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Uninsurable Risks: Uncertainty in Production, the Value of Information and Price Dispersion

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This article digresses over the interaction of uncertainty with the firm's optimal decisions in a simple framework: a standard price-taking (short-run restricted) single-input and output unit, subject to the interaction with a zero-mean Bernoulli lottery of variable dispersion.

The firm is always considered an expected profit-maximizing entity. We inspect the consequences of exogenous uncertainty on the optimal allocations and on its "mean-(and)variance" valuation position. On the one hand, we contrast the effect of different sources of uncertainty on the producer's problem — input and output prices and quantities. On the other, we analyse the impact of ex-post flexibility of the decision variables.

Importance and role of measures of risk-aversion (of concavity and convexity) imbedded in the firms technology — either the production, marginal productivity or the cost function, — and potentially risk-enhancing or deterrent features of the latter in the transmission of exogenous uncertainty to the optimal profits' mean and volatility under the different scenarios are highlighted.

Key Words: Uncertainty and Production; Uncertainty and Labor Demand; Firm's Valuation; Mean-Variance; Commitment under Uncertainty; Risk-aversion; Absolute Convexity; The Value of Information/ Flexibility to a Firm; Statistical Discrimination.

JEL Classification Numbers: D80, J23, J24, J71, L14, L15.

1. INTRODUCTION

The foundations of the theory of uncertainty are centred in the role of consumers' preferences and behaviour. Risk-aversion measures, in the Arrow (1965) and Pratt (1964) sense[1], have wide recognition in the in-

[1]Prudence, Kimball (1990). Temperance, Gollier and Pratt (1996) .

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formation literature[2]. The impact of exogenous dispersion on producers' decisions is less unified. It is the purpose of this research to provide general conclusions in this field, confronting different scenarios for the source of uncertainty, and also for the reaction ability of the decision-maker.

The positive effect of (input[3] as output[4]) price dispersion on expected profits with ex- post flexibility is well-known in the literature. Technically, it is a consequence of the convexity of the profit function in output and input prices[5]. Effects of such uncertainty on expected profits stem standardly from comparable measures and indicators, usually oriented towards expected utility — of background uncertainty in other prices can be inferred from other literature[6]. More revealing is an ex-ante decision commitment scenario[7] — that if combined in some literature[8], has not been directly contrasted with it. This comparison is particularly important, once it gives the value the firm attributes to early information — or to flexibility[9]. Legally, it connects with the expected minimum deterrent penalty against contract default. Moreover, if we extend the analysis to wage rates, effects — and/or reverse causes — of the rise in its dispersion observed in the last decade may come forward. On another angle, we can see the firm accomplishing some volatility itself towards the output price through quantity discount or more or less regular product promotion policies.

Technology uncertainty has also been staged[10]; one can consider its consequences as similar to those of random input quality — as price uncertainty can simulate random output quality settings, of unplanned better (worse) product that can (must) be sold at a higher (lower) price. The effect of labour quality dispersion would belong to statistical discrimination[11] — generating (or not) an explanation for the employer's preference for lower quality dispersion (even if with the same mean) of the group hired, implying a higher equilibrium wage for its members. However, a link between both effects is not generally found. And importance of risk-aversion measures for factor demand or output supply response in the (either) context rarely disentangled.

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[2]See Gollier (2001) for a recent survey on the theory of uncertainty. Also, Laffont (1989), Karni and Schmeidler (1991), Hirshleifer and Riley (1992).

[3]Rothemberg and Smith (1971).

[4]Oi (1961).

[5]See Varian (1992) for references. It is indifferent whether the price argument affects positively or negatively the objective function — see Martins (2004), for example, for the definition of the premium to a risk added to variable but defined in anothers metric.

[6]See Martins (2004), for example.

[7]Sandmo (1971), relying on a cost function approach only. Batra and Ullah (1974), Ha rtman (1975). These authors contemplated price uncertainty.

[8]Rothemberg and Smith (1971), Hartman (1976).

[9]Of course, the concept would also apply to consumer theory.

[10]See Feldstein (1971), for example.

[11]See Phelps (1972).

Aiginger (1987) contains the most thorough survey of producer's theory under uncertainty that we know of[12]. He identifies benchmark results obtained by different authors for several market organization scenarios (competition/monopoly), distinguishing types of uncertainty (price/quantity; additive/multiplicative), firms reaction ability (ex-ante/ex-post), and inspected effects (prices/factors/output; in general, expected profits). It was our purpose to simulate the addition of a zero-mean randomness to each of the firm's decision terms and infer and contrast the effect of increasing its dispersion simultaneously on all the relevant economic aggregates under the different assumptions. We restricted ourselves to a competitive environment and a single-output, single-input firm. Unlike the usual literature, we highlight the hiring (factor demand) and production technology — even if the complementary conditions for cost functions are also cited.

We assume that firms maximize expected profits[13], and — in line with a mean-variance[14] behaviour of investors -, refer the transmission of the exogenous uncertainty to the variance of the optimal profits, a feature usually neglected in standard theory.

We stage a simple random environment: a Bernoulli lottery — two states of nature — with zero mean added, linearly or proportionately, to a relevant aggregate. The inspection of the impact of a rise in the spread between the two possible outcomes has generated the same consequences for the interpretation of risk-aversion measures as those revealed by general probability distributions in more complex scenarios than usual[15]. Moreover, we wanted to replicate the usefulness of these indicators as reflecting the role of concavity and convexity on the producer's side. And indeed we were able to.

The exposition proceeds as follows: in section 2, we advance general notation and highlight some properties of the uncertainty environments simulated in the text. Section 3, considers the impact of quantity and/or quality uncertainty. Section 4 explores the opportunities opened to the firm by ex-post flexibility in quantity decisions. Section 5 generates conclusions for a firm that faces price dispersion. Some considerations on the effects of (also) randomly affected control variables are advanced in section 6. The exposition ends with some concluding remarks.

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[12]Drèze (1987) also approaches the subject, but relying on a cost function approach only. He focuses on demand uncertainty and concludes on the relation between the equilibrium number and size of competitive firms in the market and uncertainty.

[13]That is, we will always assume "von Neumann-Morgenstern" profits.

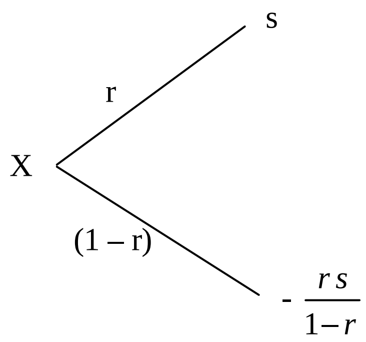
[14]See Tobin (1958), for example.

[15]See Martins (2004), for example.

2. NOTATION: RANDOMNESS EXPOSURE AND THE FIRM

We admit a firm that maximizes profits, facing output (q) price P and hiring labor L at unit cost w. It has a production function $q = f(L)$, continuous, increasing, concave and differentiable to several orders in L. It enjoys a cost function $C(q)$ continuous, increasing, convex and differentiable to several orders in q. If only L is adjustable — in the short-run —, $C(q) = wf^{-1}(q) + F$, where $f^{-1}(q)$ denotes the inverse production function and F fixed costs.

We consider a lottery that with probability r generates the reward s and with probability $(1 - r)$ the loss s', having null expected value — i.e., $rs - (1 - r)s' = 0$; then $s' = \frac{rs}{1-r}$. Diagrammatically:



The lottery will have variance $Var(X) = rs^2 + (1 - r)\left(-\frac{rs}{1-r}\right)^2 = \frac{rs^2}{1-r}$. The larger s (for given r) if positive, the higher the variance of the lottery — although its expected value remains zero: the more distant will the two possible outcomes be from each other; if negative, the larger s (that is, the less negative), the smaller the variance, that is, the uncertainty represented by the lottery — the closest will the two possible outcomes be. Then, we can represent an increase in uncertainty by a rise in s if $s > 0$, a decrease in s if $s < 0$. We shall analyse the effects of additive uncertainty on a given variable over other parameters by inspecting how these react to s when we add the lottery X to the former variable in accordance with those principles[16].

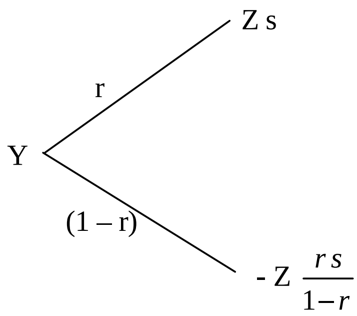
Yet, for given s, $Var(X)$ also increases with r. But r is a parameter that we would rather inspect if we wanted to study asymmetry, in particular 3rd centered moments. It is easy to show that $E\{(XCE[X])^3\} = \frac{r(1-2r)s^3}{(1-r)^2} = \frac{(1-2r)s}{1-r}Var(X)$. One can show that $E\{(XCE[X])^3\}$ decreases (increases)

[16]Such simulation with s reproduces Rothschild and Stiglitz (1970) notion of "mean-preserving spreads".

with r iff $r > (<)\frac{1}{3}$ -hence, around $\frac{1}{2}$. Skewness of the distribution of X increases if some measure related to and with the sign of $E\{(XCE[X])^3\}$ (say, the measure of asymmetry $E\{(XCE[X])^3\}/Var(X)^{3/2}$) becomes more positive when $r < \frac{1}{2}$, more negative when $r > \frac{1}{2}$.

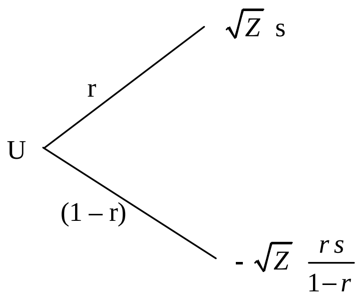
The lottery will never be symmetric nor $E\{(XCE[X])^3\}$ independent of $Var(X)$ unless $r = \frac{1}{2}$: we could, therefore, specify results for this special case. Nevertheless, our sign results 2 revealed themselves as invariant to r.

In some contexts, the addition of a "proportional" lottery may be more appropriate — generating multiplicative uncertainty relative to a deterministic variable Z. Then, it is as if Z is added of the lottery $Y = ZX$:



It is still the case that $E[Y] = 0$. Y will have now variance $r(Zs)^2 + (1-r)\left(-Z\frac{rs}{1-r}\right)^2 = Z^2\frac{rs^2}{1-r}$ and this reacts qualitatively to s, for given Z, according to the same principles as before. Measures of relative risk-aversion will have relevance in this context — see Appendix A for the confrontation of the risk-premium for additive and multiplicative lotteries.

The variance of the sum of n independently identically distributed random variables, is n times their unitary variance. Therefore, and with analogy with such context, we may consider the null expected value lottery U to be added to Z as:



Its variance will be $r(\sqrt{Z}s)^2 + (1-r)\left(-\sqrt{Z}\frac{rs}{1-r}\right)^2 = Z\frac{rs^2}{1-r}$ and proportional to Z as required. We will experiment briefly with the addition of this randomness and term it factored or unitary uncertainty — see in Appendix A the relation between the risk-premium for such a lottery and properties of the consumer's utility function.

3. QUANTITY/QUALITY UNCERTAINTY UNDER COMMITMENT

3.1. Factor Uncertainty

1) Additive Uncertainty

Admit ex-ante uncertainty — non-observed dispersion - with respect to labor quality, or in the technical efficiency with which it is used, in such a way that the output obtained with L labor units is $f(L+s)$ with probability r and $f(L-\frac{rs}{1-r})$ with probability $(1-r)$. The firm hires L — pays its unitary cost w — but it can either get $f(L+s)$ as $f(L-\frac{rs}{1-r})$ of output with it. The firm maximizes:

$$\max_L E\pi(L) = rPf(L+s) + (1Cr)Pf(L - \frac{rs}{1-r}) - wL \qquad (1)$$

The F.O.C. requires that the optimal hiring will be L^* such that the expected value of the value of the marginal product of labour equals the wage rate:

$$rPf'(L^*+s) + (1Cr)Pf'(L^* - \frac{rs}{1-r}) = w \qquad (2)$$

We can infer the effect of an increase in uncertainty over the optimal $L^* = L(P, W, s, r)$ simulating a variation in s:

$$rPf''(L^*+s)(dL^*+ds) + (1Cr)Pf''(L^* - \frac{rs}{1-r})(dL^* - \frac{r}{1-r}ds) = 0$$

Then:

$$\frac{dL^*}{ds} = r\frac{f''(L^* - \frac{rs}{1-r}) - f''(L^*+s)}{rf''(L^*+s) + (1-r)f''(L^* - \frac{rs}{1-r})} \qquad (3)$$

For S.O.C. to be observed, the denominator must be negative (which will be satisfied if $f''(.) < 0$).

