ANNALS OF ECONOMICS AND FINANCE **9-2**, 397–399 (2008)

Mean Reversion and Consumption Smoothing: A Comment*

Fang Zhou

School of Economics and Management, Wuhan University, 430072, China

Liutang Gong

Guanghua School of Management, Peking University, Beijing, 100871, China

and

Yaping Wang

Guanghua School of Management, Peking University, Beijing, 100871, China
E-mail: ypwang@gsm.pku.edu.cn

This comment restudies Black's (1990) [Black F., 1990. Mean reversion and consumption smoothing. *Review of Financial Studies* 3,107-114.] paper and shows that the conclusions in Black (1990) are not right.

Key Words: Mean reversion; Portfolio selection.
JEL Classification Number: O41.

Black (1990) allowed wealth variability and the market's risk premium to vary with wealth and time to explain consumption smoothing and equity risk premium in a conventional model. In his paper, the most important technology is to let the risk aversion of the direct utility function and the indirect utility function be different. Upon on this set up, he explained the "consumption smoothing puzzle" and "equity premium puzzle". We find there are serious mistakes at his paper. Following his process, the risk aversion of the direct utility function and the indirect utility function must be equal, thus it reduces to the conclusions Merton (1971) have presented.

Following Black's (1990) framework, the instantaneous direct utility function is specified as

$$u(c, W) = \frac{c^{1-\delta}}{1-\delta}, \tag{1}$$

*Project 70725006 supported by National Science Foundation for Distinguished Young Scholars of China.

1529-7373/2008

where $\delta > 0$ is the risk aversion of direct utility function.

Suppose the investor's wealth is W, he chooses his asset holdings x and his consumption c to maximize his discounted utility

$$\max E_0 \int_0^\infty u(c)e^{-\rho t}dt$$

subject to

$$dW = (rW + ax - c)dt + sxdz \tag{2}$$

with the given initial wealth $W(0)$.

Where z is the Brownian motion, it is further assumed that dz is temporally independent, normally distributed, and

$$E(dz) = 0, \quad Var(dz) = dt,$$

and r is the interest rate; a is the risk premium on assets; s is the wealth volatility.

To solve the optimization problem, we introduce the value function

$$V(W(t), t) = \max_{c,x} E_t \int_t^\infty u(c_s, W_s)e^{-\rho s}ds \tag{3}$$

subject to the budget constraint (2).

Associated with the above optimization problem, we have the recursive equation[1]

$$\max_{c,x}\{u(c)e^{-\rho t} + V_t + V_W(rW + ax - c) + \frac{1}{2}s^2x^2V_{WW}\} = 0. \tag{4}$$

The first-order conditions are:

$$u_c(c, W) = V_W, \tag{5}$$

$$aV_W + s^2xV_{WW} = 0, \tag{6}$$

and the Bellman equation

$$u(c, W)e^{-\rho t} + V_t + V_W(rW + ax - c) + \frac{1}{2}s^2x^2V_{WW} = 0.$$

[1]Black (1990) mistook this equation as

$$\max_{c,x}\{u(c) - \rho V + V_t + V_W(rW + ax - c) + \frac{1}{2}s^2x^2V_{WW}\} = 0$$

To derive the explicit solution, Black (1990) postulated the value function

$$V(W,t) = \frac{be^{-\rho t}W^{1-\gamma}}{1-\gamma}, \tag{7}$$

where b is a constant, which will be determined below; γ is the relative risk aversion of indirect utility function.

From equations (5), (6), and (7), we have

$$c(W,t) = b^{-1/\delta}W^{\gamma/\delta} \equiv b^{-1/\delta}W^{k},$$

$$x = -\frac{aV_W}{s^2V_{WW}} = \frac{aW^{-\gamma}}{\gamma s^2W^{-\gamma-1}} = \frac{a}{\gamma s^2}W,$$

where we denote $k = \gamma/\delta$.

Substituting the above equations into the Bellman equation, we have

$$(\frac{1}{1-\delta} - 1)b^{-1/\delta}W^{k-1} + (-\rho\frac{1}{1-\gamma} + r + \frac{1}{2}\frac{a^2}{\gamma s^2}) = 0.$$

Because b is a constant[2], we must have $k = 1$, thus $\gamma = \delta$, this reduces to the Merton's (1971) model, the conclusions presented by Black (1990) will not hold yet!

REFERENCES

Black F., 1990. Mean reversion and consumption smoothing. *Review of Financial Studies* **3**, 107-114.

Merton R. C., 1971. Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory* **4**, 373-413.

[2]Black (1990) subsituted the equlibrium condition $x = W$ into the above equation, thus, he did not impose $r = \delta$ even if b is a constant. But the equilibrium condition cannot be used when we consider the consumer's optimization problem.