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Skilled Labor, Economic Transition and Income Differences: A Dynamic Approach*

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We propose a dynamic model of economic transition in which the supply constraint of skilled labor and skill premium are the focus. We argue that the constraint of skilled labor affect both the beginning date and the subsequent path of modern growth. The model matches the observed multiple paths of income inequality, such as "U-shaped", "inverted U-shaped" or "N-shaped" paths. Hence, the model requires faster technology change and more investment on skill formation to account for the current income differences relative to models that focus only on steady states.

Key Words: Skilled labor; Economic transition; Income inequality.
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1. INTRODUCTION

Ever since the pioneering work of Kuznets (1955), economic growth and income inequality have occupied worldwide attention and led to huge controversy. Although Kuznets (1955) propose the famous inverted U-shape hypothesis, that the income inequality may increase in the early period of

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growth, and turn to decrease as the economy grows, many empirics of different countries and/or different periods provide widely divergent results. Most literatures in new growth theory emphasize the effect of specialized human capital, spillover effect of R&D, technology innovation and adoption on long-run growth, but make the assumption of homogenous labor (Romer, 1986; Lucas, 1988; Aghion & Howitt, 1992; Parente & Prescott, 1999). Thus these literatures neglect a very important development fact: countries that have experienced modern growth (a sustained increase in per capita output) also experienced multiple evolutionary paths of income inequality overtime. Other literatures study skill differences and skill premium, and point out that skill differences make huge difference in dynamics of economic growth and result in significant inequality (Acemoglu, 2002). However, skill premium is usually defined as the ratio of returns to labor with different education attainments in industrial countries, and thus cannot be used to analyze how skill differences affect the dynamics of growth and inequality in LDCs.

In this paper, we introduce the endogenous supply of skilled labor into the Hansen-Prescott (2002) model of transition, and show that the disparity of technology between modern industry and traditional agriculture, and the supply constraint of skilled labor affect the economy's turning point from traditional to modern growth, and subsequent growth path. The initial inadequacy in skilled labor and its low growth rate delay the transition and result in sustained skill premium and disparity in per capita income between sectors, which in turn present multiple possible paths (inverted U-shaped curve as an example) of the dynamic change in inequality.

Historically, instead of following the unique inverted U-shaped path, the dynamics of inequality turn out to change significantly with the pace of transition and differ across countries and periods. In U.K. during 1688-1995, the inequality spreads out an N-shaped curve, which can be divided into an inverted-U-shaped period in 1688-1900 and a U-shaped one in 1911-1995. In U.S.A. from 1913 to 1994, the change in inequality is less dramatic and pursues a relatively flatter U-shaped path. While in Japan whose transition starts later, the inequality increases sharply from 1886 to late 1930s, declines to a much lower level after WWII, and then remains relatively low inequality ever since[1].

There are two empirical and theoretical approaches to growth and inequality. One is to test the hypothesis that inequality is an inverted U-shaped function of the level of output. Many provide positive evidence (Ahluwalia, 1976; Deininger & Squire, 1996), while others prove there is not enough evidence for the hypothesis (Anand & Kanbur, 1993a,1993b).

[1]Readers are referred to Appendix A for a description of the dynamics of inequality in the U.K., U.S.A. and Japan during a long period for detail.

Some empirics find out that inequality is a U-shaped, or cubic function of output level (Fields & Jakubson, 1995; Francois & Rojas-Romagosa, 2008). Another parallel approach is to test whether more inequality enhances or reduces growth rate. Many researchers obtain a negative relationship (Alesina & Rodrik, 1994; Persson & Tabellini, 1994; Perotti, 1996)[2]. Other researchers conclude that the effect of inequality on growth is positive or indeterminate. Li & Zou (1998) argue with recent empirical findings that inequality is not harmful to growth. Forbes (2000) illustrates that a positive relationship presents only in the short-run. Barro (2000) contrasts the negative effect in poor countries with the positive effect in the rich[3]. Lundberg & Squire (2003) find out a number of factors that potentially influence both growth and inequality and test their joint significance. Nevertheless, it remains an open question why, as output per capita increase, the path of inequality turns out to be multiple in different economies and periods.

Another theoretical challenge is to explain: what is the possible influence of economic transition on income inequality overtime. Hansen & Prescott (2002) construct a two-period OLG model, which includes a Malthus sector and a Solow sector. The former is traditional agriculture sector that uses capital, labor and land as inputs, while the latter is modern industry using capital and labor. The model establishes a crucial condition for the occurrence of industry. As long as the condition is satisfied, industry with higher technology level will mobilize labor out of agriculture. They assume homogenous labor and smooth rural-urban migration, The transition from agriculture to modern industry will be fulfilled with the sustained increase of proportions of industry in both total employment and output. The model provides an explanation for both the stagnant growth in pre-1700 period in Europe, and the significant increase in productivity and per capita output after industrial revolution. However, with labor indifference assumption, there is continuous labor migration from agriculture to industry as long as technology progress permits. In equilibrium, wage rates equalize across sectors leaving no skill premium or income differences. Therefore this model, which explains the economic transition and growth in centuries, does not illustrate the dynamics of income differences accompanying transition.

Wang & Xie (2003) consider three factors in the activation of modern industry: skill requirements, economy of scale, and subsistence consump-

[2]The various explanations for negative effect include: inequality has negative political effect (Alesina & Rodrik); inequality may hurt investment in physical and human capital (Galor & Zeira, 2003; Aghion & Bolton, 1997); inequality reinforces the unequal distribution of natural resources (Gylfason & Zoega, 2003).

[3]The possible reasons for positive relationship include: inequality enhances public investment (financing through taxation) or private investment (Saint-Paul & Verdier, 1993), social capital formation (Benabou, 1996a) and technological change (Galor & Teisson, 1997).

tion level. They compare the high growth in the NICs with the stagnant growth in some African, South Asian countries. They propose that the activation of modern industry requires not only technology but also skilled labor. Moreover, the simultaneous development of industry is required to produce reciprocal demand and industry-wide spillovers. Thus the barrier may come from the inadequate supply of skilled labor. But they do not study the possible multiple paths of income distribution. Ngai(2004) shows that the barriers to private investment may delay the turning point and affect the subsequent growth. She assumes labor indifference and only explains the invert U-shaped curve of income inequality.

Empirical evidences have accumulated for skill differences and skill premium. Many researchers calculate skill premium with the ratio of wage of college graduates to that of high school graduates. Murphy & Welch (1992) find out that although there are more college graduates in industries, the skill premium for them increases overtime. Acemoglu (1998) shows that the increase of skilled labor in total labor forces means a larger skill-complementary technology market and more monopoly rents, which in turn provide incentive for firms to increase productivity of skilled labor. With the increase of skilled labor, skill premium may decrease initially (the substitution effect), then induce skill-biased technology change (SBTC, the economy of scale effect), which in turn increase skill premium to a level even higher than its original level. Acemoglu (2002) formalizes the mechanism for SBTC. Most researchers focus on skill premium across industries in developed economy, thus cannot specify the supply of skilled labor or trace the income inequality in economic transition.

We introduce skill differences of labor into two-sector framework (Hansen & Prescott, 2002), and analyze the dynamic transition. We show that the technology disparity across sectors, as well as the supply constraint of skilled labor affect the turning point of transition, and lead to multiple possible paths of income inequality, including invert U-shaped, U-shaped, N-shaped curves. The reminder of the paper is organized as follows. Section 2 presents the OLG transition model with skill differences of labor. Section 3 discusses the effect of supply constraint of skilled labor on the turning point of transition, the pace of development, and skill premium. The quantitative studies are in Section 4 to show the potential of the model to account for different paths of income inequality, and a conclusion follows in Section 5.