Being $s > 0$, an increase in s performs an increase in uncertainty. It will generate a:

-rise in L^* (and in costs) iff $\frac{dL^*}{ds} > 0$, which requires $f''(L^* - \frac{rs}{1-r}) < f''(L^*+s)$. This will hold iff $f''(.)$ rises with the argument, that is, if $f'''(.) > 0$ — or the marginal product of labor, $f'(.)$, (the inverse firm's labor demand under certainty; as the direct labor demand, as the function has only one argument and is negatively sloped) is convex. That will be the case of constant-elasticity demand functions, for example.

-decrease in L^* iff $\frac{dL^*}{ds} < 0$, requiring $f''(L^* - \frac{rs}{1-r}) > f''(L^* + s)$. Then, $f'''(.) < 0$ — the marginal productivity of labor, $f'(.)$ (firm's labor demand or its inverse) is concave.

Being $s < 0$, a rise uncertainty correspond to a decrease in s. It implies:

-a rise in L^* iff $\frac{dL^*}{ds} < 0$, requiring $f''(L^* - \frac{rs}{1-r}) > f''(L^* + s)$. As s is negative, that will occur iff $f'''(.) > 0$ — that is, if $f'(.)$ is convex.

-a decrease in L^* iff $\frac{dL^*}{ds} > 0$, requiring $f''(L^* - \frac{rs}{1-r}) < f''(L^* + s)$, compatible with $f'''(.) < 0$ — marginal productivity, $f'(.)$ concave.

Therefore, in any case, a rise in uncertainty will decrease firm (employment) size if the (short-run) labor demand is concave, it will rise it if it is convex. The intuition is simple if one contrasts the setting with another without uncertainty: the firm equates the expected value of marginal product of labor to the wage. Being the value of marginal product a concave function, its expected value is smaller than the value of the expected argument (of $L^* + X$, which is always L^*); being the marginal product a negative function of the argument, to insure a given fixed expected value of marginal product $w^{\#}$, L^*, the expected value of the argument under uncertainty, must be lower than the certain quality $L^{\#}$ chosen. Graphically — Fig. 1 —, L^* is chosen such that the line that connects the points $[L_1, Pf(L_1)]$ and $[L_2, Pf(L_2)]$, where $L_1 = L^* - \frac{rs}{1-r}$ and $L_2 = L^* + s$, intersects $w^{\#}$ at the horizontal axis value L^*; of course, such line is below $Pf'(L)$ if this is concave; hence $L^{\#}$, the demand for certain quality hired at wage $w^{\#}$, read on $Pf'(L)$, must be to the right of L^*: uncertainty in quality decreases employment.

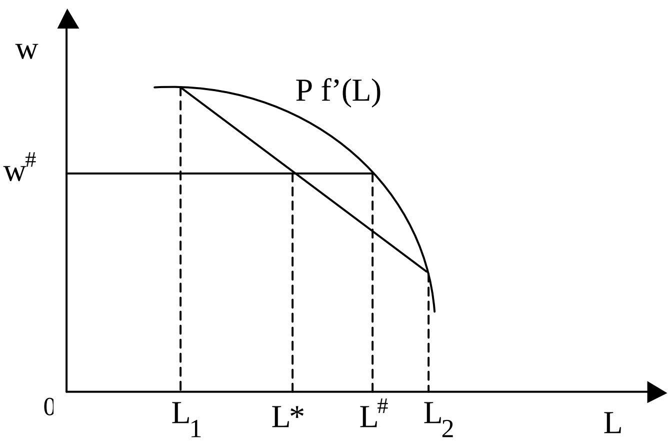


FIG. 1.

If $f'(L)$ is convex, we infer the opposite effect.

One can visualize the expansion of the numerator of (3) to the first order as $-f'''(L^*)(s+\frac{rs}{1-r})$. Then, $\frac{dL^*}{ds}$ approximates $(s+\frac{rs}{1-r})$ times $-\frac{f'''(L^*)}{f''(L^*)}$: $\frac{dL^*}{ds}$ would be directly proportional to absolute prudence[17] imbedded on the production function - the absolute risk-aversion measure of the marginal product function (Yet, as $f'(L)$ is negatively sloped, one could sustain that risk aversion embedded in the function $f'(L)$ would more appropriately be measured by plus $\frac{f'''(L)}{f''(L)}$: a higher premium in the metric of and added to L will be borne by a maximizer of $f'(L)$ to avoid randomness in L — see Appendix A). We recover, thus, an application of Kimball's (1990) statements — absolute prudence captures the "sensitivity of the optimal choice of a decision variable to risk".

We can also conclude that optimal expected output, $E[q^*] = rf(L^* + s) + (1Cr)f(L^* - \frac{rs}{1-r})$, reacts to s according to:

$$\begin{aligned}
&\frac{dE[q^*]}{ds}\\
=&[rf'(L^*+s)+(1Cr)f'(L^*-\frac{rs}{1-r})]\frac{dL^*}{ds}+r[f'(L^*+s)-f'(L^*-\frac{rs}{1-r})]\\
=&\frac{w}{P}\frac{dL^*}{ds}+r[f'(L^*+s)-f'(L^*-\frac{rs}{1-r})]
\end{aligned}$$

If $s > 0$, as $f''(L) < 0$, the last term is negative; even if $\frac{dL^*}{ds} > 0$, $E[q^*]$ may decrease with uncertainty. Developing the first expression, we arrive at:

$$\frac{dE[q^*]}{ds} = r\frac{f'(L^*+s)f''(L*-\frac{rs}{1-r}) - f'(L^*-\frac{rs}{1-r})f''(L^*+s)}{rf''(L^*+s)+(1-r)f''(L^*-\frac{rs}{1-r})} \tag{4}$$

Being $s > 0$, $\frac{dE[q^*]}{ds} < 0$ and $E[q^*]$ decreases with uncertainty iff $-\frac{f''(L^*+s)}{f'(L^*+s)} > -\frac{f''(L^*-\frac{sr}{1-r})}{f'(L^*-\frac{sr}{1-r})}$. That is, if the Arrow-Pratt measure of absolute risk-aversion (measuring the concavity) of the production function, $f(L)$ (which is increasing in L: $f'(L) > 0$), $r(L) = -\frac{f''(L)}{f'(L)}$, is increasing in the argument. q^* increases with uncertainty iff $-\frac{f''(L)}{f'(L)}$ is decreasing in L — this is the case of (concave) constant elasticity production functions (in fact, Cobb-Douglas technologies).

For $s < 0$, the last term is positive: even if $\frac{dL^*}{ds} < 0$ (L^* rises with uncertainty), $\frac{dq^*}{ds}$ may be positive — and a rise in uncertainty (a fall of

[17]Defined by Kimball (1990). Its importance is recognized in intertemporal contexts of consumer decisionmaking — see Carroll and Kimball (1996) for a recent example.

s) have a negative impact of output's expected value. $\frac{dE[q^*]}{ds} > 0$ and q^* decreases with uncertainty iff $-\frac{f''(L^*+s)}{f'(L^*+s)} < -\frac{f''(L^*-\frac{sr}{1-r})}{f'(L^*-\frac{sr}{1-r})}$; as now $s < 0$, we still require the Arrow-Pratt measure of risk aversion of the production function, $z(L) = -\frac{f''(L)}{f'(L)}$, to be increasing with the argument.

A further qualification — and an intuition — for the assertion can be made by the development of the meaning of increasing and decreasing risk-aversion. Risk aversion embedded in $f(L)$ will increase with the argument, L, iff:

$$-\frac{f''(L)}{f'(L)} > -\frac{f'''(L)}{f''(L)} \tag{5}$$

if absolute risk aversion is larger than absolute prudence; that is, if the production function is more concave than the marginal product function is (once $f''(L) < 0$) convex. The effect of uncertainty on $E[q^*]$ would be a composition of two effects: an indirect effect determining the effect of uncertainty, X, on L^*, working through — as we saw above — the convexity of $f'(.)$, and a direct additive effect of uncertainty around L on q, both compounded through the concavity of $f(.)$. If $f'(.)$ is concave, L^* decreases with uncertainty — the more negative $f'''(L)$ is - suggesting a decrease in q^*; then the indirect affect counteracts (and powers) the direct one more intensely the more concave the production function is.

The measure of absolute risk-aversion also corresponds to the symmetric of the semi-elasticity of the marginal product function. We can rewrite $\frac{dE[q^*]}{ds}$ as approximately proportional to $-f'(L^*)\frac{\frac{d[-\frac{f''(L^*)}{f'(L^*)}]}{dL^*}}{-\frac{f''(L^*)}{f'(L^*)}}$, that is, to the symmetric of the semi-elasticity of the absolute risk-aversion measure with respect to the argument times the marginal product function. Or to $f'(L^*)\{-\frac{f'''(L^*)}{f''(L^*)}[-\frac{f''(L^*)}{f'(L^*)}]\}$: the difference between prudence and risk-aversion of the production function, factored by the marginal product; that is, to the difference of the semi-elasticities of the marginal product and of $f''(L^*)$, factored by the value of marginal product.

With respect to the optimal expected profits:

$$\begin{aligned}\frac{dE[\pi^*]}{ds} &= P\{rf'(L^*+s)+(1-r)f'(L^*-\frac{rs}{1-r})]\frac{dL^*}{ds} \\ &+ r[f'(L^*+s)-f'(L^*-\frac{rd}{1-r})]\} - w\frac{dL^*}{ds} \\ &= Pr[f'(L^*+s)-f'(L^*-\frac{rs}{1-r})]\end{aligned} \tag{6}$$

Once $f(.)$ is concave, $f''(.) < 0$, optimal expected profits decrease with uncertainty: $\frac{dE[\pi^*]}{ds} < 0$ if $s > 0$; $\frac{dE[\pi^*]}{ds} > 0$ if $s < 0$.

Expanding (1.6) by Taylor's series around L^*:

$$\frac{dE[\pi^*]}{ds} = Pr\{(s + \frac{rs}{1-r})f''(L^*) + [1 - \left(\frac{r}{1-r}\right)^2]\frac{s^2}{2}f'''(L^*) + \cdots\}$$

We conclude that expected profits sensitivity to the outside turbulence will be higher ($\frac{dE[\pi^*]}{ds}$ will be smaller — more negative — for $s > 0$), the higher the output price P on the one hand, and the more distant $f'(L^* + s)$ and $f'(L^* - \frac{rs}{1-r})$ are on the other; the latter is compatible with a more negative $f''(L)$ — a more concave production function, more negatively sloped marginal product function, less negatively "sloped" firm's (short run) demand for the factor. And/or a smaller size (a smaller L^*) if the marginal product function is convex and $f''(L)$ rises (becomes less negative) with the argument — if L^* rises with uncertainty; a larger size if the marginal product function is concave, that is, L^* decreases with uncertainty.

One can infer that the variance of profits:

$$\begin{aligned}
& Var(\pi^*) \\
= \; & rP^2[f(L^*+s) - rf(L^*+s) - (1-r)f(L^* - \frac{rs}{1-r})]^2 \\
+ \; & (1-r)P^2[f(L^* - \frac{rs}{1-r}) - rf(L^*+s) - (1-r)f(L^* - \frac{rs}{1-r})]^2 \\
= \; & P^2(1-r)r[f(L^*+s) - f(L^* - \frac{rs}{1-r})]^2 \\
= \; & P^2(1-r)r\{(s + \frac{rs}{1-r})f'(L^*) + [1 - \left(\frac{r}{1-r}\right)^2]\frac{s^2}{2}f''(L^*) + \cdots\}^2 \qquad (7) \\
& \frac{dVar[\pi^*]}{ds} \\
= \; & 2P^2(1-r)r[f(L^*+s) - f(L^* - \frac{rs}{1-r})] \\
\cdot \; & \{[f'(L^*+s) - f'(L^* - \frac{rs}{1-r})]\frac{dL^*}{ds} + [f'(L^*+s) + \frac{r}{1-r}f'(L^* - \frac{rs}{1-r})]\} \\
= \; & 2P^2r[f(L^*+s) - f(L^* - \frac{rs}{1-r})] \\
\cdot \; & \frac{(1-r)f'(L^*+s)f''(L^* - \frac{rs}{1-r}) + rf'(L^* - \frac{rs}{1-r})f''(L^*+s)}{rf''(L^*+s) + (1-r)f''(L^* - \frac{rs}{1-r})} \qquad (8)
\end{aligned}$$

For $s > 0$, $\frac{dVar[\pi^*]}{ds} > 0$ and $\frac{dVar[\pi^*]}{ds} < 0$ if $s < 0$: the variance of profits always rises with uncertainty.

Inspection of the two last expressions suggests that outside turbulence will imply higher profit volatility the higher the marginal product — ultimately, the higher the wage rate, and in a proportional relation to its square. Possibly, it is not so sensitive to the output price in the context: as the expected marginal productivity is equated to $\frac{w}{P}$, P^2 tends to cut with the effect of this denominator.

