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Another Look at the Boom and Bust of Financial Bubbles

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A rational bubble is explained through the covariance between the marginal rate of substitution and the future price. Surprisingly, in the present literature, this quantity has always been set equal to zero either because of a first-order Taylor approximation, or because of a risk-neutrality assumption. One first shows that the intrinsic bubble of Froot and Obstfeld (1991) is a re-parameterization of the quantity in question. One then shows how this quantity depends on economic shocks after introducing a Taylor rule-based monetary policy. Some empirical evidence is also presented.

Key Words: Bubbles; Present-value model; Monetary rule.
JEL Classification Numbers: G12, E44.

1. INTRODUCTION

Financial bubbles remain a difficult phenomenon to explain and to predict. Surprisingly, all theoretical models aimed at performing this task, even when reaching a very high degree of sophistication, rely on the same simplifying assumption embedded in the present value models: the discounting factor of the fundamental component and of the bubble component is the same. See Cuthbertson and Nitzsche (2005), chapter 17, for a survey, or Galì (2013) for one of the most recent articles. This may be acceptable only under risk neutrality of market participants or under a deterministic linear relationship between the two components. This article relaxes these assumptions and shows how the covariance between the bubble component and the marginal rate of substitution may play an important role in determining the formation and the burst of the bubble. One shows how the intrinsic bubbles of Froot and Obstfeld (1991) are indeed a re-parameterization of the micro-founded bubble term found in this article. One also shows how demand and supply shocks affect the bubble term.

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2. THE MODEL

Assume a representative agent optimizing her expected, present value of utility over N periods. Her utility depends both on consumption and on real money balances $E_0[\sum_{j=0}^{N} \beta^j U(C_{t+j}, M_{t+j}/Q_{t+j})]$ where C_{t+j} and M_{t+j}/Q_{t+j} denote real consumption and real money balances respectively at period $t+j$. Q_{t+j} denotes the price level. She may invest in an asset paying stochastic dividends D_{t+j} over an arbitrary period $T(T : T = t + 1, \dots, N)$. She may also invest in the one-period risk-free interest rate i_{t+j}: i_{t+j} is set in $t+j$. Then the price of this risky asset is determined as:

$$P_t = \sum_{j=t}^{T} E_t[MRS_{t,t+j} D_{t+j}] + E_t[MRS_{t,t+T} P_{t+T}] \tag{1}$$

where $MRS_{t,t+j} = \beta \frac{U'_{C_{t+j}}}{U'_{c_t}} \frac{Q_t}{Q_{t+j}}$ with $U'_{C_{t+j}} = \frac{\partial U(C_{t+j}, M_{t+j}/Q_{t+j})}{\partial C_{t+j}}$. Note that if P_{t+T} follows a different stochastic process with respect to that of dividends[1] and if $MRS_{t,t+i}$ is not constant $\forall j : 0, \dots, T$, the present value formula omits the evaluation of the covariance term embedded in the second addend of eq. (1). This quantity can be defined as the bubble term (decomposed as):

$$B_t = E_t[MRS_{t,t+T} P_{t+T}] = E_t[MRS_{t,t+T}] E_t[P_{t+T}] + \mathrm{cov}_t[MRS_{t,t+T}, P_{t+T}] \tag{2}$$

The first addend of the right-hand side corresponds to the bubble term assumed so far in the literature[2]. The evolution of $\mathrm{cov}_t[MRS_{t+T}, P_{t+T}]$ also determines the bubble term and which addend prevails is only an empirical matter. Assume for the rest of the article a power utility with parameter $\lambda : MRS_{t,t+j} = \beta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\lambda} \frac{Q_t}{Q_{t+j}}$.

3. MICROFOUNDING INTRINSIC BUBBLES

The relationship between the present price P_t and $\mathrm{cov}_t[MRS_{t,t+T}, P_{t+T}]$ may also explain why the intrinsic bubbles of Froot and Obstfeld (1991) although not micro-founded, are verified at the empirical level[3] and hence become popular.

In fact, their article assumes a present value model for the present price determination but also an exogenous (not micro-founded) relationship be-

[1] Implying that there is no linear relationship between the two components.

