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Is Standard Deviation a Good Measure of Volatility? the Case of African Markets with Price Limits

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Investment decisions are often based on the analysis of two main investment components: risk and return. In many instances risk is measured by the standard deviation of the asset returns. For portfolio managers who are trading cross different assets and countries, the exercice could be tricky as price limits could vary from a country to another. When price limits are imposed, the observed prices are truncated and the equilibrium prices are unobservable. This adds a bias to the estimation of standard deviation and hence the volatility. In this paper, we tackle this issue of biasness by proposing a methodology that correct this bias in order to get more efficient risk estimates. Two approaches are proposed. The first one is based on stochastic volatility models and the second one on options pricing. We perform a step by step numerical application that displays a clear bias coming from price limits.

Key Words: Stochastic Volatility; Price Limits; Truncated Time Series; Censored Variables.

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1. INTRODUCTION

In the last thirty years, volatility was always of a major concern to the financial community. The main turning point that put the volatility importance forward is probably the crash of October 1987, commonly known as Black Monday. On Monday October 19th 1987, the Dow Jones Industrial Average dropped by 508 points, a daily drop of 22.61%, causing the halt of trading at the New York Stock Exchange. The crash also led to proposals for increased regulation to control price volatility and imposing price limits. A vast literature has flourished to describe the behaviour of volatility, starting with the seminal work of Engle (1982), extended by Bollerslev (1986) to define the GARCH model. Historically, academicians focused their empirical research on some specific markets. Engle (1982) studied the UK inflation, Bollerslev (1990) looked at the short-run nominal exchange rates, Lien and Yang (2008) analyzed the commodity markets, Leeves (2007) focused on the Asian crisis and Singh et al. (2010) studied the volatility spillovers across North American, European, and Asian stock markets.

Some other researchers had interest on the African stock markets. Using an EGARCH model which incorporates asymmetric volatility, Appiah-Kusi and Pescetto (1998) analyze the issue of volatility spill-overs in the African markets. Collins and Biekpe (2003) study the extent of market integration by measuring the degree of contagion between African equity markets and global emerging markets. Smith et al. (2002) use multiple variance ratio tests to test whether african stock markets price indices follow a random walk process. Leon (2007) study the relation between the stock return volatility and trading volume in the region stock exchange of the West African Economic and Monetary Union. Mlambo and Biekpe (2007) test the efficient market hypothesis for ten African stock markets. Alagidede (2008) investigate the day of the week anomaly in Africa's largest stock markets by looking at the two first moments of returns.

The African continent has some interesting microstructure features that always brought the interest of the financial community. Among these features, we can cite the summer effect, week day effect and Price Limits. In the African stock markets price limits vary from a country to another, but the average is usually between 5% and 15%. For instance in Tunisia, the price of each stock is restricted to a 3% ceiling and floor from its previous closing price. When the limits are reached, the stock trading is halted for 15 minutes and the window of trading is increased by 1.5% from each side (e.g. after the first halt, the new ceiling and floor reach 4.5%), and this up to 6.09%. In Kenya, for example, those price limits are larger and it is possible to trade up to 10% ceiling and floor from its previous closing price.

The debate whether to have price limits is a long run debate that dragged the financial community for decades. On one hand, price limit advocates claim that price limits decrease stock price volatility, counter overreaction, and do not interfere with trading activity. On the other hand, price limit critics advance that price limits cause several negative effects: higher volatility levels on subsequent days (volatility spillover hypothesis), prevent prices from efficiently reaching their equilibrium level (delayed price discovery hypothesis), and interfere with trading due to limitations imposed by price limits (trading interference hypothesis). Tooma (2003) investigated the impact of price limits on volatility dynamics in the Egyptian Stock Exchange using GARCH type models. Deb et al. (2010) proposed a flexible price limit system based on the predicted likelihood of improper price limit imposition to undermine the cost related to price limits.

The purpose of this paper is to look at the volatility from a fund manager standpoint. A fund or portfolio manager with an African focus will have to pick stocks through the entire region between markets who don't have necessarily the same trading rules. As mentioned above, the change of price limits policy has an impact on the market volatility. Let's suppose will look to every stock from a risk premium point of view. He will need to assess the return and volatility of each component of his portfolio. The question we can ask, how can he compare the volatility of stocks coming from markets with different price limits. It is clear that a market where the returns has a support of $[-3\%3\%]$ is different from a market with returns having a support of $[-10\%10\%]$.

Mathematically, imposing price limits on a stock price boils down to truncating or censoring a time series. Practioners commonly disregard censored data cases which often result into biased estimate. Park et al. (2007) presented a remedy for handling autocorrelated censored data based on an imputation method well suited for fitting Autoregessive Moving Average (ARMA) models. Chou (1999) proposed the use of two-limit truncated and Tobit regression models to analyze regression models whose dependent variable is subject to price limits. Wei (2002) proposed a censored-GARCH model to tackle the price limits issue and developed a Bayesian approach to this model. Wei and Chiang (2004) derived a generalized method of moments (GMM) estimator for variance in markets with daily price limits. Hsieh and Yang (2009) developed a censored stochastic volatility model to reconstruct a return series censored by price limits.