2. THE MODEL

2.1. Technology

Consider a two-sector economy with agriculture (a) and industry (i). Both technologies are subject to exogenous change and both have constant returns to scale. Industry uses only skilled labor, while agriculture can use skilled and unskilled labor. The two production functions are as follows:

$$Y_{at} = A_a \gamma_a^t K_{at}^{\phi} N_{at}^{\mu} L_{at}^{1-\phi-\mu} \tag{1}$$
$$Y_{it} = A_i \gamma_i^t K_{it}^{\theta} N_{it}^{1-\theta} \tag{2}$$

where K_{jt}, N_{jt} are capital and labor used at time t in technology j ($j = a, i$). Land is used only in agriculture, and thus normalized, i.e. $L_{at} = 1$. $\phi \in (0,1)$ is the capital share, $\mu \in (0,1)$ is the labor share and $(1-\phi-\mu) \in (0,1)$ the land share in agriculture. $\theta \in (0,1)$ is the capital share in industry. A_a and A_i are initial level of TFP in both sectors, and $\gamma_a^t > 1$ and $\gamma_i^t > 1$ are the growth rates of technology. The capital intensity in industry is assumed to be higher, $\theta > \phi$. Capital is assumed to depreciate completely each period[4]. Outputs of the two sectors are identical and can be used for consumption or investment. Feasibility requires:

$$C_t + X_{at} + X_{it} = Y_{at} + Y_{it} \tag{3}$$

where C_t is aggregate consumption, while X_{at} and X_{it} are aggregate investments.

We take into account the skill differences of labor. Industry technology matches skilled labor, while any labor can be used in agriculture. Let total labor be N_t, the supply of skilled labor be Q_t. We have:

$$N_{at} + N_{it} = N_t \tag{4}$$

$$N_{it} \leq Q_i, \quad N_{at}/N_t \leq 1 \tag{5}$$

(4) is the total labor supply constraint. (5) means that industry cannot employ more labor than the supply of skilled labor, while agriculture can use up all labor in the economy.

In traditional economy, all labor is involved in agriculture. Only when the supply of skilled labor surpasses a certain critical level, can modern industry be established to employ skilled labor with skill premium. Thus in economic transition, the supply of skilled labor is an increasing function of skill premium, and all labor will be skilled labor with the completion of transition. The supply of skilled labor is defined as:

$$Q_t = \begin{cases} N_t(1-\xi), & \text{if } \hat{\xi} \leq \xi \leq 1 \\ N_t(1-\xi(z)\lambda^t), & \text{if } \xi > \hat{\xi} \end{cases} \tag{6}$$

[4]In the quantitative studies a period is interpreted to be 30 years, so this assumption is empirically reasonable.

where ξ is the proportion of unskilled labor in total labor, $(1-\hat{\xi})$ is the minimum requirement of skilled labor to activate transition. If ξ is higher than $\hat{\xi}$, then all labor are involved in agriculture earning equal wage rate. Economic transition gets started if $\xi \leq \hat{\xi}$, and the supply of skilled labor will increase as a function of skill premium ever since. Meanwhile, unskilled labor reduces ($\xi' < 0, \xi'' < 0$), and in the limit, skilled labor approaches to total labor. We suppose each individual born in agriculture sector is endowed with the same piece of land and labor time, he earns income by labor work and capital-renting, which is expended to consume, repay rents to capital or land, and accumulate capital. Thus the profit maximization problem is:

$$\max_{N_{at},K_{at},L_{at}} Y_{at} - w_{at}N_{at} - r_{Kt}K_{at} - r_{Lt}L_{at} \quad \text{s.t. } (1) \tag{7}$$

where w_{at} is the wage rate of agriculture, r_{Kt} and r_{Lt} are rents of capital and land respectively. Land is normalized. Let $k_{at} = K_{at}/N_{at}$. The FOCs of (7) are:

$$r_{Kt} = \phi A_a \gamma_a^t K_{at}^{\phi-1} N_{at}^{\mu+\phi-1} = \phi A_{at} k_{at}^{\phi-1} N_{at}^{\mu+\phi-1} \tag{8}$$

$$w_{at} = (1-\phi) A_a \gamma_a^t k_{at}^{\phi} N_{at}^{\mu+1} = (1-\phi) A_{at} k_{at}^{\phi} N_{at}^{\mu+\phi-1} \tag{9}$$

Because of the property of CRS in production (2), we aggregate all industry firms. Given technology, interest rate and wage rate of industry (w_{it}), the profit maximization problem is:

$$\max_{K_{it},N_{it}} Y_{it} - r_{Kt}K_{it} - w_{it}N_{it} \quad \text{if} \quad N_{it} \leq Q_t \tag{10}$$

or:

$$\max_{K_{it}} Y_{it} - r_{Kt}K_{it} - w_{it}Q_t \quad \text{if } N_{it} > Q_t \tag{11}$$

The FOCs of (10) and (11) are respectively:

$$r_{Kt} = \theta A_i \gamma_i^t K_{it}^{\theta-1} N_{it}^{1-\theta}; \qquad w_{it} = (1-\theta) A_i \gamma_i^t K_{it}^{\theta} N_{it}^{-\theta} \tag{12}$$

$$r_{Kt} = \theta A_i \gamma_i^t K_{it}^{\theta-1} Q_t^{1-\theta}; \qquad w'_{it} = (1-\theta) A_i \gamma_i^t K_{it}^{\theta} Q_t^{-\theta} \tag{13}$$

In (13), because of the supply constraint of skilled labor, wage rate in industry presents higher skill premium: $w'_{it} = (1-\theta)A_{it}(K_{it}/Q_t)^{\theta} > (1-\theta)A_{it}(K_{it}/N_{it})^{\theta} = w_{it}$. We will discuss this point in more detail later.

2.2. Household sector

The model has two-period overlapping generations (Diamond, 1965). Let N_t be the number of young agents and c_{1t} be the consumption level for the young agents in period t. Population dynamics are given by $N_{t+1} = g(c_{1t})N_t$, where $g(.)$ is an exogenous function that will be specified later. In period 0, there are N_{-1} old agents, each is endowed with K_0/N_{-1} units of capital and L/N_{-1} units of land. Young agents are born with one unit of labor time, which they supply elastically. Different from former models, only skilled labor is demanded by industry. They make a consumption-saving decision on how much land and capital to purchase. They become old in the second period where their sources of income are from renting land and capital to firms and from the sale of land to the next generation. For each generation t, young agents choose consumption (c_{1t}, c_{2t+1}) and investment $(x_{at}, x_{it}, l_{t+1})$ to maximize lifetime utility:

$$u(c_{1t}, c_{2t+1}) = \ln c_{1t} + \beta \ln c_{2t+1} \tag{14}$$

subject to the budget constraints:

$$c_{1t} = w_{jt} - (x_{it} + x_{at} + q_t l_{t+1}), \quad j = i, a \tag{15}$$

$$c_{2t+1} = r_{Kt}(x_{it} + x_{at}) + (q_{t+1} + r_{l,t+1})l_{t+1} \tag{16}$$

where is β discount factor and q_t is the price of land in period t. Rate of return to capital equalizes in both sectors, while wage rates differ during transition as a result of skill premium.

2.3. Equilibrium

Following Hansen & Prescott (2002) and Ngai (2004), we introduce the endogenous skill differences of labor. The competitive equilibrium and the dynamics are established[5]. We investigate the equilibrium where dynamics of the model are characterized by three development stages. Stage 1 is the traditional growth stage where modern technology is not used and the economy is on the traditional balanced growth path (TBGP). The exogenous population growth function is chosen such that the growth in agriculture is absorbed by population growth. Hence, there is no increase in per capita income or output. Stage 2 is the transition stage where the level of TFP in industry is sufficiently high relative to agriculture and the supply of skilled labor has surpassed the minimum requirement. It becomes profitable for industry to use modern technology and employ skilled labor with skill premium. The economy is in transition to modern growth, and more and more labor has been cultivated as skilled labor and mobilized to industrial sector. In Stage 3, only modern technology is used, all labor forces are skilled labor and the economy converges to a modern balanced growth path (MBGP).

[5]Readers are referred to Appendix B for precise definition and proofs.

The dynamics of the model shows that only when the supply of skilled labor surpasses a minimum critical level and the industrial technology is high enough to pay for skilled labor with a skill premium, can the economy starts its transition to modern economy. Meanwhile, even if the transition gets started, if the supply of skilled labor is limited, modern industry cannot employ enough skilled labor, which in turn results in higher skill premium. High skill premium provides incentive for accumulating skills, and the increased supply of skilled labor can absorbed by modern industrial sector in the long run. Finally, with more and more skilled labor being allocated in industrial sector, the economy converges to a balanced growth path where output per worker is growing at a constant rate.

