ANNALS OF ECONOMICS AND FINANCE **1**, 145–156 (2006)

Elasticity of Substitution, Capital Inflow and Government Size

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A number of recent studies have attempted to identify the determinants of government size. It is well known that the size of government has implications for welfare and economic growth. This paper shows that the size of the fixed cost involving public good provision affects the magnitude of capital inflow induced changes in government size and welfare. By making use of a simulation exercise, it is argued that capital inflow can decrease (increase) the size of government and welfare if the elasticity of substitution is sufficiently large (small).

Key Words: Producer Services; Public Goods; Capital Inflow; Elasticity of Substitution.

JEL Classification Numbers: F20, H19, H41.

1. INTRODUCTION

The role of government in market economies can be divided into two broad categories: (i) provision of public goods and services and (ii) redistribution of income. Provision of public goods such as highways, parks and social services involve significant fixed cost. A number of recent studies have examined the determinants of government size and its impact on productivity and growth. For example, Lee and Lin (1994) have shown that demographic variables also affect the size of government. Commander, Davoodi and Lee (1997) found no robust relationship between government size and income growth. While examining the link between government size and economic growth, Vedder and Gallaway (1998) have argued that the growth of government in emerging economies has contributed to increased output. By making use of OECD data, Dar and Khalkhali (2002) found that on the average total factor productivity growth and the productivity

* This paper has greatly benefited from comments from the participants of 2005 Conference of Economists held at the Unievrsity of Melbourne.

1529-7373/2006

of capital is weak in countries with larger government size. DiPietro (2003) examined the relationship between corruption and the size of public sector. Andres, Domenech and Fatas (2004) argue that in the presence of some nominal and real rigidities, an increase in government size is associated with decreased output volatility. Anwar (2005) has shown that variations in the supply of factors of production can affect the government size. By making use of the Taiwanese data, Chen and Lee (2005) have shown that that excessive government size can contribute to negative growth. Garen and Trask (2005) have extended the earlier work of Rodik (1998) by showing that terms of trade risk affects the size of government. Hanson (2005) has shown that uncertainty can affect the size of government. By making use of an intertemporal model, Guo and Harrison (2005) have considered the link between government size and macroeconomic stability. Tridimas, and Winer (2005) have argued that the size of government and the structure of taxation jointly depend on the demand for publicly provided goods.

Governments, in most real economies, spend a significant amount of money on social programs that can be viewed as public goods. The degree of substitution between social services and private goods can also influence the size of government. However, none of the available studies appear to have considered the role of the elasticity of substitution between the public and private good consumption. In addition, the fact that provision of many public goods and services involves significant fixed cost has not been taken into account.

The past two decades have witnessed a significant increase in international capital mobility. Capital inflow affects a number of economic variables including the supply of public goods and services, which affects the size of government. While examining capital inflow induced changes in production, welfare and government size, this paper focuses on the role of the size of elasticity of substitution between public and private good consumption. Unlike the existing literature, this paper utilises a model where the provision of a public good involves significant fixed cost. A simulation exercise is conducted to highlight the role of the elasticity of substitution between private and public good consumption.

The rest of the paper is organised as follows. A simple general equilibrium model is developed in section two. The impact of exogenous capital inflow on production, government size and welfare is examined by means of a simulation exercise in section three. The last section offers some concluding remarks.

2. A SIMPLE GENERAL EQUILIBRIUM MODEL

Considers an economy that produces a composite final good (Y) by means of capital, labour and a large number of varieties of an intermediate

good (X). The intermediate good sector can be viewed as providing producer services; for example consulting, auditing, engineering, architectural, etc. Such services are widely utilised by producers in all real economies.[1] The production function for Y is as follows:[2]

$$Y = K_y^{\beta(1-\alpha)} L_y^{(1-\alpha)(1-\beta)} \left(\sum_{i=1}^{n} x_i^{\delta} \right)^{\frac{\alpha}{\delta}} \tag{1}$$

Where α, β and δ are parameters in the range $[0, 1]$; x_i is the output of the i-th variety produced by industry X; n is the number of varieties produced; L_y and K_y respectively are labour and capital used in the production of Y. Each variety of the intermediate good is produced by means of capital and labour. The total cost of production consists of fixed and variable cost as follows where r and w respectively are the price of capital and the wage rate.

