Stock Market Manipulation in the Presence of Fund Flows

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We study the manipulation of stock market prices by fund managers in the presence of potential future fund flows. As investors will make further investment as long as the asset price is not fully revealing, the informed manager has incentives to prevent the asset value to be revealed too early, in order to maximise the size of fund flows. Hence in the early trading round, the informed manager always buys the asset even when it is overpriced based on her private information, and the uninformed manager follows suit. Subsequently, the informed manager trades based her private information, and the uninformed one trades based on a mixed strategy. The investors’ decisions to invest arise endogenously within the model.

Key Words: Asymmetric information asset pricing; Stock market manipulation; Delegated portfolio management.

JEL Classification Numbers: G12, G14, G11.

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1. INTRODUCTION

In the fund management industry, the compensation to money managers is normally a fixed proportion of asset management. This simple payoff structure creates incentives for the managers to maximize the fund size rather than trading profit. Thus, perceiving the possibility that investors might make further investment to the funds, managers are reluctant to trade on the basis of their private information. The intuition is that when the information is fully incorporated in the price, investors will not invest further because there is no room for fund managers to make any profit in the future.

In order to study the above incentives in a theoretical framework, we develop a financial equilibrium with an informed (good) fund manager and an uninformed (bad) fund manager. The price of the asset may not be fully-revealing due to the presence of the uninformed manager. Because investors do not have access to the financial market\(^1\), they can only delegate their money to the fund managers based on their beliefs about the quality of the managers. In addition, they can choose to make further investments into the funds at a later stage, if they expect that the fund managers to make profits in the future.

The fund managers aim to maximize the size of their fund, including both trading profits and future inflows. Therefore the informed manager has incentives to prevent the asset value to be revealed too early, despite the trading loss that may result from her actions. Assuming that the fund flows are large enough, the informed manager will always buy the asset at the early stage in equilibrium, even when the asset is overpriced based on her private information, and the bad manager will choose the same action. At the later stage, the informed manager starts trading based on her private information and the uninformed manager trades based on a mixed strategy.

Our paper falls into the category of delegated portfolio management and financial market manipulation. For the delegated portfolio management literature, the most related papers would be Dow & Gorton (1997) and Dasgupta & Prat (2006). In the Dow and Gorton’s paper, they design an optimal contract to rule out the bad managers, but the good manager will trade randomly (churn) without private information given the contract. In our paper, the good (informed) manager trades incorrectly even if she has perfect information. Dasgupta & Prat shows that the bad manager will work as the noise trader when facing the Career Concerns, which is similar to the behaviors of the uninformed manager in our model. However, our focus is the informed manager’s behavior. Moreover, Allen and Gale (1993) show that the bubble can exist because the fund managers will churn if they do not have any information\(^1\)

\(^1\)This can be referred to the literature of limited market participation.
For the literature of financial market manipulation, the most related papers are Fishman and Hagerty (1995) and John and Narayanan (1997). Fishman and Hagerty show that disclosure of insider’s trading position can sometimes increase insider’s profit. The reason is that if the insider is actually not informed, she knows the price set by the market maker is not correct, hence can make a profit. John and Narayanan take one step further to show that disclosure can even lead the insider to trade against information because she might maintain her information advantage for longer period. Our paper differs from those papers by provide a different source of incentives (fund flows) for market manipulation.

The rest of the paper is organized as follows. We introduce the model setup in section 2, and develop a financial equilibrium in section 3. Finally, we conclude in section 4.

2. THE MODEL

The model can be described by the following timeline:

![FIG. 1. The Timeline](image)

There are three discrete time periods: time 0, 1 and 2 as illustrated above. There is one risky asset that pays $V$ at time 2, where $P(V = H) = P(V = L) = \frac{1}{2}$ and $E(V) = \frac{1}{2}(H + L)$. The price of the asset is $P_0$ at time 0 and $P_1$ at time 1. There is also a riskless asset with payoff normalized to 1. There are two mutual funds that trade on behalf of their investors at time 0 and time 1. Assume that fund managers can only invest in this risky asset by submitting a fixed order as in Kyle (1985) and is subject to the short sale constraint. At time 0 and time 1, a fund manager can either buy $d$ assets, do nothing, or sell $d$ assets if she holds $d$ assets. Note that the individual trading actions and positions of the fund managers are not immediately observable to either the market maker or the investors and are only disclosed to them in the following period\(^2\).

Suppose that there is a good manager who knows the value of $V$ at time 0, and a bad manager who does not know $V$ until time 2. Assume that at the start of time 0, each fund consists of the same amount of cash and $2d$ of assets valued at $E(V)\(^3\)$. Each manager will be paid a salary proportional to

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\(^2\)We believe that this assumption is realistic as mutual fund disclosures are usually made with a delay.

