundation (NSF) which does not knowingly give grants of the U.S. taxpayers' money for pseudo-science or junk science. Yet the NSF gave at least eight awards of grants totaling more than one million U.S. dollars to Fama that resulted in at least nine published articles that turned out to be unscientific research containing undisclosed embedded asset pricing circular SES.

3. DIVIDENDS

An example of direct circular SES appears in Fama and French (1988, 2002) in the form of dividend yield defined as total annual dividends per share divided by share price. Fama and French (1988) investigates the power of dividend yields to forecast stock returns, measured by the coefficient of determination (R^2) in linear regression. Fama and French (2002) investigates the equity premium. Yet the authors are silent on the logically prior issue of the direct circular SES that results from specifying dividend yield (D/P) in a return model. Fama and French (1988, 2002) violate the crucial independence assumption of the CLRM behind the Gauss-Markov Theorem and thus seriously deviate from standard scientific methodological practice. Fama and French (1988, 2002) are neither scientifically interesting nor important.

The dividend yield variable is both dividend-entailing and price-entailing. Any factor that includes the dividends variable either in theory or in its operational definition is dividend-entailing. The payout ratio, defined as dividends divided by earnings (D/E), could be specified instead of dividend yield in an asset pricing model of return, and the model would still be a direct circular SES.

The unscientific research reported in Fama and French (1988) was supported by the NSF (Fama), CRSP (French), Batterymarch Financial Management (French), and it included comments especially from the editor, G. William Schwert and the referee, James Poterba. Disclosing the sources of support in cash or in kind for the research work serves as an alert of any conflicts of interests that could bias or taint the research. This is especially important to all persons who believe, act on and trust the published research findings.

4. PRICE

R.D. Coleman (2005) shows that the most prominent example of circular SES in asset pricing, the Fama-French Three-Factor (FF3F) model of return [Fama and French (1992, 1993), Fama, French, Booth and Sinquefield (1993)] specifies two logically circular risk factors, size and value. Size (market equity) and value (book-to-market equity ratio) each renders the FF3F model a direct circular SES twice over, due to price and due to shares. The unscientific research reported in Fama and French (1992, 1993) and in Fama, French, Booth and Sinquefield (1993) was supported by the NSF (Fama) and CRSP (French).

Size is emphasized in Booth and Fama (1992) with asset-class portfolios for small-cap and very-small-cap stocks as well as S&P500 (proxy for U.S. large-cap), U.S. small-cap, international large-cap and international small-cap stocks. Sinquefield (1991) argues that a return to smallness can be earned on stocks in both hypothetical simulations and actual practice in spite of relatively higher transaction costs, and he argues against Fouse (1989) who says the return to smallness is a hoax due to the costs of trading illiquid stocks. The pertinent issue here is not transaction costs, which is a diversion, but rather the true issue of concern is scientific validity. The conclusion of Fouse is correct, but his analysis fails to convince because it does not include basic logic, inferential statistics, econometrics or the diagnostics of hypothesis testing.

The two price-entailing risk factors in FF3F, size and value or book-equity yield, are not valid in an asset pricing model of price-entailing return estimated and tested by scientific statistical methods. In addition the Fama-French split-sample ad hoc diagnostic test of FF3F presented in Fama and French (1993) is an indirect circular SES due to the same two price-entailing risk factors being used to sort the data and thus is not scientifically valid.

Davis, Fama and French (2000, p. 397) says: "Like the tests in Fama and French (1993, 1996), Table II unmasks the three-factor model for what it is, a model, and so necessarily false. But the model does provide a reasonable approximation for the returns on portfolios formed on size and BE/ME characteristics. ... Moreover, since all models are false, the three-factor model should only be discarded in favor of a better model." The statement that "all models are false" is equivalent to the statement that "all language is false". Both statements are true in a trivial sense, i.e., models and words are not identical to the reality they describe or refer to. Both statements explain everything and thereby explain nothing. The quoted statement is a distraction, and it misses entirely the most important point, about which the authors are silent. The main point is not "truth versus falsity" but rather it is "validity versus non-validity." Furthermore, the three-factor model can not provide a reasonable approximation for returns, because it is circular reasoning and thus meaningless. On scientific grounds, the Fama-French Three-Factor Model must be discarded due to the fatal fallacy of circular reasoning.

It is noteworthy that Fama, French, Booth and Sinquefield (1993) is the first to mention "the three-factor model" in reference to a return model with stock-pricing factors related to market, size and book-to-market equity and no bond-pricing factors, but it does not disclose that the four authors are affiliated with one and the same private investment company which was founded by Fama, Booth and Sinquefield to exploit their reported research findings.

According to Light (1993) this new private investment company had poor growth prospects because of the limited amount of money that could be invested profitably in very-small-cap stocks with their low liquidity and relative high transaction costs. The size-factor SES increased the use of CRSP data, and the addition of the value-factor SES in the FF3F model further increased the use of CRSP data. More importantly the introduction of the FF3F model created virtually unlimited growth opportunities for the private investment company.

Fama, French, Booth and Sinquefield each may be a qualified "academic" who shares in revenues at the investment company, which offers to share ten percent of product revenues with any academic who writes a research paper which becomes the basis of one or more of their new products. According to Light (1993, p. 5) and to a document entitled Academics with a link labeled Academic Ties at the investment company website, the company rewards authors who first publish a paper demonstrating "genuine risk factors" that are significantly priced, both in time-series and in cross-section, in asset pricing models. Fama designed the investment company's fixed income strategies and created the statistical model that it uses to structure portfolios and investment strategies. Fama and French designed the investment company's value strategies: small-cap value-style, large-cap value-style, and high value or high book-to-market equity (B/M).