$$c(w, r, x_i) = r\mu + w(\lambda x_i) \tag{2}$$

μ and λx_i respectively is the capital and labour used in the production of each variety. Because of the presence of fixed cost, the production of each variety of the intermediate good x_i is subject to internal economies of scale. Due to identical production functions and an equalisation of factor prices between sectors, all varieties produced are equally priced. It is customary to consider a symmetric equilibrium where the aggregate production $X = nx$. Accordingly, the production function for the final good can be written as

$$Y = L_y^{1-\alpha-\beta+\alpha\beta} K_y^{\beta(1-\alpha)} X^{\alpha} n^{\frac{\alpha(1-\delta)}{\delta}} \tag{3}$$

From the point of view of each firm in Y industry, the number of varieties supplied is given. Accordingly, there are constant returns at the firm level but for the industry as a whole there are external economies of scale.[3] The external economies of scale in Y-industry are compatible with perfect competition. Within the intermediate good industry, a large number of differentiated goods are produced, the price elasticity of demand for each differentiated good is $1/(1-\delta)$.[4]

[1] For an analysis of the role of the services sector in open economies, see Markusen (1989), Melvin (1989), Marrewijk, Sitobra, Vaal and Viaene (1997) and Markusen and Venables (2000).

[2] Except for the inclusion of a public good, the above model closely resembles Rivera-Batiz and Rivera-Batiz (1991) and Rodrik (1996).

[3] $\alpha(1-\delta)/\delta$ is assumed to be positive but less than unity.

[4] This and similar assumptions are widely used in the existing literature. See Wong (1995), Bhagwati, Panagariya and Srinivassan (1998) and Feenstra (2004) for a review of related literature.

The economy also produces a public consumption good (G) by means of capital and labour. Provision of most public goods in real life involves significant fixed cost which is ignored by most existing studies. This paper explicitly assumes that the provision of the public good involves fixed as well as variable cost as follows where θ and σ are positive.

$$c(w, r, G) = r\theta + w(\sigma G) \tag{4}$$

The above cost function indicates that capital enters as a fixed input where as labour enters as a variable input in the production of the public good (i.e., $r\theta$ is the fixed cost whereas $w(\sigma G)$ is the variable cost).

The following condition determines the profit maximising output of the final good where p is the price of x and Ψ is a positive.

$$1 = \Psi w^{(1-\beta)(1-\alpha)} r^{\alpha(1-\beta)} p^{\alpha} n^{\frac{-\alpha(1-\delta)}{\delta}} \tag{5}$$

The right-hand side of equation (5) is the unit cost of production whereas the left-hand side is the unit price, which has been set equal to unity. The productivity of the final good sector is linked to the number of varieties of the intermediate good available. An increase in the number of varieties available decreases the unit cost of production in the final good sector.

The presence of internal economies of scale in the intermediate good sector implies that a single firm under monopolistic competition will produce each variety. The following first order condition determines the profit maximising output of each variety where the right hand side is the marginal cost and the left had side is the marginal revenue.

$$\delta p = \lambda w \tag{6}$$

Because of free entry and exit, the price of each variety of the intermediate good in the long-run equilibrium will just cover average cost as follows:

$$p = \frac{r\mu}{(1-\delta)x} \tag{7}$$

The market clearing condition for labour, which is assumed to be in fixed supply, is as follows:

$$\sigma G + n(\lambda x) + \Psi(1-\alpha)(1-\beta)\left[\frac{w}{r}\right]^{-\beta(1-\alpha)}\left[\frac{w}{p}\right]^{-\alpha} n^{\frac{-\alpha(1-\delta)}{\delta}} Y = L \tag{8}$$

The left had side of equation (8) is the demand for labour whereas the right hand side is the supply of labour. The market clearing condition for

capital, which is assumed to be in fixed supply, is as follows:

$$\theta + n(\mu) + \Psi\beta(1-\alpha)\left[\frac{w}{r}\right]^{1-\beta(1-\alpha)}\left[\frac{w}{p}\right]^{-\alpha} n^{\frac{-\alpha(1-\delta)}{\delta}} Y = K \tag{9}$$

The left had side of equation (9) is the demand for capital whereas the right hand side is the supply of capital. The market clearing condition for the intermediate good is as follows:

$$\alpha\Psi\left[\frac{w}{r}\right]^{-\beta(1-\alpha)}\left[\frac{w}{p}\right]^{1-\alpha} n^{\frac{-\alpha(1-\delta)}{\delta}} Y = nx \tag{10}$$

The left-hand side of the above equation is the demand for the intermediate good in Y-industry whereas the right hand side is the supply.