3. ECONOMIC TRANSITION AND THE DYNAMICS OF INCOME INEQUALITY

We focus on the endogenous skill differences and the role of skill premium in the three stages of development to examine the dynamics of income inequality. In TBGP, all labor is allocated in agriculture, whose technology A_{at} matches unskilled labor. Hence, wage rate is w_{at} regardless of the skill levels. Let ν_a be the agriculture capital-output ratio, the output per worker is:

$$\hat{y}_a = [A_a \gamma_a^t \nu_a^\phi N_t^{\mu+\phi-1}]^{1/(1-\phi)} \tag{17}$$

which is constant given population growth rate of $\gamma_a^{1/(1-\mu-\phi)}$. Along TBGP, the rents and price of land grow at a rate of $\gamma_a^{1/(1-\mu-\phi)}$, while wage rate $\hat{w}_a$ and interest rate of capital $\hat{r}_K$ are constant. On TBGP, firms determine when is profitable to start using modern industrial technology given the prices of production inputs at their TBGP levels.

If there is enough supply of skilled labor in the economy, and $N_{it} \le Q_t$, the condition of firms to start using modern technology is:

$$A_{it} = A_i \gamma_i^t \ge z_1^{1-\theta} \left(\frac{\hat{w}_a}{1-\theta}\right)^{1-\theta} \left(\frac{\hat{r}_K}{\theta}\right)^\theta \tag{18}$$

where z_1 is the skill premium in industrial sector ($w_{it} = z_1 w_a$). It turns out that the higher the level of technology in industrial sector (A_{it}), the lower the skill premium, the more possible for the economy to start its economic transition.

If the skilled labor stock has surpassed the critical minimum level, but $N_{it} > Q_t$, then the condition of starting modern industrial sector is:

$$A_{it} = A_i \gamma_i^t \ge z_2^{1-\theta} \left(\frac{\hat{w}_a}{1-\theta}\right)^{1-\theta} \left(\frac{\hat{r}_K}{\theta}\right)^\theta \tag{19}$$

where the wage rate in industrial sector is $w'_{it} = z_2 w_a$. (12) and (13) imply $z_2 > z_1$. Therefore, given the TFP in industrial sectors, the supply constraint of skilled labor makes it more difficult for firms to adopt industrial technology, which delays economic transition.

Defining the turning point t^* to be the period that the industrial technology is first used, (18) and (19) imply the following conditions.

$$\begin{aligned} A_i\gamma_i^{t^*} \geq & Bz_1^{1-\theta}\left(\frac{1}{N_0}\right)^{(1-\mu-\phi)(1-\theta)/(1-\phi)} > A_i\gamma_i^{t^*-1} \text{ if } N_{it} \leq Q_t; \\ A_i\gamma_i^{t^*} \geq & Bz_2^{1-\theta}\left(\frac{1}{N_0}\right)^{(1-\mu-\phi)(1-\theta)/(1-\phi)} > A_i\gamma_i^{t^*-1} \text{ if } N_{it} > Q_t \end{aligned} \tag{20}$$

where $B = \left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}(\nu_a^{(\phi-\theta)}A_a^{(1-\theta)})^{1/(1-\phi)}$, is a function of technologies and preference parameters. Since the threshold is constant, (20) will be satisfied and the industrial technology will get used at some point as long as it keeps growing. Therefore, no matter what is the difference in technologies between two sectors, the transition to modern growth is inevitable in all economies. However, even if they have access to the same technologies, the turning points can still be different, depending on the growth rate of modern technologies in each economy, and whether there is supply constraint of skilled labor and resulting levels of skill premium. Thus we have the following Proposition 1.

PROPOSITION 1. *The turning point of transition to a modern economy depends on: (1) the accessibility to and growth rate of modern technology; (2) whether there is abundant supply of skilled labor and the level of skill premium.*

Skill premium affects economic transition in two ways. On one hand, with relatively higher supply of skilled labor and lower skill premium, the transition is easier to activate. However, because of the relative surplus of skilled labor, part of skilled labor $(Q_t - N_{it})$ has to work in agriculture with no skill premium. The fact provides disincentive for skill formation, which may result in shortage of skilled labor at some later time during transition. On the other hand, the inadequacy in skilled labor and the higher skill premium makes it more difficult to activate transition. Nevertheless, higher premium motivates more skill formation and an upgrading of average skill level, which can facilitate transition.

Consider two economies, where the supply of skilled labor is abundant in the first one, and inadequate in the second. From (20), the difference of turning points of the two economies is:

$$t_2^* - t_1^* = \frac{(1-\theta)\ln(z_2/z_1)}{\ln\gamma_i} \tag{21}$$

Given technology available in industrial sector, the turning point in the second economy is delayed by $\frac{1-\theta}{\ln\gamma_i}$ period.

When the economy is on its transition to modern economy, the allocation of factors requires equalization of marginal products across sectors. Let n_{at}^* be the equilibrium fraction of labor in agriculture, it solves:

$$f(n_{at}) = \frac{\phi}{\theta}\left(\frac{\varphi}{z_1}\right)^{\phi-1}(1-(1-\varphi/z_1)n_{at})^{\theta-\phi} - \frac{A_{it}}{A_{at}}I_t^{\theta-\phi}N_t^{1-\theta-\mu}n_{at}^{1-\phi-\mu} = 0 \tag{22}$$

where $\varphi = \frac{(1-\theta)\phi}{\theta\mu}$, I_t is the total value of investment by the young generation at time $(t-1)$. Assume that in modern industrial sectors the capital intensity is higher $(\theta > \phi)$, and technology grows faster $(A_{it} > A_{at})$, the fraction of labor in agriculture is decreasing and converges to zero. Meanwhile the fraction of skilled labor is increasing until all labor is skilled labor $(Q_t \to N_t)$. In (22), skill premium z_1 serves as an accelerator to the process[6]. If there is shortage in supply of skilled labor, then skill premium (z_2) will be higher, which will further reinforce the above effect. Therefore we have Corollary 1.

Corollary 1. *Given the technology change in modern industrial sector, the shortage in supply of skilled labor delays the turning point of transition, but accelerates the process in the long run.*

With more skilled labor reallocated to industrial sectors, asymptotically the economy behaves like a one-sector modern economy. Assume the population growth rate converges to a constant rate, the economy then converges to MBGP. The capital-output ratio along MBGP is $\nu_i = \frac{\beta(1-\theta)}{(1+\beta)\gamma_i^{1/(1-\theta)}}$, thus the output per worker is $y_{it} = (A_i\gamma_i^t\nu_i^\theta)^{1/(1-\theta)}$. Along MBGP all labor will be skilled labor, and wage and consumption per worker will grow at rate of $\gamma_i^{1/(1-\theta)}$.

We now examine the dynamic change of supply in skilled labor and income inequality. Let the technology difference across sectors be $E(t) = \frac{A_i\gamma_i^t}{A_a\gamma_a^t} = \frac{A_{it}}{A_{at}}$, we consider how the skill premium changes given the supply constraint of skilled labor.

[6] $z_1 > 1$ makes the first term in (22) smaller, and the second term larger, which results in faster convergence of n_{at}^* towards 0. The readers are referred to the Appendix B for detail.