There are two faces to the FF3F model, and they are seen differently by academia and by Wall Street. The academic face is the Fama-French Three-Factor model of return for stock pricing. The view by perceptive financial economists is that the FF3F model is not motivated by theory but rather is atheoretical *ad hoc* empiricism, that the FF3F model is a fatal fallacy of circular reasoning, and that the FF3F model is a circular SES and thus is meaningless, non-interpretable, indeterminate and not scientifically valid. No scientific statement can be made that the FF3F model earns consistent long-term average expected returns in excess of general stock market proxies adjusted for risk. Table 1 presents the academic view.

Perceptive financial economists further observe that in the FF3F model both the size-related and the value-related risk factors have a direct linear relationship with price. Thus these two factors are perfectly multicollinear, and their contributions to the explanation of expected return are conflated and confounded with no possibility of disentanglement. The separate contributions of size and value are indeterminate, meaningless and non-interpretable.

Perceptive financial economists also observe the limitations of statistical estimation. Fama and French (2004) reviews asset pricing but is silent about econometric simultaneity and perfect multicollinearity. It focuses on the statistical estimation of factor premiums and says:

> For perspective, the average value of the market premium RMt - Rft [market return minus risk-free rate] for 1927-2003 [77 years] is 8.3 percent per year, which is 3.5 standard errors from zero. The average values of SMBt [small minus big size-related factor] and HMLt [high minus low value-related factor] are 3.6 percent and 5.0 percent per year, and they are 2.1 and 3.1 standard errors from zero. All three premiums are volatile, with annual standard deviations of 21.0 percent (RMt - Rft), 14.6 percent (SMBt) and 14.2 percent (HMLt) per year. Although the average values of the premiums are large, high volatility implies substantial uncertainty about the true expected premiums.

Econometric simultaneity renders superfluous the point about perfect multicollinearity. Simultaneity and multicollinearity each renders superfluous the points about large standard errors relative to the estimates and about high volatility relative to the averages. Standard errors and volatility in this case each suffices to make the estimates meaningless.

TABLE 1A.

FF3F MODEL: ACADEMIC VIEW: THEORY, SES AND PMC

Risk Factor Description	**Risk Factor Name**	**Motivation: Theory or Ad Hoc Empirical**	**Circular SES Single-Equation Simultaneity**	**Perfect Multicollinearity PMC**
Market Excess	RM-Rf	Ad hoc empirical	No	No
Size-related	SMB	Ad hoc empirical	Yes - entails Size	Yes - entails Size
Value-related	HML	Ad hoc empirical	Yes - entails Size	Yes - entails Size

Notes:
1. Due to the joint-hypothesis problem, it is impossible to test (prove or disprove) market equilibrium and market efficiency. One must choose to test for either equilibrium or efficiency, conditional on the assumption of the other, and the choice is totally arbitrary. It is also not possible to disambiguate so-called equilibrium variables and efficiency variables; thus interpretation of an estimated stock pricing model is quite arbitrary and speculative at best.
2. The theoretical two-parameter risk-return relationship in equation form is:

$$E(R_i) = R_F + [E(R_M) - R_F](\beta_{iM}), i = 1, \ldots, n.$$

In words, the expected return on any security i is the risk-free borrowing and lending rate R_F plus a risk premium which is the risk measure, market beta or β_{iM}, multiplied by the difference between the expected return on the market portfolio M and R_F.
The excess return form of the equation for the two-parameter risk-return relationship is:

$$E(R_i) - R_F = [E(R_M) - R_F](\beta_{iM}), i = 1, \ldots, n.$$

3. A market proxy, market return or market excess return is not a theory-based market beta. Size is market equity, and value is the ratio of book equity to market equity.
4. The FF3F market-excess risk factor is the market portfolio return minus the risk-free rate of return. The size-related FF3F risk factor is SMB or Small minus Big (market equity), and the value-related FF3F risk factor is HML or High minus Low (book-to-market equity ratio). Both SMB and HML are zero-net-investment arbitrage portfolios.

TABLE 1B.

FF3F MODEL: ACADEMIC VIEW: ERROR AND VOLATILITY

Risk Factor Name	**Premium % PA**	**Standard Error (SE) %**	**SE's from Zero**	$-/+2$ **SE Range %**	**Volatility Gaussian % PA**	**Volatility Paretian % PA**
RM-Rf	8.3	2.4	3.5	3.5 – 13.1	21.0	∞
SMB	3.6	1.7	2.1	0.2 – 7.0	14.6	∞
HML	5.0	1.6	3.1	1.8 – 8.2	14.2	∞

Source: Fama and French (2004), pp. 38-39, 44 fn 7.

Notes:
1. Premium is average risk factor value from 1927 to 2003 (77-year sample period) in percent per annum (% pa). Fama and French combine the standard error of estimate of the slope coefficient of each risk factor with the mean of that risk factor. The slope coefficients are unitless for all three factors, and all three factors are measured in units of percent per annum.
2. These statistics are based on assumed Gaussian or normal probability distributions of returns. Gaussian distributions have finite variance. The actual distributions of returns are not well approximated by Gaussian distributions, but they are well approximated by the non-Gaussian members of the Stable Paretian family of probability distributions. The non-Gaussian members have infinite variance.
3. Volatility or variability is measured by standard deviation, which is equal to the square root of variance. Infinite variance means infinite standard deviation. The infinite volatility of factor risk premiums renders them meaningless.

