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Do the Economies of Specialization Justify the Work Ethics? A Further Examination of Buchanan'S Hypothesis*

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Ng & Ng (2003) provide a qualified support for Buchanan's (1991, 1994) hypothesis on work ethics by showing that a decrease in preference for leisure (a higher work ethics) by an individual benefits her trading partners by improving the terms of trade of the latter. Moreover, the higher the degree of the economies of specialization, the larger is this beneficial effect. Using a similar model, the present paper shows that a simultaneous artificial decrease in preference for leisure by all individuals decreases intrinsic utility evaluated at the original preference. However, using a more realistic model allowing for both home and firm/market production developed by Ng & Zhang (2005), a stronger support is provided for Buchanan's hypothesis as a shift in preference by everyone from leisure to market goods produced under increasing returns and average-cost pricing increases utility even if evaluated in accordance with the original preference.

Key Words: Work; Leisure; Increasing returns; Specialization; Ethics; Buchanan.
JEL Classification Numbers: D60, J20.

1. INTRODUCTION

Buchanan (1991, 1994) proposes an interesting hypothesis explaining the prevalence of an ethic encouraging more work. Of course, the encouragement of the work ethics may be due to different reasons, including reducing the social costs possibly partly created by laziness and overcoming the problem of motivating employees especially in the presence of imperfections in identifying contributions. (On the motivation of employees and solders by identity creation, see Akerlof & Kranton 2005.) Rather, Buchanan abstracts away from all these factors and concentrate on the role of the

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economies of specialization. The economies of specialization mean that more division of labor may increase productivity. If most people work more, this increases the extent of the market which enables a higher degree of division of labor and leads to higher productivity. Thus, individual choice between leisure and work results in a sub-optimal level of work. One way to counteract this is to maintain a work ethic. In fact, Buchanan & Yoon (1994) construct a specific model and show that an equilibrium constrained by a higher work ethics may be superior. In a recent paper (Ng & Ng 2003), the validity and significance of this hypothesis is examined. Instead of modeling the work ethics as an additional constraint, it examines the effect of a higher preference for work (or a lower preference for leisure). It concludes with a qualified support for the Buchanan hypothesis by showing that a decrease in preference for leisure (a higher work ethics) by an individual benefits her trading partners by improving the terms of trade of the latter. Moreover, the higher the degree of the economies of specialization, the larger is this beneficial effect on others. The qualifications consist of two parts. First, it suggests that the Buchanan hypothesis probably has more relevance in ancient times when the work ethics originated but is less significant in the current world of global trade where the billions of individuals involved is sufficient to sustain specialization without artificial encouragement of additional work effort. Secondly, competition for relative standing, the materialistic bias caused by our accumulation instinct and advertising, and the environmental disruption of material production and consumption suggest that the discouragement of long working week may be more conducive to welfare.

The second qualification is important. (See Ng 2003 for an analysis of the welfare implications of competition for relative standing, the materialistic bias, and environmental disruption.) However, in the main analysis of the current paper, those considerations will be abstracted away to focus more on the relationship between economies of specialization (or increasing returns) and work ethics as such. In this more traditional economic setting, this paper examines the support for the Buchanan hypothesis as well as the first qualification. It is shown in the next section that both contain some misleading elements and that the analysis by Ng & Ng has to be extended to get a less misleading picture.

Essentially, the artificial decrease in the preference for leisure (and the correspondingly higher production and hence higher desire to trade) by an individual may benefit her trading partners but actually harms the individual herself, at least if valued at her original or intrinsic preference. Thus, it is unclear that this benefit (even if it is larger at a higher degree of economies of specialization) justifies the work ethics. Ng & Ng (2003) did not pursue this point since they focused on the effect on the terms of trade rather than on utility or welfare to avoid the tricky questions of interpersonal comparison of utility and welfare evaluation in the presence of preference changes. In this paper, a way is found to do the analysis despite these tricky questions.

Section 2 below shows that a simultaneous artificial decrease in preference for leisure by all individuals decreases the intrinsic utility (the utility derived from using the original utility function with the new set of objective variables) of all individuals in a simple model of ex-ante identical individuals. (In a model of heterogeneous individuals, it is possible to construct specific cases where some individuals may gain but usually others will lose more, still failing to provide a case for the artificial encouragement of work ethics at the societal level.) Rather, if the original population size is not large enough to allow full specialization, an increase in population increases utility by allowing more specialization.

Moreover, using a more realistic model allowing for both home and firm/market production developed by Ng & Zhang (2005), a stronger support (than Ng & Ng's) is provided for Buchanan's hypothesis by showing that a shift in preference by everyone from leisure to market goods produced under increasing returns and average-cost pricing increases utility even if evaluated in accordance with the original preference. However, a shift in preference from leisure to home goods (even if also produced under increasing returns) decreases utility evaluated in accordance with the original preference. The contrast between market and home goods is due to the fact that the effect of increasing returns in home production is already fully taken into account by each individual (or household) while that of firm production has not been fully taken into account. Market goods are priced at average costs, consisting of the average fixed cost plus a constant marginal cost. This make the fixed cost component possesses some publicness aspect which is not fully accounted for by individual maximization. This contrast is explained more fully in the concluding section.

