The Informational and Strategic Impacts of Real Earnings Management

Shirley J. Ho

Department of Economics, National Chengchi University, Taiwan
E-mail: sjho@nccu.edu.tw

and

Hao-Chang Sung

Department of Money and Banking, National Chengchi University, Taiwan
E-mail: 94352505@nccu.edu.tw

We address the informational and strategic impacts of real earnings management (REM) in a two-period oligopoly model with one-sided information. For the strategic impacts of REM, once the demand falls short of expectation, a firm should raise the price instead of cutting it to reach the earnings target. For the informational impacts, to maintain opponents’ uncertainty, the privately informed firm could conceal its identity by taking a mixed strategy and setting the first period price to be higher than in the separating equilibrium. Finally, the presence of tunnelling from cross-shareholding firm will enhance the price cut in the second period.

Key Words: Real earnings management; Incomplete information; Price manipulation; Cross-shareholding; Tunnelling.

classD4; D83; G3.

1. INTRODUCTION

The issue of real earnings management (REM) has attracted increasing attention, especially following the outbreaks of several financial crises when there are calls for tightening the accounting standards to reduce management’s discretion in financial reporting. REM refers to management’s discretion in structuring real activities that deviate from normal business practices, including the manipulation of operating and investing activities as well as the manipulation of financial activities. Accruals manipulation is more likely to draw the auditor’s or regulator’s attention than real
activities on pricing and production. Hence, although real activities manipulation potentially imposes greater long-term impacts on the company, managers are more willing to manipulate earnings through real activities than to manipulate accruals (see Roychowdhury, 2006).

The REM manipulation, however, will create further informational and strategic impacts on the real activities. First, as described, REM is gaining in popularity because it can be mixed with normal real activities and hence is difficult for auditor or even the opponent to detect. For the opponent, the suspicion that a firm might be dishonest on earnings report will create additional uncertainty that can affect the opponent’s real activities. Under such uncertain circumstance, it is interesting to ask if it is possible for the firm to take advantage of its opponent’s uncertainty, to seduce the opponent to react less aggressively, and to achieve the desired earnings target? If the answer is yes, then in order to maintain the opponent’s uncertainty, should some strategic actions be taken before the demand uncertainty realizes to be short of expectation? The empirical research by Burgstahler and Eames (2003) also mentioned this uncertainty. They studied whether analysts are able to identify which specific firms engage in earnings management, using data from Zacks Investment Research, and concluded that analysts are unable to consistently identify the specific firms that engage in earnings management to avoid small losses. Our study will further explore this informational impact on the opponent’s decisions in real activities.

Second, REM activities might induce strategic reactions from the opponents. For example, when uncertain market demand falls short of expectation, unilateral operating activities such as a price cut can increase a single firm’s return that helps achieving the earnings target (see Jiang, 2008). But, this is only part of the story. It is well known that since firms are strategic complements with price competition, the opponent’s best reply to a price cut is to cut its own price (see Shy, 1997). As a result, all firms end up with lower prices and lower profits in equilibrium. For the firm that initiates a price cut to meet its earnings target, if this REM activity is fully anticipated, the situation can only become worse off.

To examine the informational and strategic impacts of REM activities, we build up a two-period oligopoly model where the opponent is uncertain about a firm’s objective type (profit-maximizing or target-reaching), and characterize the perfect Bayesian equilibria where a firm’s price can partially reveal its objective type. The private information on a firm’s objective type is assumed to capture the opponent’s suspicion about the firm’s honesty on earnings report. Regarding to the strategic impacts of REM, we conclude that once the demand falls short of expectation, a targeting

1By profit maximization, since the market prices have deviated from the profit maximizing prices, the resulting profits will be lower.
type of firm should raise the price, instead of cutting it, so that the profit-reducing price war can be avoided. Next, for the informational impacts, we show in a hybrid equilibrium that in order to maintain the opponent’s uncertainty about a firm’s objective type, the privately informed firm has an incentive to conceal its identity by taking a mixed strategy and setting the first period price to be higher than in the separating equilibrium. This result illustrates the evidence that firms cut their prices around the fiscal quarter-end to reach the earnings target (Oyer, 1998; DeGeorge, et. al, 1999; Chapman and Steenburgh, 2010). Gunny (2010) also concluded that real activities manipulation is positively associated with firms just meeting earnings benchmarks (zero and last year’s earnings).

Our study is related to the literature on earnings management in competitive markets. As explained, the presence of competition can change the results that are intuitively correct in the case of a single firm. Although many articles have examined earnings management and income smoothing in a single firm framework (Lambert, 1984; Dye, 1988; Fudenberg and Tirole, 1995), only a few papers address earnings management in competitive markets, and most of them dealt with the issues of accruals management. Our study is different from the existing literature in focusing on the informational impacts of REM. The manipulation on real activities will provide a noisy signal for the signaling firm’s strategic and informational incentives, and this complexity might give a better picture why REM imposes a greater long term impact. Moreover, the private information in our model is about the signaling firm’s ”objective type”. The uncertainty about the objective type reflects the opponent’s doubt about the signaling firm’s honesty (on earnings report). We are interested in how each side (the informed and uninformed) takes advantage of this incomplete information and how it is related to the REM activities.

Since earnings management often occurs in pyramids or family-owned firms, the model can be slightly extended to consider the impacts of cross-shareholding on a firm’s REM activities. In this respect, our study is related to Riyanto and Toolsema (2008), who presented a model of tunneling in a pyramidal ownership structure. Tunneling refers to controlling shareholders shifting resources from one firm to another in the same pyramid. They compared the pyramidal ownership structure with the horizontal ownership structure, and showed that tunneling may justify the pyramidal structure only in the presence of myopic investors or in combination with propping. By taking a different approach, our study shows that the presence of tunneling from cross-shareholding firm will enhance price cuts in the second

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period. Tunnelling from a lower-end firm in the pyramidal chain can reduce REM activities’ strategic impacts on its own industry, which hence permits more aggressive actions. Our focus on REM through prices and that we consider the informational and strategic impacts of REM on non-member competitors are both absent in Riyanto and Toolsema’s model.

The remainder of this paper proceeds as follows. In Section 2 we describe a two-period oligopoly model with asymmetric information. The private information is about the signaling firm’s objective type: profit-maximizing or target-reaching. The objective function for each type of the signaling firm and the sequence of actions are provided here. Section 3 characterizes the separating and hybrid Bayesian equilibria of the game. The hybrid equilibrium suggests that the targeting type may take actions to deliberately maintain the opponent’s asymmetric information. Section 4 contains the concluding remarks. For ease of presentation, all long derivations and proofs are relegated to the Appendix.

2. THE MODEL

In order to examine the strategic and informational impacts of REM, this section describes a two-period oligopoly model with one-sided incomplete information. The private information concerns with a firm’s honesty on financial reporting. We will demonstrate that with this uncertainty, the REM activities will not be fully anticipated by the opponents and hence it is possible for a firm to take advantage of this uncertainty and reach the desired earnings target.

The Environment Specifically, we consider three firms in two industries with uncertain demands. Firm 1 and firm 2 provide differentiated products in the first industry, and firm 3 is the monopolist in the second industry. Following Dixit (1979) and Singh and Vives (1984), we assume the following (inverse) demand structures. For the first industry, the demand is given by

\[ Q_i(p_i, p_j, \epsilon) = a - p_i - \beta p_j + \epsilon, \]

where \( p_i \) and \( p_j \), for \( i, j = 1, 2 \), denote firm \( i \) and \( j \)’s price, respectively. \( \beta \) represents the degree of substitution; The two products are complements for \( \beta > 0 \), and they are substitutes for \( \beta < 0 \). The random term \( \epsilon \) catches the market uncertainty, which is assumed to be distributed over \([-\bar{\epsilon}, \bar{\epsilon}]\) according to a nondecreasing distribution function \( F(\epsilon) \), with a density \( f(\epsilon) \).

\[4\] We have assumed specific function forms to better illustrate the market equilibrium. More general assumptions will not change our main results.
To examine the REM impacts on pyramids or family-owned businesses (see Wang, 2006), we assume that firm 1 holds a proportion $\rho$ of stock shares in firm 3. Firm 3 is the monopolist in the second industry, whose demand is given by

$$Q_3(p_3, \varepsilon) = a - p_3 + \varepsilon.$$  

Without loss of generality, we assume a linear cost function for all three firms: $c_iQ_i$, for $i = 1, 2, 3$.