The best known Wall Street face of the FF3F model is the size-and-value three-by-three equity style box with size categories of small-, mid- and large-cap stocks and value categories of value-, blend- and growth-style stocks. The view by investors interested in investment strategies and financial products derived from the FF3F model is that expected return is generated by three factors related to market, size and value, and that the FF3F model earns consistent long-term average expected returns of two or more percentage points greater than conventional market indexes on a risk-adjusted basis. The formulation of the FF3F model for prospective investors effectively conceals the embedded circular SES and the fatal fallacy of circular reasoning, and thus it is much less transparent and revealing than the formulation for academic financial economists. Table 2 presents the Wall Street view.

Bernstein (2000), silent about econometric simultaneity, perfect multicollinearity and the standard error of estimates, asserts that the three FF3F model risk factors have superior risk-adjusted returns or risk premiums — market, size, and value (book-to-market equity ratio) — and have been extensively researched by Fama and French and others, who have allegedly shown that all three risk premiums exist in the U.S. stock market and many other stock markets over a very long time. Bernstein further asserts that the market, size and value factors averaged 5.65% per annum, 1.71% per annum and 3.77% per annum over the 36-year sample period ending in

TABLE 2.

FF3F MODEL: WALL STREET VIEW

Premium Name	Premium	Contribution	Consistency
Market	5.65 % pa	51 %	78 %
Size	1.71 % pa	15 %	53 %
Value	3.77 % pa	34 %	87 %
Total	11.13 % pa	100 %	n/a

Source: Bernstein (2000).

Notes:
1. The premium is the annualized return in percent per annum (% pa) over the full 36-year sample period from 1964 to 1999.
2. The market premium is defined as the return on the risky security or market portfolio minus the riskless rate such as the rate on U.S. government treasury bills (T-bills).
3. The size or small-stock premium is defined as the return of the smallest half of companies on the NYSE minus the largest half.
4. The value premium is defined as defined as the return of the stocks with the lowest P/B ratios (value-style stocks) minus the returns of the stocks with the highest P/B ratios (growth-style stocks). Note that the lowest price-to-book per share ratio (P/B) is the highest book per share-to-price ratio (B/P) or book-to-market equity ratio (B/M).
5. Consistency is the percentage of rolling 5-year periods throughout the full 36-year sample period that the premium was positive.

1999. Thus the market, size and value factors allegedly contributed about one-half, one-sixth and one-third of total return of 11.13% per annum, and the combination of the size and value factors allegedly contributed 5.48% per annum or about one-half of total return. In a caption under each of three graphs of five-year annualized premiums for market, size and value plotted by years, Bernstein cites his data source as “Ken French/DFA” in fine print without identifying DFA.

In addition Fama and French (1996, 2004) allege that the unscientific FF3F model provides a better description of average returns than the scientific CAPM and captures most of the average-return anomalies missed by the CAPM. The authors are silent about scientific validity.

The same persons, who boast to investors of their scientific academic ties while in their industry executive positions, are silent in their journal articles about their industry ties while in their academic science faculty positions. Their two-faced position in Wall Street and academia is a result of asymmetric information and impression management, and it supports what is known as an iron triangle among the vertices of academic finance, finance journals and the financial-services industry, which in effect achieves so-called regulatory capture at all levels of government worldwide. It is quite difficult if not impossible for securities administrators to regulate securities derived from financial fallacies that are designed to avoid being detected and are not understood by impercipient financial economists in the academic finance guild.

5. PHASES AND PATTERNS

The most prominent examples of asset pricing circular SES include SES with each of the three generic variables in the definition of return. The definition of return includes price in the numerator and denominator. A price yield has share price in the denominator, and this complements size with share price in the numerator and an implicit denominator equal to one. Thus the combination of size and value or book-equity yield in a model of return increases the alleged explanatory power of the simultaneity in the model because price is entailed by size in the numerator and by yield in the denominator. Not all firms pay dividends, and sometimes earnings are zero or negative, so dividend yield and earnings yield are not available or convenient metrics for all firms. Book-to-market value is a price yield that has the advantage of being greater than zero and existing for every firm in the sample.

TABLE 3.

ASSET PRICING CIRCULAR SIMULTANEITY

Article By Year and Topic	**Component**	**Circular SES**
Fama, Fisher, Jensen and Roll (1969) Event: Stock Splits	Shares	Indirect
Fama, Fisher, Jensen and Roll (1969) Event: Cash Dividends	Dividends	Indirect
Banz (1978, 1979, 1981) Risk Factor: Size	Price	Direct
Reinganum (1979a, 1979b, 1981) Risk Factor: Size and Earnings Yield	Price	Direct
Fama and French (1988) Risk Factor: Dividend Yield	Dividends	Direct
Fama and French (1992) Return Model: FF3F Size and Book-Equity Yield	Price	Direct
Fama and French (1993) Return Model: FF3F Ad Hoc Diagnostic Test	Price	Indirect

Notes: 1. The most prominent examples of asset pricing circular SES (single-equation simultaneity) for each of the components of return are listed in chronological order by component. 2. Direct circular SES is specification of the logically circular variable as a factor in a model. Indirect circular SES is sorting the data by a logically circular variable. The shares component is the number of common stock shares issued and outstanding in the total capitalization of a firm. Earnings yield is the inverse of the price-earnings ratio. Book-equity yield is book value of equity to market value of equity ratio or book value of equity per share divided by share price. FF3F is the Fama-French Three-Factor stock pricing model of return.