Like the papers by Buchanan, Yoon, Ng and Ng, the present paper is only concerned with the potential benefits of having a work ethic but not concerned with the coordination and/or game-strategic process in arriving at the accepted ethic, work or otherwise. On some aspect of the latter and related issues, see, e.g. Gauthier (1986) who argues that ethics may emerge from the agreement by rational individuals based on the mutual interest of observance. Similarly, the present study is not concerned with the crowding out of intrinsic motivation by incentive payments or other aspects of the principal-agent problem; on this crowding-out of work ethics, see Grepperud and Pedersen (2001).

2. DIVISION OF LABOR BY IDENTICAL INDIVIDUALS

Since Buchanan relates work ethics to economies of specialization from the division of labor, his case is examined by using models (Ng & Ng use a most simple model of two symmetrical goods and two ex-ante identical individuals with no non-labor inputs) of a framework designed for analyzing the division of labor by Yang and Ng (1993), but with leisure added to examine the role of work ethics. In this section, that model is expanded to allow for three goods (in addition to leisure) X, Y, and Z (in order to examine the role of population size later; the number of individuals remains at two at the moment; we can easily collapse this model into the one of two goods similar to Ng & Ng's paper by taking Z out of the utility function and make l_z, the amount of labor used to produce Z, equal to zero.) It will be shown in this model of division of labor with economies of specialization that an artificial shift in preference from leisure to goods by all individuals decreases the utility of the representative individual evaluated at the original preference.

Each individual has T units of time which may be used for leisure or the production of either one or more of the three goods X, Y and Z. Thus, each individual is faced with the time constraint

$$T = \ell + l_x + l_y + l_z \tag{1}$$

Where $\ell =$ leisure time, l_i is the amount of time used in the production of good i. The production functions are

$$X = l_x^a; \qquad Y = l_y^a; Z = l_z^a \tag{2}$$

where $a > 1$ indicates the presence of economies of specialization. Individuals are taken as ex-ante identical and production functions are symmetrical to emphasize the point that gain from trade may arise from the economies of specialization alone.

A Cobb-Douglas utility function is used for simplicity. Since we are considering a change in preference only for leisure, we let only leisure to carry a preference variable. Those for x, y and z (using lower case to indicate the amount consumed) may be normalized to unity by scaling the utility function since the preferences are symmetric over the goods.

$$U = xyzl^{\alpha} \tag{3}$$

Consider first the case of autarky where the individual self-produces all goods. The maximization of (3) subject to (1) and (2) gives the solution[1]

$$\begin{aligned} l_x &= l_y = l_z = aT/(3a+\alpha), \\ \ell &= \alpha T/(3a+\alpha), \\ U^A &= a^{3a}\alpha^{\alpha}[T/(3a+\alpha)]^{3a+\alpha}, \end{aligned} \tag{4}$$

where the superscript A indicates autarky.

Now consider the case of specialization where an individual specializes in producing either X or Y and exchange with the other individual who specializes in the other good, and both individuals self-produce and consume Z. Due to symmetry, we need only to consider the case where the individual specializes in X. From her output of X, she supplies x^s in exchange for y at a price of p (price of X in terms of Y). However, market transactions is supposed to incur a transactions cost of $1-k$. Thus, while the amount of Y he bought is equal to px^s, the amount he actually consumes equals only kpx^s. His utility function may thus be written as

$$U = [(T-\ell-l_z)^a - x^s]kpx^s l_z^a \ell^{\alpha} \tag{5}$$

The maximization of (5) with respect to ℓ, l_z and x^s gives the following solution

$$\begin{aligned} \ell &= \alpha T/(3a+\alpha), \\ l_x &= [2aT/(3a+\alpha)]^a \\ x &= x^s = [2aT/(3a+\alpha)]^a/2, \\ y &= kp[2aT/(3a+\alpha)]^a/2, \\ U_2^D &= 2^{2(a-1)}kpa^{3a}\alpha^{\alpha}[T/(3a+\alpha)]^{3a+\alpha} \end{aligned} \tag{6}$$

where the superscript D indicates division of labor and the subscript 2 indicates the number of individuals or the number of traded goods. From (5) and (6), we have for the symmetrical case with $p=1$ (as required by the assumption of free choice of occupation with initially identical individuals),

$$U_2^D/U^A = 2^{2(a-1)}k \tag{7}$$

Thus, division of labor yields a higher utility if and only if $2^{2(a-1)}k > 1$, i.e. if the economies of specialization (measured by $a-1$) is sufficiently large to offset the effect of transactions costs (measured by $1-k$). For