In this paper, we try to answer the following questions: How can we take into account the impact of price limits on volatility? How can we deal with truncated time series ? The remainder of this paper is organized as follows. Section 2 describes the african stock markets sepcificities. Section 3 goes through the issue of truncated and censored times series. Section 4 sets up the stochastic volatility model. Section 5 shows some empirical and simulation results. We propose a different approach based on options pricing in section 6 and we conclude in section 7.

2. AFRICAN STOCK MARKETS SPECIFICS

2.1. Tunis Stock Exchange

The Tunis Stock Exchange was created in 1969. It is a private company that is owned by the 23 licensed stock brokers. There are 59 listed stocks with a market of capitalization of about 16 billion dinars (1.1 billion USD). The trading hours on a normal session (by opposition of summer and Ramadan sessions) lie from 9 am to 2:10 pm, with a pre-opening session from 9 to 10 am, and pre-closing session from 2 to 2:05 pm. During the continuous trading hours, i.e. from 10 am to 2 pm, trading is limited to a specific price window. First, at 10 am the opening price has to be within $+/-$ 3% window of the previous closing price. Second, once a stock started trading, it can only trade within $+/-$ 3% price window with respect of the opening price, beyond which trading is halted for 15 minutes and then both the ceiling and floor are increased by an extra 1.5 percent. This mechanism is repeated until we reach the $+/-$ 6.09% limits with respect to yesterday's closing.

2.2. Egyptian Stock Exchange: Nilex

The Egyptian Stock Echange (ESE) comprises two exchanges Cairo and Alexandria. both governed by the same board and share the same trading, clearing and settlement systems. The Alexandria stock exchange was officially established in 1883, followed by Cairo SE in 1903. There are 381 listed stocks and a market capitalization of about 60 billion dollars. Stocks trading hours for a normal session last from 10.30 am to 2.30 pm. In February 1997, ESE adopted the price limit tool in order to stabilize stock prices and minimize the losses due to high volatility in prices. All ESE stocks where subject to a 5 percent daily price limit. Since May 2002 however, ESE exempted some stocks from the price fluctuation limits. Among the main criteria for the company to have the price limit rule lifted are that 15% of the capital is free float and the stock traded for at least 220 days. According the new trading halt mechanism, the stock price trades within an opening range of $+/-$ 10% beyond which trading would be halted for 30 minutes. Then, the stock trades within the range of $+/-$ 20% beyond which trading is suspended for the rest of the day.

2.3. Nigeria Stock Exchange

The Nigerian Stock Exchange (NSE) was founded in 1960. It has about 200 listed companies for a total market capitalization of about 50 billion dollars. Stock trading hours last from 10am to 4pm local time. Price limits in Nigeria were set to be +/− 5% movements from yesterday's closing. However, few years ago, in order to curb the propagation of the global crisis, a number of measures were taken. One of those measures was the introduction of a 1 per cent maximum downward limit on daily price movement and 5 per cent on upward price movement. This was met by criticism from stakeholders, including operators in the market, on the basis that it was bias upwards and thus was not efficient and fair, and the price discovery process was distorted. This policy was later changed back to the 5% either way from the end October 2008. More recently, since September 18 2012, the NSE introduced market making on sixteen blue-chip shares. Since it is difficult to make markets on tight margins, the daily price limits where increased from 5 percent to 10%.

2.4. Casablanca Stock Exchange

The Casablanca Stock Exchange (CSE) was established in 1929 and currently has 17 members of about 80 listed stocks for a total capitalization of about 50 billion dollars. On a normal session, the trading hours last from 9 am to 3h30 pm local time. Price limits in CSE are set to +/− 5% movements from yesterday's closing.

2.5. Bourse Régionale des Valeurs Mobilières (BRVM)

The BRVM is a regional stock exchange that includes the following West African countries : Benin, Burkina Faso, Guinea Bissau, Cote d'Ivoire, Mali, Niger, Senegal, Togo. Headquartered in Abidjan, the BRVM was founded in 1996, has 36 listed stocks with a large dominance of securities from Cote d'Ivoire (more than 85% of the stocks) and has a market capitalization of about 5 billion dollars.

3. CENSORED TIME SERIES AND STATISTICAL METHODOLOGY

Time series is a chronological ordered sequence of data points usually measured at uniform time intervals. Hence, a daily record of stock price over a period of time is a time series. However, when analyzing a time series, it is possible to have some missing values in the data sequence. Missing data can lead to distorted values and wrong conclusions.

Censored time series is a time series where the value of the observations are only partially known, meaning the values are only observable under certain conditions. Time series are either Right censored (when we only know the minimum value of the variable), Interval censored (when we only know that the value of the variable lies between a certain minimum or maximum) or Left-censored (when we only know the maximum value of the variable).

Truncated time series is a time series where observations are totally missing or never recorded. Thus the value of the variables are unknown.