In order to focus on the impact of capital mobility on factor prices, production, welfare and government size, this paper considers a situation where in the initial equilibrium the economy is not involved in international trade. This implies that $C_y = Y \div N$ is the amount of final good consumed by the representative consumer where N is the population size. Each consumer is endowed with one unit of labour which is supplied inelastically so that $L = N$. The utility function of the representative consumer is as follows where ϕ lies in the range $[1, -\infty]$.

$$U = \left[C_y^{\phi} + G^{\phi}\right]^{\frac{1}{\phi}} \tag{11}$$

The above utility function shows that the entire amount of the public good is available to each consumer (i.e., G is a pure public good). $1/(1-\phi)$ is the elasticity of substitution between the public and the private goods. The above utility function given by equation (11) can be used to determine the optimal provision of the public good. As indicated by equation (4), the public good industry is charatersied by internal economies of scale. This paper views public good as being provided by a public firm that is not aiming to maximise profit. The optimal provision of the public good is determined by average cost pricing as follows:

$$L\left[\frac{C_y^{1-\phi}}{G^{1-\phi}}\right] = \left[\frac{r\theta + w(\sigma G)}{G}\right] \tag{12}$$

Equation (12) is a zero profit condition where the right-hand side is average cost and the left-hand side is the price of the public good. Price of the public good equals the aggregate marginal rate of substitution (MRS) between the public and private goods derived from equation (11).

This completes the description of the model where equations (5) to (12) are eight equilibrium conditions in eight endogenous variables; Y, G, X, U, n, w, r and p.

3. EXOGENOUS CAPITAL INFLOW, FACTOR PRICES AND PRODUCTION

This section deals with the impact of exogenous capital inflow on prices and production. An increase in the supply of capital in the present case is akin to exogenous capital inflow. There is no trade or factor mobility in the initial equilibrium. The impact of an exogenous capital inflow on the number of varieties of the intermediate good produced is as follows:

$$\left[\frac{\partial n}{\partial K}\right]\left[\frac{K}{n}\right] = \frac{K}{K-\theta} > 0 \tag{13}$$

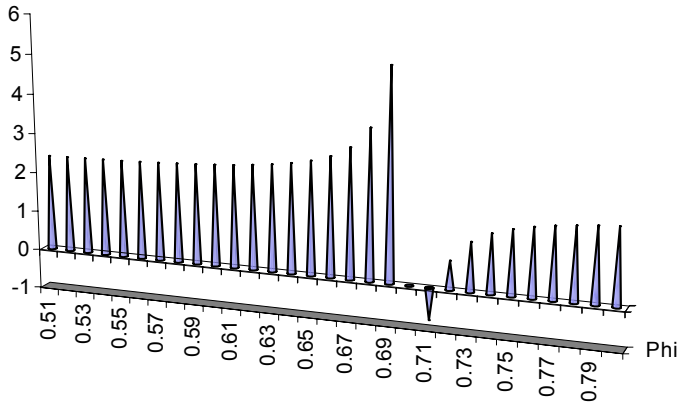
The above equation shows that Capital inflow increases the number varieties produced. This follows from the fact that capital inflow decreases the fixed cost which encourages entry into the intermediate good industry. Since each firm produces a distinct product, the number of available varieties increases. As indicated by equation (6) and (7), there is an inverse relationship between the production of each variety and $w-r$ ratio. Capital inflow decreases (increases) the production of each variety when its impact on $w-r$ ratio is positive (negative). The impact of capital inflow on $w-r$ ratio is as follows:

$$\begin{aligned}&\left[\frac{\partial(\frac{w}{r})}{\partial K}\right]\left[\frac{K}{(\frac{w}{r})}\right]\\ =\ & -L\left[(1-\phi)-\phi\frac{\alpha(1-\delta)}{\delta}\left(\frac{L_g}{L}\right)-\left(\frac{r\theta}{r\theta+w(\sigma G)}\right)\left(1-\frac{L_g}{L}\right)\right]\div\Delta\end{aligned} \tag{14}$$

$$\Delta = L\left\{\phi\left[1-\beta(1-\alpha)\left(\frac{L_g}{L}\right)\right]-\left[\frac{w(\sigma G)}{r\theta+w(\sigma G)}\right]\right\}(K-\theta)$$