Proposition 2. *Given supply constraint of skilled labor (i.e. $\xi \le \hat{\xi}$, and $N_{it} > Q_i$), and the condition of economic transition is satisfied: $A_{it} = A_i\gamma_i^t \ge z_2^{1-\theta}\left(\frac{\hat{w}_a}{1-\theta}\right)^{1-\theta}\left(\frac{\hat{r}_K}{\theta}\right)^{\theta}$, then we have $\partial z_2/\partial E(t) > 0, \partial z_2/\partial Q_i < 0$.*

Proof. By condition of profit maximization, we have:

$$
\begin{aligned}
z_2 =& \frac{w'_{it}}{w_{at}} = \frac{(1-\theta)A_{it}K_{it}^{\theta}Q_t^{-\theta}}{(1-\phi)A_{at}K_{at}^{\phi}(N_t-Q_t)^{-\phi}} \\
=& \frac{(1-\theta)A_{it}K_{it}^{\theta}Q^{-\theta}}{(1-\phi)A_{at}K_t^{\phi}(N_t-Q_t)^{-\phi}} \cdot \frac{\phi K_{at}^{\phi-1}(N_t-Q)^{1-\phi}}{\theta K_{it}^{\theta-1}Q_t^{1-\theta}} = \frac{(1-\theta)\phi}{(1-\phi)\theta} \cdot \frac{k_{it}}{k_{at}}
\end{aligned}
\tag{23}
$$

The third equation uses the condition of equalization of marginal return of capital across sectors. Because physical capital investment is homogenous, market-clearing condition requires: $Q_t k_{it} + (N_t - Q_t)k_{at} = I_t$. Taking derivatives on both sides with respect to $E(t)$, we get:

$$
Q_t \frac{\partial k_{it}}{\partial E(t)} + (N_t - Q_t)\frac{\partial k_{at}}{\partial E(t)} = 0 \tag{24}
$$

The equalization of marginal product of capital across sectors means: $\theta E(t) k_{it}^{\theta-1} = \phi k_t^{\phi-1}$. Taking natural logarithm and taking derivatives with respect to $E(t)$ on both sides will give us:

$$
-\frac{1}{E(t)} + (1-\theta)\frac{1}{k_{it}} \cdot \frac{\partial k_{it}}{\partial E(t)} = (1-\phi)\frac{1}{k_{at}} \cdot \frac{\partial k_{at}}{\partial E(t)} \tag{25}
$$

(24) implies that $\partial k_{it}/\partial E(t)$ and $\partial k_{at}/\partial E(t)$ must be of opposite signs. Combining (25) leaves us $\partial k_{it}/\partial E(t) \ge 0$, $\partial k_{at}/\partial E(t) \le 0$[7], which means as the technology difference between two sectors enlarges, there is capital deepening in industrial sector, while capital-labor ratio decreases in agricultural sector. Using this result, we get from (23): $\partial z_2/\partial E(t) > 0$.

(23) also implies $\frac{k_{it}}{k_{at}} > \frac{(1-\phi)\theta}{(1-\theta)\phi} > 1$ (because of $\theta > \phi$). Based on $Q_t k_{it} + (N_t - Q_t)k_{at} = I_t$, if supply of skilled labor Q_t increases, then it must be $\partial k_{it}/\partial Q_t \le 0$ and $\partial k_{at}/\partial Q_i \ge 0$. We then get the following result from (23): $\partial z_2/\partial Q_t < 0$. ∎

Proposition 2 shows that under supply constraint of skilled labor, skill premium will enlarge with the increase of technology difference between

[7]Equation (25) does not hold if $\partial k_{it}/\partial E(t) \le 0$ and $\partial k_{at}/\partial E(t) \ge 0$. Moreover, these conditions implies that the widening gap of technology across sectors will lead to lower capital-labor ratio in industrial sector, and higher capital-labor ratio in agricultural sector, which apparently does not fit the reality.

sectors, and shrink with the increase in supply of skilled labor. Let the ratio of output per worker between sectors be $\tau_{yt} = \frac{y_{it}}{y_{at}}$. We then, based on (9), (13) and $w'_{it} = z_2 w_a$, obtain $\tau_{yt} = \frac{1-\phi}{1-\theta} \cdot z_2 > z_2$, which is summarized in Corollary 2.

Corollary 2. *Under supply constraint of skilled labor, the difference in output per worker between industrial and agricultural sectors is larger than skill premium during economic transition.*

4. QUANTITATIVE ANALYSIS

Based on our model, the differences of technology between sectors and supply constraint of skilled labor affect skill premium, which in turn influences the turning point and the subsequent pace of development. We thus establish calibration to analyze how income inequality is affected. We will provide an explanation why, instead of pursuing a unique path, the dynamics of income inequality during transition differs across countries.

In our calibration, a period in the economy is 30 years in real time. Agents will live 60 years working for the first 30 years in their lifespan. The initial conditions, A_i, A_a, L and N_0 are set to 1 arbitrarily. Given N_0, K_0 is determined such that the economy is initially on TBGP, and all labor work in agriculture. The capital share in modern industry is $\theta = 0.6$, while the capital share in agriculture is $\phi = 0.1$. For simplicity, assume population growth rate is 0, which will not bring meaningful change to the main results. Let discount factor β be 1.

4.1. Constraint of skilled labor supply and the beginning date of transition

As can be found in (6), economic transition can get started only when the supply of skilled labor surpasses a certain critical level. On one hand, different countries start transition at different dates, when the industrial technology available and the minimum requirement of skilled labor differ. On the other hand, due to the differences in skill premium, the increases in skilled labor supply differ across countries during transition and the subsequent pace of development. Figure 1 presents how the minimum requirement of skilled labor and the initial supply of skilled labor affect the beginning date of transition. Suppose technology change rate is 1 in agriculture, but 1.2 (0.61% per year) in industry, which corresponds to the skilled labor demand curve.

In Figure 1, the curves of skilled labor supply 1 and 2 come from (6), the corresponding parameter pairs (ξ, λ) are $(0.9, 0.5)$ and $(0.999, 0.5)$ respectively. Suppose the minimum requirement of skilled labor for transition is

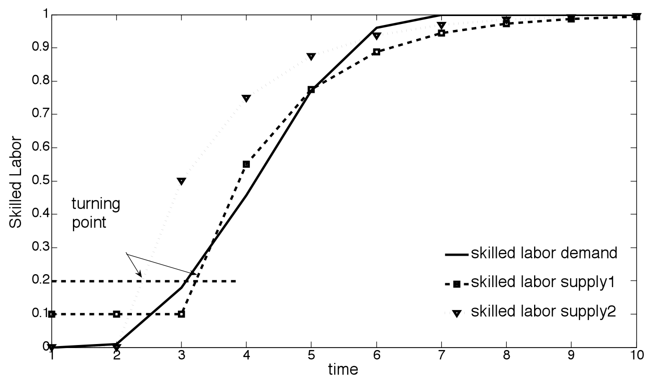

FIG. 1. Initial skilled labor, minimum requirement of skilled labor and the beginning date of transition
Note: Skilled labor demand corresponds to industrial technology change at rate of 1.2. The curves of "skill labor supply" 1 and 2 come from (6), with initial shares of skill labor of 0.1 and 0.001 respectively, the corresponding parameter pairs of (ξ, λ) are $(0.9, 0.5)$ and $(0.999, 0.5)$ respectively. The minimum requirement of skilled labor for transition is defined by $\hat{\xi} = 0.8$.

$\hat{\xi} = 0.8$. The initial supply of skilled labor is very low (0.001) for "skilled labor supply 2", which leads to skilled labor shortage and higher skill premium when the industrial technology is low. The supply of skilled labor is stimulated to increase faster at period 2 and reaches the minimum requirement of skilled labor at around period 2.4. As for "skilled labor supply 1", the initial share of skilled labor is higher (but not as high as 0.2, the minimum requirement for transition), and the supply of skilled labor moves to a quick increase around period 3, when the industrial technology is higher, and reaches the turning point of transition at around period 3.2.

Given technology change rate of 1.2, Figure 1 presents a situation that the economy with lower initial skilled labor may take off earlier than the economy with higher initial skilled labor. Why is the case? As Proposition 1 and (21) show, the low initial skilled labor in an economy may delay its economic take-off (level effect), however, the supply of skilled labor may start to increase faster at earlier data because the skill premium is higher (growth effect). If the growth effect dominates, the economy with lower initial skilled labor can take off earlier than an economy with higher initial skilled labor. Nevertheless, two implied prerequisites work: (1) there is minimum requirement of skilled labor for take-off; (2) the supply of skilled labor should be elastic with respect to skill premium. Also in Figure 1, if the minimum requirement of skilled labor share is lower (say, 0.1), then the economy with higher initial skilled labor (skill labor supply 1) will take off earlier (at period 1), while the economy with lower initial skilled labor (skill labor supply 2) will wait at least 30 years to take off. Whether skilled labor supply is sensitive to skill premium depends on many factors, including education system, culture, history, as well as technology growth rate.