The idea of censoring is not to be confused with the idea of truncation. Hausman and Wose (1977) insist on the distinction between censored data, eventually censored time series, and truncation. They describe censored data as *piled up* at a censoring point, and truncation as the data generated by a relevant subset of the population. Heckman (1976) also made strong distinction between a truncated and censored samples : "In a truncated sample one cannot use the available data to estimate the probability that an observation has complete data. In a censored sample, one can". In general, when the observations result either in knowing the exact value that applies, or in knowing that the value lies within an interval we talk about censoring. But, when values outside the range are never seen or never recorded if they are seen, we are talking about truncation. Lubès (1992) gives more explanations and reaches the conclusion that censorship characterizes the sample while truncation is a property of the probability distribution. Truncated laws and censored samples can be defined in the same way left (or inferiorly), or right (or superiorly), or superiorly and at the bottom (or right and left).

When handling censored data, one can discard the censored observations or delete them from the sample. Both approaches produce a bias of measures. In order to correct this bias, we must find new parameter estimation methodologies to handle censored or truncated time series. In the field of statistical analysis, Helsel (1990) replaced the censored values with a upper and lower limit constant. This has resulted in underestimating the censoring rate and the effect on the inference. Robinson (1980) estimated the censored values through their conditional expectations knowing the totally observed values. However, the method cant be applied for multiple consecutive censored observations. Zeger and Brookmeyer (1986) considered both a full likelihood estimation and an approximation approach for an autoregressive time series model. Noticing that the full likelihood method may not be feasible when the censoring rate is very high, the authors suggested the use of pseudo likelihood estimation in order to get over this limitation. Shaw (1988) suggested normal and Poisson regression models to analyze truncated samples of count data. Grogger and Carson (1991) extended his work and showed the greart importance of over dispersion when it comes to estimating truncated count models. Their Monte Carlo results showed that the bias of measure can be very important if over dispersion is not taken into accounted. Hopke et al. (2001) used multiple imputation based on a Bayesian approach, but did not provide enough explanation about the estimators unbiaseness and efficiency.

Dealing with censored data occurs at each time we deal with time series whether we are in the field of signal processing, mathematical finance, marketing, communication engineering, survival analysis, etc. In the field of survival analysis and life testing, Kalbfleisch and Prentice (1980) discussed several censoring schemes by proposing an extension to censored data of the Wilcoxon test. Miller Jr (1981) designated three types of censoring as type 1 (fixed time termination), type 2 (termination of experiment at r-th failure), and random censoring. Nelson (1972) presented the theory and applications of a simple graphical method for multiple censored data on equipment service life. Nelson (1982) studied the failure time of diesel generator fans commonly known as reliability theory). Therneau (2000) extended the Cox model by proposing diagnostic plots for identifying the functional form of covariates. Lee and Wang (2003) described and illustrate several useful nonparametric and parametric statistical methods to analyze survival data. Hosmer Jr et al. (2011) applied regression models on real-world examples and case studies in survival analysis.

In mathematical finance, censored time series and truncated distributions were very useful in modeling price limit. Researchers and practitioners studied the effect of price limit on market volatilities and proposed several methods for handling the price limit bias.

Kodres (1988) developed a censored regression model with a lagged latent dependent variable. Taking account of conditional heteroskedasticity, Kodres (1993) updated her previous model and built a model that later led to the development of Tobit-GARCH models Lee (1999). However, the numerical complexity of these models made them very difficult to implement.

Chou (1999) proposed the use of two-limit truncated and Tobit regression models to analyze regression models whose dependent variable is subject to price limits. Wei (2002) proposed a censored-GARCH model to tackle the price limits issue and developed a Bayesian approach to this model. However, the model is too complex and the parameter estimation demands a estimation process, especially when more parameters and/or price limit moves appear. Park et al. (2007) presented a remedy for handling autocorrelated censored data based on a class of Gaussian ARMA models by introducing an imputation method that fits the ARMA models.

Hsieh and Yang (2009) developed a censored stochastic volatility model (CSV) to reconstruct a return series censored by price limits. The CSV model recovers censored returns and gives an estimate of standard deviation with less than 1% error. The results suggest that the model outperforms other approaches with respect to the estimation of model parameters, the unconditional means, and the standard deviations.

Compared to other models, the CSV is easier to implement. In a nutshell, the CSV model proposed by Hsieh and Yang (2009) overcomes problems faced by previous models for instance model complexity and implementation time.

Based on the work of Hsieh and Yang (2009), this paper studies the effect of price limit on asset prices and volatility. The data is treated as series censored by price limits.

4. MODEL SET UP

4.1. The Stochastic Volatility Model

Hsieh and Yang (2009) used a Stochastic Volatility (SV) model to tackle the issue of price-limited variables unobservability.

Under Kim et al. (1998) and Hsieh and Yang (2009) framework, the return at time t, under the (SV) model, is given by the differential equation:

$$r_t = e^{h_t/2}\varepsilon_t$$

for $t = 1, \ldots, T$, where $\varepsilon_t \sim \mathcal{N}(0,1)$. Hence, the conditional variance of y_t is $Var(r_t|h_t) = e^{h_t}$.

The states h_t are assumed to evolve according to the stationary process

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \zeta_t$$

for $t = 2, \ldots, T$, where $\zeta_t \sim \mathcal{N}(0, \sigma_h^2)$ and is independent of ε_t. h_t is a Ornstein-Uhlenbeck process, thus, a mean-reverting process as it tends to drift towards its long-term mean.