\(^3\)We make this assumption so that the fund managers can sell even in presence of short sale constraints. Note that the fund managers will make a loss on their holdings if they sell the assets at a price below.
the total size of his fund at the end of time 2. There is also a competitive market maker, who will observe the aggregate market orders from the two managers and set the price of the asset to its expected payoff based on this information. The trading mechanism is assumed to be sequential, i.e. the fund managers cannot base their trading actions on information inferred from price.

There are also a continuum of investors have delegated their wealth to the fund managers (the fund will be liquidated to repay the investors at time 2). Each investor receives an equal amount of income at time 1, which can be invested in the funds at the end of \( t = 1 \), just after the fund managers have traded. Assume that, at time 0, the investors believe that the probability that one manager is good is \( q \), where \( q \) is uniformly distributed on the interval \([0, 1]\). Denote the total amount of income of the investors by \( 2n \). Finally, we assume that every player in our model is risk neutral.

3. EQUILIBRIUM

The equilibrium can be described by the following proposition.

**Proposition 1.** Assume \( n \geq \frac{1}{2}d(H - L) \). Then there exists a mixed strategy Nash equilibrium in which

- At time 0: both managers buy
- At time 1: the good manager buys if \( V = H \) and sells otherwise and the bad manager buys with probability \( \frac{1}{2} \) and sells with probability \( \frac{1}{2} \); the investors invest \( n \) in the fund if \( P = E(V) \) and nothing otherwise

Therefore, the investors’ strategy is such that the fund managers will receive cash flows in the interim date. We will first derive the equilibrium strategy of the managers based on the above strategy of the investors, and then show that the above strategy is consistent with the investors’ incentives.

3.1. The equilibrium strategy of the managers

Before exploring the strategy of the managers, we need to specify the beliefs of the market maker.

The belief of the market maker is specified as follows:

At time 1: The market maker will set the price \( P_1 = H \) if he observes \( \{+d,+d\} \), \( P_1 = L \) if he observes \( \{-d,-d\} \), and \( P_1 = pL + (1 - p)H \) if he observes \( \{d,-d\} \), where \( p \) is the probability that the bad manager buys the asset, and \( 1 - p \) is the probability that the bad manager sells the asset. This is based on the (consistent) belief that the good manager will buy if
$V = H$ and sell if $V = L$ at time 1. Out-of-equilibrium beliefs: He will set the price to $\frac{1}{2}(H + E(V))$ if he observes $\{+d, 0\}$, and he will set the price to $\frac{1}{2}(L + E(V))$ if he observes $\{-d, 0\}$ (since he does not know where the zero order comes from).

At time 0: The market maker will set the price $P_0 = E(V)$ if he observes $\{+d, +d\}$ at time 0, since both managers buy at equilibrium. Out-of-equilibrium beliefs: He will set the price to $\frac{1}{2}(L + E(V))$ if he observes $\{+d, -d\}$ or $\{+d, 0\}$, since he does not know which manager deviated from the equilibrium strategy$^4$. He will set the price to $L$ if he observes anything else at time 0 (since he knows for sure that the good manager deviated and hence $V$ cannot be $H$).

Given the above described market maker beliefs, the following lemma shows the possible equilibrium strategies of two managers at time 1.

**Lemma 1.** Suppose that both managers bought the asset at time 0, then given the above market maker’s beliefs, there exists three Nash equilibriums in the sub-game at time 1 in which the good manager trades correctly according to his information (i.e. he buys if $V = H$ and sells otherwise) and the bad manager (i) always buys; (ii) always sells or (iii) buys with probability $\frac{1}{2}$ and sells with probability $\frac{1}{2}$, respectively.

**Proof.** Let’s denote the payoff of the managers by $\Phi$. We start by showing that the good manager does not have an incentive to deviate from the above strategy.

At time 1, when $V = H$, the payoff of the good manager is ($n > 0$ if $P = E(V)$, and $n = 0$ otherwise)

<table>
<thead>
<tr>
<th></th>
<th>Bad manager buys</th>
<th>Bad manager sells</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$H$</td>
<td>$E(V)$</td>
</tr>
<tr>
<td>Payoff</td>
<td>$3d(H - E(V))$</td>
<td>$E(V)$</td>
</tr>
<tr>
<td>Inflow</td>
<td>$0$</td>
<td>$4d(H - E(V))$</td>
</tr>
<tr>
<td>Price</td>
<td>$E(V)$</td>
<td>$L$</td>
</tr>
<tr>
<td>Payoff</td>
<td>$2d(H - E(V))$</td>
<td>$d(H - E(V))$</td>
</tr>
<tr>
<td>Inflow</td>
<td>$n$</td>
<td>$0$</td>
</tr>
<tr>
<td>Price</td>
<td>$\frac{1}{2}(H + E(V))$</td>
<td>$\frac{1}{2}(L + E(V))$</td>
</tr>
<tr>
<td>Payoff</td>
<td>$3d(H - E(V))$</td>
<td>$3d(H - E(V))$</td>
</tr>
<tr>
<td>Inflow</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