(In Appendix B we develop some considerations about the effect of outside uncertainty on the expected value and variance of a function the argument of which is added of a generic random variable. We note, however, that the context here differs: we are inspecting the optimal profits after internalizing the effect of uncertainty on the control variables themselves. The peculiarities of our distribution, potentially asymmetric, may also be important for the effects on the variance.)

One concludes that the trade-off between $Var(\pi^*)$ and $E[\pi^*]$ is always negative and varies according to:

$$\begin{aligned} &\frac{dVar[\pi^*]}{dE[\pi^*]} \\ &= 2P\frac{f(L^*+s)-f(L^*-\frac{rs}{1-r})}{f'(L^*+s)-f'(L^*-\frac{rs}{1-r})} \\ &\cdot \frac{(1-r)f'(L^*+s)f''(L^*-\frac{rs}{1-r})+rf'(L^*-\frac{rs}{1-r})f''(L^*+s)}{rf''(L^*+s)+(1-r)f''(L^*-\frac{rs}{1-r})} < 0 \end{aligned} \quad (9)$$

By accepting higher uncertainty in factor quality the firm cannot improve its expected profit position.

Finally, it is straightforward to show that the variability of supply is

$$Var(q^*) = (1-r)r[f(L^*+s)-f(L^*-\frac{rs}{1-r})]^2 \quad (10)$$

and powered by the level of the marginal product function, that is, of the wage price ratio $\frac{w}{P}$, squared.

PROPOSITION 1. *Consider a standard price-taking, expected profit maximizing firm subject to ex-ante commitment in hiring decisions and additive uncertainty in input quality. After a rise in such uncertainty, the firm's:*

1.labor demand decreases (increases) if the marginal product function is concave(convex) in the argument.

2.expected output or supply decreases (increases) if concavity of the production function is more (less) pronounced than the convexity of the marginal product one as measured by the Arrow-Pratt measure of absolute risk aversion. Equivalently, if this measure for the production function rises (decreases) with the argument, the factor.

3.expected profits decrease — more pronouncedly, the higher the output price and the more concave is the production function. Their volatility increases — more intensely, the higher the (square of the) wage rate.

4.variance of output supply increases, more intensely, the higher the (square of the) marginal product or wage-price ratio.

2) Multiplicative Uncertainty
Multiplicative uncertainty has similar qualitative effects. Suppose the n that the firm suffers a multiplicative impact of the lottery X relative to the labor force it hires, that is the X is a proportional deviation of efficient-units that a given L employed generates, and hence, the production function is really $f(L+LX)=f[L(1+X)]$. The firms problem becomes:

$$\max_L E\pi(L)=rPf[L(1+s)]+(1-r)Pf[L(1-\frac{rs}{1-r})]-wL \qquad (11)$$

F.O.C. requires still require that the expected value of the value of the marginal product of labour equals the wage rate:

$$rPf'[L^*(1+s)](1+s)+(1-r)Pf'[L^*(1-\frac{rs}{1-r})](1-\frac{rs}{1-r})=w \qquad (12)$$

Then:

$$\frac{dL^*}{ds}= \qquad (13)$$
$$r\frac{f'[L^*(1-\frac{rs}{1-r})]+f''[L^*(1-\frac{rs}{1-r})]L^*(1-\frac{rs}{1-r})-f'[L^*(1+s)]-f''[L^*(1+s)]L^*(1+s)}{rf''[L^*(1+s)](1+s)^2+(1-r)f''[L^*(1-\frac{rs}{1-r})](1-\frac{rs}{1-r})^2}$$

The denominator is negative by S.O.C. L^* will decrease with uncertainty iff $g(L)=f'(L)+Lf''(L)$ is decreasing in L. That is to say, if $G(L)=Lf'(L)$, the marginal product times the factor, is a concave function. Or that $2f''(L)+Lf'''(L)<0$ and — once $f''(L)<0$ — the Arrow-Pratt measure of relative risk aversion of the marginal product function $f'(L)$, $-\frac{Lf'''(L)}{f''(L)}$ (again, we note that, once $f''(L)<0$, relative risk-aversion of $f'(L)$ could be inferred by plus $\frac{f'''(L)L}{f"(L)}$) is smaller than 2:

$$-\frac{Lf'''(L)}{f''(L)}<2 \qquad (14)$$

This will be the case for constant-elasticity — Cobb-Douglas type-production functions.

Expected supply reacts to s according to:

$$
\begin{aligned}
\frac{dE[q^*]}{ds} &= \{rf'[L^*(1+s)](1+s) + (1-r)f'[L^*(1-\frac{rs}{1-r})](1-\frac{rs}{1-r})\}\frac{dL^*}{ds} \\
&+ rL^*\{f'[L^*(1+s)] - f'[L^*(1-\frac{rs}{1-r})]\} \qquad (15) \\
&= \frac{w}{P}\frac{dL^*}{ds} + rL^*\{f'[L^*(1+s)] - f'[L^*(1-\frac{rs}{1-r})]\} \\
&= \frac{r}{rf''[L^*(1+s)](1+s)^2 + (1-r)f''[L^*(1-\frac{rs}{1-r})](1-\frac{rs}{1-r})^2} \\
&\cdot \{f'[L^*(1+s)]f''[L^*(1-\frac{rs}{1-r})]L^*(1-\frac{rs}{1-r}) \\
&- f'[L^*(1-\frac{rs}{1-r})]f''[L^*(1+s)]L^*(1+s) \\
&- \frac{w}{P}(f'[L^*(1+s)] - f'[L^*(1-\frac{rs}{1-r})])\} \qquad (16)
\end{aligned}
$$

That is, $\frac{dE[q^*]}{ds} > 0$ for $s > 0$ and q^* will increase with uncertainty iff

$$\frac{\frac{w}{P} - f''[L^*(1-\frac{sr}{1-r})]L^*(1-\frac{sr}{1-r})}{f'[L^*(1-\frac{sr}{1-r})]} > \frac{\frac{w}{P} - f''[L^*(1+s)]L^*(1+s)}{f'[L^*(1+s)]}$$

that is $\frac{\frac{w}{P}-f''(L)L}{f'(L)}$ is decreasing with L. It will decrease with uncertainty if $\frac{\frac{w}{P}-f''(L)L}{f'(L)}$ is increasing with L. Note that near the optimal solution, $\frac{w}{P} = E[f'(L+X)]$ and close to $f'(L)$. Then, $\frac{\frac{w}{P}-f''(L)L}{f'(L)}$ reacts to L in that neighbourhood as the measure of relative risk aversion (of relative concavity) of the production function, $-\frac{f''(L)L}{f'(L)}$ does (this measure is constant for constant-elasticity — Cobb-Douglas type - production functions, for example). One can develop the concept of increasing relative risk aversion easily: $-\frac{f''(L)L}{f'(L)}$ will increase with L iff:

$$-\frac{f'''(L)L}{f''(L)} - [-\frac{f''(L)L}{f'(L)}] < 1 \qquad (17)$$

That is, if relative prudence — the relative risk aversion measure of the marginal product function — minus relative risk aversion is smaller than 1.

As for expected profits:

$$\begin{aligned} & \frac{dE[\pi^*]}{ds} \\ = & PrL^*\{f'[L^*(1+s)] - f'[L^*(1-\frac{rs}{1-r})]\} \qquad (18) \\ = & PrL^*\{(s+\frac{rs}{1-r})L^*f''(L^*) + [1-\left(\frac{r}{1-r}\right)^2]\frac{(L^*s)^2}{2}f'''(L^*) + \cdots\} \end{aligned}$$

They always decrease with uncertainty. Expanding the variance of the optimal profits:

$$\begin{aligned} & Var(\pi^*) \\ = & P^2(1-r)r\{f[L^*(1+s)] - f[L^*(1-\frac{rs}{1-r})]\}^2 \qquad (19) \\ = & P^2(1-r)r\{(s+\frac{rs}{1-r})L^*f'(L^*) + [1-\left(\frac{r}{1-r}\right)^2]\frac{(L^*s)^2}{2}f''(L^*) + \cdots\}^2 \end{aligned}$$

For $s > 0$, we expect that $\frac{dVar[\pi^*]}{ds} > 0$ and $\frac{dVar[\pi^*]}{ds} < 0$ if $s < 0$: the variance of profits rises with uncertainty. The variance of profits will be enhanced by the (square of) $PL^*f'(L^*)$ — the higher $Pf'(L^*)$, that approaches the wage, times L^*, employment. That is, by the wage bill size.

The variability of supply will equal:

$$Var(q^*) = (1-r)r\{f[L^*(1+s)] - f[L^*(1-\frac{rs}{1-r})]\}^2 \qquad (20)$$

We can conclude that:

Proposition 2. *Consider a standard price-taking, expected profit maximizing firm subject to ex-ante commitment in hiring decisions and multiplicative uncertainty in input quality. After a rise in such uncertainty, the firm's:*

1.labor demand decreases (increases) if the "factored" marginal product function — the marginal product function times the factor — is concave (convex) in the argument. That is, if the Arrow-Pratt measure of relative risk aversion of the marginal product function is smaller (larger) than 2.

2.expected output or supply decreases (increases) if the Arrow-Pratt measure of relative risk aversion for the production function rises (decreases) with the argument, the factor. That is if the difference between the Arrow-Pratt measures of relative risk aversion of the marginal product and of the production functions is smaller (larger) than 1.

3.expected profits decrease — more pronouncedly, the higher the output price, the (square of) factor demand, and the more negatively sloped is the "factored" marginal product function, the marginal product function times the factor. Their volatility increases — more int ensely, the higher the (square of the) wage bill.

4.variance of output supply increases, more intensely, the higher the (square of the) "factored" marginal product function, the marginal product function times the factor, or wage bill-price ratio.

3) "Factored" Uncertainty

Suppose that the variance of the input increases linearly with the amount hired: $\sqrt{L}X$ is added to L at the production function argument level. The firm maximizes:

$$\max_L E\pi(L) = rPf(L+\sqrt{L}s + (1-r)Pf(L-\sqrt{L}\frac{rs}{1-r}) - wL \tag{21}$$

F.O.C. requires still require that the expected value of the value of the marginal product of labour equals the wage rate:

$$rPf'(L^*+\sqrt{L^*}s)(1+\frac{1}{2\sqrt{L^*}}s)+(1-r)Pf'(L^*-\sqrt{L^*}\frac{rs}{1-r})(1-\frac{1}{2\sqrt{L^*}}\frac{rs}{1-r}) = w$$

The sign of $\frac{dL^*}{ds}$ is equal to the derivative of the left hand-side with respect to s — once the derivative with respect to L^*, the denominator, is negative by S.O.C. $\frac{dL^*}{ds}$ has the sign of:

$$\begin{aligned} &\frac{1}{2\sqrt{L^*}}rP\{f'(L^*+\sqrt{L^*}s) + f''(L^*+\sqrt{L^*}s)(2L^*+\sqrt{L^*}s) \\ - \ & f'(L^*-\sqrt{L^*}\frac{rs}{1-r}) - f''(L^*-\sqrt{L^*}\frac{rs}{1-r})(2L^*-\sqrt{L^*}\frac{rs}{1-r})\} \end{aligned} \tag{22}$$

It will be negative for $s>0$ and L^* decreases with uncertainty if, approximately, $\frac{1}{2\sqrt{L}}[f'(L)+2Lf''(L)]$ decreases with L; that is, if the function $G(L) = \sqrt{L}f(L)$ is concave. L^* increases with uncertainty if $G(L) = \sqrt{L}f'(L)$ is convex.

One can make the correspondence of a concave $G(L) = \sqrt{L}f'(L)$ to the condition, involving relative risk-aversion and relative prudence:

$$-\frac{Lf''(L)}{f'(L)}[-\frac{Lf'''(L)}{f''(L)} - 1] < \frac{1}{4} \tag{23}$$

Expected profits will react according to:

$$\begin{aligned}\frac{dE[\pi^*]}{ds} &= Pr\sqrt{L^*}[f'(L^*+\sqrt{L^*}s)-f'(L^*-\sqrt{L^*}\frac{rs}{1-r})] \\ &= Pr\sqrt{L^*}\{(s+\frac{rs}{1-r})\sqrt{L^*}f''(L^*) \\ &\quad +[1-\left(\frac{r}{1-r}\right)^2]\frac{L^*s^2}{2}f'''(L^*)+\cdots\}\end{aligned} \tag{24}$$

They always decrease with uncertainty. The variance becomes:

$$\begin{aligned}Var(\pi^*) &= P^2(1-r)r[f(L^*+\sqrt{L^*}s)-f(L^*-\sqrt{L^*}\frac{rs}{1-r})]^2 \\ &= P^2(1-r)r\{(s+\frac{rs}{1-r})\sqrt{L^*}f'(L^*) \\ &\quad +[1-\left(\frac{r}{1-r}\right)^2]\frac{L^*s^2}{2}f''(L^*)+\cdots\}^2\end{aligned} \tag{25}$$

$$Var(q^*) = (1-r)r[f(L^*+\sqrt{L^*}s)-f(L^*-\sqrt{L^*}\frac{rs}{1-r})]^2 \tag{26}$$

The rooting of L with have a correspondence in the assertions made for multiplicative uncertainty.