The SV Model is:

$$\begin{cases} r_t = e^{h_t/2}\varepsilon_t \\ h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \zeta_t \end{cases}$$

The process can be seen as a modification of the random walk in continuous time, or Wiener process. However, there is a tendency of the walk to move back towards a central location. The tendency grows as the process walks away from the centre. The Ornstein-Uhlenbeck process can also be seen as the analogue of the discrete-time AR(1) in a continuous-time process because h_t depends of h_{t-1}.

In the model we assume that $|\phi_h| < 1$ to get the stationarity property of the process, and the states are initialized

$h_0 \sim \mathcal{N}(\mu, \sigma_h^2/(1 - \phi_h^2))$ which is the stationary distribution of the process.

Note that h_t measures the amount of volatility on a trading day t, ϕ_h is interpreted as the persistence in volatility and σ_h is interpreted as the conditional volatility of log volatilities.

In the litterature, the SV models are considered as a successful alternative to the class of Autoregressive Conditionally Heteroscedastic (ARCH) models. However, because of the estimation complexity, the SV models are not as popular as the ARCH models.

Since the SV model is nonlinear, its likelihood function depends upon high dimensional integrals. In fact, the dimension of these integrals is equal to the number of returns observed. Hence, using the maximum likelihood method is very cumbersom.

In order to overcome this difficulty, several estimation methods have been proposed. Melino and Turnbull (1990) investigated the consequences of stochastic volatility for pricing spot foreign currency options by developing a Generalized Method of Moments (GMM). Nelson (1988), Ruiz (1994) and Harvey et al. (1994) developed a quasi-maximum likelihood estimation of SV models. The main idea of the quasi-maximum likelihood method is to treat non-normal disturances as if they are normal and then maximize the quasi-maximum likelihood function. Based on the GMM, Gallant et al. (1997) built the Efficient Method of Moments (EMM). While maintaining the flexibility of the GMM, the EMM estimation seeks to attain the efficiency of ML. Another method for estimating stochastic volatility, is the Markov Chain Monte Carlo procedure proposed by Jacquier et al. (1994) and improved by Kim et al. (1998).

4.2. Parameter Estimation

The SV model estimation, as developed by Chan and Hsiao (2013) consists in using the Precision Sampler for Linear Gaussian State Space Models. However since the SV model is non linear, they used the Auxiliary mixture sampler to overcome this issue.

To approximate the nonlinear stochastic volatility model using a mixture of linear Gaussian models.

Auxiliary mixture sampler

Auxiliary mixture sampling is a simple MCMC method for estimating a broad class of non-Gaussian models.

The differential equation of the SV model can be transformed into a linear model by taking the squares logarithm of observations:

$$\log y_t^2 = h_t + \log \varepsilon_t^2$$

where $E(\log \varepsilon_t^2) = -1.2704$ and $Var(\log \varepsilon_t^2) = 4.93$ as estimated by Kim et al. (1998)

Kim et al. (1998) designed an offset mixture of normal distribution to accurately approximate the exact likelihood. The approximating parametric model for the linear approximation will be an offset mixture time series model:

$$y_t^* = h_t + \varepsilon_t^*$$

where $\varepsilon_t^* = \log \varepsilon_t^2$ and $y_t^* = \log(y_t^2 + c)$ for some small constant $c = 10^{-4}$, to avoid numerical problems when y_t is close to zero. The SV model becomes a linear state space model in h_t.

Harvey et al. (1994) proved that under the assumption that ε_t^* is normal, the quasi-likelihood estimator has poor sample properties. As ε_t^* follows a $\log -\chi_1^2$ distribution, it no longer has a Gaussian distribution,

To overcome this difficulty, $f(\varepsilon_t^*)$, the density of ε_t^*, is approximated by an appropriate Gaussian mixture.

$$f(\varepsilon_t^*) \simeq \sum_{i=1}^{n} p_i f_{\mathcal{N}}(\varepsilon_t^*, \mu_i - 1.2704, \sigma_i^2),$$

where $f_{\mathcal{N}}(\varepsilon_t^*, \mu, \sigma)$ denotes the Gaussian density with mean μ and variance σ^2, p_i is the probability of the i-th mixture component, and n is the number of components.

The mixture density can be written in terms of an auxiliary random variables $s_t \epsilon \{1, \ldots, n\}$ that represent the mixture component indicator:

$$(\varepsilon_t^* \mid s_t = i) \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

$$P(s_t = i) = p_i$$

This representation makes the model linear and Gaussian conditional on the component indicator s_t. Hence, the simulation techniques for estimating the stochastic volatility parameters can be applied.

