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Multivariate Analysis of East African Currency Exchange Rate Dynamics

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The main aim of this paper is to investigate the conditional correlations between daily returns of 6 currencies of East African countries relative to the US dollar. We fitted the CCC-GARCH, DCC-GARCH and ADCC-GARCH models on the daily returns conditional covariance matrix. The findings of this paper provide evidence that the correlation parameters between the pair of exchange rate returns are significant. This shows that the conditional correlations among the six East African countries exchange rate returns change with time. Lastly, this paper provides insight into the nature of correlation among East African currency exchange rates over the sample period.

Key Words: Dynamic conditional correlations; East African currency; Exchange rate volatility; Multivariate GARCH.
JEL Classification Numbers: C32, C51, C58.

1. INTRODUCTION

Exchange rates have an impact on a country's prices, on portfolio allocation, on the production decision of firms and more generally on its competitiveness. In addition to bad business strategy, extreme exchange rates volatility could bring the loss in trading with the currency pairs. "Exchange rate modelling is interesting for businesses and policymakers who use exchange rates models as tools in their risk management and policymakers use them to acquire knowledge about the impact of economic factors on exchange rate volatility for informed policymaking" (Mojsej and Tartalova, 2013, p.1). Exchange rate dynamics have become an important subject for academics, economists and policymakers after the collapse of the Bretton Woods System agreement of fixed exchange rates. Hence, it is very important to model exchange rates and predict their future behaviour, especially in times of uncertainty and financial stress.

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The major focus in financial econometrics has been analysing volatility since Engle (1982) developed the Autoregressive Conditional Heteroscedasticity (ARCH) models. Bollerslev (1986) proposed the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model which is a benchmark model in volatility modelling.

It is well known that financial market returns likely to move together with time. Similarly, financial volatilities tend to move together with time across both assets and markets. Modeling a time-varying volatility matrix or conditional correlation matrix is crucial in many financial applications, such as asset pricing, hedging, portfolio selection, option pricing, and risk management. It is because the aforementioned financial issues are dependant on information about the covariances or correlations between the underlying returns. Therefore, it is important to recognize the multivariate relationship between financial markets. This has motivated a multivariate modelling framework rather than univariate models (Bauwens et al., 2006; Ruppert & Matteson, 2015). However, the curse of dimensionality is one of the challenges of multivariate volatility. For example, there are $n(n+1)/2$ variances and covariances for a n-dimensional process (Ruppert & Matteson, 2015).

Multivariate GARCH started to exist in the late 1980s and the beginning of 1990s. A number of multivariate GARCH models have been proposed in the literature. Bauwens et al. (2006) provide a detailed survey of multivariate GARCH models. The most commonly applied models in the literature are reviewed as follows. The Constant Conditional Correlation (CCC) GARCH model has been introduced by Bollerslev (1990). This model deals about the constant correlation matrix in order to relate univariate GARCH models to one another. The Maximum Likelihood Estimation (MLE) is applied to standardized residuals to estimate the correlation matrix. However, the assumption of constant conditional correlations may not seem realistic for many practical financial applications (see Tsui and Yu, 1999; Tse, 2000, among others). Therefore, Engle (2002) and Tse and Tsui (2002) relaxed the assumption of constant correlation by allowing time-dependent correlations. The model is called the Dynamic Conditional Correlation (DCC) GARCH model. DCC-GARCH model is a generalization of the CCC-GARCH model. This model directly estimates the conditional correlation between macroeconomic variables. Cappiello et al. (2006) extend further to incorporate the asymmetric impacts on the correlations caused by good news and bad news. This model is called the Asymmetric Dynamic Conditional Correlation (ADCC)-GARCH model.

1.1. Aim and outline of the paper

This paper addresses the internal links among the East African currency exchange rate returns using appropriate univariate GARCH and multivariate GARCH models. This paper will be able:

- to develop an appropriate univariate GARCH model to each of the East African currency exchange rate returns,
- to investigate whether the correlations between currency exchange rate returns change through time,
- to estimate the East African currency exchange rate correlation dynamics using multivariate GARCH models.

1.2. Why volatility and correlation dynamics of East African currency exchange rate returns?

There are trade treaties among East African countries. The Common Market for Eastern and Southern Africa (COMESA) aims to create free trade between members. They are also progressing to have a unified visa, free internal trade and a single currency. For instance, the East African Community (EAC) includes Kenya, Uganda, Tanzania, Rwanda and Burundi has made progress by launching an integrated East Africa tourist visa in 2014. This requires the study and understanding of the collective dynamics of East African currencies and identification of currencies that behave similarly or dissimilarly. There are few pieces of literature about volatilities in East African markets. However, most of the studies in the Sub-Saharan African countries are centred on the univariate symmetric and asymmetric GARCH models. This paper, therefore, examines the dynamic relationship between East African currency exchange rate returns using different multivariate GARCH models.