**FIG. 1.** The Sequence of Moves.

The production of the three firms lasts for two periods, for which one can consider the situation that full-year earnings consist of two half-year earnings, or consist of peak and off-peak season earnings. Figure 1 presents the sequence of actions for this game.

To illustrate the uncertainty about a firm’s honesty on earnings report, we assume that before competition, firm 1 is privately informed of its objective type $k$, which can be either a profit-maximizing ($m$) or target-reaching type ($r$). The profit-maximizing type pursues profit maximization in each period, while the target-reaching type achieves an earnings threshold. The uncertainty about firm 1’s objective type reflects the opponent’s doubt about firm 1’s honesty (on earnings report).

With asymmetric information, firm 1 and firm 2 compete in prices in the first industry, and firm 3 produces the monopoly output. At the end of period one, firm 2 updates its belief about firm 1’s objective type after observing firm 1’s first period price, and the targeting type of firm 1 will determine the extent of REM according to its realized profit. In period two, the three firms compete once again in two markets with uncertain demand, and the second period profits realize in the end.

**Information** To describe the uncertainty about firm 1’s objective type, we give more definitions as follows. First, denote $\mu^t$, $t = 1, 2$, as the opponent firms’ beliefs (prior and ex post) that firm 1 is a maximizing type, and $(1 - \mu^t)$ that it is a targeting type. Here and henceforth, we use
a superscript \( t \) to denote the variables in period \( t \). Second, let \( \pi_t^i \) represent firm \( i \)'s profit in period \( t \), where
\[
\pi_t^i(p_t^1, p_t^2, \varepsilon^t) = Q_t^i(p_t^1, p_t^2, \varepsilon^t)(p_t^i - c_i), \quad \text{for} \ i = 1, 2,
\]
and
\[
\pi_t^3(p_t^3, \varepsilon^t) = Q_t^3(p_t^3, \varepsilon^t)(p_t^3 - c_3).
\]
Third, to distinguish the pricing strategy for each type of firm 1, let \( p_t^1(k) \) denote the price set by type \( k \) \((k = m, r)\) of firm 1 in period \( t \).

Thus, firm \( i \)'s total profit at \( t \) is written as \( \Pi_t^i \), where
\[
\Pi_t^1 = \pi_1^i(p_t^1, p_t^2, \varepsilon^t) + \rho \pi_t^3(p_t^3, \varepsilon^t),
\]
\[
\Pi_t^2 = \mu \pi_t^2(p_t^1(m), p_t^2, \varepsilon^t) + (1 - \mu) \pi_t^2(p_t^1(r), p_t^2, \varepsilon^t),
\]
\[
\Pi_t^3 = (1 - \rho) \pi_t^3(p_t^3, \varepsilon^t).
\]
Remind that \( \rho \) is the proportion of firm 3’s stock shares that firm 1 holds.

Next, we describe the objective functions for each type of firm 1 and the opponent firms.

2.1. Maximizing Type of Firm 1
If firm 1 is a maximizing type, then for all three firms, the first period objectives will be
\[
\max_{p_t^i} E(\Pi_t^1 + \Pi_t^2) \quad \text{for} \quad i = 1, 2, 3, \tag{1}
\]
where \( E \) is the notion of expectation over the demand shock \( \varepsilon \). Notice that throughout this paper, we have assumed no discounting for simplification. Given the market equilibrium in the first period, the second period objectives become
\[
\max_{p_t^i} E\Pi_t^2 \quad \text{for} \quad i = 1, 2, 3. \tag{2}
\]
Remind that there is still demand uncertainty in the second period, so the notion of expectation still appears in equation (2).

2.2. Targeting Type of Firm 1
If firm 1 is a targeting type, then firm 2 and firm 3’s first period objectives remain the same as equation (1), but firm 1’s first period objective is to find a \( p_t^1 \) to reach some earnings target \( \pi \), that is,
\[
E(\Pi_t^1 + \Pi_t^2) \geq \pi. \tag{3}
\]
This earnings target can be the zero earnings, previous period’s earnings, or analysts’ forecasts (See Levitt, 1998; Graham, et. al. 2005, Zhang and Kang, 2007)

In the second period, since the first period shock \( \varepsilon^1 \) has realized, the targeting type of firm 1 needs to calculate the extent of earnings management at \( t = 2 \). Let \( \Pi_1^*(p_1^1(m), p_1^2(r), p_1^3, \varepsilon^1) \) denote firm 1’s first period realized profit. The extent of earnings management is hence \( \pi - \Pi_1^*(p_1^1(m), p_1^2(r), p_1^3, \varepsilon^1) \). For simplification, we will abbreviate this difference as \( \tilde{\pi}(p_1^1(m), p_1^2(r), \varepsilon^1) \).

There are two approaches to reach this target: firm 1 can reach the target either by manipulating prices in the oligopoly market, or by tunnelling the required profit from the partner firm 3. In countries with weak legal investors protection, tunnelling is often seen in the pyramids or family-owned business (Johnson, et. al., 2000). Here, following the notion by Jian and Wong (2010), Friedman, et. al. (2003) and Riyanto and Toolsema (2008), tunnelling refers to a transfer of resources from a lower-end firm to a higher firm in the pyramidal chain. Djankov, et. al. (2008) noted that related party transactions may provide direct opportunities for related parties to extract cash from listed companies through tunneling activities. Cheung, et. al. (2006) and Bertrand, et. al. (2002) empirically concluded that the minority shareholders in these firms seem to be subject to expropriation through tunneling. The details for the two approaches are given as follows.

2.2.1. Price Manipulation

Without tunnelling from firm 3, the targeting type of firm 1 can only strategically choose its price to reach the second period target. That is, the targeting type of firm 1 will choose a \( p_1^2 \) such that

\[
E \Pi_1^2 \geq \tilde{\pi}(p_1^1(m), p_1^2(r), \varepsilon^1). \tag{4}
\]

Firm 2 and firm 3’s objectives remain the same as equation (2).

2.2.2. Tunnelling from Firm 3

Tunnelling from a lower-end firm in the pyramidal chain can reduce REM activities’ strategic impacts on the first industry, but at the cost of the low-end firm’s profit. To capture the notion of tunnelling, denote \( D \) as the size of profit to be tunnelled from firm 3, with \( 0 \leq D \leq \tilde{\pi}(p_1^1(m), p_1^2(r), \varepsilon^1) \). Let \( c(D) \), with \( c'(D) > 0 \), denote the tunnelling cost. The optimal level of tunnelling can be endogenously determined by the marginal condition, where \( c'(D) \) is equal to the marginal benefit of tunnelling \( D \). Since our focus is on REM, we will assume for simplification that there exists a
unique value satisfying this marginal condition, and to abuse the notation, we denote this value as $D$.