Equation (14) shows that the impact of capital inflow on $w-r$ ratio cannot be unambiguously determined. An increase in the supply of capital decreases the price of capital and hence $w-r$ ratio tends to increase. However, in the present case, capital inflow also increases the number varieties of the intermediate good produced which increases the productivity of capital and labour in the final good sector. Equation (14) shows that the magnitude of capital inflow induced changes in $w-r$ depends, among other factors, on the size of ϕ, $\alpha(1-\delta)/\delta$ and θ. The sign and the size of ϕ affect the size of elasticity of substitution between the private and public

FIG. 1. Impact of Capital Inflow on $w - r$ Ratio



good consumption. A negative value of ϕ implies that the elasticity of substitution is less than unity. The elasticity of substitution is greater than unity when ϕ is positive. $\theta = 0$ implies that provision of the public goods does not involve any fixed cost. The existing literature suggests that only a small value of $\alpha(1-\delta)/\delta$ can ensure that the equilibrium is stable. It can be easily confirmed that Δ is unambiguously negative when $\theta = 0$. In the presence of non-zero fixed cost, the sign Δ cannot be unambiguously determined which has implications for the magnitude of capital inflow induced changes in production, government size and welfare.

By making use of some plausible parameter values, the impact of capital inflow on $w - r$ ratio is shown in figure 1 below when ϕ lies in the range $[0.52, 0.80]$ which implies that the elasticity of substitution is greater than unity.

The above figure shows the relationship between the magnitude of capital inflow induced changes in $w - r$ ratio and various values of ϕ (where ϕ is measured along the horizontal axis). The simulation exercise is conducted by making use of some plausible numerical values such as $\beta(1-\alpha) = 0.40$, $\alpha(1-\delta)/\delta = 0.05$ and $K/(K-\theta) = 2$. It should be noted that the general trend shown in Figures 1 to 5 would not significantly change if different numerical values were used.[5]

Figure 1 above shows that capital inflow increases $w - r$ ratio for sufficiently small values of ϕ. The elasticity of substitution between the public and private good consumption increases as $\phi \to 1$. In the case of figure 1,

[5]Simulation exercise also assumes that the proportion of the fixed cost in pubic sector and the labour force employed in the public sector as a proportion of total labour force are invariant to changes in ϕ.

as long as ϕ is less than 0.7, capital inflow increases $w-r$ ratio. In the present case, $w-r$ ratio function is undefined when $\phi = 0.7$. At a value of ϕ that is just below 0.7, capital inflow leads to a very large increase in $w-r$ ratio. On the other hand, capital inflow leads to a very large decrease in $w-r$ ratio when $\phi \rightarrow 0.7$ from the right. It is however clear that capital inflow induced changes in $w-r$ ratio are unambiguously positive as long as the elasticity of substitution is larger than 3.57 (i.e., $\phi > 0.72$).

The impact of capital inflow on production of the public and private goods is as follows:

$$\begin{aligned} & \left[\frac{\partial G}{\partial K}\right]\left[\frac{K}{G}\right] \\ = \; & L\left[\phi\left\{\frac{\alpha(1-\delta)}{\delta}+\beta(1-\alpha)\right\}-\frac{r\theta}{r\theta+w(\sigma G)}\right]\left[K\left(1-\frac{L_g}{L}\right)\right]\div\Delta \quad (15) \\ & \left[\frac{\partial Y}{\partial K}\right]\left[\frac{K}{Y}\right] \\ = \; & -\{1-\beta(1-\alpha)\}\left[\frac{\partial(\frac{w}{r})}{\partial K}\right]\left[\frac{K}{(\frac{w}{r})}\right]+\left\{1+\frac{\alpha(1-\delta)}{\delta}\right\}\left[\frac{K}{K-\theta}\right] \quad (16) \end{aligned}$$

Equations (15) and (16) highlight the role of the size of the elasticity of substitution between the public and private goods. It is clear that capital inflow increases the public good provision when the elasticity of substitution is less than unity. Equation (15) can also be used to demonstrate the role of the size of the fixed cost. For example, in the absence of the fixed cost (i.e., when $\theta = 0$), capital inflow leads to an unambiguous decrease in public good provision. However, capital inflow can increase the production of the public good when the fixed cost is non-zero. Equation (16) shows that the impact of capital inflow on production of the private good depends on the response of $w-r$ ratio. Figure 2 below shows the magnitude of capital inflow induced changes in the supply of the private good when ϕ varies in the rage $[0.51, 0.80]$ and $\theta > 0$.