3.2. Output Uncertainty

1) Additive Uncertainty

Suppose that uncertainty — lottery X — is not directly added to labour quality but to financial efficiency in factor hiring processing in such a way that the cost associated to a production level q, which the firm controls, is $C(q+X)$, that is, $C(q+s)$ with probability r and $C(q-\frac{rs}{1-r})$ with probability $1-r$. Then the firm will try to:

$$\max_q E\pi(q) = Pq - rC(q+s) - (1Cr)C(q-\frac{rs}{1-r}) \tag{27}$$

F.O C. requires that the price equals expected marginal cost:

$$P = rC'(q^*+s)+(1-r)C'(q^*-\frac{rs}{1-r}) \tag{28}$$

The effect of a change in s on the optimal decision q^* can be inferred from:

$$rC''(q^*+s)(dq^*+ds)+(1-r)C''(q^*-\frac{rs}{1-r})(dq^*-\frac{r}{1-r}ds)=0$$

Then:

$$\frac{dq^*}{ds} = r\frac{C''(q^*-\frac{rs}{1-r})-C''(q^*+s)}{rC''(q^*+s)+(1-r)C''(q^*-\frac{rs}{1-r})} \tag{29}$$

$C''(.) > 0$ — the denominator is positive (for S.O.C. to be obeyed). Being $s > 0$, a rise in uncertainty generates:

-an increase in q^* iff $\frac{dq^*}{ds} > 0$, which requires that $C''(q - \frac{rs}{1-r}) > C''(q + s)$. That will occur if $C'''(.) < 0$ — that is, marginal cost, $C'(.)$ (the firm's inverse supply under certainty) is concave (hence, the direct supply, positively sloped, convex).

-a decrease in q^* iff $\frac{dq^*}{ds} < 0$, which requires $C''(q - \frac{rs}{1-r}) < C''(q + s)$. That occurs $C'''(.) > 0$ — that is, marginal cost, $C'(.)$ (the firm's inverse supply under certainty) is convex (the firm's supply is concave).

Being $s < 0$, we will have a symmetric effect of s. We therefore conclude that: a rise in uncertainty decreases the target-output if the inverse supply is convex, it raises it if it is concave. The widely used quadratic cost function will generate invariability of q^* to such uncertainty.

An intuition can also be given for the condition found. Being the marginal cost function a convex function, its expected value is larger than the value of the expected argument (of $q^* + X$, which is always q^*); being the marginal product a positive function of the argument, to insure a given fixed expected marginal cost $P^{\#}$, q^*, the expected value of the argument under uncertainty, must be lower than the certain quality $q^{\#}$ chosen. Graphically — Fig. 2 -, q^* is chosen such that the line that connects $[q_1, C'(q_1)]$ and $[q_2, C'(q_2)]$, where $q_1 = q^* - \frac{rs}{1-r}$ and $q_2 = q^* + s$, intersects $P^{\#}$ at the horizontal axis value q^*; of course, such line is above $C'(q)$ if this is convex; hence $q^{\#}$, the output chosen under certainty at price $P^{\#}$, read on $C'(q)$, must be to the right of q^*: uncertainty in quality decreased the target output.

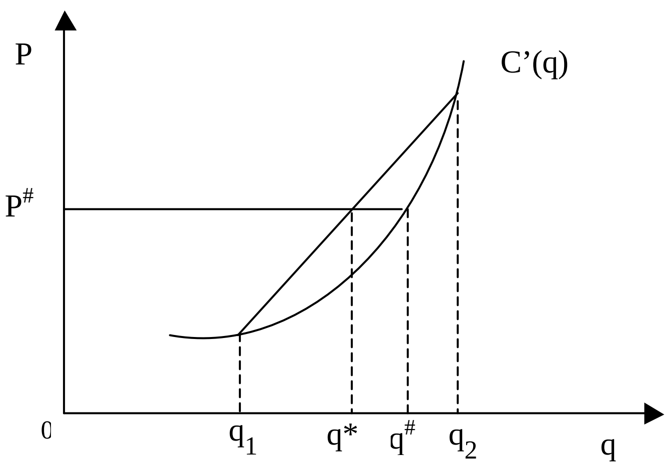


FIG. 2.

With a concave marginal cost function, we would arrive at the opposite conclusion.

We can visualize $\frac{dq^*}{ds}$ as directly proportional to the measure of prudence of the cost function, i.e., to $-\frac{C'''(q^*)}{C''(q^*)}$.

Expected costs react according to:

$$\begin{aligned}\frac{dE[C(q^*)]}{ds} &= [rC'(q^*+s)+(1-r)C'(q^*-\frac{rs}{1-r})]\frac{dq^*}{ds}\\ &+ r[C'(q^*+s)-C'(q^*-\frac{rs}{1-r})]\\ &= P\frac{dq^*}{ds}+r[C'(q^*+s)-C'(q^*-\frac{rs}{1-r})]\end{aligned}$$

For $s>0$, the last term is positive, once marginal cost is increasing with the argument: even if $\frac{dq^*}{ds}<0$, costs may rise with uncertainty. Developing the second expression we can derive that:

$$\frac{dE[C(q^*)]}{ds} = r\frac{C'(q^*+s)C''(q^*-\frac{rs}{1-r})-C'(q^*-\frac{rs}{1-r})C''(q^*+s)}{rC''(q^*+s)+(1-r)C''(q^*-\frac{rs}{1-r})} \tag{30}$$

For $s>0$, it will be positive iff $\frac{C''(q^*+s)}{C'(q^*+s)} < \frac{C''(q^*-\frac{sr}{1-r})}{C'(q^*-\frac{sr}{1-r})}$: minimum expected cost rises with uncertainty if the degree of absolute convexity of the cost function $C(q)$ (increasing in $q: C'(q)>0$), $b(q)=\frac{C''(q)}{C'(q)}$, is decreasing in the argument. That is a property, for example, of the conventional quadratic cost function.

The convexity of $C(q)$ as measured by $\frac{C''(q)}{C'(q)}$ will decrease with the argument iff:

$$\frac{C''(q)}{C'(q)} > \frac{C'''(q)}{C''(q)} \tag{31}$$

If the absolute convexity of the cost function is larger than that of the marginal cost function, expected costs rise with uncertainty. The effect of uncertainty on $E[C^*]$ would be a composition of two effects: an indirect effect determining the effect of uncertainty, X, on q^*, working through — as we saw above — the convexity of $C'(.)$, and an direct effect of uncertainty around q on C, working through the convexity of $C(.)$. If $C'(.)$ is convex, q^* decreases with uncertainty suggesting a decrease in C; then, the direct effect counteracts the other more intensely the more convex the cost function is.

We can visualize $\frac{dE[C(q^*)]}{ds}$ as approximately proportional to $-C'(q^*)\frac{\frac{d\left[\frac{C'''(q^*)}{C'(q^*)}\right]}{dq^*}}{\frac{C''(q^*)}{C'(q^*)}}$, that is, to the symmetric of the semi-elasticity of the absolute "convexity" measure with respect to the argument times the marginal cost function. Or to $-C'(q^*)[\frac{C'''(q^*)}{C''(q^*)} - \frac{C''(q^*)}{C'(q^*)}]$

We could still deduct that:

$$\begin{aligned}\frac{dE[\pi^*]}{ds} &= r[C'(q^* - \frac{rs}{1-r}) - C'(q^* + s)] \\ &= -r\{(s + \frac{rs}{1-r})C''(q^*) + [1 - \left(\frac{r}{1-r}\right)^2]\frac{s^2}{2}C'''(q^*) + \cdots\}\end{aligned} \tag{32}$$

$C(.)$ is convex — by S.O.C.; then, for $s > 0$, $\frac{d\pi^*}{ds} < 0$ and for $s < 0$, $\frac{d\pi^*}{ds} > 0$. Again, the optimal expected profits decrease with uncertainty.

For $s > 0$, $\frac{dE[\pi^*]}{ds}$ will be larger in absolute value, the larger is $C''(q^*)$ — the more convex the cost function is, the higher the slope of (the steeper) the firm's inverse supply, lower the "slope" of the firm's (short run) supply. And/or a larger size (q^*) if the marginal cost function is convex and $C''(q)$ rises with the argument — if q^* decreases with uncertainty; a smaller size if the marginal cost function is convex, that is, q^* increases with uncertainty.

$$\begin{aligned}Var(\pi^*) &= r[C(q^* + s) - rC(q^* + s) - (1-r)C(q^* - \frac{rs}{1-r})]^2 \\ &+ (1-r)[C(q^* - \frac{rs}{1-r}) - rC(q^* + s) - (1-r)C(q^* - \frac{rs}{1-r})]^2 \\ &= (1-r)r[C(q^* + s) - C(q^* - \frac{rs}{1-r})]^2 \qquad (33) \\ &= (1-r)r\{(s + \frac{rs}{1-r})C'(q^*) + [1 - \left(\frac{r}{1-r}\right)^2]\frac{s^2}{2}C''(q^*) + \cdots\}^2\end{aligned}$$

$$\begin{aligned}\frac{dVar[\pi^*]}{ds} &= 2(1-r)r[C(q^* + s) - C(q^* - \frac{rs}{1-r})] \\ &\cdot \{[C'(q^* + s) - C'(q^* - \frac{rs}{1-r})]\frac{dq^*}{ds} \\ &+ [C'(q^* + s) + \frac{r}{1-r}C'(q^* - \frac{rs}{1-r})]\} \\ &= 2[C(q^* + s) - C(q^* - \frac{rs}{1-r})] \qquad (34) \\ &\cdot \frac{(1-r)C'(q^* + s)C''(q^* - \frac{rs}{1-r}) + rC'(q^* - \frac{rs}{1-r})C''(q^* + s)}{rC''(q^* + s) + (1-r)C''(q^* - \frac{rs}{1-r})}\end{aligned}$$

If $s > 0$, $\frac{dVar[\pi^*]}{ds} > 0$; $s < 0$, $\frac{dVar[\pi^*]}{ds} < 0$ — instability in the optimal profits always rises with uncertainty.

Inspection of the two last expressions suggests that exogenous uncertainty in output will imply higher profit volatility the higher the marginal cost — ultimately, the higher the output price, and in a proportional relation to its square.

One concludes that the trade-off between $Var(\pi^*)$ and $E[\pi^*]$ is always negative and varies according to:

$$\frac{dVar[\pi^*]}{dE[\pi^*]} = 2\frac{C(q^*+s) - C(q^* - \frac{rs}{1-r})}{C'(q^* - \frac{rs}{1-r}) - C'(q^*+s)} \cdot \frac{(1-r)C''(q^*+s)C''(q^* - \frac{rs}{1-r}) + rC'(q^* - \frac{rs}{1-r})C''(q^*+s)}{rC''(q^*+s) + (1-r)C''(q^* - \frac{rs}{1-r})} < 0 \quad (35)$$

$Var[C^*]$ replicates the optimal profits variance.

Proposition 3. *Consider a standard price-taking, expected profit maximizing firm subject to ex-ante targeting decision context with respect to output and additive uncertainty around the cost function argument. After a rise in such uncertainty, the firm's:*

1. output supply decreases (increases) if the marginal cost function is convex (concave) in the argument.

2. expected costs decrease (increase) if convexity of the cost function is less (more) pronounced than that of the marginal cost one as measured by the symmetric measure to the Arrow-Pratt measure of absolute risk aversion. Equivalently, if this measure for the cost function rises (decreases) with the argument, the output.

3. expected profits decrease — more intensely the more convex is the cost function - and their volatility increases — more intensely the larger (the square of) marginal cost and (the square of) output price.

We note that the slope of an inverse function has the same sign as the slope of the original one. The sign of the convexity of an inverse function coincides with that of the direct one if negatively sloped, it is symmetric if positively sloped. On the other hand, in our simple single-input scenario, $C(q) = wf^{-1}(q) + F$; if we ignore fixed costs and normalize price to 1, we recover in $C(q)$ the inverse production function and marginal cost is one over the marginal product. Hence, the conditions for quantity effects on the convexity of $C(q)$ or $C'(q)$ in Proposition 3 have correspondence to those on the concavity of the $f(L)$ in Proposition 1 (except for those based on results scaled by second derivatives).