2. EMPIRICAL LITERATURE REVIEW

Exchange rate volatility has received considerable attention in the literature in the last three decades because of its significant impact on key macroeconomic variables (Kibiy & Nasieku, 2016). Abdalla (2012) considered nineteen Arab countries in order to examine the daily exchange rate returns using univariate GARCH models. Marreh et al. (2014) applied the GARCH(1,1) process to model the daily exchange rates of the Gambian Dalasi against the Euro and US dollars. Similarly, Narsoo (2015) modeled the daily Mauritian Rupee against the US dollars by using different GARCH type of models. Emenike (2016) employed symmetric and asymmetric GARCH models to estimate and compare volatilities of official, interbank and bureaux de change markets of Nigerian Naira against US dollars.

Figure 1 shows that currency exchange rates in East African markets fell and rose significantly in the last ten years. Currently, there is a fall in currencies and high costs of living in the East African Region (Proti, 2013). The brief review of the currencies of some of the East African countries considered in this paper is given as follows.

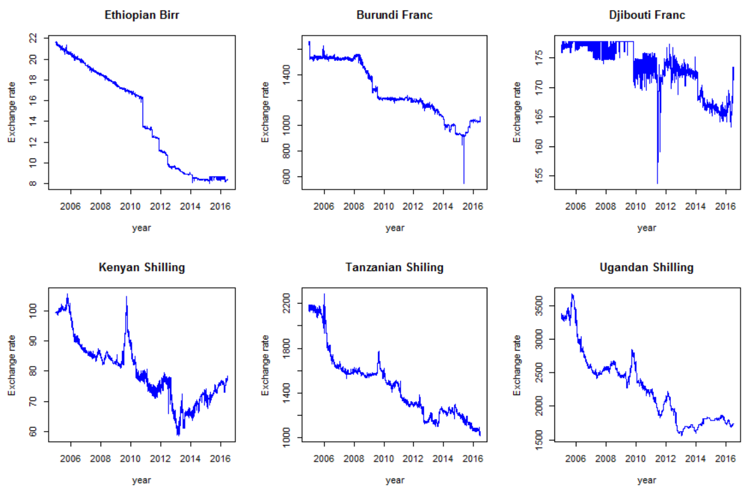

FIG. 1. Time plots of the daily exchange rates (2005 — 2016).

The Ethiopian Birr has been consistently depreciating in nominal terms from year to year since the devaluation of the Birr on October 1, 1992. The new policy regime helps to open the economy to foreign competition with a view of benefiting the economy from expanded markets (Mehare & Edriss, 2012). On 30^{th} September 2016 the spot interbank market average nominal exchange rate was 21.9795 Birr per US dollar with a depreciation of about 334 per cent compared to the 1992, 5.0 Birr per USD. The need for greater interest rate and exchange rate flexibility including through exchange rate adjustment was underscored by the recent meeting of International Monetary Fund (IMF) country report (IMF, 2014).

Kenya has also experienced exchange rate fluctuations and volatility. These have had an impact on the country's competitiveness, international trade, inflation and general economic growth. There has been a continuous trend of unpredictable fluctuations of the Shillings (Kibiy & Nasieku, 2016). The Kenyan financial market has been affected by the exchange rate volatilities. This fluctuation affects the stock market, foreign exchange market and international trade (Kirui et al., 2014; Kibiy & Nasieku, 2016).

Commercial banks were allowed to open foreign currency in the third quarter of 1992 for both residents and non-residents. This dramatically increased foreign currency deposits by 15 per cent within a year (Kessy, 2011). On the other hand, this might cause financial instability if the banks and residents deposits and do not hedge themselves against exchange rate risk. The Tanzanian Shilling fell by 50% from 1999 to 2011. The fall of currency has a lot of impacts on society and the economy as a whole (Proti, 2013).

The exchange rate market was liberalized in the Uganda market in the early 1990's as part of wider economic reforms. Since then demand and supply are the determinant forces of exchange rates. The recent paper by Katusiime et al. (2016) argued that exchange rate volatility positively affects economic growth in Uganda in both the short run and the long run. However, similar to other East African countries, the Uganda shilling has been depreciating over the last few years. For example, the Uganda shillings had hit the 3,000 marks by the second week of March 2015; 20 per cent depreciation only a year earlier.