[1]It is not difficult to check that the second-order condition is satisfied; the demonstration is available from the author. For the more general question on the existence of equilibrium for this types of models, see Yang & Ng (2003), Sun, Yang & Zhou (2004).

example, if $a = 1.1$, k has to be no smaller than 0.87055 for division of labor to dominate autarky in this simple model. It may be noted that, in this model, the relative superiority of division of labor vs. autarky does not depend on the preference for leisure. However, if division of labor is chosen, then the willingness to trade decreases with higher preference for leisure; x^s is a decreasing function of α. Thus, to the extent that other people may benefit from one's willingness to trade, a higher degree of work ethics (reduced preference for leisure) may be regarded as favourable. This may be shown more formally by bringing in the other person who produces Y into the picture. His situation is exactly the same as the person producing X depicted above, except that his preference parameter for leisure is indicated by β. Then his supply y^s is the same as the third equation in (6) for x^s except that α is replaced by β. Then, from the market equilibrium requirement $px^s = y^s$ (since the price of Y in terms of X is the inverse of the price of X in terms of Y), we may solve for the equilibrium price of X in terms of Y as

$$p = [(3a + \alpha)/(3a + \beta)]^a \tag{8}$$

which clearly increases with α and decreases with β. Thus, the person producing X benefits from the higher work ethics of the person producing Y and vice versa.

Now, let us examine the effect of a higher degree of the economies of specialization (a larger a) on the benefits of the higher work ethics of others. To concentrate on the increase in work effort rather than the complication of an initial interpersonal difference in preferences for leisure, we evaluate the effect of an increase in a on the effect of a change in α (or β, the two are symmetrical) on p at an initial position when $\alpha = \beta$. We then have,

$$\begin{aligned} \frac{\partial^2 p}{\partial\alpha\partial a} &= \frac{(3a+\alpha)^{a-1}}{(3a+\beta)^a}\left\{1 - \frac{3a}{3a+\alpha} + a[\ln(3a+\alpha) - \ln(3a+\beta)\right. \\ &\quad \left. - \frac{3a^2(\alpha-\beta)}{(3a+\alpha)(3a+\beta)}\right\} \end{aligned} \tag{9}$$

Since a, α, and β are all positive, the right hand side of (9) is positive at $\alpha = \beta$. This means that, the higher the degree of economies of specialization, the larger is the beneficial effect of a higher work ethics on the trading partner. From this (but using a model of only two goods), Ng & Ng (2003) conclude that Buchanan's conjecture has validity and is related to the economies of specialization. However, as already noted in the previous section, the artificial decrease in the preference for leisure by an individual actually harms the individual herself, at least if valued at her original or intrinsic preference. Thus, it is unclear that this benefit (even if it is larger at a higher degree of economies of specialization) justifies the work ethics. To support Buchanan's hypothesis, we need to show that, if all individuals decrease their preference for leisure, they will all benefit in accordance to their original preference, at least in a model of identical individuals. A way of examining whether this is true is provided below. The use of a model of identical individuals not only abstracts away the complications of interpersonal differences, it also allows us to avoid having to make interpersonal utility comparisons.

Using the above model (the same result obtains when using a similar model with different numbers of individuals and goods, including the simpler model of two goods used in Ng & Ng), we let each individual adopt objective variables ℓ, x, etc. in accordance to those that would obtain if the preference for leisure parameter α were lower at α'. (Since we are now letting all or both individuals adopt this change simultaneously, we do not have to distinguish two different preference-for-leisure parameters α and β). From (6), but replacing α by α', we have

$$\begin{aligned} \ell' &= \alpha' T/(3a+\alpha'), \\ x' &= x^{s\prime} = [2aT/(3a+\alpha')]^a/2, \\ y' &= kp[2aT/(3a+\alpha')]^a/2, \\ U' &= 2^{2(a-1)}kpa^{3a}(\alpha')^\alpha[T/(3a+\alpha')]^{3a+\alpha} \end{aligned} \tag{6'}$$

where U' is the value of utility if the values of leisure, x, and y are given by ℓ', x', y' respectively but the utility function remains at the original (3), in particular with the preference for leisure parameter still at α.

Define $U'/U \equiv R$, where U is as given in (6) with variables in accordance to the original preference, we have

$$R = (\alpha')^\alpha(3a+\alpha)^{3a+\alpha}/[\alpha^\alpha(3a+\alpha')^{3a+\alpha}] \tag{10}$$

From which we have

$$\partial R/\partial\alpha' = 3a\alpha^\alpha(\alpha')^{\alpha-1}(3a+\alpha)^{3a+\alpha}(3a+\alpha')^{3a+\alpha-1}(\alpha-\alpha') \tag{11}$$

over a perfect square.