Thus, given the level of $D$, the objective function of the targeting type is to find a $p^1_2$ to reach the target, i.e.,

$$E(\pi_2^2(p^1_1(r), p^2_2, \varepsilon^2) + \rho[\pi_3^2(p^3_1, \varepsilon^2) - D] - c(D) \geq \hat{\pi}(p^1_1(m), p^1_1(r), \varepsilon^1) - D).$$

(5)

Firm 2’s objective remains the same as equation (2). But, for firm 3, the objective becomes

$$\max_{p^3_2} \{\Pi_3^2 - D\}.$$

2.3. Beliefs

Following the literature, we assume that the first period belief $\mu^1$ with $0 < \mu^1 < 1$ is exogenously given as prior belief. The second period belief $\mu^2$ will be endogenously determined by the prior belief $\mu^1$ and firm 1’s first period pricing strategies $(p^1_1(m), p^1_1(r))$. We will discuss the off-equilibrium path beliefs shortly while characterizing the equilibrium.

3. CHARACTERIZATION OF EQUILIBRIUM

In this section, we first characterize the second period market equilibrium $(p^2_2(m), p^2_2(r), p^2_3, p^3_2)$, for a given level of posterior belief $\mu^2$ and the first period random shock $\varepsilon^1$. Then we discuss the first period market equilibrium $(p^1_1(m), p^1_1(r), p^1_2, p^1_3)$, and interpret the setting of on and off-equilibrium path beliefs $\mu^2$. Notice that, in addition to considering the opponent’s reaction to earnings management in the second period, we are interested in knowing whether strategic actions have to be taken in the first period when firm 1 wants to take advantage of the private information to mitigate the damage from a price war. As mentioned earlier, the pricing strategy can be interpreted as a result of earnings management or of a purely strategic concern. If a price cut is recognized as a purely strategic action, then it can induce retaliation from the opponent. Hence, the targeting type of firm 1 may have an incentive to signal itself out from the maximizing type at $t = 1$.

3.1. Market Equilibrium for $t = 2$

The second period market equilibrium is different from that of the first period in mainly three aspects. (i) Since it is the last period, firm 1’s pricing strategy has no signaling indication; (ii) Firm 2 uses the posterior
belief $\mu^2$ instead of the prior belief to calculate the expected profit $\Pi_2^2$.

(iii) The targeting type of firm 1 now needs to reach a target depending on the first period market equilibrium and the random demand shock $\epsilon^1$. As noticed earlier, firm 1 can reach the target either through manipulating prices in the market or by tunnelling from partner firm 3. Here we will provide the detailed characterization for each alternative, and then derive the first period market equilibrium.

1) Price Manipulation

In the beginning of the second period, all firms can observe the first period market equilibrium and the realized random shock $\epsilon^1$, both of which will determine the threshold $\pi(p_1^1(m), p_1^1(r), \epsilon^1)$. As for the second period belief $\mu^2$ (on and off-equilibrium path), we will discuss in more details when we characterize the first period equilibrium, and for the moment the value of $\mu^2$ is treated as constant. The two types of firm 1, firm 2 and firm 3 determine $(p_2^1(m), p_1^1(r), p_2^2, p_3^2)$ simultaneously in the second period.

First, for the maximizing type of firm 1, let

$$p_2^1(m) = \arg \max_{p_2^1} \mathbb{E}(\pi_2^1 + \rho \pi_2^3),$$

where

$$\mathbb{E}(\pi_2^1 + \rho \pi_2^3) = \int_{-\varepsilon}^{\varepsilon} (a - p_2^1 - \beta p_2^2 + \varepsilon)(p_2^2 - c_1) \frac{1}{2\pi} d\varepsilon + \int_{-\varepsilon}^{\varepsilon} \rho \pi_2^3 \frac{1}{2\pi} d\varepsilon.$$

The maximizing type’s best reply to $p_2^2$ is:

$$p_2^1(m) = \frac{1}{2}(a - \beta p_2^2 + c_1). \quad (6)$$

For the targeting type of firm 1, denote $p_1^1(r)$ as the price to satisfy equation (4), i.e.,

$$p_1^1(r) \in \left\{ p_1^1 \int_{-\varepsilon}^{\varepsilon} (a - p_2^1 - \beta p_2^2 + \varepsilon)(p_2^2 - c_1) \frac{1}{2\pi} d\varepsilon + \int_{-\varepsilon}^{\varepsilon} \rho \pi_2^3 \frac{1}{2\pi} d\varepsilon \geq \pi_2^1(m, p_1^1(m), \epsilon^1) \right\}. \quad (7)$$

Next, firm 2 chooses $p_2^2$ to maximize $E(\mu^2 \pi_2^3(p_1^1(m), p_2^2, \epsilon^2) + (1-\mu^2) \pi_2^2(p_1^1(r), p_2^2, \epsilon^2))$, i.e.,

$$\max_{p_2^2} \left\{ \int_{-\varepsilon}^{\varepsilon} (a - p_2^2 - \beta (\mu^2 p_2^2(m) + (1-\mu^2) p_1^1(r)) + \varepsilon)(p_2^2 - c_2) \frac{1}{2\pi} d\varepsilon \right\}. \quad (7)$$
Firm 2’s best reply to $p_1^2(m)$ and $p_1^2(r)$ is hence:

$$p^2_2 = \frac{1}{2} \left( c_2 + \alpha \beta \mu^2 p_1^2(m) + (1 - \mu^2) p_1^2(r) \right). \quad (8)$$

Figure 2 depicts the best replies of firm 2 and the targeting type, given a level of $p_1^2(m)$. The reason for taking $p_1^2(m)$ as given is because this value will be uniquely determined by equation (6). Since the targeting type of firm 1 can choose among a range of feasible prices to reach the target, the best replies of $p_1^2(r)$ are indicated by the shadow area in the diagram. Accordingly, one can expect many equilibrium prices in the second period. Since there is no obvious criterion to rule out any equilibrium, we will focus on those equilibria which can be supported by the evidence.

Finally, firm 3 chooses $p_3^2$ to maximize $E(1 - \rho)\pi^2_3$, i.e.,

$$\max_{p_3^2} (1 - \rho) \int_{-\epsilon}^{\epsilon} (a - p_3^2 + \epsilon)(p_3^2 - c_3) \frac{1}{2\epsilon} d\epsilon,$$

and the optimal price is:

$$p_3^2 = \frac{1}{2} (a + c_3). \quad (9)$$

The second period market equilibrium is determined by equations (6)–(9) simultaneously. To describe the equilibrium properties, notice first that the first period equilibrium will affect the continuation payoff through the Bayesian updating for $\mu^2$, as well as through the level of $\hat{\pi}(p_1^1(m), p_1^1(r), \epsilon^1)$, which will determine $p_1^2(r)$. Second, as a benchmark of comparison, we
denote \((p_1^*, p_2^*)\) as the price which maximize firm i’s one-shot profit, i.e.,

\[
p_i^* \equiv \arg \max_{p_i} E(\pi_i^t(p_i, p_i^t, \varepsilon^t)),
\]

and \(\pi_i^*\) as the respective profit.

**Lemma 1.** For the target type of firm 1 to reach a \(\pi_1^2 > \pi_1^*\) in the second period, it needs to set a \(p_2^r(r)\) higher than \(p_1^*\) for both \(\beta < 0\) and \(\beta > 0\).

*Proof.* Notice that \(\pi_1^2(\mu^2, p_2^2(r), p_2^1, \varepsilon^2)\) is concave in \(p_2^2(r)\) and decreasing in \(\beta p_2^2\). If the targeting type of firm 1 is to set a profit higher than \(\pi_1^*\), it requires a \(p_2^2\) higher than \(p_2^*\) for \(\beta < 0\). Since firm 2 faces an expected price \(\mu^2 p_1^2(m) + (1 - \mu^2) p_2^2(r)\) and \(p_1^2(m)\) will be the same as \(p_1^*\) when \(p_2^2 = p_2^*\), it requires a \(p_2^1(r)\) higher than \(p_2^*\). Similarly, we can argue for the case \(\beta > 0\). \(\blacksquare\)

Lemma 1 shows that once the market demand falls short of expectation, a targeting type of firm should raise the price, instead of cutting it, so that the profit-reducing price war can be avoided. However, this does not violate the empirical results that firms cut their prices to reach the earnings target. We will demonstrate shortly that, due to the informational concern, there exist equilibria where firms raise prices in the first period, and then cut the prices in the second period.