Precision Sampler for Linear Gaussian State Space Models

The Auxiliary Mixture Sampler transformed the SV model into the conditionnally linear Gaussian model in:

$$\begin{cases} h_t = \mu + \phi_h(h_{t-1} - \mu) + \zeta_t \\ \log y_t^2 = h_t + \log \varepsilon_t^2 \\ (\varepsilon_t^* \mid s_t = i) \sim \mathcal{N}(\mu_i, \sigma_i^2) \\ P(s_t = i) = p_i \end{cases},$$

Given the prior distributions of μ, ϕ and σ^2, the SV model parameters can be estimated using standard MCMC techniques. However, Chan and Hsiao (2013) exploited the special structure of the model to estimate the joint distribution of the log-volatilities $p(h|y^*, s, \mu, \phi, \sigma^2)$. Chan and Hsiao (2013) showed that the precision matrix-inverse of the covariance matrix of $p(h|y^*, s, \mu, \phi, \sigma^2)$ is a band matrix. Containing a small number of nonzero elements along a diagonal band, the computation of $\{h_1, h_2, \ldots, h_n\}$ will speed up. The other conditional densities are calculated based on the prior distributions and the conditional density $p(h|y^*, s, \mu, \phi, \sigma^2)$.

The algorithm of the parameter estimation cycles through:

1. $p(h|y^*, s, \mu, \phi, \sigma^2)$;
2. $p(s|y^*, h, \mu, \phi, \sigma^2) = p(s|y^*, h)$;
3. $p(\mu|y, h, s, \phi, \sigma^2) = p(\mu|h, \phi, \sigma^2)$;
4. $p(\phi|y, h, s, \mu, \sigma^2) = p(\phi|h, \mu, \sigma^2)$;
5. $p(\sigma^2|y, h, s, \mu, \phi) = p(\sigma^2|h, \mu, \phi)$.

4.5. The Censored Stochastic Volatility Model

The CSV model is a SV Model that takes into account the effect of price limit, i.e. unobservability of returns.

Mathematically, price limits can be described as follows:

$$r_t^0 = \begin{cases} u_t & \text{if } r_t \geq u_t \\ r_t & \text{if } d_t < r_t < u_t \\ d_t & \text{if } r_t \leq d_t \end{cases},$$

where r_t^0 and r_t are the continuously compounded observed and equilibrium log returns of the asset respectively; u_t and d_t are the upper and lower return limits derived from the price limit rules. In fact, the return limits u_t and d_t may vary or be constant depending on the price limit rules: we take constant limits $u_t = \log(1 + limit)$ and $d_t = \log(1 - limit)$.

The relationship between the equilibrium and observed returns and prices can be derived through simple algebra (e.g., (1) and (2) in Wei (2002)):

$$p_t = \begin{cases} p_{t-1} + a & \text{if } p_t^* \geq p_{t-1} + a \\ p_t^* & \text{if } p_{t-1} - a < p_t^* < p_{t-1} + a \\ p_{t-1} - a & \text{if } p_t^* \leq p_{t-1} - a \end{cases}, \tag{1}$$

where p_t, and p_t^* are the market observed and equilibrium prices at time t, respectively.

Define $r_t^* = \ln p_t^* - \ln p_{t-1}^*$ and $r_t = \ln p_t - \ln p_{t-1}$, the continuously compounded equilibrium and observed returns of the asset.

With some simple algebra, it is easy to prove that the two returns are related in the folowing equation:

$$r_t = \begin{cases} \bar{c_t} & \text{if } r_t^* + LO_{t-1} \geq \bar{c_t} \\ r_t^* + LO_{t-1} & \text{if } \underline{c_t} < r_t^* + LO_{t-1} < \bar{c_t} \\ \underline{c_t} & \text{if } r_t^* + LO_{t-1} \leq \underline{c_t} \end{cases}; \tag{2}$$

where

$$\bar{c_t} \equiv \ln(1 + \frac{a}{p_{t-1}}); \;\; \underline{c_t} \equiv \ln(1 - \frac{a}{p_{t-1}}) \text{ and } LO_{t-1} \equiv \ln(p^*_{t-1}/p_{t-1})$$

Given $\theta = \{\mu, \phi, h_0\} \cup \{h_t, t = 1, \ldots, n\}$, the set of observed returns $R^0 = \{r^0_1, r^0_2, \ldots, r^0_n\}$, the set of indicators as $\Delta = \delta_1, \delta_2, \ldots, \delta_n$ where $\delta_t = 1$ (or -1) if the upper (lower) limit is hit at time t and 0 otherwise and all latent returns in $R^* = \{r^*_1, r^*_2, \ldots, r^*_n\}$ except r_k (i.e $R^* - \{r_k\}$), the conditional density of r_k is a normal density truncated from below if the price hits the upper limit, but it is a normal density truncated from above if the price hits the lower limit:

$$p(r_k|\theta, r^0, r^* - \{r_k\}) = \begin{cases} f_u(r_k) = f_{\mathcal{N}}(r_k)\mathcal{I}[r^0_k, \infty) & \text{if } k \in T_u \\ f_d(r_k) = f_{\mathcal{N}}(r_k)\mathcal{I}(-\infty, r^0_k] & \text{if } k \in T_d \end{cases},$$

where $T_u = \{t : \delta_t = 1, 1 \leq t \leq n\}$, $T_d = \{t : \delta_t = -1, 1 \leq t \leq n\}$ and $f_{\mathcal{N}}$ is the normal density with a mean 0 and variance $e^{h_t/2}$. The idea behind the genration of the latent returns is simple. If the upper price limit is hit, the latent returns are sampled from a normal truncated distribution from below. If the lower price limit is hit, the latent returns are sampled from a normal truncated distriution from above.