From (11), it is clear that $\partial R/\partial\alpha'$ equals zero at $\alpha=\alpha'$ and is positive at all values of $\alpha' < \alpha$. Thus, a marginal increase (or decrease) in work ethics (a marginal decrease/increase in α to α') from the original equilibrium has no effect on utility as the original equilibrium is optimized already (the envelope result) but further increases in work ethics decreases utility in accordance to the intrinsic utility function. Thus, the artificial encouragement of work ethics does not benefit the society, though each individual will benefit from the increase in work ethics of the other individual.

It may appear odd that an increase in work ethics of one person benefits others yet an increase by all is not beneficial to all. This may be explained. An increase in work ethics of one person (or more persons in a model of more than two individuals) benefits others by improving the terms of trade for others through her attempt to sell more and buy more. An increase in work ethics of all cannot make the terms of trade better for all, hence missing the terms-of-trade effect, leaving only the distortive effect if the original situation is already efficient. Also, an increase in work ethics of a person decreases her utility evaluated at the original preference. The benefits to others are at the expense of the person herself. When everyone increases work, the utility-decreasing effect dominates the beneficial effects.

Ng & Ng (2003, p.349) explain their result that the artificial encouragement of work ethics does not benefit the society with the statement 'as the economy of specialization has already been fully utilized at the original point ... where each individual specializes in the production of only one good'. (That paper has the number of individuals equal to the number of goods). This explanation suggests that, if the number of individuals is smaller than the number of goods and hence insufficient to allow for full specialization, the encouragement of work ethics may benefit the society by providing more scope for specialization. This is in fact misleading. In the model above, we have already made the number (3) of goods larger than the number (2) of individuals. Yet, we have shown above that the artificial decrease in preference for leisure does not benefit the society, at least for the type of models where the economies of specialization is individual specific.

The reason is that, without an increase in the number of individuals, although the lower preference for leisure releases more labor time for the production of goods and hence achieve a higher productivity, the net effect on utility is negative for the following reason. The original amount of labor time allocated to the production of goods is already optimal given the price and production possibilities the individual faces. An artificial increase cannot therefore increase utility. This is not changed by considering a simultaneous increase in labor time by all individuals as such an increase does not change the relative price between goods and hence is similar to the situation faced by the optimization problem of a price-taking individual. Thus, an increase in the extent of the market through more work (a lower α increase x^s), even if leading to higher productivities in goods production through the economies of specialization, need not be utility enhancing.

Rather than increasing labor time, it is an increase in the number of individuals (which also increases the extent of the market) that can increase utility. This can be illustrated using the above model but with the number of individuals increases to three, allowing for all three goods to be produced by specialists, with no home production needed. Using a similar method as the derivation of (6) above, we have for the symmetric case with the prices of all goods equal to one and for the person specializing in the production of X (persons specializing in producing Y and Z are symmetrical to this case as well),

$$\begin{aligned} \ell &= \alpha T/(3a+\alpha), \\ l_x &= [3aT/(3a+\alpha)]^a \\ x &= [3aT/(3a+\alpha)]^a/3, \\ y &= z = k[3aT/(3a+\alpha)]^a/3, \\ U_3^D &= 3^{3(a-1)}k^2 a^{3a}\alpha^\alpha [T/(3a+\alpha)]^{3a+\alpha} \end{aligned} \tag{12}$$

where the subscript 3 indicates that the number of individuals or traded goods is now three. From (12) and (6), we have,

$$U_3^D/U_2^D = 3^{(a-1)}(3/2)^{2(a-1)}k = (3/2)^{a-1}(9/8)^{a-1}2^{2(a-1)}k$$

which is clearly larger than one for the case where the division of labor between two individuals dominates autarky, i.e. where $U_2^D/U^A = 2^{2(a-1)}k$ (see Eq.7) is larger than one. Thus, if the transaction efficiency k is large enough and/or the degree of economies of specialization (a) is large enough such that division of labor between two persons is better than autarky, division of labor between three persons is even better still. More generally, using a model with any number of goods, a larger population (up to the number of goods in models where this number is exogenously given) facilitates more specialization and increases per capita utility. Our results so far may be summarized as

PROPOSITION 1. *In our model of division of labor with economies of specialization, an artificial shift in preference from leisure to goods by all individuals decreases the utility of the representative individual evaluated at the original preference. An increase in population size increases utility by allowing more specialization if the original population size is not sufficient for full specialization.*

Could we then conclude that Buchanan's hypothesis is not applicable with respect to a decrease in preference for leisure but only applicable with respect to an increase in population size? This conclusion is yet premature since a different result may be obtained in a different model. The next section examines this.