2) Multiplicative Uncertainty

Suppose the uncertainty is multiplicative: with probability r the argument of the cost function is added of qs, with probability $(1-r)$ subtracted of $q\frac{rs}{1-r}$. The firm will try to:

$$\max_q E\pi(q) = Pq - rC[q(1+s)] - (1-r)C[q(1-\frac{rs}{1-r})] \tag{36}$$

F.O C. requires that the price equals expected marginal cost:

$$P = rC'[q(1+s)](1+s) + (1-r)C'[q(1-\frac{rs}{1-r})](1-\frac{rs}{1-r}) \tag{37}$$

The effect of a change in s on the optimal decision q^* is :

$$\frac{dq^*}{ds} = \tag{38}$$
$$r\frac{C'[q^*(1-\frac{rs}{1-r})] + C''[q^*(1-\frac{rs}{1-r})]q^*(1-\frac{rs}{1-r}) - C'[q^*(1+s)] - C''[q^*(1+s)]q^*(1+s)}{rC''[q^*(1+s)](1+s)^2 + (1-r)C''[q^*(1-\frac{rs}{1-r})](1-\frac{rs}{1-r})^2}$$

The denominator is positive by S.O.C. q^* will increase with uncertainty iff $g(q) = C'(q) + qC''(q)$ is decreasing in q. That is to say, if $G(q) = qC'(q)$, the marginal cost times the output, is a concave function. Or that $2C''(q) + qC'''(q) < 0$ and — once $C''(q) > 0$ — the symmetric of the Arrow-Pratt measure of relative risk aversion of the marginal cost function $C'(q)$, $\frac{qC'''(q)}{C''(q)}$ is smaller than minus 2:

$$\frac{qC'''(q)}{C''(q)} < -2 \tag{39}$$

q^* will decrease with uncertainty iff $g(q) = C'(q) + qC''(q)$ is increasing in q or $G(q) = qC'(q)$ is convex in q. This will be the case for a quadratic cost function.

Expected cost reacts to s according to:

$$\begin{aligned}\frac{dE[C^*]}{ds} &= \frac{r}{rC''[q^*(1+s)](1+s)^2 + (1-r)C''[q^*(1-\frac{rs}{1-r})](1-\frac{rs}{1-r})^2} \\ &\cdot \{C'[q^*(1+s)]C''[q^*(1-\frac{rs}{1-r})]q^*(1-\frac{rs}{1-r}) \\ &- C'[q^*(1-\frac{rs}{1-r})]C''[q^*(1+s)]q^*(1+s) \\ &- P(C'[q^*(1+s)] - C'[q^*(1-\frac{rs}{1-r})])\}\end{aligned} \tag{40}$$

That is, $\frac{dE[C^*]}{ds} > 0$ for $s > 0$ and $E[C^*]$ will increase with uncertainty iff

$$\frac{C''[q^*(1-\frac{sr}{1-r})]q^*(1-\frac{sr}{1-r}) - P}{C'[q^*(1-\frac{sr}{1-r})]} > \frac{C''[q^*(1+s)]q^*(1+s) - P}{C'[q^*(1+s)]},$$

that is, if $\frac{C''(q)-P}{C'(q)}$ is decreasing with q. $E[C^*]$ will decrease with uncertainty if $\frac{C''(q)q-P}{C'(q)}$ is increasing with q. Note that near the optimal solution, $P = E[C'(q+X)]$ and close to $C'(q)$. Then, $\frac{C''(q)q-P}{C'(q)}$ reacts to q in that neighbourhood as the measure of relative convexity of the production function, $\frac{C''(q)q}{C'(q)}$ does. One can develop the concept of relative convexity: $\frac{C''(q)q}{C'(q)}$ will increase with q iff:

$$\frac{C''(q)q}{C'(q)} - \frac{C'''(q)q}{C''(q)} < 1 \tag{41}$$

That is, relative convexity of the cost function minus relative convexity of the marginal cost function - is smaller than 1.

As for expected profits:

$$\begin{aligned} & \frac{dE[\pi^*]}{ds} \\ = \; & rq^*\{C'[q^*(1-\frac{rs}{1-r})] - C'[q^*(1+s)]\} \qquad (42) \\ = \; & -rq^*\{(s+\frac{rs}{1-r})q^*C''(q^*) + [1-\left(\frac{r}{1-r}\right)^2]\frac{(q^*s)^2}{2}C'''(q^*) + \cdots\} \end{aligned}$$

They always decrease with uncertainty.

$$\begin{aligned} & Var(\pi^*) \\ = \; & (1-r)r\{C[q^*(1+s)] - C[q^*(1-\frac{rs}{1-r})]\}^2 \qquad (43) \\ = \; & (1-r)r\{(s+\frac{rs}{1-r})q^*C'(q^*) + [1-\left(\frac{r}{1-r}\right)]\frac{(q^*s)^2}{2}C''(q^*) + \cdots\}^2 \end{aligned}$$

Proposition 4. *Consider a standard price-taking, expected profit maximizing firm subject to ex-ante targeting decision context with respect to output and multiplicative uncertainty around the cost function argument. After a rise in such uncertainty, the firm's:*

1.output supply decreases (increases) if the "factored" marginal cost function — the marginal cost function times the output -is convex (concave) in the argument. That is, if the symmetric of the Arrow-Pratt measure of relative risk aversion of the marginal cost function is larger (smaller) than minus 2.

2.expected costs decrease (increase) if the symmetric of the measure of relative risk aversion for the cost function rises (decreases) with the argument, the factor. That is, if the difference between the relative convexity of the cost and of the marginal cost functions is smaller (larger) than 1.

3.expected profits decrease — more pronouncedly, the higher the (square of) output supply, and the more positively sloped is the marginal cost function, the inverse supply function. Their volatility increases — more intensely, the higher the (square of the) total revenue.

4. QUANTITY/QUALITY UNCERTAINTY AND EX-POST FLEXIBILITY

4.1. Factor Uncertainty

1) Additive Uncertainty

Admit floating quality: with probability r, production reacts as if benefiting of the addition s to the input; with probability $(1-r)$, as if it has been deducted of $\frac{rs}{1-r}$.

If the firm has ex-post decision ability — it can easily adjust employment either by new hires as by dismissals -, it is going to hire L_1 when it observes s such that:

$$\max_{L_1} Pf(L+s)CwL \tag{44}$$

$$Pf'(L_1+s)=w; \text{then} \frac{dL_1}{ds}=-1 \tag{45}$$

When it faces $-\frac{rs}{1-r}$, it will hire L_2 such that:

$$\max_{L_2} Pf(L-\frac{rs}{1-r})-wL \tag{46}$$

$$Pf'(L_2-\frac{rs}{1-r})=w; \text{then } \frac{dL_2}{ds}=\frac{r}{1-r} \tag{47}$$

Let $L^{\#}$ be such that

$$Pf'(L^{\#})=w \tag{48}$$

The firm always insures the productivity — and the production $q^*=f(L^{\#})$ -associated to such $L^{\#}$; therefore, $L_1=L^{\#}-s$ and $L_2=L^{\#}+\frac{rs}{1-r}$. The expected value of the firm's demand $E[L^*]=rL_1+(1-r)L_2=L^{\#}$ and will not react to uncertainty. We will have that:

$$\frac{dE[L^*]}{ds}=\frac{dE[q^*]}{ds}=\frac{dE[\pi^*]}{ds}=0 \tag{49}$$

However, uncertainty affects (in the same direction) the variability of labor demand and of the firm's profits:

$$\begin{aligned} Var[p^*] &= (-W)^2 Var(L^*) = W^2[r(L_1 - L^{\#})^2 + (1-r)(L_2 - L^{\#})^2] \\ &= W^2[r(-s)^2 + (1-r)(\frac{rs}{1-r})^2] = W^2 Var(X) \end{aligned} \tag{50}$$

We conclude that with this type of uncertainty and ex-post decisions, we arrive at identical solutions in terms of expected value as those of a context without uncertainty (in which it is irrelevant if decisions are ex-ante or ex-post ...). Let $s > 0$. With ex-ante decisions we concluded that:

$\frac{dE[L^*]}{ds} > 0$ for $s > 0$ iff $f'''(L) > 0$ — that is, marginal productivity, $f'(.)$, is convex. Then, the firm that decides ex-post (of "lower" s — in fact, of $s = 0$) has lower expected labor demand than that of the enterprise that decides ex-ante. Se $f'''(L) < 0$, we conclude the opposite.

$\frac{dE[q^*]}{ds} < 0$ iff the Arrow-Pratt's measure of absolute risk-aversion of the production function $f(L)$, $r(L) = -\frac{f''(L)}{f'(L)}$, is increasing in the argument. In this case, the firm that decides ex-post (of "lower" s) has higher expected output supply than the one that decides ex-ante. If $-\frac{f''(L)}{f'(L)}$ is decreasing in L, we have the opposite conclusion.

$\frac{dE[\pi^*]}{ds} < 0$. Then if decisions are ex-post, expected profits are always higher than if decisions are taken ex-ante.

(If $s < 0$, signs change but the conclusions with respect to the sign effect of an increase in uncertainty.)

Proposition 5. *Consider a standard price-taking, expected profit maximizing firm with ex-post flexibility towards hiring decisions and additive uncertainty in input quality. The firm's*

1. expected employment, output and profits are invariant to quality uncertainty. Profit and employment (labor demand) variability will rise with such uncertainty, the profits being enhanced by the (square of the) wage level.

2. expected profits will be higher than for a firm restricted by ex-ante commitment.

3. expected labor demand will be higher (lower) than that of the ex-ante deciding firm under the conditions of 1. of Proposition 1.

4. expected output supply will be higher (lower) than that of the ex-ante deciding firm under the conditions of 2. of Proposition 1.

2) Multiplicative Uncertainty

Admit the multiplicative uncertainty case and ex-post flexibility. Then with probability r the firm solves:

$$\max_{L_1} Pf[L(1+s)]CwL \tag{51}$$

$$Pf'[L_1(1+s)](1+s) = w \tag{52}$$

then $\frac{dL_1}{ds} = -\frac{f'[L_1(1+s)]+L_1(1+s)f''[L_1(1+s)]}{(1+s)^2 f''[L_1(1+s)]}$
When it faces $-\frac{rs}{1-r}$, it will hire L_2 such that:

$$\max_{L_2} Pf[L(1-\frac{rs}{1-r})]CwL \tag{53}$$

$$Pf'[L_2(1-\frac{rs}{1-r})](1-\frac{rs}{1-r}) = w \tag{54}$$

then $\frac{dL_2}{ds} = \frac{r}{1-r}\frac{f'[L_2(1-\frac{rs}{1-r})]+L_2(1-\frac{rs}{1-r})f''[L_2(1-\frac{rs}{1-r})]}{(1-\frac{rs}{1-r})^2 f''[L_2(1-\frac{rs}{1-r})]}$

$$\begin{aligned} \frac{dE[L^*]}{ds} &= r\frac{dL_1}{ds} + (1-r)\frac{dL_2}{ds} = r \\ &\cdot\ \{\frac{f'[L_2(1-\frac{rs}{1-r})]+L_2(1-\frac{rs}{1-r})f''[L_2(1-\frac{rs}{1-r})]}{(1-\frac{rs}{(1-r)})^2 f''[L_2(1-\frac{rs}{1-r})]} \\ &- \frac{f'[L_1(1+s)]+L_1(1+s)f''[L_1(1+s)]}{(1+s)^2 f''[L_1(1+s)]} \end{aligned} \tag{55}$$

As $L_1(1+s) > L_2(1-\frac{rs}{1-r})$ for $s>0$, it is possible (around $s=0$) that $\frac{dE[L^*]}{ds}$ is larger or smaller than 0 under analogous conditions — see (13) -of the ex-ante scenario. The math became much more cumbersome and to circumvent the problem we rearranged the assumptions:

Admit that (multiplicative) uncertainty affects production but also surrounds the costs — see section 5. Then, multiplicative uncertainty would not altered the conclusions with respect to expectations of optimal parameters — and would compare with multiplicative uncertainty with ex- ante commitment according to the same principles. Now, target or planned employment will have variance:

$$Var[L^*] = L^{\#2}Var(X) \tag{56}$$

The profit's volatility will vanish: $Var[p^*] = 0$.

PROPOSITION 6. *Consider a standard price-taking, expected profit maximizing firm with ex-post flexibility towards hiring decisions and multiplicative uncertainty in input quality for which the firm must necessarily pay for. The firm's*

1.expected employment, output and profits are invariant to quality uncertainty. Target employment (labor demand) variability will rise with such uncertainty; the profit's will vanish.

2.expected profits will be higher than for a firm restricted by ex-ante commitment.

3.expected labor demand will be higher (lower) than that of the ex-ante deciding firm under the conditions of 1. of Proposition 2.

4.expected output supply will be higher (lower) than that of the ex-ante deciding firm under the conditions of 2. of Proposition 2.