[2] One then exploits the inverse relationship between the interest rates and the expectation of the MRS.

[3] As reported by the authors.

tween the bubble and the dividends:

$$B_t(D) = cD_t^{\lambda} \tag{3}$$

with $c > 0$ and $\lambda > 1$. Assume now that C_t is proportional to dividends D_t such that $C_t \approx mD_t$ with $m > 0$.

On the other side, the bubble term in eq.(1) can be rewritten as:

$$\begin{aligned} B_t &= E_t[MRS_{t,t+T}P_{t+T}] = E_t\left[\beta \frac{C_t^{\lambda}}{C_{t+T}^{\lambda}} \frac{Q_t}{Q_{t+j}} P_{t+T}\right] \\ &= E_t\left[\beta C_{t+T}^{-\lambda} \frac{Q_t}{Q_{t+j}} P_{t+T}\right] C_t^{\lambda} \quad j : 1, 2, \ldots \end{aligned} \tag{4}$$

It should be now clear that eq. (3) and eq. (4) are equivalent when one imposes:

$$E_t\left[\beta C_{t+T}^{-\lambda} \frac{Q_t}{Q_{t+j}} P_{t+T}\right] m^{\lambda} \equiv c$$

The latter expected value contains a scaled value of the covariance term $\mathrm{cov}_t[MRS_{t,t+j}B_{t+j}]$ (times $m^{\lambda}/C_t^{\lambda}$); according to this result the parameter c should not be constant as postulated by the authors[4]. This may explain why, Froot and Obstfeld could reject the joint hypothesis that $c = 0$ and $\lambda - 1 = 0$ but not the null that $c = 0$. Its large standard error may depend on the time-varying nature of this parameter.

All this implies that, adding an intrinsic bubble to a PVM does not overcome all the theoretical and empirical limitations of the PVM.

4. DEMAND, SUPPLY SHOCKS AND THE ROLE OF MONETARY POLICY

One shows now, how monetary policy is related to the covariance term of eq. (2). Note now that the allocation between the risky asset and the short-term bond must obey the following first order conditions:

$$E_{t+T}[U'_{C_{t+T+1}}(1 + h_{t+T})] = E_{t+T}[U'_{C_{t+T+1}}](1 + i_{t+T}) \tag{5}$$

where $1 + h_{t+T} = \frac{P_{t+T+1}+D_{t+T+1}}{P_{t+T}}$ is the holding period return of the risky asset. One obtains

$$P_{t+T} = \frac{E_{t+T}[U'_{C_{t+T+1}}(P_{t+T+1} + D_{t+T+1})]}{E_{t+T}[U'_{C_{t+T+1}}]} \frac{1}{(1 + i_{t+T})} \tag{6}$$

[4]Apart from the case where consumption growth is homoscedastic.

Assume now that monetary policy[5] is performed through a monetary rule of the form:

$$1 + i_t = (1+i)\hat{\Pi}_t^{\alpha_1}\hat{C}_t^{\alpha_2}e^{u_t} \quad \forall t : t = 1, \ldots, N \tag{7}$$

where i is the equilibrium interest rate, $\hat{\Pi}_t$ is the medium-run deviation between the actual inflation and the target inflation: $\hat{\Pi}_t = \frac{Q_t}{Q_{t-j}}/\bar{Q}_t$. $\hat{C}_t$: $\hat{C}_t = \frac{C_t}{C_{t-j}}/\bar{C}_t$ is the (medium-run) consumption gap and $j > 1$. Assume also that $\bar{Q}_t > 0$ and $\bar{C}_t > 0$; α_1 and α_2 are positive (constant) parameters and u_t is a zero-mean exogenous term independent of consumption and of prices shocks.