3. A MODEL OF MIXED HOME AND FIRM/MARKET PRODUCTION

The Yang-Ng framework analyses division of labor from the level of individuals with individual-specific economies of specialization. This has the advantage of tackling the problem at the most basic level of individual choice. The framework has also been used to analyse the emergence of firms, industrialization, urbanization, and other problems (for a survey, see Yang 2003). However, the increasing returns (due to the economies of specialization) involved do not go beyond the individual level. The majority of production in most advanced economies is undertaken by firms with increasing returns prevailing over the whole relevant range of production, but also with individual home production still taking place. Ng & Zhang (2005) combines the Yang-Ng analysis of economies of specialization at the individual level with the Dixit & Stiglitz (1977) analysis of the free-entry (average-cost pricing) monopolistic production by firms. (For an earlier model combining home production with market production by firms emphasizing the role of the number of intermediate goods and different stages of production, see Locay 1990. Here, the complications due to intermediate goods and stages in production are ignored.) In this section, the model developed by Ng & Zhang (2005) is used to examine the role of work ethics.

Consider an economy with M identical consumers, each with the following decision problem for consumption which includes the set R of goods bought in the market from the firms and the set J of goods home produced by the individual herself.

$$\begin{aligned} &\text{Max:} u = l^{1-\alpha-\beta}\left[\sum_{r\in R} x_r^{\rho_1}\right]^{\alpha/\rho_1}\left[\sum_{j\in J} x_j^{\rho_2}\right]^{\beta/\rho_2} \quad \text{(utility function)} \\ &s.t. \sum_{r\in R} p_r x_r = 1 - l - \sum_{j\in J} l_j \ \text{(budget constraint)} \qquad (13) \\ &x_j = \frac{l_j - a}{c} \ \text{(home production function)} \end{aligned}$$

where p_r is the price (all prices are relative to the price of labor which is used as the numeraire) of market good r, x_r is the amount of good r that is purchased from the market, R is the set of market goods, x_j is the amount of home good j, l_j is the amount of labor used in producing home good j, $a < 1$ is a fixed cost parameter which stands for economy of specialization in home production, c is the marginal cost in home production. (For the utility function in Eq. 13 to make sense, a also has to be positive. With a zero, the preference for diversity implied in the utility function will make the home number of goods m go to infinity and the amount of each home good approach zero.) J is the set of home goods, l is leisure, $\left(1 - l - \sum_{j \in J} l_j\right)$ is the amount of labor hired by firms, $\rho_i \in (0, 1)$ is the parameter of elasticity of substitution between each pair of consumption goods, α, β is a preference parameter, and u is utility level. The amount of time each individual has is normalized to unity. Each individual is a price taker and her decision variables are l, l_j and x_r. It is assumed that the elasticity of substitution $1/(1 - \rho) > 1$, or $1 > \rho > 0$ as usual.

The problem in (13) gives the following demand function and home labor utilization, and the number of home goods (for the symmetrical case where all market goods are symmetrical and similarly for all home goods; see Ng & Zhang 2005 for details)[2],

$$x_r = \frac{\alpha\rho_2}{[\rho_2 + \beta(1 - \rho_2)]np} \tag{14}$$

$$l_j = \frac{a}{1 - \rho_2} \tag{15}$$

$$m = \frac{\beta(1 - \rho_2)}{a[\rho_2 + \beta(1 - \rho_2)]} \tag{16}$$

where n is the number of market goods, m is the number of home goods, and p is the price of all the symmetrical market goods.

For the business sector (with symmetrical firms producing for market sales), a model with monopolistic competition with free entry and zero profit (average-cost pricing) similar to the Dixit-Stiglitz model is used. The production function of market good r is

$$X_r = (l_r - A)/b$$

so that the labor cost function of good r is

$$l_r = bX_r + A$$

where X_r is the output of market good r (note that $X_r = Mx_r$), A is the fixed cost, and b the constant marginal cost. (Again A has to be positive, otherwise a profit maximization equilibrium with a constant marginal cost is inconsistent with the zero profit condition, as the demand curve for each good is downward sloping.) This gives rise to increasing returns or decreasing average costs. With the zero profit condition and the profit maximization condition for each firm, together with utility maximization by each individual described above, the general equilibrium of the system

[2] In particular, the Yang-Heijdra (1993) exact formula is used instead of the Dixit-Stiglitz approximation. Hence a slight difference with the Dixit-Stiglitz result is involved.

is given by the following results (dropping the subscript r)

$$\begin{aligned}
p &= \frac{b(n-\rho_1)}{\rho_1(n-1)}, \text{ (price of goods produced by firms)}\\
X &= \frac{\rho_1 A(n-1)}{bn(1-\rho_1)}, \text{ (output of a firm)}\\
x &= \frac{\rho_1 A(n-1)}{bn(1-\rho_1)M}, \text{ (individual consumption of a good produced by a firm)}\\
l &= \frac{\rho_2(1-\alpha-\beta)}{\rho_2+\beta(1-\rho_2)} \text{ (amount of leisure of an individual)} \qquad (17)\\
x_j &= \rho_2 a/(1-\rho_2)c, \text{ (amount of a home produced good)}\\
n &= \frac{M\alpha\rho_2(1-\rho_1)}{A[\rho_2+\beta(1-\rho_2)]} + \rho_1, \text{ (number of goods produced by firms)}\\
m &= \frac{\beta(1-\rho_2)}{a[\rho_2+\beta(1-\rho_2)]}. \text{ (number of home goods produced by an individual)}
\end{aligned}$$