4.2. Output Uncertainty

1) Additive Uncertainty

Consider the case where we analyzing additive uncertainty at the cost function argument level. If the firm has ex-post decision ability — it can easily adjust output -, it is going to chose q_1 when it observes s such that:

$$\max_{q_1} Pq - C(q+s) \tag{57}$$

$$P = C/(q_1+s); \text{then } \frac{dq_1}{ds} = -1 \tag{58}$$

When it faces $-\frac{rs}{1-r}$, it will target q_2 such that:

$$\max_{q_2} Pq - C(q - \frac{rs}{1-r}) \tag{59}$$

$$P = C'(q_2 - \frac{rs}{1-r}); \quad \text{then } \frac{dq_2}{ds} = \frac{r}{1-r} \tag{60}$$

Let $q^{\#}$ be such that

$$P = C'(q^{\#}) \tag{61}$$

The firm always insures the marginal cost — and the minimum cost $C^* = C(q^{\#})$ - associated to such $q^{\#}$; therefore, $q_1 = q^{\#} - s$ and $q_2 = q^{\#} + \frac{rs}{1-r}$. The expected value of the firm's supply $E[q^*] = rq_1 + (1-r)q_2 = q^{\#}$ and will not react to uncertainty. We will have that:

$$\frac{dE[C^*]}{ds} = \frac{dE[q^*]}{ds} = \frac{dE[\pi^*]}{ds} = 0 \tag{62}$$

However, uncertainty affects (in the same direction) the variability of output supply and of the firm's profits — the latter proportionately to the square of the output price level:

$$\begin{aligned} Var[p^*] &= P^2Var(q^*) = P^2[r(q_1 - q^{\#})^2 + (1-r)(q_2 - q^{\#})^2] \\ &= P^2[r(-s)^2 + (1-r)(\frac{rs}{1-r})^2] = P^2Var(X^*) \end{aligned} \quad (63)$$

We conclude that with this type of uncertainty and ex-post decisions, we arrive at identical solutions in terms of expected value as those of a context without uncertainty (in which it is irrelevant if decisions are ex-ante or ex-post ...). Let $s > 0$. With ex-ante decisions we concluded that:

q^* rises with uncertainty ($\frac{dq^*}{ds} > 0$ for $s > 0$, $\frac{dq^*}{ds} < 0$ for $s < 0$) if $C'''(.) < 0$ — that is, marginal cost, $C'(.)$ is concave; Then, the firm that decides ex-post (of "lower" s — in fact, of an equivalent case to $s = 0$) has lower expected supply than that of the enterprise that decides ex-ante. If $C'''(L) > 0$ and $C'(.)$ is convex, we conclude the opposite.

$E[C^*]$ rise with uncertainty ($\frac{dE[C^*]}{ds} > 0$ iff $s > 0$; $\frac{dE[C^*]}{ds} < 0$ iff $s < 0$), if the measure of absolute convexity of the cost function $C(q)$, $r(q) = \frac{C''(q)}{C'(q)}$, is decreasing in the argument. In this case, the firm that decides ex-post (of "lower" s) has lower expected costs than the one that decides ex-ante. If $\frac{C''(q)}{C'(q)}$ is increasing in q, we have the opposite conclusion.

$\frac{dE[\pi^*]}{ds} < 0$ if $s > 0$; $\frac{dE[\pi^*]}{ds} > 0$ if $s < 0$ and profits decrease with uncertainty. Then, if decisions are ex-post, expected profits are always higher than if decisions are taken ex-ante.

In this context:

$$Var(q^*) = Var(X) \quad (64)$$

Supply totally cushions exogenous quantity fluctuations, and has the same variance as X.

PROPOSITION 7. *Consider a standard price-taking, expected profit maximizing firm with ex-post flexibility towards output decisions and additive uncertainty around the cost function argument. The firm's*

1.expected output, costs and profits are invariant to such uncertainty. Profit and output (supply) variability will rise with it — supply in a one to one relation; the profit's being enhanced by the (square of the) output price level.

2.expected profits will be higher than for a firm restricted by ex-ante commitment.

3.expected output supply will be higher (lower) than that of the ex-ante deciding firm under the conditions of 1. of Proposition 3.

4.expected total costs will be higher (lower) than that of the ex-ante deciding firm under the conditions of 2. of Proposition 3.

2) Multiplicative Uncertainty

Under multiplicative uncertainty, such that for output level q costs can either be $C[q(1+s)]$, which occurs with probability r, or $C[q(1-\frac{rs}{1-r})]$, with probability $(1-r)$, admitting that unplanned costs in fact generated output that can be sold in the market — see section 5 -, one can easily show that the $q^{\#}$ that solves the additive uncertainty case $-P = C'(q^{\#})$ -is recovered, $q_1 = q^{\#}(1-s)$ and $q_2 = q^{\#}(1+\frac{rs}{1-r})$. Still:

$$\frac{dE[C^*]}{ds} = \frac{dE[q^*]}{ds} = \frac{dE[\pi^*]}{ds} = 0 \tag{65}$$

However, uncertainty affects (in the same direction) the variability of target output supply, even if corrected ex-post:

$$Var(q^*) = [r(q_1 - q^{\#})^2 + (1-r)(q_2 - q^{\#})^2] = q^{\#2} Var(X) \tag{66}$$

Profits' variability is driven down to zero.

Proposition 8. *Consider a standard price-taking, expected profit maximizing firm with ex-post flexibility towards output decisions and multiplicative uncertainty around the cost function argument, which can, nevertheless, be sold in the market. The firm's*

1.expected output, costs and profits are invariant to such uncertainty. Target supply variability will rise with it — supply, enhanced by (the square of the) output size. The profit's variability will disappear.

2.expected profits will be higher than for a firm restricted by ex-ante commitment.

3.expected output supply will be higher (lower) than that of the ex-ante deciding firm under the conditions of 1. of Proposition 4.

4.expected total costs will be higher (lower) than that of the ex-ante deciding firm under the conditions of 2. of Proposition 4.

5. PRICE UNCERTAINTY

5.1. Wage Rate Dispersion

1) "Additive" Uncertainty

Admit wage fluctuations, with the Bernoulli behaviour: with probability r the wage rate is added of s; with probability $(1-r)$, deducted of $\frac{rs}{1-r}$. If a decision must be taken ex-ante, uncertainty in the wage has no effects on the optimal quantities or expected values; it is straight-forward to infer that, in this case:

$$Var[p^*] = {L^*}^2 Var(X) \tag{67}$$

profits become more unstable with wage uncertainty, factored by the square of optimal employment — hence, possibly attenuated by the wage level (once demands are negatively sloped).

PROPOSITION 9. *Consider a standard price-taking, expected profit maximizing firm with ex-ante commitment towards hiring decisions and additive uncertainty in the wage rate.*

1.The firm's expected employment, output and profits are invariant to price uncertainty.

2.Profit variability will rise with such uncertainty, being enhanced by the (square of the) employment level.

If the firm has ex-post decision ability — it can easily adjust employment -, it will hire L_1 when it observes $W + s$ such that:

$$\max_{L_1} Pf(L) - (w + s)L \tag{68}$$

$$Pf'(L_1) = w + s; \quad \text{then} \quad \frac{dL_1}{ds} = \frac{1}{Pf''(L_1)} \tag{69}$$

It will hire L_2, when it faces $w - \frac{rs}{1-r}$ such that:

$$\max_{L_2} Pf(L) - (w - \frac{rs}{1-r})L \tag{70}$$

$$Pf'(L_2) = w - \frac{rs}{1-r}; \quad \text{then} \quad \frac{dL_2}{ds} = -\frac{1}{Pf''(L_2)} \frac{r}{1-r} \tag{71}$$

Then, $L_1 = L^D(\frac{w+s}{P})$ and $L_2 = L^D(\frac{w-\frac{rs}{1-r}}{P})$, where $L^D(\frac{w}{P})$ denotes the firm's (optimal) labor demand as a function of the observed wage divided by output price, corresponding to the inverse marginal product function evaluated at $\frac{w}{P}$.

Let $s > 0$.

As $f''(L) < 0$, $L_2 > L_1$. With probability r, the firm will employ L_1, with $(1 - r)$, L_2. The expected value of demand $E[L^*] = rL_1 + (1 - r)L_2$, reacts to s according to:

$$\begin{aligned}\frac{dE[L^*]}{ds} &= r\frac{dL_1}{ds} + (1-r)\frac{dL_2}{ds} = \frac{r}{P}[\frac{1}{f''(L_1)} - \frac{1}{f''(L_2)}] \\ &= \frac{r}{Pf''(L_1)f''(L_2)}[f''(L_2) - f''(L_1)] \end{aligned} \tag{72}$$

If $f''(L)$ is increasing in L, that is, if the marginal product function, $f'(L)$, is convex ($f'''(L) > 0$), $E[L^*]$ increases with uncertainty — and is higher for a firm that can react ex-post than for the one that decides ex-ante (for an equivalent $s = 0$, necessarily smaller). Being concave, $E[L^*]$ decreases with uncertainty — and is lower for a firm that responds ex-post.

As for output:

$$\begin{aligned}\frac{dE[q^*]}{ds} &= rf'(L_1)\frac{dL_1}{ds} + (1-r)f'(L_2)\frac{dL_2}{ds} \\ &= \frac{r}{P}[\frac{f'(L_1)}{f''(L_1)} - \frac{f'(L_2)}{f''(L_2)}] \\ &= \frac{rf'(L_1)f'(L_2}{Pf''(L_1)f''(L_2)}[\frac{f''(L_2)}{f'(L_2)} - \frac{f''(L_1)}{f'(L_1)}] \end{aligned} \tag{73}$$

If absolute risk-aversion , $-\frac{f''(L)}{f'(L)}$, is increasing in L, as $L_2 > L_1 (s > 0)$, $\frac{dE[q^*]}{ds} < 0$ and expected output decreases with uncertainty — and is lower for the firm that can react ex-post. Being decreasing with L, $\frac{dE[q^*]}{ds} > 0$ — and $E[q^*]$ is higher for the firm reacting ex-post.

$$\begin{aligned}\frac{dE[\pi^*]}{ds} &= r[Pf'(L_1) - (w+s)]\frac{dL_1}{ds} + (1-r)[Pf'(L_2) - (w - \frac{rs}{1-r})]\frac{dL_2}{ds} \\ &\quad -rL_1 + (1-r)\frac{r}{1-r}L_2 \\ &= r(L_2 - L_1) \\ &= r[L^D(\frac{w - \frac{rs}{1-r}}{P}) - L^D(\frac{w+s}{P})] \\ &= -r\{\frac{s + \frac{rs}{1-r}}{P}L^{D'}(\frac{w}{P}) + [1 - \left(\frac{r}{1-r}\right)^2]\frac{s^2}{2P^2}L^{D''}(\frac{w}{P}) + \cdots\} \end{aligned} \tag{74}$$

For $s > 0$, $\frac{dE[\pi^*]}{ds} > 0$: anticipated wage dispersion favours expected profits — and ex-post decisions are profitable compared to ex-ante employment commitment. $\frac{dE[\pi^*]}{ds}$ will be larger, the more negatively sloped is the firm's factor demand — and the less negatively sloped is the inverse factor demand (the value of marginal product function) and, thus, the less concave is the firm's production function. (This conclusion would be conformable with the standard properties of an optimal profit function[18] — the benefits from uncertainty around the wage more profit-enhancing the more convex the profit function is with respect to it; or the higher its second partial derivative with respect to the wage, with correspondence to the symmetric of the slope of 19 the derived factor demand function[19].)

However, we can show that profits become more unstable with uncertainty:

$$\begin{aligned} & Var[p^*] \\ = \ & (1-r)r\{[Pf(L_1)-(w+s)L_1]-[Pf(L_2)-(w-\frac{rs}{1-r})L_2]\}^2 \quad (75) \\ = \ & (1-r)r\{\frac{s+\frac{rs}{1-r}}{P}L^D(\frac{w}{P})+[1-\left(\frac{r}{1-r}\right)^2]\frac{s^2}{2P^2}L^{D'}(\frac{w}{P})+\cdots\}^2 \end{aligned}$$

Then:

$$\begin{aligned} \frac{dVar[\pi^*]}{ds} = \ & 2(1-r)r\{[Pf(L_1)-(w+s)L_1] \quad (76) \\ & -[Pf(L_2)-(w-\frac{rs}{1-r})L_2]\}\{-L_1-\frac{r}{1-r}L_2\} \end{aligned}$$

$\frac{dVar[\pi^*]}{ds} > 0$ if $s > 0$; $\frac{dVar[\pi^*]}{ds} < 0$ if $s < 0$. Exogenous uncertainty will imply higher profit volatility the higher the firm's labor demand, in a proportional relation to its square — ultimately, the lower the wage rate.