Next, since the set of \(p_2^2(r)\) is affected by the level of \(\hat{\pi}(p_1^2(m), p_1^1(r), \varepsilon^1)\), we rewrite the second period payoff \(\Pi_2^r\) as \(\Pi_2^r(\mu^2, \hat{\pi}(p_1^2(m), p_1^1(r), \varepsilon^1), p_2^2(m), p_2^1(r))\).

The following lemma describes the properties of the equilibrium payoff when the targeting type of firm 1 needs to reach a target higher than \(\pi_1^*\).

**Lemma 2.** If the targeting type of firm 1 needs to reach a target higher than \(\pi_1^*\), then (i) \(\Pi_2^r(\mu^2, \hat{\pi}(p_1^2(m), p_1^1(r), \varepsilon^1), p_2^2(m), p_2^1(r))\) decreases with \(\mu^2\) for \(\beta < 0\), and increases with \(\mu^2\) for \(\beta > 0\). (ii) The lower bound of \(\Pi_2^r(\mu^2, \hat{\pi}(p_1^2(m), p_1^1(r), \varepsilon^1), p_2^2(m), p_2^1(r))\) for the targeting type will increase with \(\hat{\pi}\), and decrease with \(\varepsilon^1\).

*Proof.* (i) Notice first that \(\pi_2^1(\mu^2, p_2^1(r), p_2^2, \varepsilon^2)\) decreases with \(\beta p_2^2\). Next, as described, \(p_2^r(r)\) is higher than \(p_2^*\) when \(\beta < 0\), and thus \(\mu^2 p_2^1(m) + (1 - \mu^2) p_2^1(r)\) will decrease with \(\mu^2\). The same argument applies to \(\beta > 0\). (ii) Notice that \(\Pi_2^1\) increases with \(\varepsilon^1\), and by definition, \(\hat{\pi}\) will decrease with \(\varepsilon^1\). \(\blacksquare\)

Lemma 2 explains how the equilibrium profit is affected by the second period target and the posterior belief. Since both of them are influenced
by the targeting type’s first period price, this result is important for the targeting type’s strategic concern in the first period. Next, the relation between firm 2’s beliefs and firm 1’s equilibrium payoff seems to vary with the degree of substitution between two products. In particular, for $\beta < 0$, we have $\Pi^1_2(0, \hat{\pi}, p^1_2(m), p^2_2(r)) > \Pi^1_2(1, \hat{\pi}, p^1_2(m), p^2_2(r))$, and for $\beta > 0$, the inequality is reversed.

Finally, consider two target levels: $\hat{\pi}$ and $\hat{\pi}'$ with $\hat{\pi} < \hat{\pi}'$. This lemma says that the lower bound of $\Pi^1_2(\mu^2, \pi, p^1_2(m), p^2_2(r))$ is smaller than $\Pi^1_2(\mu^2, \hat{\pi}', p^1_2(m), p^2_2(r))$. Moreover, if $\beta < 0$, the lower bound of $\Pi^1_2(1, \hat{\pi}, p^1_2(m), p^2_2(r))$ is smaller than $\Pi^1_2(1, \hat{\pi}', p^1_2(m), p^2_2(r))$. Since the cross effect is smaller than the own effect, the lower bound of $\Pi^1_2(0, \hat{\pi}, p^1_2(m), p^2_2(r))$ is smaller than the lower bound of $\Pi^1_2(1, \hat{\pi}', p^1_2(m), p^2_2(r))$.

**Lemma 3.** The lower bound and feasible set of $p^2_1(r)$ will increase with $\hat{\pi}(p^1_1(m), p^1_1(r), \varepsilon^1)$ and decrease with $\varepsilon^1$.

**Proof.** First, notice that $\hat{\pi}$ will decrease with $\varepsilon^1$. Next, by Lemma 2, as $\hat{\pi}$ increases, the lower bound of profit for the targeting type $\Pi^1_2(\cdot)$ will increase. Finally, according to Lemma 1, as the difference between targeting type’s target and $\pi^1_1$ increases, the difference between $p^2_1(r)$ and $p^2_1$ will also increase.

Remind that $\hat{\pi}(\cdot) \equiv \pi - \Pi^1_2(\cdot)$. Lemma 1 and Lemma 3 indicate that as $\hat{\pi}$ increases, the lower bound and the feasible set of $p^2_1(r)$ will increase. Moreover, since $\Pi^1_2(\cdot)$ is concave in $p^1_1(k)$, $k = m, r$, the impact of $p^1_1(k)$ on $\hat{\pi}$ will depend on the relative sizes of $p^1_1(k)$ and $p^1_1$.

Lemma 2 and Lemma 3 describe that the degree of REM will decrease with the business state. Cohen and Zarowin (2007) found empirical evidence that the tendency for firms to meet or beat earnings benchmarks is significantly related to the $P/E$ ratio.

**2) Tunnelling from Firm 3**

With tunneling from firm 3, the two types of firm 1, firm 2 and firm 3 are facing the following problems. First, the best replies for the maximizing type of firm 1 and firm 2 are the same as equations (6) and (8). Second, the targeting type of firm 1 needs to choose $p^2_2(r)$ to satisfy

$$E(\pi^1_2(p^2_2(r), p^2_2, \varepsilon^2) + p(\pi^3_3(p^3_3, \varepsilon^3) - D)) \geq \hat{\pi}(p^1_1(m), p^1_1(r), \varepsilon^1) - D, \quad (10)$$

where we have assumed $D$ to be the optimal level determined by the marginal condition, and to simplify the analysis, we assume that $c'(D) < $
(1-\(\rho\)). The set of feasible \(p_{11}^2(\tau)\) is given by

\[
p_{11}^2(\tau) \in \left\{ p_{11}^2 \left| \int_{-\pi}^{\pi} (a-p_{2}^2-\beta p_{3}^2+\varepsilon)(\tau^2-c_1) \frac{1}{2\pi} d\varepsilon + \int_{-\pi}^{\pi} \rho \pi^2 \frac{1}{2\pi} d\varepsilon \geq \hat{\pi}(p_{11}^1(m), p_{11}^1(\tau), \varepsilon^1) + c(D)-(1-\rho)D \right\}
\]

(11)

The term \(c(D)-(1-\rho)D\) is decreasing with \(D\), and hence the lower bound of the targeting type will decrease with \(D\).

Finally, firm 3 chooses \(p_{23}^3\) to maximize \(E((1-\rho)[\pi_{23}^3(p_{23}^3, \varepsilon^2)-D])\). Since the presence of \(D\) does not affect the decision of optimal price, \(p_{23}^3\) remains the same as equation (9).

Tunnelling from a low-end firm in the pyramidal chain can reduce REM’s strategic impacts on the first industry, but at the cost of the low-end firm’s profit. The required profit to be manipulated by the targeting type of firm 1 will decrease with \(D\). If \(D\) is high enough, then according to Lemma 1, it is possible for the targeting type to change the second period price to be a more aggressive level. Since \(\hat{\pi}\) decreases with \(D\), the impact of \(D\) on the equilibrium profit will be negatively related to those of \(\hat{\pi}\) (see Lemma 2 and 3).

3.2. Market Equilibrium for \(t = 1\)

The first period prices of the two types of firm 1 have two effects. (i) They relate directly to the first period profits, which together with the random shock, indirectly determine the required profits for the targeting type of firm 1 in the next period. These will change the second period prices for the targeting type of firm 1, which, in turn, affects \(p_{23}^3\). (ii) They will change firm 2’s belief about firm 1’s objective type, the impact of which, according to Lemma 1 and 2, will depend on whether the targeting type of firm 1 needs to set a level higher than \(\pi_1^*\). In other words, there is coordination between firm 1’s first period prices and its second period prices.