Similar to other East African currencies, the Burundian franc has been depreciating over the past few years. For instance, the Burundian franc (BIF) depreciated by 5% against the US dollar (USD) from January to December 2012. Similarly, the Burundian franc lost 7.8% of its value against the US dollar in 2013. This slight depreciation helped the Bank of the Republic of Burundi to maintain reserves equal to 3.8 months of imports in 2013 (AfDB, OECD & UNDP, 2014). In 2015, the Burundian franc depreciated by only 1.6 per cent in relation to the US dollar.

The remainder of this article is organized as follows. Section 3 presents the data. In Section 4 we briefly describe the different univariate GARCH and multivariate GARCH models. Section 5 presents the empirical results of estimation of the univariate GARCH and the multivariate GARCH models. Section 6 contains concluding remarks.

3. DATA DESCRIPTION

The data relevant to this study has taken from secondary source recorded data from www.oanda.com/currency/historical-rates/. The time series data used in this paper consists of the daily returns of exchange rate on the Ethiopian (Birr), Burundi (Franc), Kenyan (Shilling), Djibouti (Franc), Tanzanian (Shilling), and Ugandan (Shilling), all against the US dollar. The sample covers from 1^{st} January 2005 to 30^{th} June 2016 G.C. and has a total of 4199 observations. For example, if x_t is the value of exchange rate at time t, then the return or relative gain, y_t, of the exchange rate at time t is

$$y_t = \frac{x_t - x_{t-1}}{x_{t-1}} \implies x_t = (1 + y_t)x_{t-1}.$$

The return y_t does not have a constant variance. We use $\nabla[\ln(x_t)]$ or $\frac{x_t - x_{t-1}}{x_{t-1}}$ to model exchange rate volatility and correlation dynamics.

4. METHODOLOGY

In this chapter, we illustrate some univariate and multivariate time series models. The univariate models considered are the GARCH, the GJR-GARCH, Exponential GARCH (EGARCH), and the univariate stochastic volatility (SV) models. GJR-GARCH and EGARCH models are nonnested and can capture asymmetry and leverage. The multivariate models considered in this paper are CCC-GARCH, DCC-GARCH, DCC-EGARCH, DCC-GJR-GARCH, ADCC-GARCH, ADCC-EGARCH and ADCC-GJR-GARCH models.

4.1. The GARCH Models

The standard linear GARCH(1,1) model can be specified as follows:

$$r_t = \mu_t + y_t, \quad y_t = \sigma_t \epsilon_t; \quad \sigma_t^2 = \alpha_0 + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{1}$$

where $\{\epsilon_t\}$ is a sequence of identically independently distributed (iid) random variables with mean 0 and variance 1, $\alpha_0 > 0, \alpha \geq 0$ and $\beta \geq 0$ and $\alpha + \beta < 1$ ensures covariance stationarity. The sum $\alpha + \beta$ measures the persistence of a shock to the conditional variance in equation.

4.2. Asymmetric Volatility Models

These kinds of models belong to the class of "asymmetric" or "leverage" volatility models. In this paper, we consider two different asymmetric volatility models. These are GJR-GARCH and EGARCH models.

4.2.1. GJR-GARCH models

Glosten et al. (1993) propose the nonlinear GJR-GARCH model in order to consider the asymmetric leverage effect to volatility. The GJR-GARCH(1,1) model can be written as

$$\sigma_t^2 = \alpha_0 + \left[\alpha + \gamma I_{\{y_{t-1}>0\}}\right] y_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{2}$$

where $I_{\{y_{t-1}>0\}}$ is an indicator function, and

$$I_{\{y_{t-1}>0\}} = \begin{cases} 1 & y_{t-1} > 0 \\ 0 & otherwise. \end{cases}$$

4.2.2. EGARCH models

GARCH model assumes that the effect of positive and negative information is symmetric, which may not completely accord with the market situation. Nelson (1991) introduced the EGARCH model to consider the asymmetric feature of asset price volatility. The EGARCH(1,1) model can be expressed as

$$\log(\sigma_t^2) = \alpha_0 + \alpha y_{t-1} + \gamma\left[|y_{t-1}| - E|y_{t-1}|\right] + \beta \log(\sigma_{t-1}^2), \tag{3}$$

where $|\beta| < 1$ to avoid explosive variance patterns.