Then, by varying s, we register the always positive trade-off:

$$\begin{aligned} \frac{dVar[\pi^*]}{dE[\pi^*]} = \ & 2\{[Pf(L_2)-(w-\frac{rs}{1-r})L_2] \quad (77) \\ & -[Pf(L_1)-(w+s)L_1]\}\frac{(1-r)L_1+rL_2}{L_2-L_1} > 0 \end{aligned}$$

The variance of factor demand, of course, rises with uncertainty -more pronouncedly the more negatively sloped labor demand is:

$$Var(L^*) = r(1-r)(L_1-L_2)^2 \quad (78)$$

That of supply will also rise with uncertainty - more intensely the more negatively sloped labor demand is and the larger marginal product; possibly higher, the lower the (square of the) measure of risk-aversion of the

[18] A consequence of Hotelling's lemma. See Varian (1992), for example.

[19] In fact, this correspondence is also apparent in the expression (3.9) below; we also refer the reader to formula (A.1) in Appendix 1 for the analog of (3.8), and to the first term of (B.1) in Appendix 2 for the recognition of similarities to (3.9).

production function:

$$Var(q^*) = r(1-r)[f(L_1) - f(L_2)]^2 \tag{79}$$

PROPOSITION 10. *Consider a standard price-taking, expected profit maximizing firm with ex-post flexibility towards hiring decisions and additive uncertainty in the wage rate. After a rise in such uncertainty, the firm's:*

1.expected labor demand decreases (increases) if the marginal product function is concave (convex) in the argument — in which case is lower (higher) for the firm with ex-post flexibility.

2.expected output or supply decreases (increases) if concavity of the production function is more (less) pronounced than the convexity of the marginal product one as measured by the Arrow-Pratt measure of absolute risk aversion — in which case is lower (higher) for the firm with ex-post flexibility. Equivalently, if this measure for the production function rises (decreases) with the argument, the factor.

3.expected profits increase — more strongly, the less concave is the production function - and their volatility also — enhanced by employment size and deterred by the wage rate. Expected profits are higher for the firm with ex-post reaction ability.

2) "Factored" Uncertainty

One may suggest that the additive uncertainty in prices considered has really a multiplicative effect on total costs. Let us suppose uncertainty is such that the lottery added to total costs is of the type $w\sqrt{L}X$ and no longer linear in L. If a decision must be taken ex-ante, uncertainty in the wage has no effects on the optimal quantities or expected values; it is straight-forward to infer that, in this case:

$$Var[p^*] = L^* Var(X). \tag{80}$$

If the firm has ex-post decision ability — it can easily adjust employment -, it will hire L_1 when it observes $W + s$ such that:

$$\max_{L_1} Pf(L) - w(L + s\sqrt{L}) \tag{81}$$

$$Pf'(L_1) = w + s\frac{1}{2\sqrt{L_1}}; \tag{82}$$

then

$$\begin{aligned}\frac{dL_1}{ds} &= \frac{\sqrt{L_1}}{P[2L_1 f''(L_1) + f'(L_1)] - w}\\ &= \frac{2L_1}{4PL_1^{\frac{3}{2}} f''(L_1) + s} = \frac{1}{2P\sqrt{L_1} f''(L_1) + \frac{1}{2L_1} s}\end{aligned}$$

It will hire L_2 when it faces $w - \frac{rs}{1-r}$ such that:

$$\max_{L_2} Pf(L) - w(L - \frac{rs}{1-r}\sqrt{L}) \tag{83}$$

$$Pf'(L_2) = w - \frac{rs}{1-r}\frac{1}{2\sqrt{L_2}}; \tag{84}$$

then

$$\begin{aligned}\frac{dL_2}{ds} &= -\frac{\sqrt{L_2}}{P[2L_2 f''(L_2) + f'(L_2)] - w}\frac{r}{1-r}\\ &= -\frac{2L_2}{4PL_2^{\frac{3}{2}} f''(L_2) - \frac{rs}{1-r}}\frac{r}{1-r} = -\frac{1}{2P\sqrt{L_2} f''(L_2) - \frac{1}{2L_2}\frac{rs}{1-r}}\frac{r}{1-r}\end{aligned}$$

Let $s > 0$.

As $f''(L) < 0$, $L_2 > L_1$. With probability r, it will hire L_1, with $(1-r)$, L_2. The expected value of demand $E[L^*] = rL_1 + (1-r)L_2$, reacts to s according to:

$$\begin{aligned}\frac{dE[L^*]}{ds} &= r\frac{dL_1}{ds} + (1Cr)\frac{dL_2}{ds} \qquad (85)\\ &= \frac{r}{P}[\frac{\sqrt{L_1}}{2L_1 f''(L_1) + f'(L_1) - \frac{w}{P}} - \frac{\sqrt{L_2}}{2L_2 f''(L_2) + f'(L_2) - \frac{w}{P}}]\end{aligned}$$

If $\frac{2Lf''(L)+f'(L)-\frac{w}{P}}{\sqrt{L}}$ is increasing in L, that is, approximately, if the function, $\sqrt{L}f''(L)$, is increasing in L — if $-\sqrt{L}f''(L)$, plain concavity of $f(L)$ factored by $\sqrt{L}$, is decreasing in L -, $E[L^*]$ increases with uncertainty — and is higher for a firm that can react ex-post than for the one that decides ex-ante (for an equivalent $s = 0$, necessarily smaller). Being decreasing, $E[L^*]$ decreases with uncertainty — and is lower for a firm that responds ex-post.

Expected profits react to s according to:

$$\frac{dE[\pi^*]}{ds} = rw(\sqrt{L_2} - \sqrt{L_1}) \tag{86}$$

$$Var[p^*] = (1Cr)r\{[Pf(L_1)-(wL_1+s\sqrt{L_1})]-[Pf(L_2)-(wL_2+\frac{rs}{1-r}\sqrt{L_2})]\}^2 \tag{87}$$

Then:

$$\frac{dVar[\pi^*]}{ds} = 2(1Cr)r\{[Pf(L_1)-(wL_1+s\sqrt{L_1})] \tag{88}$$
$$- [Pf(L_2)-(wL_2+\frac{rs}{1-r}\sqrt{L_2})]\}\{-\sqrt{L_1}-\frac{r}{1-r}\sqrt{L_2}\}$$

Uncertainty in the output price generates mostly the same conclusions as uncertainty in the wage does. We will therefore not repeat the exercise and rather perform it below for a firm with a given cost function.

5.2. Output Price Uncertainty

Admit now output price fluctuation: with probability r the price P is added of s; with probability $(1-r)$, deducted of $\frac{rs}{1-r}$. If a decision must be taken ex-ante, uncertainty in the output price has no effects on the optimal quantities or expected values; it is straight-forward to infer that, in this case:

$$Var[p^*] = q^{*^2}Var(X) \tag{89}$$

profits become more unstable with price uncertainty, factored by the square of optimal output.

Proposition 11. *Consider a standard price-taking, expected profit maximizing firm with ex-ante commitment towards hiring decisions and additive uncertainty in the output price.*

1.The firm's expected employment, output and profits are invariant to price uncertainty.

2.Profit variability will rise with such uncertainty, being enhanced by the (square of the) output size.

If the firm has ex-post decision ability — it can easily adjust output -, it will produce q_1 when it observes $P+s$ such that:

$$\max_{q_1}(P+s)q - C(q) \tag{90}$$

$$P+s = C'(q_1); \quad \text{then} \quad \frac{dq_1}{ds} = \frac{1}{C''(q_1)} \tag{91}$$

It will target q_2 when it faces $P - \frac{rs}{1-r}$ such that:

$$\max_{q_2}(P - \frac{rs}{1-r})q - C(q) \tag{92}$$

$$P - \frac{rs}{1-r} = C'(q_2); \quad \text{then} \quad \frac{dq_2}{ds} = -\frac{1}{C''(q_2)}\frac{r}{1-r} \tag{93}$$

Denote $q^S(P)$ the inverse marginal cost function — the firm's supply at output price P. Then $q_1 = q^S(P+s)$ and $q_2 = q^S(P - \frac{rs}{1-r})$.

Let $s > 0$.

As $C''(q) > 0$, $q_1 > q_2$. With probability r, the firm will chose q_1, with $(1-r)$, q_2. The expected value of supply $E[q^*] = rq_1 + (1-r)q_2$, reacts to s according to:

$$\begin{aligned} \frac{dE[q^*]}{ds} &= r\frac{dq_1}{ds} + (1-r)\frac{dq_2}{ds} = r[\frac{1}{C''(q_1)} - \frac{1}{C''(q_2)}] \\ &= \frac{r}{C''(q_1)C''(q_2)}[C''(q_2) - C''(q_1)] \end{aligned} \tag{94}$$

If $C''(q)$ is increasing in q, that is, if the marginal cost function, $C'(q)$, is convex ($C'''(q) > 0$), $E[q^*]$ decreases with uncertainty — and is lower for a firm that can react ex-post than for the one that decides ex-ante (for an equivalent $s = 0$, necessarily smaller). Being concave, $E[q^*]$ increases with uncertainty — and is higher for a firm that responds ex-post.

As for expected costs:

$$\begin{aligned} \frac{dE[C^*]}{ds} &= rC'(q_1)\frac{dq_1}{ds} + (1-r)C'(q_2)\frac{dq_2}{ds} = r[\frac{C'(q_1)}{C''(q_1)} - \frac{C'(q_2)}{C''(q_2)}] \\ &= r\frac{C'(q_1)C'(q_2)}{PC''(q_1)C''(q_2)}[\frac{C''(q_2)}{C'(q_2)} - \frac{C''(q_1)}{C'(q_1)}] \end{aligned} \tag{95}$$

If absolute convexity of $C(q)$, $\frac{C''(q)}{C'(q)}$, is increasing in q, as $q_1 > q_2$ for $s > 0$, $\frac{dE[C^*]}{ds} < 0$ and expected costs decrease with uncertainty — and are lower for the firm that can react ex-post. Being decreasing with q, $\frac{dE[q^*]}{ds} > 0$ — and are higher for the firm adjusting ex-post.

$$\begin{aligned} \frac{dE[\pi^*]}{ds} &= r[P + s - C'(q_1)]\frac{dq_1}{ds} + (1-r)[P - \frac{rs}{1-r} - C'(q_2)]\frac{dq_2}{ds} \\ &+ rq_1 - (1-r)\frac{r}{1-r}q_2 = r(q_1 - q_2) = r[q^S(P+s) - q^S(P - \frac{rs}{1-r})] \\ &= r\{(s + \frac{rs}{1-r})q^{S'}(P) + [1 - \left(\frac{r}{1-r}\right)^2]\frac{s^2}{2}q^{S''}(P) + \cdots\} \end{aligned} \tag{96}$$

For $s > 0$, $\frac{dE[\pi^*]}{ds} > 0$: anticipated output price volatility favours expected profits — and ex-post decisions are profitable compared to exante output targeting. $\frac{dE[\pi^*]}{ds}$ will be larger, the higher the slope of the firm's output supply — the lower the slope of the firm's inverse supply, $C''(q^*)$, and the less convex is the firm's cost function. (This conclusion would be conformable with the standard properties of an optimal profit function[20] — the benefits from uncertainty around the output price more profit-enhancing the more convex the profit function is with respect to it; or the higher its second partial derivative with respect to the price, with correspondence to the slope of the output supply function[21].)

However, we can show that profits become more unstable with uncertainty:

$$\begin{aligned} Var[p^*] &= (1-r)r\{[(P+s)q_1 - C(q_1)] - [(P - \frac{rs}{1-r})q_2 - C(q_2)]\}^2 \qquad (97) \\ &= (1-r)r\{(s + \frac{rs}{1-r})q^S(P) + [1 - \left(\frac{r}{1-r}\right)^2]\frac{s^2}{2}q^{S'}(P) + \cdots\}^2 \end{aligned}$$

Then:

$$\begin{aligned} \frac{dVar[\pi^*]}{ds} &= 2(1-r)r\{[(P+s)q_1 - C(q_1)] \\ &- [(P - \frac{rs}{1-r})q_2 - C(q_2)]\}\{q_1 + \frac{r}{1-r}q_2\} \qquad (98) \end{aligned}$$

$\frac{dVar[\pi *]}{ds} > 0$ if $s > 0$; $\frac{dVar[\pi^*]}{ds} < 0$ if $s < 0$. $\frac{dVar[\pi^*]}{ds} > 0$ if $s > 0$; $\frac{dVar[\pi^*]}{ds} < 0$ if $s < 0$. Exogenous uncertainty will imply higher profit volatility the higher the firm's output supply, in a proportional relation to its square — ultimately, the higher the output price.

By varying s, we register the always positive trade-off:

$$\frac{dVar[\pi^*]}{dE[\pi^*]} = 2\{[(P+s)q_1 - C(q_1)] - [(P - \frac{rs}{1-r})q_2 - C(q_2)]\}\frac{(1-r)q_1 + rq_2}{q_1 - q_2} > 0 \qquad (99)$$

The variance of output will rise with uncertainty -more pronouncedly the more positively sloped labor supply is:

$$Var(q^*) = r(1-r)(q_1 - q_2)^2 \qquad (100)$$

The variance of total costs also rises with uncertainty - more intensely the more positively sloped labor supply is and the larger the marginal cost

[20] A consequence of Hotelling's lemma. See Varian (1992), for example.