Figure 2 shows that capital inflow increases the private good production as long as ϕ is sufficiently smaller than 0.7. The function is undefined when ϕ equals 0.7. It is however clear that capital inflow can decrease the private good production. In the small neighbourhood of $\phi = 0.7$ (i.e., when the elasticity of substitution is either just above or just below 3.33), capital inflow can lead to a very large change in the production of the final good. The impact of capital inflow on public good provision is shown in figure 3 below when ϕ varies in the rage $[0.51, 0.80]$ and $\theta > 0$.

Figure 3 shows that capital inflow leads to an unambiguous increase in provision of the public good as long as ϕ is smaller than 0.7. An increase in the supply of capital can decrease public good provision if the elasticity of substitution is sufficiently large i.e., $\phi > 0.7$.

FIG. 2. Impact of Capital Inflow on Private Good Production

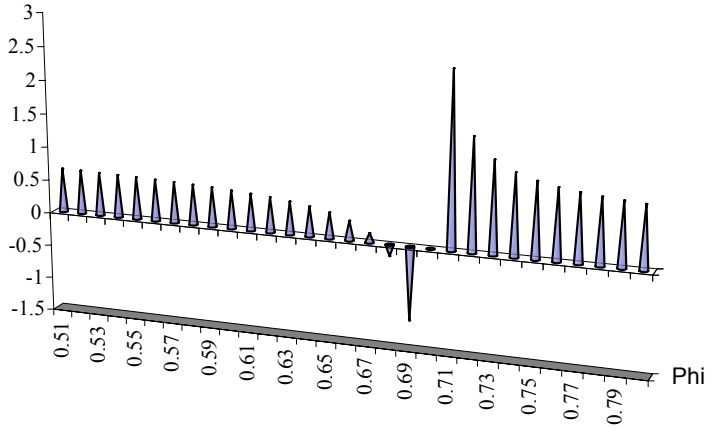
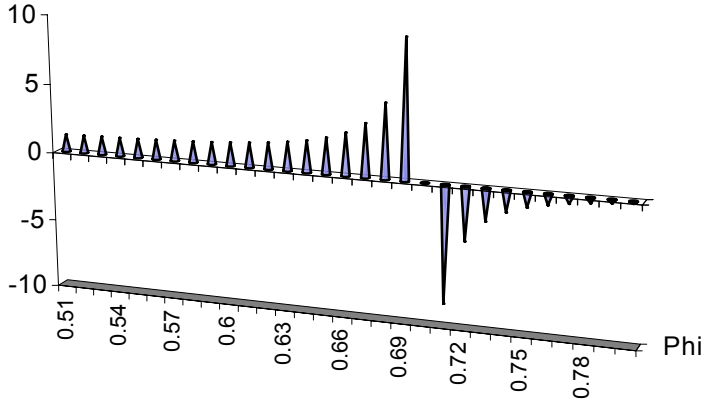


FIG. 3. Impact of Captial Inflow on Public Good Provision



3.1. Capital Inflow and the Size of Government

The size of government (Z) is defined as follows where p_g is the price of the public good which equals $L(C_y/G)^{1-\phi}$.

$$Z = \frac{p_g G}{p_g G + Y} \tag{17}$$

The impact of capital inflow on the size of government can be determined by differentiating the above equation with respect to K as follows:

$$\left[\frac{\partial Z}{\partial K}\right]\left[\frac{K}{Z}\right] = \phi\left[\frac{Y}{Z}\right]\left[\left\{\frac{\partial G}{\partial K}\right\}\left\{\frac{K}{G}\right\} - \left\{\frac{\partial Y}{\partial K}\right\}\left\{\frac{K}{Y}\right\}\right] \tag{18}$$

Equation (17) shows that capital inflow affects government size by through changes in the production of the public and private goods. This implies that the magnitude of capital inflow induced changes in government size is affected by the same factors that influence the production of the public and private goods — namely, the size of the elasticity of substitution and the size of the fixed cost involving public good production. The impact of capital inflow on government size is shown in figure 4 below when ϕ varies in the rage $[0.51, 0.80]$ and $\theta > 0$.