5. RESULTS

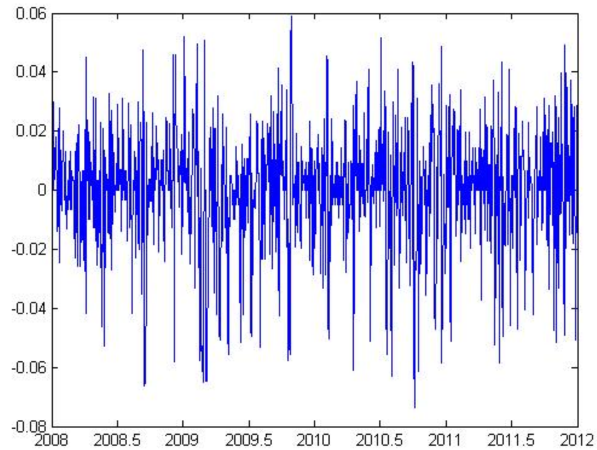
5.1. Data

In order to test the model, we used daily observations on Tunisian stocks to estimate the model parameters. We chose Societe Moderne de Cramique (SOMOCER) because it is considered as one of the most liquid stocks on the Tunisian Stock Exchange (TSE).

Somocer is a corporation, nationally governed by Tunisian law. It was established on July third, 1985. The company main business lies in manufacturing tiles, sandstone and baths. The data used spans a period of 5 years from 2008 to 2012. We chose to include in the data the year 2009 to analyse the global financial crisis impact and the year 2011 to encompass the arab spring revolution effect. Figure 1 shows the Somocer daily returns from January 2008 to January 2012.

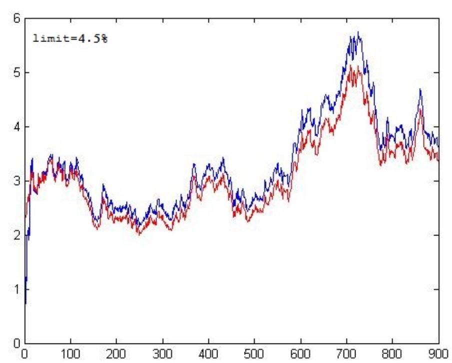
Let S_t be the sample value of the price S at day t. Daily returns could be computed either in a discrete form ($r_t = \frac{S_t - S_{t-1}}{S_{t-1}}$) or a continuous form ($r_t = \log(\frac{S_i}{S_{i-1}})$). Obviously both forms are linked since $\log(\frac{S_i}{S_{i-1}}) = \log(x + 1) \sim x$ where $x = \frac{S_t - S_{t-1}}{S_{t-1}}$ (and $x \sim 0$). For the purpose of the research we chose the discrete form as its more suited for a non liquid market as Tunisia. The result of our estimation gives the following vector

FIG. 1. Somocer daily returns from January 2008 to April 2012



$(\mu, \phi, \sigma) = (-7.57, 0.9158, 0.054)$. Asset prices are simulated from the SV model in figure 2.

FIG. 2. Simulated asset prices where a 4.5% symmetric price limit is imposed



5.2. Simulation results

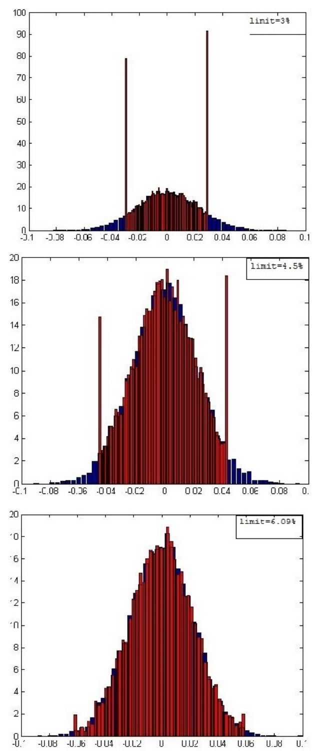
Figure 3 shows the result of our Monte Carlo simulation where an Euler scheme of discretization was used:

$$\begin{cases} y_{t\Delta} = e^{h_t/2}(B_{t\Delta} - B_{(t-1)\Delta}) \\ h_{t\Delta} = \mu + \phi_h(h_{(t-1)\Delta} - \mu) + \sigma(W_{t\Delta} - W_{(t-1)\Delta}) \end{cases}$$

where B and W are two independant Brownian motions. Δ is the distance between two observations. As the data have a daily frequency, $\Delta = 1$.

The histograms in figure 3 confirm the results found by Hsieh and Yang (2009): the price-limited data have a truncated normal distribution. Figure 3 shows the two peaks in the distribution extremes that we commonly see in truncated time series. In the financial literature it is called price limits magnets.