[21] In fact, this correspondence is also apparent in the expression (3.31) below; we also refer the reader to formula (A.1) in Appendix 1 for the analog of (3.30), and to the first term of (B.1) in Appendix 2 for the recognition of similarities to (3.31).

at which the firm is operating; possibly higher, the lower the measure of convexity (symmetric to risk-aversion) of the cost function:

$$Var(C^*) = r(1-r)[C(q_1) - C(q_2)]^2 \tag{101}$$

Proposition 12. *Consider a standard price-taking, expected profit maximizing firm with ex-post flexibility towards output decisions and additive uncertainty in the output price. After a rise in such uncertainty, the firm's:*

1.expected output supply decreases (increases) if the marginal cost function is convex (concave) in the argument — in which case is lower (higher) for the firm with ex-post flexibility.

2.expected costs decrease (increase) if convexity of the cost function is less (more) pronounced than that of the marginal cost one as measured by the symmetric measure to the Arrow-Pratt measure of absolute risk aversion — in which case is lower (higher) for the firm with ex-post flexibility. Equivalently, if this measure for the cost function rises (decreases) with the argument, the output.

3.expected profits increase — more intensely the less convex is the cost function - and their volatility as well — enhanced, as that of supply, by output size and price. Expected profits are higher for the firm with ex-post reaction ability.

6. DECISION UNCERTAINTY

In the models advanced we generally considered the addition of uncertainty only at one of the relevant features of the firm profit function. It is conceivable that firm's decision targets may be affected by inadequate transmission — due to organizational complexity, lack of authority, faulty management, uncooperative resources.

Suppose decisions are surrounded by uncertainty X, so that even if the decision process targets $L(q)$, the effective employment (supply) is in fact $L + X(q + X)$. Then, the conclusions drawn for the expected values of the several aggregates of the profit maximizing firms deciding ex-ante and facing quantity uncertainty — of section 2 -would remain valid. However, the variance of the optimal profits would be higher than found before.

Ex-post flexibility would allow to correct for all possible "mistakes" or deviations. Then, the conclusions of section 3 would still hold for expected values (under additive as multiplicative uncertainty if we restrict ourselves for the latter to the cases of simultaneously random wage bill or revenues) — remaining invariant to the dispersion -but, as expected, the variance of profitability would be driven down to zero. Simulation with additional costs, increasing function of, say, s^2, would provide an interesting source of compensation mechanism to (for further) study.

7. CONCLUSION

A unified vision of the effects of exogenous randomnesses on the different levels of the competitive firm's economic activity is now possible:

1. Effects of a rise in uncertainty on expected factor demand and supply (supply and costs) added to the input (output) under ex-ante commitment are analogous to those of uncertainty added to its price in a context of total flexibility. Such correspondence is useful, once the properties of optimal profit functions are well-studied in producer's theory — quantity responses to simulation under any environment may be inferred more straight-forwardly by staging the analogous conditions under which those functions, of prices, can be used directly.

Those effects are conditional on the concavity or convexity features of the firm's technology — of marginal product and marginal cost -, and relations with risk-aversion measures were invariably encountered.

2. In general, an increase in exogenous uncertainty increases profits volatility.

3. An increase in exogenous uncertainty in quantities decreases expected profits with ex-ante commitment but has no effects on expected values with ex-post flexibility. An increase in exogenous uncertainty in prices has no effects on expected profits (or other variables) with ex-ante commitment but increases expected profitability with ex-post flexibility. In this last decision environment, it may (then) be possible for the firm to trade positions in the mean-variance coordinates of profitability by exposure to the adequate price diversification (or wage dispersion).

Of course, flexibility — a move towards ex-post adjustable decisions — increases expected profits. But, it induces control variables' fluctuations.

4. Concavity or convexity of production or cost function determine the size of the impact of the outside turbulence on firm's expected profits. Under quantity uncertainty, the variability of profits would depend more closely on the (usually the square of the) first derivative of the technology elements (production or cost function) — that is, of marginal product or of marginal cost at which the firm operates —, generally with correspondence to a price; under price uncertainty, on the (square of the) size (quantity) of a control variable. However — see Appendix 2 —, these last conclusions may be less immune to a change in the distribution of the exogenous randomness.

5. The simulation of multiplicative instead of additive uncertainty did not alter qualitatively the conclusions (in ex-post quantity uncertainty, mild

adjustments were considered to the assumptions, though, reproducing general uncertainty in the decision variable). The role of absolute risk-aversion measures was replaced by relative ones. Importance of "factored" marginal product and marginal cost functions emerged in its presence. Proportionality of results to prices or their squares found for additive scenarios was frequently replaced by either revenues or costs.

APPENDIX A

Taylor's expansion to the second order of any function $F(.)$ around neighbourhood X of a given level L generates

$$F(L+X) = F(L) + F'(L)X + \frac{F''(L)}{2!}X^2 + \cdots \quad \text{(A.1)}$$

Let X be a random variable added to L, being L independent of X's variability (but not of moments of X's distribution, necessarily). Then:

$$E[F(L+X)] = F(L) + F'(L)E[X] + \frac{F''(L)}{2}\{Var(X) + E[X]^2\} \quad \text{(A.2)}$$

Admit a consumer with utility function derived from income $F(.)$, with $F(.)' > 0$, being income $L+X$ where L represents a deterministic part and X a random variable with null expected value. The consumer is more averse to income's volatility the higher the amount m — the risk-premium — that he is willing to pay to avoid the randomness X; that is, m such that:

$$F(L-m) = E[F(L+X)]$$

Take Taylor's expansion of $F(L-m)$ around L but only to the first order and equalize to that of the right hand-side — this to the second order — for that lottery of null expected value:

$$F(L) - F'(L)m = F(L) + \frac{F''(L)}{2}Var(X)$$

Then: $m = -\frac{F''(L)}{F'(L)}\frac{Var(X)}{2}$

The higher $-\frac{F''(L)}{F'(L)}$, the Arrow-Pratt measure of absolute risk-aversion, the larger the consumer's sensitivity to risk, that is, to the variance of X (as to that of total income, $L+X$).

Suppose we have a multiplicative lottery: $X = LZ$ and $E[Z] = 0$. Then, $Var(X) = L^2 Var(Z)$. The risk premium m' defined per unit of L that the consumer is willing to pay to avoid risk Z will be such that:

$$F(L-m) = F(L-m'L) = E[F(L+LZ)]$$

Hence: $m = m'L = -\frac{F''(L)}{F'(L)}\frac{L^2Var(Z)}{2}$ and: $m' = -\frac{LF''(L)}{F'(L)}\frac{Var(Z)}{2}$

The larger $-\frac{L^2F''(L)}{F'(L)}$, the larger m will be — the larger $\frac{dm}{dVar(Z)}$-, and the larger the consumer's sensitivity to the randomness Z. The larger $-\frac{LF''(L)}{F'(L)}$, the Arrow-Pratt measure of relative risk-aversion, the larger the premium per unit of income L that the consumer is willing to pay to eliminate the variance that Z brings to his utility.

Finally, take the factored lottery $X = \sqrt{L}Z$ with $E[Z] = 0$. Then, $Var(X) = LVar(Z)$. The premium m to pay to avoid Z will be such that:

$$F(L-m) = E[F(L+\sqrt{L}Z]$$

Hence: $m = -\frac{LF''(L)}{F'(L)}\frac{Var(Z)}{2}$ $m' = \frac{m}{L} = -\frac{F''(L)}{F'(L)}\frac{Var(Z)}{2}$

The larger $-\frac{LF''(L)}{F'(L)}$, the Arrow-Pratt measure of relative risk-aversion, the higher the consumer's reaction to the variability Z, and the larger the premium he is willing to pay to avoid it — the larger $\frac{dm}{dVar(Z)}$. The larger $-\frac{F''(L)}{F'(L)}$, the Arrow-Pratt measure of absolute risk-aversion, the higher, m', the premium per unit of income L that the consumer is willing to pay.

Note that if we were analysing aversion to a risk surrounding a negatively valued argument of $F(.)$, i.e., if $F'(.) < 0$, we could propose to measure it by how much of a certain amount of that argument, denote it by n, the maximizer of $F(.)$ will be willing endure to avoid it:

$$F(L+n) = E[F(L+X)]$$

Then: $n = \frac{F''(L)}{F'(L)}\frac{Var(X)}{2}$

The higher n, the more risk-averse to X the maximizer of $F(.)$ would be. The higher $\frac{F''(L)}{F'(L)}$ and not its symmetric, the larger would n be, and thus the measure of aversion to a risk added to L - see Martins 2004, for example.

APPENDIX B

Depart from Taylor's expansion of $F(L+X)$ in (A.1) to the second order, and its expectation derived in (A.2). Consider an increase in $E[X]$ at a fixed variance level. Then

$$\frac{\partial E[F(L+X)]}{\partial E[X]} = F'(L) + F''(L)E[X]$$

$\frac{\partial E[F(L+X)]}{\partial E[X]} > 0$ provided that, for $F'(L) > 0$, $-\frac{F''(L)}{F'(L)}E[X] < 1$. Of course, around $E[X] = 0$, $\frac{\partial E[F(L+X)]}{\partial E[X]}$ is determined by $F'(L)$. Consider the effect of a change in $Var(X)$ — that is compatible with a fixed $E[X]$:

$$\frac{\partial E[F(L+X)]}{\partial Var(X)} = \frac{F''(L)}{2}$$

The effect is signed according to the convexity of $F(L)$ — positive if convex, negative if concave, as is well-known.

Admit instead that we would want to analyze the transmission of the variance of X to that of the variance and not the expected value of the function $F(L+X)$. We should now inspect:

$$\begin{aligned} & Var[F(L+X)] \\ = \; & F'(L)^2 Var(X) + \frac{FL''(L)^2}{4} Var(X^2) + F'(L)F''(L)Cov(X,X^2) \end{aligned} \tag{B.1}$$

Of course,

$$\begin{aligned} Var(X) &= E[(X-E[X])^2] = E[X^2] - E[X]^2 \\ Var(X^2) &= E[(X^2-E[X^2])^2] = E[X^4] - E[X^2]^2 \\ Cov(X,X^2) &= E[(X-E[X])(X^2-E[X^2])] = E[X^3] - E[X]E[X^2] \end{aligned}$$

Now, 3rd and 4th moments of the distribution of X become important to classify the impact of a change in the parameters of the distribution in the volatility of $F(L+X)$. Take for simplicity, the null expected value noise, that is, $E[X]=0$. Then, $Var(X) = E[X^2]$; $Cov(X,X^2) = E[X^3]$ and we can replace in (B.1):

$$\begin{aligned} & Var[F(L+X)] \\ = \; & F'(L)^2 E[X^2] + \frac{FL''(L)^2}{4}[E[X^4] - E[X^2]^2] + F'(L)F''(L)E[X^3] \end{aligned} \tag{B.2}$$

Interestingly, the last expression allows as to infer the effect of asymmetry in the distribution of X — through the effect of $E[X^3]$. And of kurtosis, through the effect of $E[X^4]$. In the common distribution families, it may be difficult to disentangle all the effects, once four different parameters would be required to produce a correspondence to the four aspects of the probability distribution of X that are now relevant: location (mean), dispersion (variance), asymmetry and kurtosis. Moreover, changing, say, the variance of a particular distribution usually implies a change in the other measures. Take an example:

Admit that X has a normal distribution of null mean; then, $E[X^3] = 0$ and $E[X^4] = 3E[X^2]^2$. Replacing above:

$$Var[F(L+X)] = F'(L)^2 E[X^2] + \frac{FL''(L)^2}{2} E[X^2]^2 \quad \text{(B.3)}$$

and $\frac{\partial Var[F(L+X)]}{\partial E[X^2]} = F'(L)^2 + F''(L)^2 E[X^2] > 0$

Interestingly, the effect is always positive and enhanced by the absolute values of both $F'(L)$ and $F''(L)$ — whereas (B.1) could suggest an importance only of the former.

Under the last assumptions, changing the variance of the distribution of X, we can accomp lish the trade-off between the expected value of $F(L+X)$ and its variance:

$$\frac{\partial Var[F(L+X)]}{\partial E[F(L+X)]} = 2\frac{F'(L)^2 + F''(L)^2 Var(X)}{F''(L)}$$

Being $F(L)$ convex, the trade-off is positive and the inverse of the ratio decreases with $Var(X)$.

(Note that L is assumed independent of $E[X^2]$. Likewise, we can only admit that an "optimal" mean-variance trade-off is forwarded in the analysis for ex-post scenarios.)

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