The calculations of the above payoffs are straightforward. For example, if the good manager buys while the bad manager also buys, the price will be equal to $H$ based on the market maker’s beliefs. Note that before $t = 1$

$^4$Although we have assumed a probability of 1/2 for simplicity, it can be shown that the equilibrium holds for other out-of-equilibrium beliefs.

$^5$The implausible out-of-equilibrium belief that the good manager may sell even when the asset’s payoff is high can be ruled out using the intuitive criterion.
the good manager already holds 3d of assets, bought at $E(V)$ (2d of assets held from the beginning plus d of assets bought at $t = 0$). This implies a profit of $3d(H - E(V))$ at $t = 2$. In addition, the manager will not make any profits on her trading at $t = 1$, because the purchase price is equal to the asset value of $H$. Therefore the total profit that the good manager will make is equal to $3d(H - E(V))$ in this case. Finally, the inflow will be 0 based on the equilibrium strategy of the investors specified earlier.

Similarly, if the good manager buys while the bad manager sells, the price will be $E(V)$. The good manager holds 4d of assets in total, all bought at $E(V)$. Therefore the total profit in this case is $4d(H - E(V))$.

If the good manager sells while the bad manager buys, the price will also be $E(V)$. The good manager makes no profit on this particular trade, as the sell price is equal to the purchase price of $E(V)$. She holds 2d of assets in total, implying a total profit of $2d(H - E(V))$.

If the good manager does nothing, her total profit will be $3d(H - E(V))$ regardless of the price.

Hence

$$E(\Phi^{\text{buy}}) = \frac{7}{2}d(H - E(V)) + \frac{n}{2}$$
$$E(\Phi^{\text{sell}}) = \frac{3}{2}d(H - E(V)) + \frac{n}{2} \text{ and } E(\Phi^{\text{nothing}}) = 3d(H - E(V)).$$

We see that $E(\Phi^{\text{sell}}) < E(\Phi^{\text{buy}})$ and $E(\Phi^{\text{nothing}}) < E(\Phi^{\text{buy}})$. So the good manager will always buy when $V = H$.

When $V = L$, we have

<table>
<thead>
<tr>
<th>TABLE 2.</th>
<th>Payoffs for good managers when $V = L$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bad manager buys</strong></td>
<td><strong>Bad manager sells</strong></td>
</tr>
<tr>
<td><strong>Price</strong></td>
<td><strong>Payoff</strong></td>
</tr>
<tr>
<td>Buy</td>
<td>$H$</td>
</tr>
<tr>
<td>Sell</td>
<td>$E(V)$</td>
</tr>
<tr>
<td>Do nothing</td>
<td>$\frac{4}{3}(H + E(V))$</td>
</tr>
</tbody>
</table>

The calculations of the above payoffs are similar to what explained in detail earlier.

So $E(\Phi^{\text{buy}}) = \frac{4}{3}d(L - E(V)) + \frac{n}{2}$, $E(\Phi^{\text{sell}}) = \frac{5}{3}d(L - E(V)) + \frac{n}{2}$, and $E(\Phi^{\text{nothing}}) = 3d(L - E(V))$. So $E(\Phi^{\text{sell}}) > E(\Phi^{\text{buy}})$ and $E(\Phi^{\text{nothing}}) < E(\Phi^{\text{sell}})$. So the good manager will always sell when $V = L$.

Recall that the bad manager buys with probability $\frac{1}{2}$ and sells with probability $\frac{1}{2}$. Her payoff will be
TABLE 3.
Payoffs for bad managers

<table>
<thead>
<tr>
<th></th>
<th>Good manager buys or $V = H$</th>
<th>Good manager sells or $V = L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Payoff</td>
<td>Inflow</td>
</tr>
<tr>
<td>Buy $H$</td>
<td>$3d(H - E(V))$</td>
<td>0</td>
</tr>
<tr>
<td>Sell $E(V)$</td>
<td>$2d(H - E(V))$</td>
<td>$n$</td>
</tr>
<tr>
<td>Do nothing</td>
<td>$\frac{1}{2}(H + E(V))$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Again, the calculations of the above payoffs are similar to what explained in detail earlier.