In this section, we will consider two groups of perfect Bayesian equilibria: separating and hybrid equilibria (see for example Gibbons, 1992). In the separating equilibrium, each type of firm 1 is willing to express its identity, so that in the second period, firm 2 will charge a price best fit to each type, instead of a price that is best reply to a weighted sum of \(p_{11}^2(m)\) and \(p_{11}^2(\tau)\). In the hybrid equilibrium, a certain type of firm 1 is not willing to express its identity, so that in the second period, this particular type of firm 1 can take advantage of the impact on \(\mu^2\), and of the fact that firm 2 will charge a price best replying to a weighted sum of \(p_{11}^2(m)\) and \(p_{11}^2(\tau)\). Different from the standard signaling game literature, here the incentive constraints for the equilibrium are not given exogenously; They will be en-
dogenously determined by the market equilibrium. In what follows, we will first characterize the incentive constraints for each equilibrium, and then check if there exists any market equilibrium to satisfy these constraints.

First of all, given the second period equilibrium \((p_2^1(m), p_2^1(r), p_2^2, p_2^3)\) as characterized above, each firm’s intertemporal profits are given as follows. Since both \(\hat{\pi}\) and \(p_2^1(r)\) are affected by \(\epsilon^1\), we will rewrite the expected continuation payoff as: \(\Pi_2^i(\mu^2, \hat{\pi}(p_1^i(m), p_1^i(r), \epsilon^1), p_2^1(m), p_2^1(r))\) in order to capture the impact from the first period shock.

The maximizing type of firm 1 needs to find a \(p_1^1(m)\) to solve the following problem

\[
\max_{p_1^1} \int \left\{ (a - p_1^1 \cdot \beta p_2^1 + \epsilon)(p_1^1 - c_1) + \rho \pi_3^1 + \Pi_2^1(\mu^2, \hat{\pi}(p_1^i(m), p_1^i(r), \epsilon), p_1^1(m), p_2^1(r)) \right\} \frac{1}{2\pi} d\epsilon.
\]

(12)

On the other hand, the targeting type of firm 1 will seek a \(p_1^1(r)\) to reach the target \(E(\Pi_1^1 + \Pi_2^1) \geq \pi\), that is,

\[
p_1^1(r) \in \{p_1^1 | \int \left\{ (a - p_1^1 \cdot \beta p_2^1 + \epsilon)(p_1^1 - c_1) + \rho \pi_3^1 + \Pi_2^1(\mu^2, \hat{\pi}(p_1^i(m), p_1^i(r), \epsilon), p_1^1(m), p_2^1(r)) \right\} \frac{1}{2\pi} d\epsilon \geq \pi \}.
\]

(13)

Meanwhile, firm 2 chooses \(p_2^1\) to solve the following problem, given the prior belief \(\mu^1\)

\[
\max_{p_2^1} \int \left\{ (a - p_2^1 \cdot \beta p_1^1(m) + \epsilon)(p_2^1 - c_2) + (1 - \mu^1)(a - p_2^1 \cdot \beta p_1^1(r) + \epsilon)(p_2^1 - c_2) \right\} \frac{1}{2\pi} d\epsilon,
\]

\[
+ \Pi_2^1(\mu^2, \hat{\pi}(p_1^i(m), p_1^i(r), \epsilon), p_2^1(m), p_2^1(r)) \right\} \frac{1}{2\pi} d\epsilon,
\]

which can be rewritten as

\[
\max_{p_2^1} \int \left\{ (a - p_2^1 \cdot \beta \mu^1 p_1^1(m) + (1 - \mu^1) p_1^1(r) + \epsilon)(p_2^1 - c_2) \right\} \frac{1}{2\pi} d\epsilon + \Pi_2^1(\mu^2, \hat{\pi}(p_1^i(m), p_1^i(r), \epsilon), p_2^1(m), p_2^1(r)) \frac{1}{2\pi} d\epsilon.
\]

(14)
Finally, firm 3 seeks for a \( p_3^1 \) to solve the following problem

\[
\max_{p_3^1} (1-\rho) \int \left\{ \left( (a-p_3^1+\varepsilon)(p_3^1-c_3) + \Pi_2^2(\mu^2, \tilde{\pi}(p_1^1(m), p_1^1(r), \varepsilon)) \right) p_2^1(m, p_1^2(r)) \right\} \frac{1}{2\varepsilon} d\varepsilon.
\]

(15)

As described in Lemma 3, the level of \( \tilde{\pi} \) will affect the lower bound of \( p_2^1(r) \), and \( \tilde{\pi} \) is affected by the first period equilibrium and the random shock. Since \( \pi_1^1 \) is increasing in \( \varepsilon \), let \( \varepsilon(p_1^1(m), p_1^1(r), \mu^2) \) denote the threshold value of \( \varepsilon \) such that \( \pi_1^1 = \pi_1^* \). Hence, for \( \varepsilon > \varepsilon(p_1^1(m), p_1^1(r), \mu^2) \), the profit target for the targeting type of firm 1 is less than \( \pi_1^* \), and according to Lemma 1, it will set a \( p_2^1(r) \) less than \( p_1^* \) for both \( \beta < 0 \) and \( \beta > 0 \).

**Lemma 4.** \( \varepsilon(p_1^1(m), p_1^1(r), \mu^2) \) will increase with \( \beta p_2^1 \) and increase with \( p_1^1 \) if \( p_1^1 > p_1^* \).

**Proof.** Notice that \( \pi_1^1 \) decreases with \( \beta p_2^1 \) and increases with \( \varepsilon \). As \( \beta p_2^1 \) increases, it requires a higher \( \varepsilon \) to keep the profit fixed at \( \pi_1^* \), and hence we have the result. For the second part, if \( p_1^1 > p_1^* \), then \( \pi_1^1 \) decreases with \( p_1^1 \), and hence \( \varepsilon(p_1^1(m), p_1^1(r), \mu^2) \) increases with \( p_1^1 \).

**1) Separating Equilibrium**

In the separating equilibrium, each type of firm 1 is willing to express its identity, so that in the second period, firm 2 will charge a price best fit to each type, instead of a price that is best reply to a weighted sum of \( p_2^1(m) \) and \( p_2^1(r) \). As described earlier, the first period equilibrium will affect the continuation payoff in three aspects: (i) It determines \( \mu^2 \); (ii) The equilibrium payoff \( \pi_1^1 \) affects the level of \( \tilde{\pi} \), which in turn affects the setting of \( p_2^1(r) \); (iii) It affects the possibility that the targeting type of firm 1 could set a price less than \( p_1^* \) in the second period. We are interested in equilibria where the targeting type of firm 1 sets a price higher than \( p_1^* \), and cuts the price in the second period, which hence explains the evidence of seasonal price cuts.

Let \( (p_1^1(m), p_1^1(r)) \) with \( p_1^1(r) > p_1^1(m) > p_1^* \) denote the equilibrium prices for the maximizing type and targeting type, respectively. Remind that \( p_1^1 \) is determined by equation (14). We consider the following beliefs for the second period:

\[
\mu^2 = \begin{cases} 
0 & \text{for } p_1^1 \geq p_1^1(r), \\
1 & \text{for } p_1^1 < p_1^1(r).
\end{cases}
\]
This setting includes the on-equilibrium path belief which is calculated by Bayes’ rule, and the setup for the off-equilibrium path belief is referred to Gibbons (1992).