Now, one can substitute in $E_t[MRS_{t,t+T}P_{t+T}]$ of eq. (2) the definition of $MRS_{t,t+T}$ and eq. (6) and (7), yielding:

$$\begin{aligned} & E_t[MRS_{t,t+T}P_{t+T}] \\ = \; & E_t\left[\beta^j\left(\frac{C_{t+T}}{C_t}\right)^{-\lambda}\frac{Q_t}{Q_{t+T}}\frac{E_{t+T}[U'_{C_{t+T+1}}(P_{t+T+1}+D_{t+T+1})]}{E_{t+T}[U'_{C_{t+T+1}}]}(1+i)^{-1}\hat{\Pi}_{t+T}^{-\alpha_1}\hat{C}_{t+T}^{-\alpha_2}e^{-u_{t+T}}\right] \\ = \; & \beta^j(1+i)^{-1}\bar{Q}_t^{\alpha_1}\bar{C}_t^{\alpha_2}E_t\left[\frac{E_{t+T}[U'_{C_{t+T+1}}(P_{t+T+1}+D_{t+T+1})]}{E_{t+T}[U'_{C_{t+T+1}}]}\left(\frac{C_{t+T}}{C_t}\right)^{-\lambda-\alpha_2}\left(\frac{Q_{t+T}}{Q_t}\right)^{-1-\alpha_1}\right] \end{aligned} \tag{8}$$

Appendix A shows that the left-hand side of eq. (8) can be approximated by a linear combination of $E_t\left[\left(\frac{C_{t+T}}{C_t}\right)^{-\lambda-\alpha_2}\left(\frac{Q_{t+T}}{Q_t}\right)^{-1-\alpha_1}\right]$; thus,

$$B_t \equiv E_t[MRS_{t,t+T}P_{t+T}] \propto E_t\left[\left(\frac{C_{t+T}}{C_t}\right)^{-\lambda-\alpha_2}\left(\frac{Q_{t+T}}{Q_t}\right)^{-1-\alpha_1}\right] \tag{9}$$

Note secondly, that the joint expectation of eq. (9) embeds not only a covariance term but also the product of expectations:

$$\begin{aligned} & E_t\left[\left(\frac{C_{t+T}}{C_t}\right)^{-\lambda-\alpha_2}\left(\frac{Q_{t+T}}{Q_t}\right)^{-1-\alpha_1}\right] \\ \simeq \; & M_{t+T}\mathrm{cov}_t\left[\frac{C_{t+T}}{C_t}, \frac{Q_{t+T}}{Q_t}\right] + \left(E_t\left[\frac{C_{t+T}}{C_t}\right]\right)^{-\lambda-\alpha_2}\left(E_t\left[\frac{Q_{t+T}}{Q_t}\right]\right)^{-1-\alpha_1} \end{aligned} \tag{10}$$

with $M_{t+T} = (-\lambda-\alpha_2)(-1-\alpha_1)\left(E_t\left[\frac{C_{t+T}}{C_t}\right]\right)^{-\lambda-\alpha_2-1}\left(E_t\left[\frac{Q_{t+T}}{Q_t}\right]\right)^{-2-\alpha_1} > 0$, having (first-order) approximated the factors of eq. (10) around their conditional expected values.

[5]The economy is assumed to be affected by demand and supply shocks and to have the usual imperfections (rigidities, etc.) such that monetary policy is not neutral in the short run.

Combining Eq. (9) and eq. (10), one is able to state that, if market participants expect, in period t, a demand shock[6] then the bubble term is positive.

This is also consistent with the following interpretation. Suppose that positive (negative) shocks for both consumption and prices occur. This decreases (increases) the marginal rate of substitution and, at the same time, causes a monetary policy reaction yielding an increase (decrease) in the interest rates. This, through the portfolio rebalancing showed in eq. (5) and (6), causes a decrease (increase) in future stock prices.

Given a supply shock[7], the left-hand side of eq. (10) is in principle undetermined because the covariance term is negative and the second addend of eq. (10) remains positive. However large supply shocks make the negative covariance dominate the other term and cause a decrease of the bubble term.

Now, note that combining eq. (2) with eq. (9) and (10), one has a functional relationship for the covariance term omitted by the present value models and the covariance term measuring the shocks of the economy:

$$\mathrm{cov}_t[MRS_{t,t+T}, P_{t+T}] \propto \mathrm{cov}_t \left[\frac{C_{t+T}}{C_t}, \frac{Q_{t+T}}{Q_t} \right] \tag{11}$$

To summarize, due to the latter relationship, demand shocks increase the bubble term and sufficiently large supply shocks decrease that.

In other words, eq. (11) describes another channel through which the economic shocks affect the bubble term. This channel is completed neglected by the present literature.