FIG. 4. Impact of Capital Inflow on the Size of Government

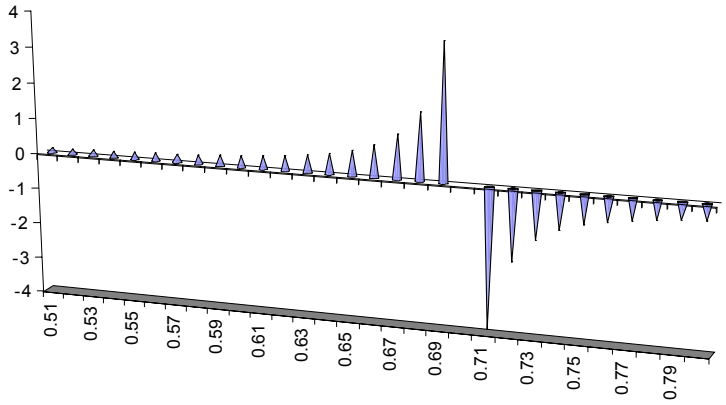


Figure 4 shows that capital inflow leads to a larger increase in government size if ϕ is closer to 0.7. Capital inflow decreases government size when $\phi > 0.7$. It is clear that the magnitude of capital inflow induced change in government size is small when the elasticity of substitution is either too large or too small.

3.2. Welfare implications

The impact of capital inflow on welfare can be examined by differentiating equation (11) as follows:

$$\begin{aligned}\left[\frac{\partial U}{\partial K}\right]\left[\frac{K}{U}\right] &= \left\{\frac{G^{\phi}}{G^{\phi}+(\frac{Y}{L})^{\phi}}\right\}\left[\frac{\partial G}{\partial K}\right]\left[\frac{K}{G}\right] \\ &+ \left\{\frac{(\frac{Y}{L})^{\phi}}{G^{\phi}+(\frac{Y}{L})^{\phi}}\right\}\left[\frac{\partial Y}{\partial K}\right]\left[\frac{K}{Y}\right] \qquad (19)\end{aligned}$$

Equations (19) shows that the impact of capital inflow on welfare depends on its impact on production of the private and public goods indexed for their importance in overall welfare. The following figure shows the impact

of capital inflow on welfare when both goods are assigned an equal weight of 0.5, ϕ varies in the rage $[0.51, 0.80]$ and $\theta > 0$.

FIG. 5. Impact of Capital Inflow on Welfare

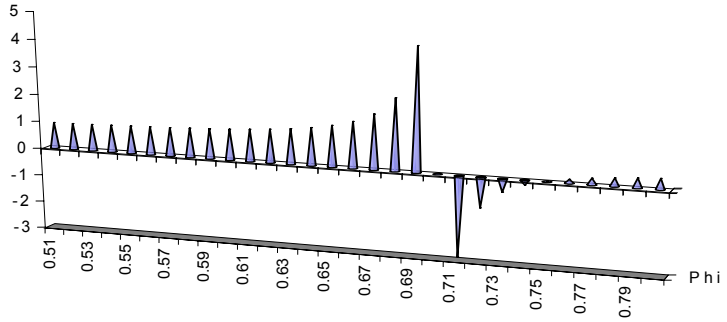


Figure 5 shows that capital inflow leads to a larger increase in welfare when the elasticity of substation is large but less than 3.33 (i.e., $\phi < 0.7$). It is interesting to note that capital inflow induced changes in welfare can be negative when $\phi > 0.7$. However, for very large values of the elasticity of substitution, capital inflow leads to a much smaller increase in welfare.

4. CONCLUDING REMARKS

A number of recent studies have attempted to identify the determinants of government size and its impact on productivity and economic growth. However, none of the available studies appear to have considered the implications of the size of the elasticity of substitution between private and public good consumption and the size of the fixed cost associated with public good provision. In all real economies, provision of public goods and services involves significant fixed cost. While focusing on the impact of capital inflow induced changes in production, government size and welfare, this paper highlights the role of the size of the elasticity of substitution between public and private good consumption and the magnitude of the fixed cost associated with provision of the public good. The paper shows that the presence of non-zero fixed cost significantly affects the magnitude of capital inflow induced changes in government size and welfare. The role of the size of the elasticity of substitution is examined by means of a simulation exercise. It is shown that capital inflow induced changes in welfare and government size can be negative if the elasticity of substitution between private and public good consumption is sufficiently greater than unity.

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