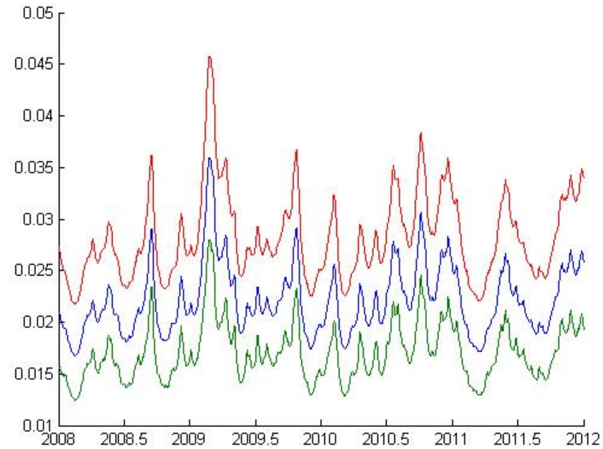
FIG. 3. Histograms of simulated returns and truncated returns.The price limit is set to 3%, 4.5% and 6.09%



The estimation of SV model using the SOMOCER data is depicted in figure 4. The figure displays the posterior means and quantiles of the time-varying standard deviation $exp(h_t/2)$ of Somocer daily returns.

As the figure show, there is substantial time-variation in the volatility. In particlar we notice two peaks, one in the late 2008 and the second in early 2011. Away from these peaks, the estimated standard deviation mostly fluctuates around 0.035%. It increases in late 2008 and almost reaches 0.05% in early 2009. This peaks may be explained by the fact that the TSE took time to get affected by the global financial crisis and that the Tunisian economy is slightly integrated in the worldwide economy.

FIG. 4. Posterior means (blue) and 90% credible intervals (red and green) of the time-varying standard deviation; Somocer daily return data



Dabbou and Silem (2013) claim that the Tunisian traders were affected psychologically by the 2007-2008 crisis through the media. In this case, downward price limit saved the TSE from a potential crash.

Figure 5 shows that when limit is set to 3% more data is truncated that when it is set at 6%. The narrower price limit is, the more frequently price limits are hit. When price limit is narrower, more price are subject to price limit truncation and hence the biasness in volatility measures tends to be larger. The figures also shows that, during crisis, price limits are more likely to be hit and the difference between real and latent returns tends to be higher.

6. OPTIONS APPROACH

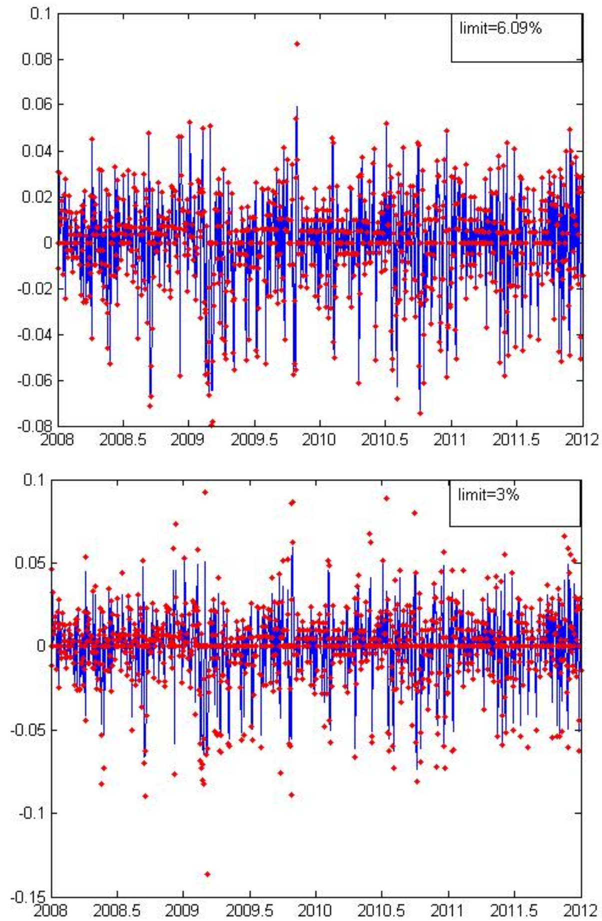
Options are contracts, that give the buyer the right, but not the obligation, to buy or sell an underlying asset at a specific price on or before a certain date. Just like a stock or bond, an option is a security.

Option markets are very important in the financial world. They provide valuable information about the future course of the financial asset and investors' expectations. The option's value consists in its exercicing chance if the asset price is above/beyond a certain strike price.

Since the investors' actual risk preferences are embedded in the price of the underlying asset, the derivative security can be priced relative to the underlying asset under the risk-neutralized probability distribution.

Since Shimko (1993), reasearchers started to pay more attention to the problem of extracting Risk Neutral Densities (RND) from option prices. A

FIG. 5. Predictive return series. **Note:** The figure plots the predicitve return series for SOMOCER. The vertical lines represent the observed returns while little points represent the predictive returns from CSV model.



large number of methods to extract RNDs from option prices have been derived. By ruling out arbitrage possibilities, Cox and Ross (1976) stated the options can be priced as if investors were risk neutrals. The price of a European call can then be computed as the discounted value of the option's expected return under risk neutrality, with respect to the equivalent martingale measure Q is:

$$
\begin{aligned}
C(S_t, K) &= e^{-r(T-t)} E^Q[\max(S_T - K, 0)|S_t] \\
&= e^{-rT} \int_K^\infty \max(S_T - K, 0) f(S_T) dS_T
\end{aligned}
\tag{3}
$$

where C is the option price, S_T is the price of the underlying asset at time T, the expiry date of the option, r is the risk-free rate of interest, K is the strike price and E^Q is the expectations operator with respect to the risk neutral density of the future price, $f(S_T)$.