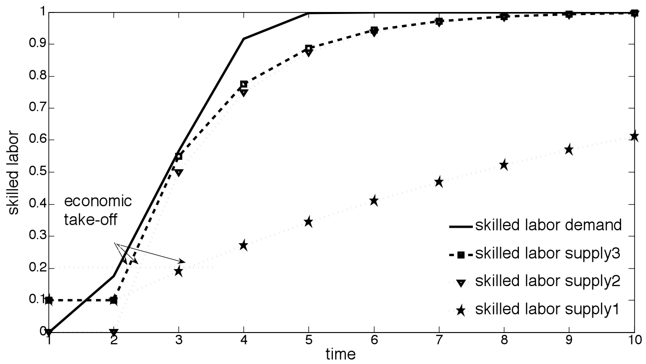

FIG. 2. Technology change, skill premium and economic take-off
Note: Skilled labor demand corresponds to industrial technology change at rate of 1.3. The curves of "skill labor supply" 1, 2 and 3 come from (6), the corresponding parameter pairs of (ξ, λ) are $(0.9, 0.9)$ and $(0.999, 0.5)$ and $(0.9, 0.5)$ respectively. The minimum requirement of skilled labor for transition is defined by $\hat{\xi} = 0.8$.

In Figure 2, skilled labor demand represents a higher rate of technology change (1.3 per period or 0.88% per year). "Skilled labor supply 2" corresponds to low initial level and high growth rate of skilled labor. The initial skilled labor is the same for "skilled labor supply 1" and "skilled labor supply 3", but the growth rate for the latter is much higher. Given this higher technology change, skill premium occurs earlier even for the economy with relatively higher initial skilled labor. The economy represented by "skilled labor supply 3" enlarges its supply of skilled labor at around period 2, and reaches the turning point of transition at around 2.4 period (earlier than the economy represented by "skilled labor supply 2"). We thus conclude that higher technology change is a necessary condition for an economy with relative abundant skilled labor to take the lead in economic transition. Meanwhile, we find that an economy represented by "skilled labor supply 1" will not reach its turning point until around 3.2 period, more than 30 years later than economy with "skilled labor supply 3", and about 20 years later than economy with "skilled labor supply 2". We again conclude that a prerequisite for an economy with relatively abundant skilled labor to take off earlier is that its supply of skilled labor should be sensitive to skill premium.

4.2. Skill premium, economic transition and the dynamics of inequality

We now focus on how the constraint of skilled labor affects the dynamics of inequality during transition. The technology of agriculture is set to 1. We consider a higher technology change rate of 1.4 (1.13% annually) and a lower technology change rate of 1.2 (0.61% annually). In order to concentrate on the pace of transition, we adjust the initial capital stock to make sure that an economy without any constraint of skilled labor reaches turning point at the beginning of period 1. The supply of skilled labor is thus simplified as:

$$Q_t = 1 - \xi\lambda^t \tag{6'}$$

where $\lambda (0 \leq \lambda \leq 1)$ is the change rate of unskilled labor, and initially $Q_0 = 1 - \xi$. We consider three combinations of skilled labor supply at the turning point. The curve "skilled labor supply 1" with parameters of $\xi = 0.1$and $\lambda = 0.9$ corresponds with the case of high initial level and low growth rate of skilled labor. Curve "skilled labor supply 2" with parameters of $\xi = 0.9$ and $\lambda = 0.5$ corresponds with the case of low initial level and high growth rate of skilled labor. For the case of low initial level and low growth rate of skilled labor, we set $\xi = 0.9$ and $\lambda = 0.9$ in "skilled labor supply 3".

In comparison with (6'), we consider a linear function of skilled labor supply, and an arctan function of skilled labor supply. The latter means when the amount of skilled labor is initially low, the increase in its supply is slow; the increase in supply of skilled labor accelerates as the fraction of skilled labor increases and finally slows down again when the fraction of skilled labor is high enough. The functions are set as follows:

$$Q(t) = -0.2 + 0.15 * t \tag{26}$$

$$Q(t) = \frac{1}{3}\arctan(2.5 * t - 10) + 0.5 \tag{27}$$

Figure 3 depicts the dynamic change of the demand for and supply of skilled labor during economic transition. The demand curves and correspond to low and high growth rates of technology respectively. In Figure 3(a), given low rate of technology change, the supply constraint of skilled labor occurs the latest in "supply of skilled labor 1" (around period 3.5), but occurs earlier in "supply of skilled labor 2" (around period2.5), and even earlier in "supply of skilled labor 3" (about period 0.3, or 10 years after the turning point). The comparison between Figure 3 (c) and (a) shows that with higher rate of technology change in industrial sectors, the supply constraint of skilled labor will not occur until later time given the

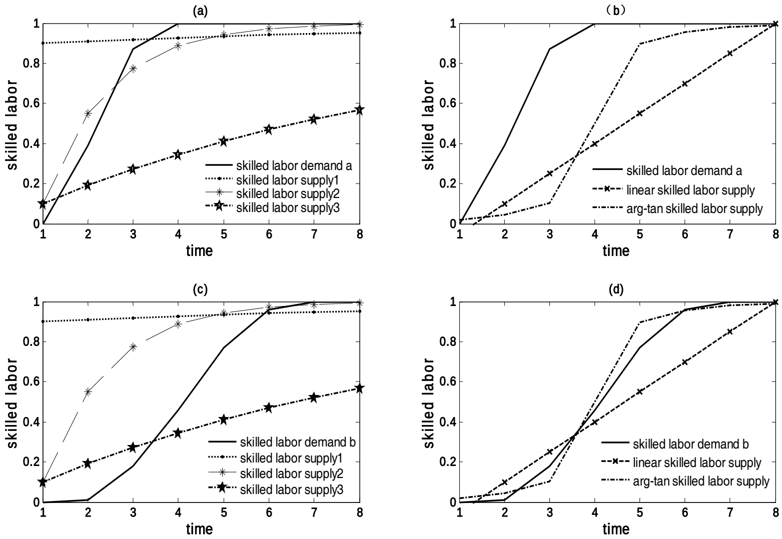

FIG. 3. Demand for and supply of skilled labor

Note: Skilled labor demand curves a and b correspond to low (1.2) and high (1.4) technology changes. Curves of skilled labor supply 1, 2, 3 correspond to the parameter pairs (ξ, λ) of $(0.1, 0.9)$, $(0.9, 0.5)$, and $(0.9, 0.9)$ respectively. Linear supply of skilled labor is defined by (26), while arctan function of supply of skilled labor defined by (27).

above three supply curves of skilled labor. For example, the supply constraint of skilled labor does not occur until around 3.5 periods in "supply of skilled labor 3". Similarly, in Figure 3(b) where technology change is slow in industry, either linear or arctan supply function of skilled labor can meet the demand for skilled labor during the whole pace of transition. While in Figure 3(d), where technology change is faster, supply constraint of skilled labor occurs around 3.5 periods after the turning point for linear supply function, the arctan function of supply of skilled labor intersects the demand for skilled labor three times representing a multi-phrase path with a change from shortage (around period 3.5) to surplus, and another shortage starting from around period 6.

We then examine the effect of supply constraint of skilled labor on the pace of transition. In Figure 4, assuming the technology change rate is 1.2 per period, we depict the benchmark of "transition with no supply constraint of skilled labor". In Figure 4(a), Transition path with "skilled labor supply 1" fits the whole benchmark path quite well[8]. Transition path with

[8]Early developed European countries present a good example for "skilled labor supply 1". In countries such as U.K., France, Germany, average education and skill levels are relatively high at their beginning of economic transition. The stable growth of skilled labor ensures a smooth development ever since the Industrial Revolution. Japan is

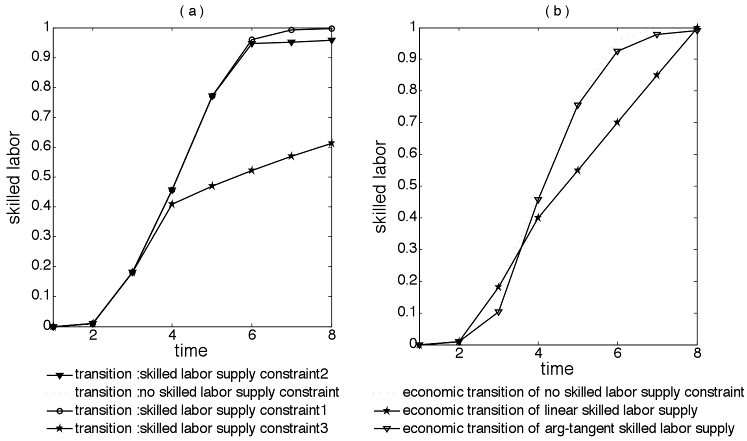

FIG. 4. Supply constraint of skilled labor and economic transition
Note: Technology change in industrial sector is 1.2 per period. The curves of skilled labor supply 1, 2, 3 correspond to the parameter pairs (ξ, λ) of $(0.1, 0.9)$, $(0.9, 0.5)$, and $(0.9, 0.9)$ respectively. Linear supply of skilled labor is set by (26), while arctan supply function of skilled labor is set by (27).