As shown in Table 3 the most prominent asset pricing circular SES models appear in chronological order in three phases: events, individual risk factors and multifactor models. The events often studied are capitalization changes which entailed shares and cash dividends. The individual risk factors often studied are size and yields on price such as earnings yield. The multifactor models often studied are the FF3F model and variants of the CAPM.

The most prominent asset pricing circular SES models share only one author in common. The research findings reported by Banz and Reinganum appear to be independent of those reported by Fama and thus coincidental in their similarity. A closer look indicates that the asset pricing circular SES models of Banz, Reinganum and other authors conform to several patterns including a common source of influence as shown in Appendix A.

In Banz's Ph.D. dissertation the non-independence of size is explicitly acknowledged. Banz (1978, p. 28), in Section 1 Normative Portfolio Selection Models, says:

> It is (at the present time) quite impossible to solve this general problem, since the solution would involve the use of one of the mixed integer-quadratic programming algorithms. ... Solutions have been found by making additional simplifying assumptions. ... For most, also, the market model is used as the return-generating process which leads to a very simple covariance structure. ... Some go even further and assume that market equilibrium is described by the CAPM.

Furthermore Banz (1978, p. 28), in footnote 2, says:

> Note that this formulation of the market model requires that $E(e_i, e_j) = 0$, where e_i is residual return of security i. Of course, this independence assumption is not compatible with another known property of the market model, namely $X'e = 0$ which implies linear dependence [Fama (1973)]. Thus, the market model used here needs assumptions that go beyond the properties of the model implied by multivariate normality of security returns [Fama (1976, Chaps. 3-4)].

It is important to observe the distinction between residuals and the error term. The residuals are e_i and e_j. In matrix notation, $X'e = 0$, where X is the market capitalization or size risk factor as explanatory variable and e is the error term. The error term or disturbance term is the stochastic or nondeterministic part of a linear regression equation. The residuals are the difference between the explanatory variable data observations and the calculated OLS best-fitting values of the explanatory variable, or the difference between actual and predicted values of the explanatory variable. The crucial assumptions of the CLRM in matrix notation and the rudiments of matrix algebra are presented in Gujarati (1988, pp. 249-251, 658-671).

It is critical to appreciate the import of this disclosure. In this one sentence in one footnote at the bottom of page 28 of his 120-page dissertation, Banz effectively credits Fama, a member of his dissertation committee, with knowledge and approval of an explanatory variable that violates the crucial independence assumption of classical statistical inference, the CLRM behind the Gauss-Markov Theorem for best linear unbiased estimators, and thereby seriously deviates from standard scientific methodological practice. Banz (1978), a Ph.D. dissertation, violates crucial assumptions and is thereby neither scientifically interesting nor important.

Banz also presents the market proportion as a risk factor directly related to size. Banz (1978, pp. 32-33), in Section 4 Portfolios of Securities with the Largest Market Values, says:

> Equations (3.6) and (3.7), despite their limitations, can be used to suggest a measurable characteristic of assets of interest to investors subject to holding costs. ... Since the beta is already included in the CAPM, *the market proportion of an asset by itself should serve as a surrogate for the likelihood of inclusion in a small portfolio and hence as a potential determinant of expected return.* The market proportion or market value of a firm may be a characteristic worth study for reasons other than holding cost. According to popular belief, a two-tier market has existed for the past decade or so which resulted in “depressed” stock prices for small, “neglected” firms and high prices for stocks of large firms favored by institutional investors-itself the result of limited diversification and quite compatible with the results of Chapter II. If there is such a two-tier market, an effect on mean returns during that period should be observed. [Italics added.]
>
> Thus, the performance of a portfolio of the N firms with the largest market capitalizations (henceforth CAP) is of some interest and will be examined in the empirical part of this chapter.

The CAP variable in Banz (1978) appears to be the ultimate origin of the size factor and the related size effect in asset pricing models of return. In these two paragraphs on pages 32 and 33 in his 1978 Ph.D. dissertation Banz states his rationalizations for the size factor. Fama and French (1992, p. 427) gives Banz (1981) credit for the size effect with their approval: “The most prominent [empirical contradiction] is the size effect of Banz (1981)” and omits the usual disclaimer in the acknowledgements about responsibility for errors. Reinganum (1979a, 1979b, 1981) are alleged to independently confirm the size effect of Banz (1978, 1979, 1981).

Kun (1995) reports: “Banz had studied 54 years of New York Stock Exchange (NYSE) data and concluded that small companies outperformed large ones by 3 percent annually. ‘What Rolf found was that it looked like over long periods of time the rates of return on small stocks, risk adjusted, were higher than the rates of return on other stocks,’ explains Robert S. Hamada, dean and Edward Eagle Brown Distinguished Service Professor of Finance. ‘That caused a major uproar on Wall Street.”’

Banz (1981, pp. 3-18), derived from Banz’s Ph.D. dissertation, expresses gratitude especially to committee members Fama and Miller for advice and comments, as shown in Appendix A.

Fama appears to have been a major influence on the origin of market capitalization as the size factor in the Three-Factor Model. What is the origin of book-to-market equity ratio as the value factor in the Three-Factor Model? Fama and French (1992, 1993, 2004) cite the original source of book-to-market equity ratio for their value risk factor as Stattman (1980). This article in *The Chicago MBA* has no acknowledgements, but the references include Banz (1979, CRSP Working Paper #29), Basu (1977, Journal of Finance), Fama (1970, Journal of Finance), Fama (1976, Foundations of Finance, Basic Books) and Reinganum (1979, CRSP Seminar Proceedings Working Paper #6).