4.3. The SV Model

Let $\mathbf{y} = (y_1, y_2, \cdots, y_n)^T$ be a vector of exchange rate returns with mean zero. Let h_t^2 represent the latent volatility on the day t, ϕ be a correlation coefficient. The hierarchical SV model can be given in the following form:

$$\begin{aligned} y_t|h_t &\sim N\left(0, \exp\ h_t\right), \\ h_t|h_{t-1} &\sim N\left(\alpha + \phi(h_{t-1} - \alpha), \tau^2\right), \\ h_0|\alpha, \phi, \tau &\sim N\left(\alpha, \frac{\tau^2}{1-\phi^2}\right), \end{aligned}$$

where $h_0 \sim N(\alpha, \tau^2)$. The vector of parameters $\boldsymbol{\theta} = \left(\alpha, \phi, \tau^2\right)^T$: α is interpreted as the level of log-variance, ϕ is usually interpreted as so called the persistence of log-variance, while τ is interpreted as the volatility of log-variance. A prior distribution for the parameter vector $\boldsymbol{\theta}$ needs to be specified in order to complete the model setup. Therefore, α has the usual Normal prior, τ has the inverse gamma distribution, while the persistence parameter $\phi \in (-1, 1)$, we chose $(\phi + 1)/2$ has the Beta distribution. (For details of the SV model, see Kastner, 2014).

4.4. Multivariate GARCH Models

4.4.1. The CCC model

The CCC model was introduced by Bollerslev in 1990 to model the time-invariant conditional correlation matrix. The conditional correlation model is defined as

$$\begin{cases} H_t = \mathbf{D}_t R \mathbf{D}_t \\ \mathbf{D}_t = diag\left(h_{11t}^{1/2} \cdots h_{nnt}^{1/2}\right) \end{cases} \tag{4}$$

where h_{iit} can be defined as a univariate GARCH model and $\mathbf{D}_t$ is a diagonal matrix with positive diagonal entries that are the conditional variances specified by univariate GARCH models. $\mathbf{R}$ is a positive definite correlation matrix with a typical element:

$$\rho_{ijt} = \frac{cov_{t-1}(u_{it}, u_{jt})}{var_{t-1}(u_{it})^{1/2} var_{t-1}(u_{jt})^{1/2}} \tag{5}$$

with $\rho_{ii} = 1$, for all $i = 1, \cdots, n$. h_{iit} in the matrix D_t is the conditional variances and can be defined as any univariate GARCH model. Obviously, the assumption that conditional correlations are constant over time, is unrealistic. As an improvement on the CCC-GARCH model, certain modifications were made by Engle (2002) and Tse and Tsui (2002).

4.4.2. The DCC model

Both Tse and Tsui (2002) and Engle (2002) generalize the constant correlation model of Bollerslev (1990) to allow for such DCCs. The DCC-GARCH model belongs to the family of multivariate GARCH models. The Engle's (2002) DCC model is as follows:

$$\begin{cases} \mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \\ \mathbf{D}_t = diag\left(h_{11t}^{1/2}, \cdots, h_{nnt}^{1/2}\right) \\ \mathbf{R}_t = \left\{diag(\mathbf{Q}_t)^{-1/2}\right\} \mathbf{Q}_t \left\{diag(\mathbf{Q}_t)^{-1/2}\right\}, \end{cases} \tag{6}$$

where $\mathbf{Q}_t = (q_{ijt})$ is the $n \times n$ symmetric positive definite matrix. The Engle's (2002) specification of dynamic correlation structure for the set of returns

$$\mathbf{Q}_t = (1 - \theta_1 - \theta_2)\bar{\mathbf{Q}} + \theta_1(u_{t-1}u'_{t-1}) + \theta_2 \mathbf{Q}_{t-1} \tag{7}$$

θ_1 and θ_2 are non-negative scalar parameters satisfying $\theta_1 + \theta_2 < 1$, $u_{t-1}u'_{t-1}$ is the lagged function of the standardized residuals. $\bar{\mathbf{Q}}$ is the $(n \times n)$ unconditional covariance matrix composed from the standardized residuals resulting from the first step estimation, and $\mathbf{Q}_t$ is the unconditional variance between series i, and j. The off diagonal elements in the matrix R_t will take the form

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}, \tag{8}$$

where $\rho_{ij,t}$ is the conditional correlation between series 1 and series 2.