After replacing $\mu^2$ with the above setting, we rewrite the intertemporal profit for the maximizing type of firm 1 as:

\[
\int_{-\varepsilon}^{\varepsilon} \{(a - p_1^1(m) - \beta p_2^1 + \varepsilon)(p_1^1(m)-c_1) + \rho \pi_3^1 \} \frac{1}{2\varepsilon} d\varepsilon
\]

\[
+ \int_{-\varepsilon}^{\varepsilon} \Pi_2^2(1, \hat{\pi}(p_1^1(m), p_1^1(r), \varepsilon), p_1^2(m), p_1^2(r)) \frac{1}{2\varepsilon} d\varepsilon
\]

\[
+ \int_{-\varepsilon}^{\varepsilon} \Pi_2^2(1, \hat{\pi}(p_1^1(m), p_1^1(r), \varepsilon), p_1^2(m), p_1^2(r)) \frac{1}{2\varepsilon} d\varepsilon. \quad (16)
\]

Here and henceforth, we denote $p_1^2(r)$ as the targeting type’s second period price for cases where the targeted profit is higher than $\pi_1^*$. Similarly, denote $\hat{p}_1^2(r)$ as the targeting type’s second period price for cases where the targeted profit is less than $\pi_1^*$.

Next, denote $\Phi$ as the intertemporal profit for the targeting type of firm 1, where

\[
\Phi \equiv \int_{-\varepsilon}^{\varepsilon} \{(a - p_1^1(r) - \beta p_2^1 + \varepsilon)(p_1^1(r)-c_1) + \rho \pi_3^1 \} \frac{1}{2\varepsilon} d\varepsilon
\]

\[
+ \int_{-\varepsilon}^{\varepsilon} \Pi_2^2(0, \hat{\pi}(p_1^1(m), p_1^1(r), \varepsilon), p_1^2(m), p_1^2(r)) \frac{1}{2\varepsilon} d\varepsilon
\]

\[
+ \int_{-\varepsilon}^{\varepsilon} \Pi_2^2(0, \hat{\pi}(p_1^1(m), p_1^1(r), \varepsilon), p_1^2(m), p_1^2(r)) \frac{1}{2\varepsilon} d\varepsilon.
\]

**Maximizing Type of Firm 1** For $(p_1^1(m), p_1^1(r))$ to be a separating equilibrium, it is required that for the maximizing type of firm 1, (i) $p_1^1(m)$ maximizes the intertemporal profit in (16), meaning that the equilibrium profit is higher than any $(p_1^1'(m), p_1^1(r))$ with $p_1^1'(m) \neq p_1^1(m)$ and $\mu^2 = 1$; (ii) The equilibrium profit is at least greater than that of mimicking the
targeting type and setting prices and belief to be \((p_1^1(r), p_1^1(r))\) and \(\mu^2 = \mu^1\). In our terms, the first condition is equivalent to the marginal condition that the partial derivation of (16) with respect to \(p_1^1(m)\) is equal to zero. The second condition requires the profit in (16) to be at least as great as the following term:

\[
\int_{-\varepsilon}^{\varepsilon} \left\{ (a-p_1^1(r)-\beta p_2^1 r + \varepsilon)(p_1^1(r)-c_1) + \rho p_1^1(r) \right\} \frac{1}{2\varepsilon} d\varepsilon + \\
\int_{-\varepsilon}^{\varepsilon} \Pi_1^2(\mu^1, \hat{\pi}(p_1^1(r), p_1^1(r), \varepsilon), p_2^1(m), p_2^1(r)) \frac{1}{2\varepsilon} d\varepsilon + \\
\int_{-\varepsilon}^{\varepsilon} \Pi_1^2(\mu^1, \hat{\pi}(p_1^1(m), p_1^1(r), \varepsilon), p_2^1(m), p_2^2(r)) \frac{1}{2\varepsilon} d\varepsilon.
\]

Note that \(p_1^1(r) > p_1^1(m)\), and lemma 4 describes that \(\varepsilon(p_1^1(r), p_1^1(r), p_2^1) > \varepsilon(p_1^1(m), p_1^1(r), p_2^1)\). Let the above condition bind, then we have

\[
\Pi_1^1(1, p_1^1(m) - p_1^1(r)) + \int_{-\varepsilon}^{\varepsilon} \Pi_1^2(1, p_1^2(r) - \Pi_1^2(\mu^1, p_2^1(r))) \frac{1}{2\varepsilon} d\varepsilon + \\
\int_{-\varepsilon}^{\varepsilon} [\Pi_1^2(1, p_2^2(r) - \Pi_1^2(\mu^1, p_2^1(r))] \frac{1}{2\varepsilon} d\varepsilon = 0,
\]

where we have abbreviated the continuation payoff \(\Pi_1^2(\mu^2, \hat{\pi}(p_1^1(r), p_1^1(r), \varepsilon), p_2^1(m), p_2^2(r))\) as \(\Pi_1^2(\mu^2, p_2^2(r))\) for simplification. The same abbreviation will apply to equation (19).

**Targeting Type of Firm 1** For the targeting type, it is only required that the intertemporal profit satisfies: \(E(\Pi_1^1 + \Pi_1^2) \geq \pi\). Notice that for the targeting type, as long as this target is reached, it is not necessary for this type to pursue the highest profit; Once the target is reached, it has no incentive to mimic the maximizing type for better profit. Hence, different from the traditional incentive constraint, for \(p_1^1(r)\) to be in the separating
equilibrium, we need

$$\Phi \geq \pi.$$  \hspace{1cm} (18)

Overall, the separating equilibrium is determined by the marginal condition in (16), equation (17) and equation (18). The following proposition describes the equilibrium properties for both of the pure price manipulation and tunnelling cases.

**Proposition 1.** There exists a separating equilibrium where the targeting type of firm 1 sets a price higher than the maximizing type, and then cut the price in the second period. That is, \(p_1^1(r) > p_1^1(m)\) and \(p_1^1(r) > p_2^1(r) > p_2^1(m)\).

**Proof.** See the Appendix.

This proposition addresses the strategic impact of REM. We show that when a firm has the incentive to meet the earnings target, it will alter its pricing strategy accordingly. In particular, the targeting type of firm 1 will set a high price in the first period and then cut the price in the second period to meet the earnings target. This is consistent with the empirical results that firms cut their prices around the fiscal quarter-end to reach the earnings target. For example, Oyer (1998) showed that manufacturing firms often offer price discounts to temporarily increase sales at the end of the fiscal year. Chapman and Steenburgh (2010) found that soup manufacturers increase the frequency and change the mix of marketing promotions (price discounts) at the fiscal quarter-end when they need to meet earnings target. Next, this result is compared to the cases with tunnelling from firm 3.

**Proposition 2.** The second period prices in this separating equilibrium are lower with the presence of tunnelling.

**Proof.** See the Appendix.

In Proposition 5 we characterize the equilibria where firms may raise prices in the first period and cut the prices in the second period to boost up earnings. Proposition 6 shows that the extent of such a price discount will increase with the presence of partial ownership in the partner firm. This is consistent with the empirical results on REM. Mizik and Jacobson (2007) presented evidence that firms inflate current-term earnings by
cutting marketing expenditure at the time of a seasoned equity offering. Cohen, et. al. (2009) also found that managers, on average, reduce advertising spending to avoid losses and earnings decreases. They reported that firms in the late stages of their life cycle can increase advertising to meet earnings benchmarks, and provided evidence that firms increase advertising in the third month of a fiscal quarter and in the fourth quarter to beat prior year’s earnings. Chapman and Steenburgh (2010) found that soup manufacturers increase the frequency and change the mix of marketing promotions when they need to meet earnings target.