"skilled labor supply 2" does not deviate from the benchmark path until the beginning of period 6[9]. Obviously, the transition path corresponding to "skilled labor supply 3" has been below the benchmark level ever since period 3, and the gap becomes larger after period 4. We therefore conclude that low fraction and slow growth rate of skilled labor can negatively affect the development pace[10]. From figure 4(b), we find out that the transi-

another example. As early as the Meiji Reform era, Japan made primary education universal, which was even earlier than many European countries. After World War two, although output per capita dropped too much below its pre-war level, and there was huge destruction of labor force during the war, the sustained stable accumulation of human capital provided abundant skilled labor to support its economic recovery and transition.

[9]U.S. is an example for "skilled labor supply 2". U.S. began its transition later compared with many European forerunners. The aggregate output in the U.S. ranked No.1 in 1894, while its output per capita did not surpass that of U.K. until 1913. U.K. began its transition around the era of the Civil War, when the supply of skilled labor was very low. The U.S. made primary education universal within 60 years, which provided a solid supply pool of skilled labor for economic take-off. Before the World War two, the U.S. made secondary education universal, which is about 30-40 years earlier than U.K., France and Germany. In the 20th century, the labor force at the age of 25-64 in the U.S remains the highest average education level, which for sure is a driving force for its development.(calculated based on A.Maddison (2001), Appendix A). Along MBGP, the interaction between SBTC and skill premium, in addition to immigration of human capital from abroad helps avoid severe shortage of skilled labor (Acemoglu, 2002; Murphy & Welch, 1992).

[10]An example for "skilled labor supply 3" is South-Sahara African countries. The average Human Development Index (HDI) for these countries is as low as 0.486, the

tion pace corresponding with linear supply of skilled labor is located below the benchmark path ever since around period 3.5, and the gap reaches its summit around period 6. Corresponding to arctan supply function of skilled labor, the transition path is below benchmark from period 2 to 3.5, and then it fits the benchmark locus well, it finally turns to be lower than benchmark level since the beginning of period 5 because the supply of skilled labor becomes growing slower.

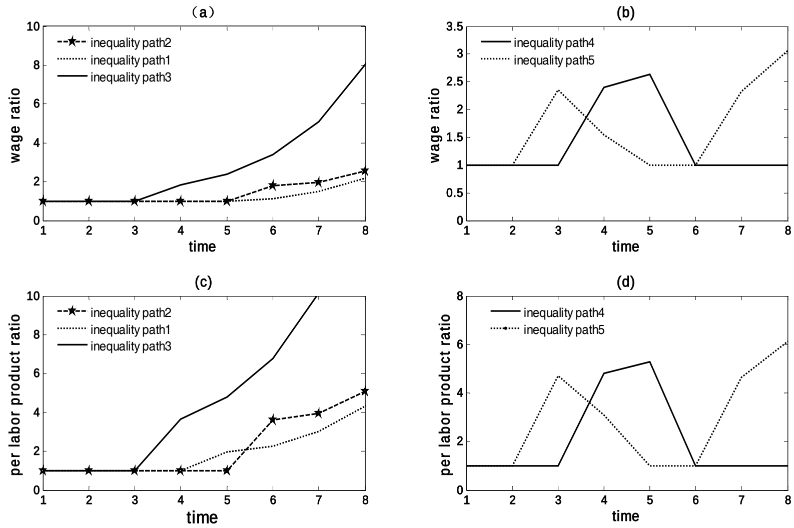

FIG. 5. The dynamics of skill premium and differences in output per worker

As Figure 5 presents, the dynamics of inequality (represented here by wage ratio) and ratio of output per worker between sectors have similar paths, while the gap in output per capita is larger as we find in Corollary 2. Figure 5(a) provides three dynamic paths of income inequality corresponding to the three combinations of parameter in equation (6'). "Skilled labor supply 1" corresponds with a U-shaped change in inequality, as what we find in U.K. during 1938-1995, in U.S.A. during 1913-1994. "Skilled labor supply 2" corresponds with a two-phrase change in inequality (skill premium keeps stable in periods 1-5, rises to a higher level around the end of period 5, and roughly remains at the new level), as what we find in Japan, where the Gini coefficient drops significantly after 1945 because

index for adult literary is 57.5, index for life expectancy at birth is 46, and index for combined gross enrollment for schools is 46. Moreover, all the indexes, which represent a very low supply of skilled labor, do not show any signal for improvement since 1975. None of these countries have fulfilled transition to modern economy, and the average index of GDP for them is as low as 0.58 (Human Development Report, UNDP,2005).

of the relative abundant human capital and extreme shortage of physical capital then. "Skilled labor supply 3" corresponds with an upward sloping and steep curve in inequality, which shows that the low initial level and low growth of supply of skilled labor lead to large and accelerating enlarging inequality overtime.

In Figure 5(b), the inequality path corresponding with linear supply of skilled labor turns out to be inverted U-shaped. Inequality increases significantly around period 3, and reaches its summit around period 5, and move into downward section until skill premium disappears along MBGP. The inequality in U.K. from 1688 to around 1900 is a typical example. As for the arctan supply function of skilled labor, the skilled labor is initially limited and growing slowly, which constrains transition from the very beginning period and make skill premium increasing. In the middle of transition, the increased supply of skilled labor surpasses the demand for it, and transition moves closer to the benchmark pace (see Figure 4(b)), which shrinks skill premium. In the later period of transition, supply of skilled labor decreases again, which leads to increasing inequality after the beginning of period 6. It thus corresponds with an N-shaped change of inequality as what we find in U.K. during 1688-1995.

5. CONCLUSIONS

Based on Hansen & Prescott (2002), we introduce skill differences in labor, and construct a dynamic model of economic transition. The models of Hansen & Prescott (2002) and Ngai (2004) make the assumption of labor indifferences so that any labor can get involved in agricultural or industrial production. We assume there is significant difference in labor skills, either unskilled or skilled labor can be employed in agriculture, while only skilled labor can match modern technology in industrial sectors. Therefore, due to the differences in the initial stock of skilled labor and its growth rate, the turning point of transition may be delayed, and the path of transition may be affected. The supply constraint affects both the beginning date and the subsequent pace of modern growth, and thus in turn affects the dynamics of income distribution.

We provide another approach to the mechanism of economic transition and long-run growth. Romer (1986) establishes an increasing-return model and explains the long-run non-deceasing growth in most European industrial countries. Other researchers explain the cross-country differences in transition and growth with externalities (Lucas, 1988), barriers to investment (Ngai, 2004), or economy of scale (Wang & Xie, 2003). In our model, the production functions of both agriculture and industry exhibit constant return to scale. We prove that, without any consideration of externalities or barriers to private investment, the differences in labor skills and in technologies across sectors make it possible for skilled labor to transform into modern industry. There will thus be continuous capital deepening in industry, which in turn results in continuous growth in output per capita. On MBGP, all labor is skilled labor, and the economy is growing at a constant rate.

Our simulation and quantitative analysis present more evidences why cross-country output per capita and income distribution are so different. To a great extent, the differences in technologies cross sectors and in the initial stock of skilled labor explain the dynamics of income distribution overtime. If the initial stock of skilled labor is low, and growing at a low rate, then there will be huge delay in the beginning date of economic transition, and income inequality will be increasing over a longer period. Given technology progress, the supply constraint and the dynamic change of skilled labor result in multiple possible paths of income distribution, such as "U-shaped", "inverted-U-shaped" or "N-shaped" ones.