Stattman (1980) analyzes the relationship between book value per share to market price ratio (BP) as of December 31 each year and risk-adjusted return (RAR) for the year beginning the following April 1. BP is mathematically identical to book-equity to market-equity ratio. RAR is CRSP-calculated returns adjusted for CAPM market beta risk. The sample time period is 15 years (1962 through 1976) and included 2,855 firms on both the Compustat and CRSP databases. The analysis uses descriptive statistics but not classical statistical inference or linear regression, the workhorse of econometrics. The nonparametric analysis makes no assumptions about the probability distributions (mean, variance, and other parameters) of random variables, and such analysis results in a loss of information.

Stattman ranks quintiles of RAR and of BP to form a 5×5 contingency table. He uses visual examination of the table and Pearson's Chi-square statistic to test the hypothesis of independence between RAR and BP, and he uses Spearman's rank correlation coefficient to measure the direction, strength and significance of the relationship, even though ranked data violate a crucial assumption of these statistics (see page 43, endnotes 3 and 4). Mere correlation is not necessarily causation, and the *ad hoc ergo propter hoc* fallacy is not addressed. Stattman is silent about the logically prior question of the logically circular type of econometric simultaneity that results when price-entailing BP ratio is an explanatory risk factor in a model of price-entailing return. This is a fatal fallacy, and it is neither scientifically interesting nor important.

Stattman does not say how he learned about and accessed the two unpublished CRSP working papers by Banz and by Reinganum. Stattman, portfolio manager of the Merrill Lynch Global Allocation Fund, did not reply to a letter from the author of this article asking him how he got the idea of studying the relationship between book-to-market price ratio and risk-adjusted return. Stattman (1980, p. 96) says: "Notes about the Authors. Dennis Stattman was awarded an MBA with concentrations in finance, statistics and international business from the Graduate School of Business of the University of Chicago in 1980." Stattman (1980) was cited in Banz (1981) as an unpublished M.B.A. honors paper at the University of Chicago.

It may be a coincidence that Banz (1981) and Reinganum (1981) are the first and second articles in the same issue of the *Journal of Financial Economics* in March 1981 for which Jensen is the sole founder and editor, Schwert is only other editor and Fama is the only advisory editor, and each article is based on one of two Ph.D. dissertations that includes Fama on the dissertation committee. It also may be a coincidence that Fama was instrumental in the founding and incorporation on May 19, 1981 in Delaware and on August 26, 1981 in Illinois of a new company that was formally co-founded and co-chaired by two former MBA students who were dropouts from the Ph.D. program. It also may be a coincidence that the new private investment company's first financial product, which began trading on December 23, 1981, was a U.S. micro-cap stock index fund derived from the size effect, and the company's equity financial products are derived from the allegedly scientific size and value factors.

Furthermore it may be a coincidence that Fama served as the head of the CRSP academic scientific research center with its database of stock prices, dividends, capital changes and calculated returns, its semi-annual two-day seminars and its limited-circulation semi-annual journal of seminar proceedings. It also may be a coincidence that CRSP information is industry supported, monopolistic, proprietary financial data used for many empirical finance studies. It also may be a coincidence that the unique selling point and competitive edge of the financial products of the private investment company, which has grown to $65 billion of assets under management worldwide, is that these investment strategies are derived from the Fama-French Three-Factor model and risk factors, are developed by prominent academic financial economists, and are claimed to be scientific and to earn excess expected returns.

Additionally it also may be a coincidence that John Heaton, member of the CRSP board of directors, who commented in Heaton (1993, pp. 36-37) on price-entailing risk factors in asset pricing return models, emphasizes in Hansen, Heaton and Luttmer (1995) statistical procedures based on specification error and volatility bounds to assess misspecified asset pricing models but is silent about the logically prior econometric fundamentals and scientifically logical methods for assessing some of the most popular asset pricing models. Hansen, Heaton and Luttmer (1995, pp. 256-257, 261) approvingly cites Snow (1991) that uses size-based portfolios to estimate bounds on the moments of various asset returns and analyzes the small-firm effect by examining whether the returns on small-capitalization stocks incrementally have more importance than the returns of large-capitalization stocks in determining volatility bounds. Snow (1991) is based on Snow's dissertation at the University of Chicago. Logically prior to such tests of misspecification in cases of circular SES models is, for example, Hausman's specification-error test modified to be an exogeneity test [Maddala (1992, pp. 506-507)].

Moreover it may be a coincidence that while in his academic positions Fama served from inception to the present the private investment company owned primarily by employees as a member of the board of directors, director of research, member of marketing committee, member of investment committee, revenue-sharing product-designing advisor, and he most likely is the major controlling shareholder. Fama reportedly remains actively involved in the day-to-day operations of the firm while serving full-time on a university finance faculty.

Furthermore it may be a coincidence that Fama, French, Booth and Sinquefield do not disclose their financial-services industry ties in articles published in scientific research journals. It also may be a coincidence that these authors do not disclose the embedded circular SES in their return models nor do they disclose the data manipulation in their estimation of these circular SES models. It also may be a coincidence that Hall and Hall (1993, p.12), clarified by Heaton (1993, pp. 36-37), which asserts that price-entailing explanatory variables such as book-to-market value, firm size, price-earnings ratios and dividend-price ratios in a model of return should be dismissed, are not found in a finance or financial economics journal that would risk exposing and losing its subscribers if it published such assertions or findings.

More important to our analysis of asset pricing circular SES, it may be coincidental that the standard checks and balances to insure the integrity of scientific research were systematically compromised and evaded in a centrally directed pattern. It also may be coincidental that there is a pattern of numerous major bias-inducing ties involving authors, scientific journals, academic institutions, industry and allegedly philanthropic gifts of money. It also may be coincidental that these coincidences, especially the positions at CRSP and at the private investment company, have a pattern of nondisclosure in the published articles by Fama and his co-authors.