Substituting the relevant variables in (17) into the utility function in (13), we have the equilibrium level of utility in terms of the parameters.

$$\begin{aligned}
u &= l^{1-\alpha-\beta} n^{\frac{\alpha}{\rho_1}} x^{\alpha} m^{\frac{\beta}{\rho_2}} x_j^{\beta}\\
&= \rho_2^{1-\alpha}\rho_1^{\alpha}\beta^{\frac{\beta}{\rho_2}} b^{-\alpha} M^{-\alpha} a^{\beta-\frac{\beta}{\rho_2}} c^{-\beta} A^{\alpha-\frac{\alpha}{\rho_1}} (1-\rho_2)^{\frac{\beta}{\rho_2}-\beta}(1-\alpha-\beta)^{(1-\alpha-\beta)}\\
&\times\ [\rho_2+\beta(1-\rho_2)]^{\alpha-\frac{\alpha}{\rho_1}+\beta-\frac{\beta}{\rho_2}-1}[M\alpha\rho_2 - A(\rho_2+\beta(1-\rho_2))]^{\alpha}\\
&\times\ \{M\alpha\rho_2(1-\rho_1) + A\rho_1[\rho_2+\beta(1-\rho_2)]\}^{\frac{\alpha}{\rho_1}-\alpha} \qquad (18)
\end{aligned}$$

Next, we use the method similar to the derivation of R given in (10) above. We let the various variables be the ones that would prevail with a lower preference for leisure (noting that, in the current model, this involves a higher α and/or β, in contrast to a lower α of the model in the previous section where α is the preference for leisure parameter) but with the utility level (denoted as U') evaluated using the original utility function. We then see how this new utility level U' compares with the original utility level U (that from variables determined by the original preferences, as given in Eq. 17) and see how the ratio $R \equiv U'/U$ changes (or how ln R changes, as R always vary in the same direction as ln R) with respect to a decrease in preference for leisure and an increase in preference for the market goods (from α to α') and/or an increase in preference for the home goods (from

β to β'). We have,

$$\begin{aligned}\frac{\partial \ln R}{\partial \alpha'} &= \frac{\alpha}{\alpha'\rho_1} - \frac{1-\alpha-\beta}{1-\alpha'-\beta'} + \frac{M\alpha\rho_2}{M\alpha'\rho_2 - A[\rho_2 + \beta'(1-\rho_2)]} \\ &- \frac{M\alpha\rho_2}{M\alpha'\rho_2 + A\rho_1[\rho_2 + \beta'(1-\rho_2)]/(1-\rho_1)}\end{aligned} \tag{19}$$

The right hand side of (19) is necessarily positive at $\alpha = \alpha'$, $\beta = \beta'$, since $1 > \rho_i > 0$ for both i, making the first term $\frac{\alpha}{\alpha'\rho_1}$ larger than one and hence exceeds the second term $\frac{1-\alpha-\beta}{1-\alpha'-\beta'}$ which is equal to one. The positive third term and the negative fourth term have the same numerator. The first terms of their denominators are also equal. The first term of the denominator of the third term is negative and that of the fourth term is positive. This means that, as long as the whole denominator of the third term remains positive, the positive third term must be larger than the negative fourth term in absolute value, making the combined third and fourth terms positive. If we substitute the solution for n into the solution for x in (17), we have

$$x = \frac{\rho_1 A\{M\alpha\rho_2 - A[\rho_2 + \beta(1-\rho_2)]\}}{bM\{M\alpha\rho_2(1-\rho_1) + A\rho_1[\rho_2 + \beta(1-\rho_2)]}$$

Since the denominator is positive and $\rho_1 A$ in the numerator is also positive, the remaining part $\{M\alpha\rho_2 - A[\rho_2 + \beta(1-\rho_2)]\}$ in the numerator must also be positive for x to be positive.[3] At $\alpha = \alpha'$, $\beta = \beta'$, this term is the same as the denominator of the third term in (19) which must thus also be positive. Thus, the right hand side of (19) must be positive at $\alpha = \alpha'$, $\beta = \beta'$. This means that, from the original position, a shift in preference from leisure to the market goods increases utility even evaluated in accordance to the original preference.

In the model of this section, given other parameters, the degree of increasing returns to scale in market production is related positively to the fixed cost parameter A. Since A occurs twice in (19) and in both case it is in the denominator and associated with a negative sign and with other associated terms positive, it is straightforward to see and show that

$$\partial^2 \ln R/\partial\alpha'\partial A > 0 \tag{20}$$

[3]Economically, the size of the fixed cost of market production A must not be too large in relation to the population size M. Otherwise the economy may not be viable if the labor of all people combined is insufficient to provide for the fixed cost of production, allowing for the necessity of producing some home goods and having some leisure.