We also argue that the income inequality during economic transition is higher than that on balanced growth paths, which means a substantial fraction of existing income differences is transitional. Therefore, during economic transition, not only faster technology progress, but also more investment on education and skill formation are recommended to reduce the supply constraint of skilled labor, facilitate economic transition and long-run growth, and decrease income inequality.

APPENDIX A

The dynamics of inequality and transition in U.K., U.S.A. and Japan

This appendix provides the dynamics of inequality in three economies: the U.K., U.S.A, and Japan to present how the changing paths of inequality are affected by economic transition and differ across economies.

Notes: (1) Inequality data of 1688-1867 covers England and Wales. Data source: Lindert & Williamson (1982, 1983) and Williamson (1985), curves of A2 and A3 refer to the income shares of top 5% and top 20% population respectively; while A1 is Gini coefficient.

(2) Later data covers the Great Britain. SPI (Survey of Personal Income) data of 1938-1974 come from Royal Commission (1977), curves of B1, B2, B3 refer to the income share of top 1%, 5% and 20% population, B4 is Gini coefficient.

(3) SPI data of 1949-1984 come from Economic Trends (1978, 1984, 1987), curves of C1, C2, C3 and C4 refer to the income share of top 1%, 5% and 20% population, and Gini coefficient respectively.

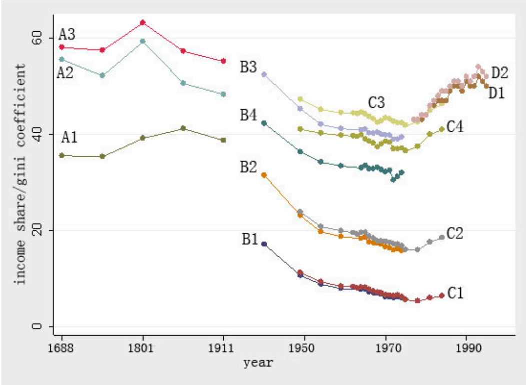

FIG. A.1 Evolution of income inequality in UK:1688-1995

(4) Data of 1977-1995 come from Economic Trends (1994, 1995, 1997), and curves of D1, D2 refer to income share of top 20% population and Gini coefficient respectively.

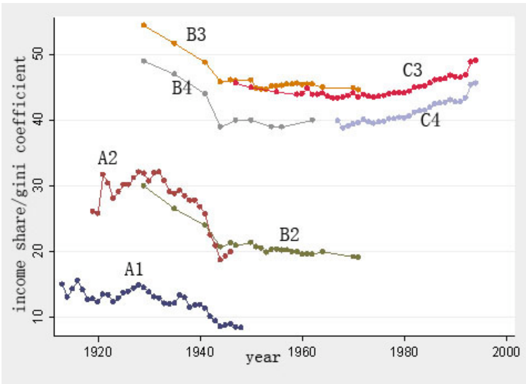

FIG. A.2 Evolution of income inequality in USA: 1913-1994

Notes: (1) Data of 1913-1948 come from Kuznets (1953), and curves A1 and A2 represent the income share of top 1% and 5% population.

(2) Data of 1929-1971 come from Goldsmith (1967) and Office of Business Economics, B2, B3, and B4 represent the income share of top 5% and 20% population, and Gini coefficient respectively. Data of 1947-1994 come from The Census Bureau and Current Population Suevey (CPS); C3 and C4 represent income share of top 20% population and Gini coefficient.

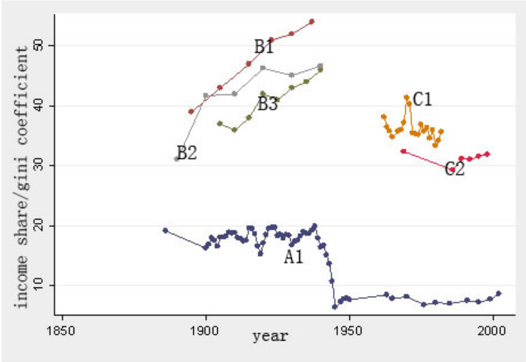

FIG. A.3 Evolution of income inequality in Japan:1886-2002

Notes: (1) A1 is based on data of 1886-2002 of the income share of top 1% population (Moriguchi & Saez, 2006).

(2) Minami (1998), Mizoguchi (1985), Ono & Watanabe (1976) provide data of Gini coefficient before WWII, which are represented by curves of B1, B2, B3 respectively.

(3) Data after WWII come from Mizoguchi & Takayama (1984) and Shirahase (2001), represented by C1 and C2 respectively.

APPENDIX B

Competitive Equilibrium of the model

This appendix derives the competitive equilibrium in our three-development-stage model under the assumptions B.1-B.6 to be specified below.

Given N_0, K_0 and L, a competitive equilibrium consists of prices vector $\{q_t, w_{it}, w_{at}, r_{Kt}, r_{Lt}\}$, firm allocation $\{K_{it}, K_{at}, N_{it}, N_{at}, L_{at}, Y_{it}, Y_{at}\}$ and household allocation $\{c_{1t}, c_{2t+1}, x_{at}, x_{it}, l_{t+1}\}$, such that: (i) given prices, household and firm allocations maximize utility and profit respectively; (ii) all markets clear, i.e. $L_{at} = 1 = N_{t-1}l_t$, $Y_{at}+Y_{it} = N_tc_{1t}+N_{t-1}c_{2t}+N_tx_t$, $N_{at} + N_{it} = N_t$, $N_{it} \leq Q_t$, $K_{at} + K_{it} = K_i$. (iii) the capital accumulation and population growth follows the laws of motion: $K_{at+1} = N_tx_{at}$, $K_{it+1} = N_tx_{it}$ and $N_{t+1} = g(c_{1t})N_t$.

Assume utility function[11] is:

$$u(c) = \ln c \tag{B.1}$$

In equilibrium, $c_{1t} = \frac{w_{jt}}{1+\beta}, j = i, a$; $R_t = \frac{q_t+r_{Lt}}{q_{t-1}}$, if $l_t > 0$; $R_t = r_{Kt}$, if $x_a > 0$ or $x_i > 0$.

[11]We can solve the model for a utility function of constant intertemporal elasticity of substitution with not much change in the key results.

B.1. TRADITIONAL BALANCED GROWTH PATH (TBGP)

On TBGP, all workers work in traditional agriculture. The function of population growth $g(\cdot)$ is chosen so that both output per capita (or per worker) ($\hat{y}_a$) and capital per capita ($\hat{k}_a$) are constant. Assume:

$$g(\hat{c}_{1a}) = \gamma_a^{1/(1-\mu-\phi)}, \quad \text{and} \quad g(c_1) > g(\hat{c}_{1a}),\ \forall c_1 \in [c_{1a}, c_{1a+\varepsilon}],\ \varepsilon > 0 \quad \text{(B.2)}$$

then output per capita will be constant: $\hat{y}_a = A_a \gamma_a^t \hat{k}_a^{\phi} N_t^{\mu+\phi-1}$. Define capital-output ratio on TBGP as $\nu_a = \hat{k}_a/\hat{y}_a$, where $\nu_a = \dfrac{(1+\beta-\mu) - \sqrt{(1+\beta-\mu)^2 - 4\mu\phi\beta(1+\beta)}}{2(1+\beta)\gamma_a^{1/(1-\mu-\phi)}}$. We thus have $\hat{y}_a = [A_a \gamma_a^t \nu_a^{\phi} N_t^{\mu+\phi-1}]^{1/(1-\phi)}$.

On TBGP, The price and rental rate of land grow at $\gamma_a^{1/(1-\mu-\phi)}$, while the wage rate and rental rate of capital are constant.