Banz (1981) and Reinganum (1981) can be seen as part of a series of items that began with Fama, Fisher, Jensen and Roll (1969) and ended with Fama, French, Booth and Sinquefield (1993) and later articles by Fama and French that introduced new investment strategies and financial products. One person alone appears in each of these items either as an author, co-author, dissertation committee member or someone mentioned in the acknowledgements. That person appears either most prominently or highly prominently in all of them. This person is the common thread through all of them and, most likely, is their common origin and source. See Appendix A for attributions that indicate this common influence. In addition the authors of the items with an undisclosed embedded asset pricing circular SES share a common institutional affiliation, and that academic institution and the private investment company have a *de facto* if not de jure joint venture partnership concerning the unscientific FF3F model and risk factors, due to either lax governance or reckless disregard for truth.

6. SCIENTIFIC VALIDITY

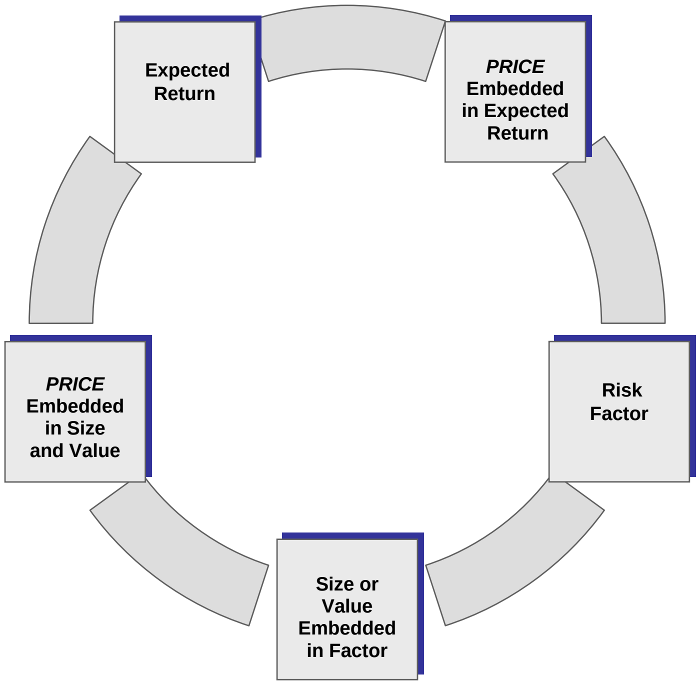
Logic, from the ancient Greek logos, means reason. As a science logic investigates and classifies the structure of statements and arguments. The scope of logic thus can be very large. Logic includes reasoning about probability and causality as well as the structure of fallacious arguments and paradoxes. Several distinctions characterize logic as a science. Logic may be formal or informal. Logic may be deductive or inductive. Logic may be of different types: Aristotlean or propositional, predicate, modal, dialectical and others. For present purposes, the most important point about logic is that if a given system of logic is scientific, then circular reasoning will not be valid in that system of logic.

It is not necessary to understand, for example, all the subtleties and nuances of the FF3F model or any other asset pricing model of return with logically circular explanatory risk factors. Likewise it is not necessary to fully understand the Fama-French split-sample ad hoc diagnostic test of the independence of factors in the FF3F model with sorted data. All that is needed is to understand that the FF3F model and the split-sample ad hoc diagnostic test are logically circular due to direct or indirect undisclosed embedded circular SES as shown in the Figure.

There is one special case where a scientifically non-valid asset pricing circular SES model can be used legitimately, and that is to serve as a benchmark for comparison with a valid asset pricing model. In rigorous asset pricing studies with all test results reported and with no data snooping, no data dredging, no circular SES or other non-scientific methodologies that spuriously induce or increase the explanatory power and the statistical significance of test results, R.D. Coleman (1995a; 1996, pp 115-116) compares the results of valid model estimation with a bracketed range between two benchmarks. Market value of equity or firm size is used solely as an upper benchmark, and lexical order of firm name is used solely as a lower benchmark. Size and lexical order each serves as the single efficiency risk factor in benchmark return models only for comparison with the explanatory power and with the number of statistically significantly priced decile portfolios in valid asset pricing model estimation, but not as a competing variable in the same model with a valid efficiency explanatory variable. Thus the problem of circular SES does not interfere with hypothesis testing. Due to circular SES, the size factor in a return model is not

FIG. 1. CIRCULAR SIMULTANEITY MODEL

$$Return = a + b(\textbf{Size-related or Value-related Risk Factor}) + e$$



Note: Beginning at the upper right and going clockwise, the first half of the cycle starts with Price in Return and ends with Price in Size or Value. Beginning at the lower left and going clockwise, the second half of the cycle starts with Price in Size or Value and ends with Price in Return at the beginning which completes the full circuit. A similar cycle to price exists for dividends and for number of shares outstanding. Either a logically circular risk factor such as size or value may be explicitly specified in the model of return (direct circular SES) or, equivalently, the data may be sorted by a logically circular risk factor (indirect circular SES).

scientifically valid, and every circular SES should be explicitly and clearly disclosed.

In summary, the circular reasoning is in the form of either direct circular SES (specification of risk factors, e.g., the FF3F model) or indirect circular SES (sorting of the data by logically circular variables before testing, e.g., split-sample ad hoc diagnostic test of the FF3F model). If the logically circular variables are not isolated or rejected from the model equation, the result is a circular SES. See Appendix B about circular reasoning and the FF3F model.