Thus, the higher the degree of increasing returns in the production of market goods, the larger is the utility-improving effect of a shift in preference from leisure to market goods. The discussion so far may be summarized as

Proposition 2. *Where the market goods are produced under conditions of increasing returns and priced at average costs, a shift in preference away from leisure towards market goods increases the utility level of the representative individual even if evaluated in accordance to the original preference. The higher the degree of increasing returns, the larger is this effect.*

It may be thought that the above proposition applies only in our model with both market and home goods. In fact, we may take the special case with $\beta = 0$ which signifies the absence of home goods (the number of home goods produced/consumed $m = 0$ from Eq. 17), the positivity of (19) and (20) is not affected. Thus, proposition 1 applies even to a model with only market goods.

Now consider a shift in preference from leisure towards the home goods. Similar to the derivation of (19), we may derive,

$$\begin{aligned}\frac{\partial \ln R}{\partial \beta'} &= \frac{\beta}{\beta' \rho_2} - \frac{\beta(1-\rho_2)}{\rho_2[\rho_2 + \beta'(1-\rho_2)]} \\ &- \frac{1-\alpha-\beta}{1-\alpha'-\beta'} - \frac{(1-\alpha-\beta)(1-\rho_2)}{\rho_2+\beta'(1-\rho_2)} - \frac{\alpha(1-\rho_2)}{\rho_1[\rho_2+\beta'(1-\rho_2)]} \\ &- \frac{\alpha A(1-\rho_2)}{M\alpha'\rho - A[\rho_2+\beta'(1-\rho_2)]} \\ &- \frac{\alpha A\rho_1(1-\rho_2)}{M\alpha'\rho_2(1-\rho_1) + A\rho_1[\rho_2+\beta'(1-\rho_2)]} \qquad (21)\end{aligned}$$

Noting that $1-\alpha-\beta$ (being of the same sign as leisure l from Eq.17), $\alpha, \beta, \rho_i, 1-\rho_i, A, M$ are all positive, it is easy to see that all terms in the right hand side of (21) except the first term are negative. (The positivity of the denominator of the second last term has already been discussed in the paragraph preceding Eq.21.) From the original position with $\beta = \beta'$, this first term equals $1/\rho_2$. It can be seen that the absolute value of the negative second term must be smaller than $1/\rho_2$, as $\rho_2 + \beta'(1-\rho_2)$ in the denominator must be larger than the term $\beta(1-\rho_2)$ in the numerator. Thus, the first two terms combined to be positive and the signing of the right hand side of (21) appears to be impossible. However, if we combine these first and second terms with the fourth and fifth terms, the combined four terms can be shown to equal

$$1 - \frac{\alpha(1-\rho_1)}{\rho_2 + \beta'(1-\rho_2)}$$

where the first term of one is exactly offset by the third term in the right hand side of (21) which equals negative one at $\alpha = \alpha'$, $\beta = \beta'$. With the negative sign, the second term of the above displayed expression is necessarily negative. Thus, all the seven terms in the right hand side of (21) must add up to be negative at $\alpha = \alpha', \beta = \beta'$. Hence from the original position, a shift in preference from leisure to the home goods decreases utility evaluated in accordance to the original preference.

PROPOSITION 3. *An artificial shift in preference from leisure towards home goods decreases utility evaluated at the original preference despite the fact that home goods are produced under conditions of increasing returns.*

The contrasting results between home goods (Proposition 3) and market goods (Proposition 2) are explained intuitively in the concluding section.

We may also examine the effect of an increase in population M holding other parameters unchanged (including the conditions of increasing returns; this may not be possible for a very large M). From (18), we have

$$\begin{aligned}\frac{\partial \ln u}{\partial M} &= \frac{A\alpha[\rho_2 + \beta(1-\rho_2)]/M}{M\alpha\rho_2 - A[\rho_2 + \beta(1-\rho_2)]} \\ &+ \frac{\alpha^2\rho_2(1-\rho_1)^2/\rho_1}{M\alpha\rho_2(1-\rho_1) + A\rho_1[\rho_2 + \beta(1-\rho_2)]} > 0 \end{aligned} \tag{22}$$

where the positivity follows from the positivity of the denominator of the first term in the right hand side as discussed in the discussion below (19). We have

PROPOSITION 4. *An increase in population, with other parameters unchanged (if feasible) increases utility by allowing the spreading of the fixed costs in market production.*

The intuition of the various propositions above is discussed in the next section.

4. CONCLUDING REMARKS

In this paper, it has been shown that

- In a model of division of labor with economies of specialization at the level of individual/household production, a shift in preference from leisure towards goods decreases individual utility evaluated in accordance to the original preference.

• If the original population size is not large enough to allow for full specialization, an increase in population increases utility by facilitating more specialization.

• In the extended model allowing for both home and firm/market production under increasing returns and with average-cost pricing for firms, a shift (if not excessive) in preference from leisure towards market goods increases individual utility even if evaluated in accordance to the original preference.