B.2. TRANSITION

B.2.1. The case when the supply of skilled labor is abundant ($N_{it} \leq Q_t$)

When a firm starts to use modern technology, its optimization problem is:

$$\Psi(r_{Kt}, w_{it}) = \max_{K_{it}, N_a} (A_i \gamma_i^t K_{it}^{\theta} N_{it}^{1-\theta} - r_{Kt} K_{it} - w_{it} N_{it})$$

The optimal decision of the firm implies: $\frac{K_{it}}{N_{it}} = \frac{\theta w_{it}}{(1-\theta) r_{Kt}}$, so the profit function becomes:

$$\Psi(r_{Kt}, w_{it}) = \max_{N_{it}} \left[A_i \gamma_i^t \left(\frac{\theta w_{it}}{(1-\theta) r_{Kt}} \right)^{\theta} - \frac{w_{it}}{1-\theta} \right] N_{it}$$

Because only skilled labor can match the modern technology used in industry, the workers that can be transformed to industry are skilled labor, who can earn "skill premium" in industrial sector. The wage rates in agriculture and industry are thus different. Assume $w_{it} = z_1 w_{at}$, where $z_1 > 1$ is the skill premium under the condition of $N_{it} < Q_t$. Therefore, the profit function can be written as:

$$\Psi(r_{Kt}, w_{at}) = \max_{N_a} \left[A_i \gamma_i^t \left(\frac{\theta z_1 w_{at}}{(1-\theta) r_{Kt}} \right)^{\theta} - \frac{z_1 w_{at}}{1-\theta} \right] N_{it}$$

Along TBGP, the firm will use modern technology if $\Psi(\hat{r}_K, \hat{w}_a) \geq 0$. The condition of transform to modern industry is:

$$A_{it} = A_i \gamma_i^t \geq z_1^{1-\theta} \left(\frac{\hat{w}_a}{1-\theta}\right)^{1-\theta} \left(\frac{\hat{r}_K}{\theta}\right)^{\theta}$$

B.2.2. The case when the supply of skilled labor is constrained ($N_{it} > Q_t$)

In this case, the optimization problem a firm faces when it starts to use modern technology is:

$$\Psi(r_{Kt}, w'_a) = \max_{K_a, N_a} (A_i \gamma_i^t K_{it}^{\theta} Q_t^{1-\theta} - r_{Kt} K_{it} - w'_{it} Q_t)$$

The optimal decision of the firm requires: $\frac{K_{it}}{Q_t} = \frac{\theta w'_a}{(1-\theta) r_{Kt}}$, and the profit function becomes:

$$\Psi(r_{Kt}, w'_{it}) = \max_{Q_t} \left[A_i \gamma_i^t \left(\frac{\theta w'_{it}}{(1-\theta) r_{Kt}} \right)^{\theta} - \frac{w'_{it}}{1-\theta} \right] Q_t$$

Due to the scarcity of skilled labor, industrial sector will provide a wage rate of $w'_{it} = z_2 w_{at}$, where z_2 is skill premium when the supply of skilled labor is constrained. Because industry has to pay more to attract skilled labor, the skill premium in this case is higher than that in the case with abundant supply of skilled labor, i.e. $z_2 > z_1$. The profit function is thus rewritten as:

$$\Psi(r_{Kt}, w_{at}) = \max_{Q_t} \left[A_i \gamma_i^t \left(\frac{\theta z_2 w_{at}}{(1-\theta) r_{Kt}} \right)^{\theta} - \frac{z_2 w_{at}}{1-\theta} \right] Q_t$$

Again, along TBGP, the firm will use modern technology if $\Psi(\hat{r}_K, \hat{w}_a) \geq 0$, and the condition of transform to modern industry is:

$$A_{it} = A_i \gamma_i^t \leq z_2^{1-\theta} \left(\frac{\hat{w}_a}{1-\theta}\right)^{1-\theta} \left(\frac{\hat{r}_K}{\theta}\right)^{\theta}.$$

Obviously, because $z_2 > z_1$, the supply constraint of skilled labor makes it even more difficult for the condition of transform to be satisfied, the economic transition is therefore expected to delay. However, the higher skill premium can provide more incentive for skill formation, and the supply of skilled labor will increase until it can satisfy the demand for skill labor in industry. Our following discussion on transition will focus on the case of $N_{it} \leq Q_t$.

B.2.3. The path of transition

Given the initial condition $K_0 = (N_0^{\mu}\nu_a)^{1/(1-\phi)}$, $\hat{r}_K$ and $\hat{w}_a$ can be represented as function of N_0. The turning point of economic transition t^* satisfies:

$$A_i\gamma_i^{t^*} \geq Bz_1^{1-\theta}\left(\frac{1}{N_0}\right)^{(1-\mu-\phi)(1-\theta)/(1-\phi)} > A_i\gamma_i^{t^*-1}$$

where $B = \left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}(\nu_a^{(\phi-\theta)}A_a^{(1-\theta)})^{1/(1-\phi)}$.

Given q_{t-1}, N_t and land normalized to 1 ($L = 1$), the total value of investment of younger generation in period $(t-1)$ is: $I_t = N_{t-1}(w_{jt-1} - c_{1t-1}) - q_{t-1}, j = i, a$. Profit maximization implies:

$$\frac{\theta Y_{it}}{K_{it}} = \frac{\phi Y_{at}}{K_{at}}, \quad w_{it} = (1-\theta)\frac{Y_{it}}{N_{it}}, \quad w_{at} = \mu\frac{Y_{at}}{N_{at}}, \quad r_{Lt} = (1-\phi-\mu)Y_{at}$$

Taking into account $w_{it} = z_1 w_{at}$, and assuming:

$$\theta > \phi \tag{B.3}$$

we have: $k_{at} = \frac{\varphi}{z_1}k_{it}$, where $k_{at} = K_{at}/N_{at}$, $k_{it} = K_{it}/N_{it}$, and $\varphi = \frac{(1-\theta)\phi}{\theta\mu} < 1$.

Market clearing requires: $k_{at} = \frac{(\varphi/z_1)\cdot(I_t/N_t)}{(1-(1-\varphi/z_1)n_{at})}$, where $n_{at} = N_{at}/N_t$. Because there are skill differences in labor, while the returns of capital are equal across sectors, we have:

$$k_{at}^{\theta-\phi} = \frac{\phi}{\theta}\frac{A_{at}}{A_{it}}\left(\frac{\varphi}{z_1}\right)^{\theta-1}N_{at}^{\mu+\phi-1}.$$

Therefore, the equilibrium n_{at}^* solves $f(n_{at}^*) = 0$, where

$$f(n_{at}) = \frac{\phi}{\theta}\left(\frac{\varphi}{z_1}\right)^{\phi-1}(1-(1-\varphi/z_1)n_{at})^{\theta-\phi} - \frac{A_{it}}{A_{at}}I_t^{\theta-\phi}N_t^{1-\theta-\mu}n_{at}^{1-\phi-\mu}$$

where $1-\mu-\phi > 0$ and $t \geq t^*$ implies $f' < 0, f(0) > 0$, and $f(1) < 0$. Thus there exists a unique $n_{at}^* \in [0,1)$.

We further assume:

$$\gamma_i > \gamma_a \tag{B.4}$$

$$\exists \bar{t}, n, \text{ s.t. } g(c_{it}) \leq n \ \ \forall t > \bar{t} \text{ if } 1-\theta < \mu \tag{B.5}$$

$$g(c_{1t}) \geq 1 \ \text{ if } \ 1-\theta \geq \mu$$

then $\frac{A_{it}}{A_{at}} I_t^{\theta-\phi} N_t^{1-\mu-\theta}$ is an increasing function of time t, and n_{at}^* converges to zero. Since the share of labor working in agriculture will converge to 0, the share of skilled labor in total labor will increase overtime i.e. in the limit, $Q_t \to N_t$.

Meanwhile, the skill premium z_1 plays a role in the above function of $f(n_{at})$ where it makes the first term smaller and the second term larger (through increasing I_t), so it helps to make the transition faster. If there is supply constrain of skilled labor in the economy, the skill premium (z_2) will be higher, which may make the above mechanism more significant.

B.3. MODERN BALANCED GROWTH PATH (MBGP)

As the economy transforms to modern economy, n_{at}^* converges to 0, both $r_{Lt} \to 0$ and $q_t \to 0$. Assume:

$$\lim_{c_1 \to \infty} g(c_1) = g \tag{B.6}$$

the economy converges to MBGP. Output per capita (y_{it}) is growing at a constant rate. The capital-output ratio of modern economy is $\nu_i = \frac{\beta(1-\theta)}{(1+\beta)\gamma_i^{1/(1-\theta)}}$, and output per capita thus equals to: $y_{it} = (A_i \gamma_i^t \nu_i^\theta)^{1/(1-\theta)}$. All labor are skilled labor, and the wage and consumption grow at a rate of $\gamma_i^{1/(1-\theta)}$.

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