7. CONCLUSIONS

Shares-entailing and dividends-entailing risk factors are not scientifically valid in a model of return estimated and tested with scientific statistical methods. Risk factors that entail either shares or dividends have been applied to practical investment strategies and financial products, but less frequently than price-entailing risk factors are so applied. On cursory review it may appear that stock splits are a counter-example to the circular single-equation simultaneity (SES). Stock splits entail shares. When included in the estimation and testing of a return model, stock splits result in a circular SES and thus are not a counter-example.

There are no exceptions, therefore, to the general statement that models of return with price-, dividends- or shares-entailing risk factors, whether directly included through model specification or indirectly included through data sorting, are circular SES and thus are fallacious, meaningless, non-interpretable, indeterminate and not scientifically valid. The fatal fallacy is irremediable, irrefutable and terminal to the argument. Circular SES models are neither scientifically interesting nor important. Circular SES models must be rejected unless the logically circular variables are either isolated or excluded. Thus these irrational and inefficient stock pricing models of return with circular SES must be rejected to avoid economic waste.

Additionally it is shown that the most prominent examples of asset pricing circular SES developed in three phases: events, risk factors and return models. Each of the circular SES conforms to one of the three phases, and the three-phase development of asset pricing circular SES culminates in the Fama-French Three-Factor stock pricing model of return.

It also is shown that the most prominent circular SES of shares-, dividends- and price-entailing risk factors in return models published in leading finance and financial economics journals all share both one common originator or dominant influence and one common academic institutional affiliation. The originator is at the center of a nexus of collaborators with multiple bias-inducing ties to scientific journals and industry. These and related patterns reveal a lack of independence among some of the most prominent and popular asset pricing models.

The combination of (1) acknowledged nonscientific research motivation, (2) numerous undisclosed embedded circular SES, (3) three circular SES phases of events, risk factors and return models, and (4) circular SES patterns of (a) common center of influence, (b) numerous major bias-inducing ties, (c) nondisclosure of these ties in scientific academic journal articles, (d) evasion of standard checks and balances on scientific research integrity, (e) large gifts of money, (f) two-faced asymmetrical information and impression management, (g) iron triangle among academia, journals and industry, and (h) effective regulatory capture through subtle complex obfuscation, is highly improbable and approaches zero likelihood. One scientifically logical explanation, therefore, for this concatenation of coincidental facts is that the Fama-French Three-Factor stock pricing model, which is a fatal fallacy, a fabrication and a falsification, is also a grand design, a Piltdown man of economic sciences, a vast expanding harmful costly wasteful hoax and an contagion spreading to bourses worldwide.

APPENDIX A

ATTRIBUTIONS

The following doctoral dissertations, scientific research seminar presentations and published scientific academic journal articles indicate they are significantly influenced by E.F. Fama.

A.1. ATTRIBUTIONS BY ROLF W. BANZ

Banz (1978, p. ii), in the Acknowledgement of the Ph.D. dissertation, says:

> I wish to thank Myron Scholes, John Gould, Roger Ibbotson and Jonathan Ingersoll for their helpful comments on earlier drafts. I am indebted to Eugene Fama and especially Merton Miller for their criticisms and improvements of numerous previous versions of this study.

Banz (1979, p. 131), in the footnote, says:

> This paper is based on part of my dissertation. I am grateful to my dissertation committee, Myron Scholes (Chairman), John P. Gould, Roger Ibbotson, Jonathan Ingersoll and especially Eugene Fama and Merton H. Miller for their support and criticisms.

Banz (1981, p. 3), in the footnote, says:

> This study is based on part of my dissertation and was completed while I was at the University of Chicago. I am grateful to my committee, Myron Scholes (chairman), John Gould, Roger Ibbotson, Jonathan Ingersoll, and especially Eugene Fama and Merton Miller, for their advice and comments. I wish to

acknowledge the valuable comments of Bill Schwert on earlier drafts of this paper.

A.2. ATTRIBUTIONS BY MARC R. REINGANUM

Reinganum (1979b, p. ii), in the Acknowledgement of the Ph.D. dissertation, says:

> I wish to thank Mike Gibbons, Pat Hess and Rob Stambaugh for engaging in many hours of constructive discussions with me. My outside committee - Jon Ingersoll, Roger Kormendi and Myron Scholes - always made themselves available to me when I needed their assistance. Members of my inside committee deserve special thanks. Eugene Fama and Robert Hamada provided me with guidance and support. Particular thanks are owed to my chairman, Jack [John] Gould, whose intellectual and personal encouragement were indispensable.

Reinganum (1979a) does not include any acknowledgements in this published scientific research journal article and scientific research seminar presentation.

Reinganum (1981, p.19), in the footnote, says:

> I wish to thank my Ph.D. committee - Eugene Fama, Jack [John] Gould, Robert Hamada, Jon Ingersoll, Roger Kormendi, and Myron Scholes - as well as David F. DeRosa, Michael Gibbons, Pat Hess, Bill Schwert, Rob Stambaugh, and Ross Watts for their helpful comments. Partial financial support for this research was provided by the University of Chicago's Center for Research in Security Prices and the Graduate School of Business. Naturally, all errors are my responsibility.