(2) Hybrid Equilibrium

In a hybrid equilibrium, a certain type of firm 1 is not willing to express its identity, so that in the second period, this particular type of firm 1 can take advantage of the impact on $\mu^2$ (which will be higher than $\mu^1$) and the fact that firm 2 will charge a price best replying to a weighted sum of $p_1^2(m)$ and $p_1^2(r)$. This benefit happens in cases with $p_2^2(m) < p_2^2(r)$, and hence $p_2^2$ will be set at least higher than $p_2^*$ (provided that $\beta < 0$). According to Lemma 1, the targeting type needs to set a price high enough to reach the target, when the random shock drops below the cutoff value $\epsilon(p_1^2(m), p_1^2(r), p_1^2)$. Hence the probability for this case is $\left(\frac{\epsilon(p_1^2(m), p_1^2(r), p_1^2) - \epsilon}{\epsilon}\right)^2$.

Let $(\theta p_1^1(m) + (1 - \theta)p_1^1(r), p_1^2)$ with $p_1^2(r) > p_1^1(m) > p_1^*$ denote the equilibrium prices, and remind that $p_2^*$ is determined by equation (14). We consider the following beliefs for the second period:

$$
\begin{align*}
\mu^2 &= \frac{\mu^1(1 - \theta)}{\mu^1(1 - \theta) + (1 - \mu^1)} \quad \text{for } p_1^1 = p_1^1(r), \\
&= 1 \quad \text{for } p_1^1 \neq p_1^1(r).
\end{align*}
$$

Maximizing Type of Firm 1 After replacing $\mu^2$ with the above setting we can rewrite the intertemporal profit for the maximizing type of firm 1. Note first that the mixed strategy between $p_1^1(m)$ and $p_1^1(r)$ indicates that the intertemporal profit for these two alternatives are the same. That is,
the profit in equation (16) can be rewritten as:

\[ \int \left\{ \left( a - p_1^1(r) - \beta p_2^1 + \varepsilon \right) \left( p_1^1(r) - c_1 \right) + \rho \pi_3^1 \right\} \frac{1}{2\varepsilon} d\varepsilon \]

\[ \epsilon(p_1^1(m), p_1^1(r), p_1^2) + \int_{\varepsilon} \Pi_1^2(\mu^2, \bar{p}(p_1^1(r), p_1^1(r), \varepsilon), p_1^2(m), \bar{p}_1^1(r)) \frac{1}{2\varepsilon} d\varepsilon \]

\[ + \int_{\varepsilon} \Pi_1^2(\mu^2, \bar{p}(p_1^1(r), p_1^1(r), \varepsilon), p_1^2(m), \bar{p}_1^1(r)) \frac{1}{2\varepsilon} d\varepsilon. \]

Let \( p_1^1(m) \) denote the price to satisfy the marginal condition of equation (16)', i.e.,

\[ \Pi_1^1(1, p_1^1(m) - p_1^1(r)) + \int_{\varepsilon} \Pi_1^2(1, p_1^2(r) - \Pi_1^2(\mu^2, \bar{p}_1^2(r)) \frac{1}{2\varepsilon} d\varepsilon \]

\[ + \int_{\varepsilon} \Pi_1^2(1, p_1^2(r) - \Pi_1^2(\mu^2, \bar{p}_1^2(r)) \frac{1}{2\varepsilon} d\varepsilon \]

\[ + \int_{\varepsilon} \Pi_1^2(1, p_1^2(r) - \Pi_1^2(\mu^2, \bar{p}_1^2(r)) \frac{1}{2\varepsilon} d\varepsilon = 0. \]

**Targeting Type of Firm 1** For the targeting type of firm 1, the intertemporal profit \( \Phi \) is given by:

\[ \hat{\Phi} \equiv \int \left\{ \left( a - p_1^1(r) - \beta p_2^1 + \varepsilon \right) \left( p_1^1(r) - c_1 \right) + \rho \pi_3^1 \right\} \frac{1}{2\varepsilon} d\varepsilon \]

\[ \epsilon(p_1^1(m), p_1^1(r), p_1^2) + \int_{\varepsilon} \Pi_1^2(\mu^2, \bar{p}(p_1^1(m), p_1^1(r), \varepsilon), p_1^2(m), \bar{p}_1^1(r)) \frac{1}{2\varepsilon} d\varepsilon \]

\[ + \int_{\varepsilon} \Pi_1^2(\mu^2, \bar{p}(p_1^1(m), p_1^1(r), \varepsilon), p_1^2(m), \bar{p}_1^1(r)) \frac{1}{2\varepsilon} d\varepsilon. \]
For the targeting type, it is only required that the intertemporal profit satisfies: $E(\Pi_1 + \Pi_2) \geq \pi$. Hence, for $p_1^1(r)$ to be the equilibrium strategy, we need

$$\hat{\Phi} \geq \pi.$$  

(20)

Overall, the hybrid equilibrium is determined by the marginal condition in (16)’, equation (19) and equation (20). The following proposition describes the equilibrium properties for both of the pure price manipulation and tunnelling cases.

**Proposition 3.** There exists a hybrid equilibrium where the maximizing type of firm 1 takes a mixed strategy, and the first period price is higher than in the separating equilibrium.

**Proof.** See the Appendix.

This proposition addresses the informational impact of REM. In order to maintain the opponent’s uncertainty about firm 1’s objective type, firm 1 has the incentive to conceal its identity by taking a mixed strategy (so that firm 2 cannot fully learn about its type) and charge a first period price higher than in the separating equilibrium. This result is accordance with the evidence of contagion in managing earnings intra-industry. Kedia, et. al. (2010) examined the GAO reports and found that firms are more likely to manage earnings after public announcement of a restatement by another firm in their industry. Also, Chapman (2011) and Karaoglu, et. al. (2006) shows that a firm’s price discount to meet earnings target can induce competitors within their industry to follow. Next, this result is compared to the cases with tunnelling from firm 3.

**Proposition 4.** The set of hybrid equilibrium decreases when there is tunnelling from the cross-shareholding firm.

**Proof.** See the Appendix.

Similar to the separating equilibrium, in Proposition 7 we characterize equilibria where firms may strategically raise prices in the first period and cut the prices in the second period to boost the earnings, and in Proposition 8 we show that the extent of price discounting could increase with the cross-shareholding of the opponent firms. Tunnelling from a lower-end firm in the pyramidal chain can reduce the strategic impacts of REM on
the first industry, which hence permits more aggressive actions. Moreover, in the hybrid equilibrium, firm 1 can take advantage of the opponent’s asymmetric information to induce a favorable response from the rival. We also show that tunneling from the affiliated firm may decrease the required extent to reach the earnings target. Johnson, et. al. (2000) showed that entrepreneurs often tunnel resources out of firms in country with weak investors protection. Chapman (2011) and Karaoglu, et. al. (2006) mentioned that firms’ marketing action (such as sales promotion, price discount) to boost earnings may induce competitors within their industry to follow. Jian and Wong (2010) found evidence that controllers of Chinese listed companies engage in tunnelling through related sales. When the listed firms have incentives to meet securities regulators’ earnings target, the increase of sales will mitigate negative industry earnings shocks.

4. CONCLUDING REMARKS

The outbreaks of several financial crises have attracted increasing attention to management’s discretion in financial reporting. REM activities, compared to accruals manipulation, are less likely to be detected by auditors and regulators, and hence managers are more willing to manipulate earnings through real activities rather than to manipulate accruals. In this article, we have addressed the informational and strategic impacts of REM on the real production activities. These issues are important but have not received much theoretical discussion. The informational impact addresses that, since REM is difficult for both auditors and the opponents to detect, the opponent’s suspicion that a firm might be dishonest on earnings reports will create an additional uncertainty which can affect the real activities. Second, REM activities can induce strategic reactions from the opponents, and the results can be worse than before the REM manipulation.