Note, however, that these covariance terms are not effect by the directions of these shocks. A negative demand shock which decreases both consumption and prices, implies a positive $\mathrm{cov}_t \left[\frac{C_{t+T}}{C_t}, \frac{Q_{t+T}}{Q_t} \right]$ as an appropriately defined positive demand shock. This however, does not mean that negative shocks cannot burst a bubble; in fact, shocks affect the bubble through their reflection on the interest rates affecting the first addend of the right-hand side of eq. (2).

One can now turn to the consistency of eq. (11) with the empirical evidence. This finding fits the results of Detken and Smets (2004) who analyzed the data of 18 OECD countries since the 1970s. They claim that: "asset price booms are typically associated with a substantial increase in output gap?(and)inflation deviations from trends rise during the boom".

They also claim that asset price booms are associated with a "relative easy monetary policy, as captured by low interest rates relative to a Taylor rule benchmark".

[6]Consumption and prices deviate with respect to their mean in the same direction.

[7]Here, it is intended as shocks of prices and consumption going in opposite directions.

In the framework of this article, a “relatively easy monetary policy” can be referred to as a negative exogenous shock u_{t+T} affecting interest rates in eq. (7) once a negative consumption shock (and a negative price shock) occurs. This implies a monetary policy setting interest rates below those predicted by the Taylor rule.

In order to see that this monetary policy causes a surge of the bubble term, one may relax the assumption that the error term u_{t+T} is independent of the economy shocks. One should then add the following expression[8] to eq. (9) and (11).

$$E_t\left[\left(\frac{C_{t+T}}{C_t}\right)^{-\lambda-\alpha_2}\left(\frac{Q_{t+T}}{Q_t}\right)^{-1-\alpha_1}(-u_{t+T})\right] \tag{12}$$

It is easy to verify that under the conditions regarding the shocks posited above, the quantity of eq. (12) is positive. Hence, the expression in eq. (12) can be interpreted as an additional bubble term once the monetary policy is implemented in a “easier” way with respect to that predicted by the Taylor rule. This confirms the empirical findings quoted above.

5. CONCLUSIONS

This article contradicts the assertion that the bubble term is a martingale. This is the straightforward implication of the present-value models or of the (micro-founded) general equilibrium models when they rely on approximations of the first order. Hence, the covariance between the marginal rate of substitution and the future price is not necessarily null but may depend on exogenous quantities. One has shown that the intrinsic bubble is a re-parameterization of this covariance (joint expected value). Furthermore, one exploited the fact that the future price level depends on the future allocation of monetary resources between the risky asset and the risk-free asset paying interest rates. In so doing, one has shown that monetary policy does influence this covariance and hence the bubble term. After considering a Taylor rule based monetary policy, the bubble ultimately depends on the shocks affecting the economy. Demand shocks create a bubble, (large) supply shocks burst it (if it is already there). One has also considered a relatively “easier” monetary policy defined, as an exogenous shock in the Taylor rule. The conditions under which this policy causes a bubble are explored. The fact that the demand (supply) shocks, but also easier (stronger) monetary shocks create (burst) a bubble are confirmed by the outcome found in the empirical literature.

[8]The following approximation has been used: $e^{-u_{t+T}} \simeq 1 - u_{t+T}$, since $E[u] = 0$.

APPENDIX A

Define in eq. (8) $m_t = \beta^j (1+i)^{-1} \bar{Q}_t^{\alpha_1} \bar{C}_t^{\alpha_2} > 0$ always. Then note that $E[xyz] = \text{Cov}[x, yz] + E[x]E[yz]$, now define $x = \frac{E_{t+T}[U'_{C_{t+T+1}}(P_{t+T+1}+D_{t+T+1})]}{E_{t+T}[U'_{C_{t+T+1}}]}$, $y = \left(\frac{C_{t+T}}{C_t}\right)^{-\lambda-\alpha_2}$ and $z = \left(\frac{Q_{t+T}}{Q_t}\right)^{-1-\alpha_1}$; then, for simplicity the covariance term $\text{Cov}[x, yz]$ can be neglected since it enters the equation additively and $E[x] > 0$ always.

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