The forme of $f(S_T)$ is never known empirically even though, equation (3) is used to price the European call option. Breeden and Litzenberger (1978) brought attention to the fact that $f(S_T)$ can be obtained from a functional expression for C and that by differentiating (3) twice with respect to the strike price K:

$$\frac{\partial^2 C(K)}{\partial K^2} = e^{-r(T-t)} f(K)$$

Increasing the exercice price by the amount dK narrows the range of stock prices S_T and reduces the payoff by the amount $-dK$ for every S_T at which the option is in the money. Taking the second derivative with respect to K however, yields the risk neutral density function at K. An approximation to the density $f(K)$ can be obtained using a finite differences scheme:

$$f(K) = e^{r(T-t)} \frac{C_{n+1} - 2C_n + Cn - 1}{\Delta K^2}$$

Our option approach consists in looking at price limited stock returns as option payoffs. In fact, the observed prices (or returns R_t are either the minimum between R_t and the upper limit l or the maximum between $-R_t$ and the lower limit $-l$. This could be translated mathematically to the following equation:

$$\overline{R}_t = \begin{cases} -\max(l - R_t, 0) + l & \text{if } R_t > l \\ \max(l - R'_t, 0) - l & \text{if } R'_t < -l \end{cases}$$

where $R'_t = -R_t$.

In the absence of arbitrage opportunities, European style options can be priced without any assumption about the underlying price process by duplicating their state dependent payoffs using the observed prices of the basis assets.

The idea behind our approach is to consider R_t as the underlying asset and extract its distribution. The shape of this distribution will be considered as the shape of the non-truncated returns and will give us a better value of the volatility. The truncated prices (or returns) will be computed as the discounted value of the option's expected return under risk neutrality, with respect to the equivalent martingale measure Q:

$$C(t,l) = \begin{cases} -e^{-rT} E^Q[\max(l - R_t, 0)] + le^{-rT} & \text{if } R_t > l \\ e^{-rT} E^Q[\max(l - R_t, 0)] - le^{-rT} & \text{if } R'_t < -l \end{cases}$$

It is clear that the prices could be perceived as a European put plus (or minus) the term including the price limit l. Theorically, this term doesn't affect the underlying distribution function as the derivative with respect to l a second time still yields the same risk neutral density function f as if it was a European option.

The problem with the interpolation methods, is that they use finite deifferences that suppose K is a vector. In our approach the strike price K is the price limit itself, which is a constant in most stock markets. We tend then to assume as, Black and Scholes (1973) did, that the underlying asset has a lognormal distribution and evolves in line with a geometric Brownian motion (GBM) stochastic process, with a constant expected return and a constant volatility:

$$dS_t = S_t \mu dt + S_t \sigma dB_t$$

where S_t is the price of the underlying asset at time t, dS_t denotes instantaneous price change, μ is the expected return, σ is the standard deviation of the price process and dB are increments from a Brownian motion process. The parameters μ and σ are assumed to be constant.

Ito's lemma applied to the the GBM results in the following result:

$$\ln S_T \sim N(\ln S_0 + (\mu - \frac{1}{2}\sigma^2 \tau, \sigma\sqrt{\tau})$$

where $N(a, b)$ is the normal distribution Nith mean a and standard deviation b. Hence, the Black-Scholes formula leads to the assumption that the RND function of underlying returns is normal with parameters μ and σ) given by:

$$f(S_T) = \frac{1}{S_T \beta \sqrt{2\pi}} exp\{-(\ln S_T - \mu)^2 / 2\sigma^2\}.$$

Now we assume we know that the underlying asset's distribution is lognormal, we can estimate it's parameters. Applied to the price-limited prices, we can get and idea about the underlying volatility of the non-truncated time series.

7. CONCLUSION

In the current financial markets and their international linkage, price limits could be challenging for asset managers dealing cross countries. For a manager trading in several markets with various price limits, the measure of volatility could be biased resulting in mispricing and inaccurate investment decisions. In this paper, we addressed thei issue of volatility biasness in markets with price limits. We use the CSV model proposed by Hsieh and Yang (2009) to model the return process of assets that are subject to price limits. We adopted the algorithm proposed by Chan and Hsiao (2013) to estimate the SV parameters. We found some interesting results applied to the Tunisian Stock Exchange and showed the extent of bias that can be seen in market with price limits. One interesting fact that we showed as well is that markets with price limits have prices that display an option look alike payoff. Hence in markets where derivatives are not developed, people are actually trading stocks with options payoffs and the common linear payoff we see in stocks. Using this feature, we proposed a second approach based on option pricing to tackle the issue of volatility underestimation in price limits. A more detailed analysis of this approach should be adressed in future research.

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