APPENDIX B

CIRCULAR REASONING AND THE FF3F MODEL

Circular reasoning can be explained in terms of classical logic. Arnauld (1964, pp. 116, 119), Part II Judgment, Chapter 5 Simple, compound and complex propositions, says:

> Complex propositions are of two kinds. A proposition may be complex (1) in virtue of the matter of the proposition-that is, the subject-idea or the attribute or both-or (2) in virtue of the form alone. A proposition is complex with respect to its matter if the proposition has (1) a complex subject-idea, (2) a complex attribute, or (3) both a complex subject-idea and a complex attribute. 1. The subject-idea of a proposition is complex when the subject-idea is expressed by a complex expression. ... 2. Often there are expressions which are doubly or triply complex, being composed of parts, each of which is complex.

A statistical model equation is the numerical form of a verbal statement or sentence. A sentence contains a proposition, which in turn contains a subject-idea and a predicate-idea. It helps to classify ideas and propositions as either simple (one subject and one predicate) or compound, and either as complex (independent and dependent or subordinate clause) or incomplex. Whether it is expressed in math or in words, the logic of the statement is the same.

Return, size and value may appear to be unrelated on the surface. Analysis of these variables reveals that they are related by common embedded components. Return is an implicitly complex subject-idea because it is a dominant idea modified by a subordinate idea. Return is also an implicitly compound subject-idea because it must be formed from its independent simple components. Return is an artificial and derived concept. Return cannot be observed directly in nature; rather it must be calculated from the components of which it is composed. Return is a composite variable: it is a combination of capital gains and dividends. Return is also a complex variable: the absolute return is converted into yield, a relative return. The one-word label "return" is a convenient shorthand expression for the more complete phrase "nominal annualized yield relative to price, which is expected from capital gains and dividends." This complete phrase reveals two parts, capital gains and dividends, both of which are qualified by the three adjectival terms: nominal, annualized, yield relative to price. The capital gains part also is an implicitly complex subject-idea. Capital gains must be calculated from the parts of which it is composed. For long positions in stocks, the basic components are buying price and selling price whether actually realized or merely recognized. Total firm return is equal to proportional share return. A nominal share adjusted for capital changes is a constant proportional share. Total firm return is expressed in terms of total market equity capitalization or size instead of individual share price. Thus return is compound and doubly complex, and it entails size, price, shares and dividends.

Size is an implicitly complex subject-idea. In the Fama-French Three-Factor (FF3F) model, size is measured by market capitalization, which cannot be observed directly. Size is derived from basic components: share price and number of shares outstanding, both of which can be observed directly. Share price expressed as the price of one share of stock at a given time is incomplex, but share price expressed as the price of a constant proportion or percentage of total outstanding shares of stock capitalization at all times as represented currently by one share of stock is complex. The latter form of share price appears in return models. Number of shares outstanding is incomplex. Thus in the FF3F model, the risk factor related to size is compound and at least doubly complex, and it entails price and shares.

Value also is an implicitly complex subject-idea. In the FF3F model, value is measured by the book value of equity to market value of equity ratio. Value is a yield on price, and it could be called book-equity yield. Value is derived from basic components: book value of equity and market value of equity. Book value of equity or net worth is a calculated variable that can be observed directly from financial statements in reports to shareholders. Market value of equity is the same thing as market capitalization or size. Size is doubly complex and is entailed by value. Thus in the FF3F model, the risk factor related to value is compound and at least triply complex, and it entails price and shares.

Arnauld (1964, pp. 247, 250, 255), Part III Reasoning, Chapter 19 Sophisms: the different ways of reasoning incorrectly, further says:

> The second sophism is to assume as true the very thing in question. This sophism Aristotle called begging the question (petitio principii). Since what serves as proof [the cause or explanation] must be clearer and better known than what we seek to prove [the effect or explained], we see easily enough that begging the question is altogether opposed to genuine reasoning.
>
> Finally, to attempt to prove [explain] the unknown on the basis of something equally or more unknown, or the uncertain by something equally or more uncertain, is to beg the question in another form.
>
> To take as a cause what is not a cause is the sophism called non causa pro causa and is a very common source of error. There are several ways to commit this sophism. ... If we argue that since one event occurs after another then the latter event must be the cause of the former, we commit this sophism in the form called *post hoc ergo propter hoc.*

Return is derived mathematically from its components of price, dividends and shares, but it does not in any causal or temporal sense follow from its component parts. If an identical variable appears on both sides of a model equation at the same time then the relationship between the two sides is not sequential in either direction, but rather it is simultaneous in both directions.

A true logical method is correct reasoning or rationality. An untrue or false logical method is incorrect reasoning or irrationality. For the purpose of identifying true and scientifically valid methods, we can designate three degrees of knowledge: lesser known, equally known and better known. Similarly we can use these three degrees to designate directions of logical movement in arguments, hypotheses and models. If the same subject-idea appears on both sides of a logical statement then the subject-idea is equally known on both sides, and this violates true method.

Arnauld (1964, p. 307), Part IV Ordering, Chapter 2 The methods of analysis and synthesis; an example of analysis, says more generally:

> In analysis, as in the method of composition, we must pass from the better known to the lesser known. No genuine [true] method can dispense with this rule.

The fallacy of circular reasoning is also known as begging the question, logical circularity, circular argument, *circulus in probando, petitio principii* and vicious circle. A viciously circular argument is one which attempts to infer a conclusion based ultimately upon that conclusion itself. Such arguments can never advance our knowledge. See Walton (1995, pp. 229-239) for a more detailed explication of this classical fallacy.

Logical circularity takes the form: In other words they are trying to tell us that X is true because X is true! In the much less transparent FF3F model, it takes the form: The dependent variable, price-entailing return, is explained by allegedly independent risk factors that have a direct linear relation to price-entailing size and value! Circular reasoning is a fatal fallacy in an argument because there is no counter-example to defend it, and it is not valid in scientific logic.

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