To examine the informational and strategic impacts of REM activities, we build up a two-period oligopoly model where the opponent is uncertain about a firm’s objective type (profit-maximizing or target-reaching), and characterize the perfect Bayesian equilibria where a firm’s price can partially reveal its objective type. The private information on a firm’s objective type is assumed to capture the opponent’s suspicion about this firm’s honesty on earnings report. For the two impacts of REM activities, we have the following conclusions. First, for the strategic impacts of REM, we conclude that once the demand falls short of expectation, the equilibrium way to reach the earnings target is to raise the price, instead of cutting it, as this can avoid the profit-reducing price war. However, this
does not violate the empirical results that firms cut their prices around the fiscal quarter-end to reach the earnings target, as there exist equilibria where firms strategically set higher prices in the first period, and then cut the prices in the second period. Second, for the informational impacts, we show in a hybrid equilibrium that in order to maintain the opponent’s uncertainty about a firm’s objective type, the privately informed firm has the incentive to conceal its identity by taking a mixed strategy, and charge a first period price higher than the separating equilibrium. Finally, we show that the presence of tunnelling from cross-shareholding firm will enhance price cuts in the second period. Tunnelling from a lower-end firm in the pyramidal chain can reduce REM activities’ strategic impacts on its own industry, which hence permits more aggressive actions.

APPENDIX A

Proof of Proposition 5

The conditions for the separating equilibrium consist of the marginal condition of (16), equations (17), and equation (18). We will demonstrate the case with $\beta < 0$ here and the explanation applies to the case with $\beta > 0$ similarly.

For $\beta < 0$, let $p_1^1(m)$ satisfy the marginal condition of (16). Suppose that $p_1^1(r) > p_1^1(m) > p_1^2$ be the equilibrium prices. By the concavity of the profits function, we have $\Pi_1^1(p_1^1(r), p_2^1, \epsilon_1^1) < \Pi_2^1(p_1^1(m), p_2^1, \epsilon_1^1)$ and by Lemma 4 we also know $\epsilon(p_1^1(r), p_1^2, p_2^2) > \epsilon(p_1^1(m), p_1^2, p_2^2)$. This indicates that the first term in equation (17) should be positive. Thus, for the equality in (17) to hold, the necessary condition is to have $\Pi_1^2(\mu^1, \hat{\pi}(p_1^1(r), p_1^1(r), \epsilon^1), p_2^1(m), p_2^2(r)) > \Pi_2^1(1, \hat{\pi}(p_1^1(m), p_1^1(r), \epsilon^1), p_2^1(m), p_2^2(r))$, for which we need $\pi_1^1(\mu^1, p_2^1(m), p_2^1(r), p_2^2) \geq \pi_1^2(1, p_2^1(m), p_2^1(r), p_2^2)$ and $\mu^1 p_2^1(m) + (1 - \mu^1) p_2^2(r) \geq p_2^2(m)$. The latter condition implies that $p_2^2(r) > p_2^2(m)$, $\forall \epsilon^1$, where $p_2^2(r)$ can be either $\hat{p}_2^2(r)$ or $p_2^2(r)$. Furthermore, notice that if $p_2^2(r) > p_2^2(m)$ and $p_2^2(r) > p_2^1(r)$, then the equality in (17) will not hold. Therefore, the prices to satisfy (17) must be $p_2^1(r) > p_2^2(r) > p_2^2(m)$, illustrating the prices cut in the second period.

As for the target type, since $\epsilon(p_1^1(r), p_1^1(r), p_2^2) > \epsilon(p_1^1(m), p_1^1(r), p_2^2)$ and $\Pi_1^1(p_1^1(r), p_2^2, \epsilon_1^1) < \Pi_2^1(p_1^1(m), p_2^2, \epsilon_1^1)$, the second period target prices $\hat{\pi}$ will be higher with $p_1^1(r)$. To satisfy (18), the target type needs need to reach a $\Pi_2^1(0, \hat{\pi}(p_1^1(r), p_1^1(r), \epsilon^1), p_2^2(m), p_2^2(r))$ higher than $\pi_2^* + \rho \pi_2^2$, for which it requires $\pi_2^3(r) \geq \pi_2^*$, and hence $p_2^2(r) > p_2^2(m)$.

Proof of Proposition 6
With the presence of cross-shareholding and tunneling from firm 3, when $D$ increases, $\pi$ decreases. Thus, according to Lemma 2 and 3, the lower bound and feasible set of $p^2_1(r)$ will decrease, which means that the second prices of the target type will be lower.

**Proof of Proposition 7**

The conditions for the hybrid equilibrium consist of the marginal condition of (16)', equations (19), and equation (20). We will demonstrate the case with $\beta < 0$ here and the explanation applies to the case with $\beta > 0$ similarly.

For $\beta < 0$, let $p_1^1(m)$ satisfy the marginal condition of (16)’. Suppose $p_1^1(r) > p_1^1(m) > p_1^2$ to be the equilibrium prices. By concavity of the profit function, we have $\Pi_1^1(p_1^1(r), p_2^1, \epsilon^1) < \Pi_1^1(p_1^1(m), p_2^1, \epsilon^1)$, which implies that $\epsilon(p_1^1(r), p_1^1(r), p_2^2) > \epsilon(p_1^1(m), p_1^1(r), p_2^2)$ according to Lemma 4. This indicates that the first term in equation (19) is positive.

Thus for the equality in (19) to hold, the necessary condition is to have $\Pi_2^1(\mu^2, \hat{\pi}(p_1^1(r), p_2^1, \epsilon^1), p_2^1(m), p_2^2(r))$ higher than $\Pi_2^1(1, \mu^2(p_1^1(m), p_1^1(r), \epsilon^1), p_2^1(m), p_2^2(r))$, for which we need $\pi_2^1(\mu^2, p_2^1(m), p_2^1(r), p_2^2) \geq \pi_2^1(1, p_2^1(m), p_2^1(r), p_2^2)$ and $\mu^2 p_2^1(m) + (1 - \mu^2) p_2^1(r) \geq p_2^1(m)$. The latter condition implies that $p_2^1(r) > p_2^1(m), \forall \epsilon^1$, where $p_2^1(r)$ can be either $\hat{\pi}_2^1(r)$ or $\pi_2^1(r)$. Furthermore, notice that if $p_2^1(r) > p_2^1(m)$ and $p_2^1(r) > p_2^1(r)$, then the equality in (19) will not hold. Therefore, the prices to satisfy (19) must be $p_1^1(r) > p_1^2(r) > p_1^2(m)$, illustrating the price cut in the second period.

As for the targeting type, since $\epsilon(p_1^1(r), p_1^1(r), p_2^1) > \epsilon(p_1^1(m), p_1^1(r), p_2^1)$ and $\Pi_1^1(p_1^1(r), p_2^1, \epsilon^1) < \Pi_1^1(p_1^1(m), p_2^1, \epsilon^1)$, the second period target $\hat{\pi}$ will be higher with $p_1^1(r)$. To satisfy (20), the targeting type needs to reach a $\Pi_2^1(\mu^2, \hat{\pi}(p_1^1(r), p_2^1, \epsilon^1), p_2^1(m), p_2^2(r))$ higher than $\pi_2^1 + \rho \pi_2^2$, for which it requires $\pi_2^1(r) \geq \pi_2^1$, and hence $p_2^1(r) > p_2^1(m)$. Finally, since the maximizing type of firm 1 takes a mixed strategy $\theta p_1^1(m) + (1 - \theta)p_1^1(r)$ at the first period, we can observe $\theta p_1^1(m) + (1 - \theta)p_1^1(r)$ higher than $p_1^1(m), \forall \theta \in (0, 1)$.

**Proof of Proposition 8**

From equation (19), we know that $\mu^2$ will affect $\Pi_2^2$, and that $\mu^2$ is calculated through Bayes’ rule. With the presence of cross-shareholding and tunneling from firm 3, when $D$ increases, $\hat{\pi}$ will decrease. According to Lemma 2 and 3, the lower bound and feasible set of $p_2^1(r)$ will decrease, and the set of hybrid equilibrium will decrease accordingly.
REFERENCES