Therefore we have $E(\Phi^{\text{sell}}) = \frac{1}{2}d(L - E(V)) + \frac{n}{2} = E(\Phi^{\text{buy}}) > E(\Phi^{\text{nothing}})$. Hence there is no incentive to deviate.

The time 0 equilibrium is shown in lemma 2.

**Lemma 2.** Assume $n > \frac{1}{2}d(H - L)$. Then the strategy that both managers buy at time 0 is consistent with manager incentives only in the mixed strategy equilibrium described in Lemma 1.

**Proof.** At time 0, when $V = H$, the expected payoff of the good manager will be $E(\Phi^G) = \frac{1}{2}d(H - E(V)) + \frac{n}{2}$, which is positive. Hence the good manager will always buy at time 0 if $V = H$. When $V = L$, the good manager has expected payoff $E(\Phi^G) = \frac{1}{2}d(L - E(V)) + \frac{n}{2}$. If the good manager sells at time 0, the investors would not be willing to invest in the fund at time 1 (see **Lemma 3** for proof). Hence the expected payoff of the good manager if she sells at time 0 is $E(\Phi^G) = \frac{1}{2}d(L - E(V))$. This strictly dominates the strategy of doing nothing at time 0, since by doing nothing the manager makes less trading profit, and the fund flows will be the same. Therefore the good manager will find it optimal to buy at time 0 if and only if $n > \frac{1}{2}d(H - L)$.

The payoff of the bad manager is $E(\Phi^B) = \frac{1}{2}d(L - E(V)) + \frac{n}{2}$ if she buys at time 0. If the bad manager sells at time 0, her payoff will be $\frac{1}{2}d(L - E(V))$ if $V = L$, or $\frac{1}{2}d(H - E(V))$, so the expected payoff is $E(\Phi^B) = d(L - E(V))$. If the manager does nothing at time 0, her payoff will be $2d(L - E(V))$ if $V = L$, or $2d(H - E(V))$, so the expected payoff is $E(\Phi^B) = 0$. Therefore the bad manager will find it optimal to buy at time 0 given $n > \frac{1}{2}d(H - L)$.

Therefore, there exists a mixed strategy equilibrium where both managers buy at time 0, the good manager trades correctly and the bad manager buys with probability $\frac{1}{2}$ at time 1, given $n > \frac{1}{2}d(H - L)$. •
3.2. The equilibrium strategy of the investors

Given the above equilibrium strategy of the managers, we now show that the investors’ strategy specified before is indeed the equilibrium strategy.

Lemma 3. Assume \( n > \frac{1}{2} d(H - L) \). Then the investors will follow the strategy below:

- At time 1: invest \( n \) in the fund if \( P = E(V) \) and nothing otherwise.

Proof. Given the equilibrium specified in Lemma 1 and 2, we show that the investors have incentives to invest at time 1 if and only if \( P = E(V) \).

At time 1, the investors have an income \( 2n \) to invest. If \( P = E(V) \), the expected profits that a good manager is going to make in the next period is \( d(H - E(V)) \), and the expected profits that a bad manager is going to make in the next period is \( d(L - E(V)) \). Hence it is optimal for each investor to invest as much as possible to get the maximum share of profits, as long as the probability that the manager is good is greater than \( \frac{1}{2} \). Given that the investors’ belief about the quality of one manager is uniformly distributed on the interval \([0, 1]\), and that their beliefs remain the same at time 1 if \( P = E(V) \), half of the investors will find it optimal to invest in one manager, and the other half will find it optimal to invest in the other manager.

If \( P = H \) or \( P = L \), the fund manager is expected to make zero profits in the next period, so it does not make a difference whether the investor invest in the fund or not. Thus the investors do not have incentives to deviate.

Finally, if a manager did not buy at time 0, we show that the investors do not have incentives to invest in the funds. This is because, for a given investor, if the fund manager is bad, there is no incentive to invest since the expected profits of a bad fund manager is negative; if the fund manager is good and did not buy at time 0, this implies that \( V \) cannot be \( H \), and hence there is no incentive to invest in the fund.

Therefore, the investors do not have an incentive to deviate and we have an equilibrium with the strategy specified.

4. CONCLUSION

This paper studies price manipulation of fund managers induced by the potential fund flows from investors. We develop an equilibrium in which both informed and uninformed managers buy the asset in the first trading round even when it is overvalued, and the informed manager trades on the
basis of her information in the second trading round and the uninformed manager trades randomly. When the price is not fully revealing, investors make further investment to the funds, which is the source of the incentive for managers to manipulate the market.

REFERENCES