• A shift in preference from leisure towards home goods decreases individual utility evaluated in accordance to the original preference.

• An increase in population with other parameters remaining unchanged (if feasible) increases utility by allowing the spreading of the fixed costs in market production.

The intuitive reasons for some of the above results may be mentioned. In the presence of increasing returns, the average cost of producing a good is falling with output and the marginal cost is below average cost. With average-cost pricing of firms (due to free entry), market goods are consumed until marginal valuation equals price which equals average cost. However, efficiency requires the equality of marginal valuation and marginal cost. Thus, market goods are under consumed. A shift in preference from leisure towards market goods may thus increase efficiency. Although home goods are also produced under the condition of increasing returns in the model of Section 3 above, the problem does not arise. Home production is consumed by the individual/household and the decision on how much to produce/consume concerns the individual only. Hence, the individual takes the effect of increasing returns fully into account and optimize accordingly. There is thus no under production/consumption in home goods. An artificial shift in preference from leisure towards home goods is thus inefficient evaluated in accordance to the original preference. For market goods produced by firms, the price determined in accordance to average cost is common to all individuals consuming the good. In our model, increasing returns arise from a fixed cost component with a constant marginal cost. The fixed cost component is shared by all consumers and spread over all units of the good produced. It thus processes the essential element of publicness as in a public good. An increase in population size helps to reduce the average cost of producing such a good and hence increases the utility of an existing individual. Also, with increasing returns, higher consumption helps to reduce the average cost for other individuals. This possesses the aspect of external benefits not taken into account by the optimization decisions of individuals. This explains the beneficial effects of an increase in preference for market goods by all individuals on all individuals even in accordance to the original preferences.[4]

The above explanation, while relevant, is yet incomplete, at least for the long run. In the short run, efficiency requires the equation of marginal valuation with the short run marginal cost and the above explanation is adequate. However, in the long run, efficiency requires the equation of marginal valuation with the long run marginal cost. With free entry and average-cost pricing, the long-run marginal cost is determined more by the average than by the (short-run) marginal cost. In the model in Section 3, the costs (relative to the price of labor) of firms are assumed to be independent of the number of firms. A shift in preference from leisure to market goods increases the demand for the product of each firm, given the number of firms in the short run. As the cost curve/function of the firm remains unchanged, this increases the profits of firms and induces entry. In the long-run equilibrium, profits become zero again. What is the gain? The gain consists of two parts. First, the larger number of firms/products increases utility by providing a wider range of choice. The individual utility function posited values variety and hence utility is increased this way. Secondly, with the larger number of firms, the demand for the product of each firm becomes more elastic and hence the demand curve is tangent to the unchanged cost curve at a lower price. Thus, despite the fact that the number of firms/products increases, the quantity produced by each firm also increases. This helps the average cost of production to be lowered due to the spreading of the fixed cost component over more units. This can be confirmed in the model of Section 3 by showing that the shift of preference from leisure to market goods will lower the price and increase the output of each market good.

APPENDIX

Here, we use a specific case (with the relevant parameters assuming some specific values) of the model in Section 3 to verify a result there. The assumed parametric values are: $\alpha = \beta = 1/4$, $\rho_1 = \rho_2 = 1/2$, $M = 49,500$, $A = 100$, $b = 0.0001$, $a = 0.01$, $c = 0.001$. We may then calculate the equilibrium values of the relevant variables from (17),

$$m = 20, x_j = 10, n = 50, x = 19.79798, l = 0.4. \qquad \text{(A.1)}$$

It may also be checked that these values satisfy the budget constraint in (13). For the symmetrical case, the utility function in (13) may be written

[4]Further welfare implications of increasing returns are examined in Ng & Zhang (2005).

as

$$u = l^{1-\alpha-\beta} n^{\frac{\alpha}{\rho_1}} x^{\alpha} m^{\frac{\beta}{\rho_2}} x_j^{\beta} \quad \text{(A.2)}$$

Substituting the equilibrium values of the variables as given in (A1) into this utility function, we have

$$u = 75.0214. \quad \text{(A.3)}$$

However, if we let α artificially increases (for all individuals) from 0.25 to 0.3762626 with all other parameters (including β) remain unchanged (thus the change corresponds to a shift in preference from leisure to market goods), the new set of values for the relevant variables is now

$$m = 20, x_j = 10, n = 75, x = 19.93266, l = 0.2989899 \quad \text{(A.4)}$$

where n and x increase at the expense of leisure l. It may again be checked that this new set of variable values also satisfy the budget constraint in (13). Substituting this new set of values into the utility function in (A2) but with the value of α there remaining at 0.25, we have

$$u = 79.573 \quad \text{(A.5)}$$

which is higher than that in (A3). Thus, if everyone shifts preference from leisure to market goods produced under increasing returns, utility, even evaluated at the original preference, may increase for everyone.

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