4.4.3. The ADCC model

In univariate GARCH models, we use the EGARCH model to model the asymmetric return dynamics. Similar to univariate GARCH models, the ADCC-GARCH model was proposed by Capiello, et al. (2006) to incorporate an asymmetric correlation effect. The specification of $\mathbf{H}_t$ in the ADCC-GARCH model is given by the equations

$$\begin{cases} \mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \\ \mathbf{D}_t = diag\left(h_{11t}^{1/2}, \cdots, h_{nnt}^{1/2}\right) \\ \mathbf{R}_t = \left\{diag(\mathbf{Q}_t)^{-1/2}\right\} \ \mathbf{Q}_t \ \left\{diag(\mathbf{Q}_t)^{-1/2}\right\}, \end{cases} \tag{9}$$

where

$$\mathbf{Q}_t = (\bar{Q} - \mathbf{A}'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G) + A'u_{t-1}u'_{t-1}A + B'\mathbf{Q}_{t-1}B + G'\mathbf{n}_{t-1}\mathbf{n'}_{t-1}G. \tag{10}$$

The special case of (10) can be rewritten as

$$\mathbf{Q}_t = (\bar{\mathbf{Q}} - \theta_1\bar{\mathbf{Q}} - \theta_2\bar{\mathbf{Q}} - g\bar{\mathbf{N}}) + \theta_1\mathbf{u}_{t-1}\mathbf{u}'_{t-1} + \theta_2\mathbf{Q}_{t-1} + g\mathbf{n}_{t-1}\mathbf{n'}_{t-1}, \tag{11}$$

where $\theta_1+\theta_2+\delta g < 1$, where δ is the maximum eigenvalue $[\bar{\mathbf{Q}}^{-1/2}\bar{\mathbf{N}}\bar{\mathbf{Q}}^{-1/2}]$, A, B, G are diagonal parameter matrices, $\bar{\mathbf{N}} = E\left[\mathbf{n}_t\mathbf{n'}_t\right]$ is a positive semi-definite parameter matrix, and $\mathbf{n}_t = I[\mathbf{u}_t < 0] \oplus \mathbf{u}_t$.

In this paper maximum likelihood (ML) estimation method was applied to estimate the conditional correlation models.

5. EMPIRICAL RESULTS AND DISCUSSION

5.1. Descriptive statistics of the daily returns of the exchange rates series

This paper compares the currency exchange rate return volatilities of six East African countries such as Ethiopia, Kenya, Tanzania, Uganda, Burundi and Djibouti. It also provides a dynamic correlation between the currency exchange rate of these countries. Table 1 presents summary statistics for the exchange rate returns series. The skewness and kurtosis respectively measure the asymmetry and peakedness of the probability distribution of returns. The data shows positive skewness for Burundi and Tanzania currencies and negative skewness for Ethiopia, Kenya, Djibouti and Uganda currencies. The excess kurtosis statistic which is equal to 163.69, 1345.92, 12.54, 37.29, 8.27 and 37.62 for Ethiopian, Burundi, Kenyan, Djibouti, Tanzanian and Ugandan currencies, respectively indicate the Leptokurtic characteristics of the return distribution. The Jarque-Bera (JB) statistic with skewness and excess kurtosis are clearly observed for all the daily returns series which indicate violations of normality assumptions. The JB is computed as

$$JB = \frac{T}{6}\left(\widehat{skew}^2 + \frac{(\widehat{kurt}-3)^2}{4}\right), \tag{12}$$

where $\widehat{kurt}$ represents the sample kurtosis and $\widehat{skew}$ represents the sample skewness (Jarque & Bera, 1980). Furthermore, Figure 2 presents return data for the six different currencies. From this figure we can easily observe the typical characteristics seen in most foreign exchange rate data; skewness, kurtosis and potential jumps. The above facts clearly pointed out that all returns series do not conform to a normal distribution. Moreover, the skewness and excess kurtosis in the returns series of all currencies are hints for conditional heteroscedasticity.

TABLE 1.

Descriptive Statistics for the exchange rate returns series

Currencies	Mean	Min.	Max.	Std.D.	Skewness	Kurtosis	Jarque Bera	ADF	ARCH-LM
Ethiopia	-0.0002	-0.1856	0.077	0.007	-5.159	163.69	4709900^*	-72.94^*	22.68^*
Burundi	-0.0001	-0.529	0.5302	0.0129	0.0684	1346	317160000^*	-102.21^*	1908.4^*
Kenya	0.0000	-0.128	0.0815	0.01092	-0.4030	12.54	27668^*	-76.38^*	622.46^*
Djibouti	0.0000	-0.099	0.0300	0.0052	-1.889	37.29	246010^*	-83.18^*	23.847^*
Tanzania	-0.0002	-0.0575	0.06372	0.0080	0.1274	8.273	11998^*	-77.34^*	940.93^*
Uganda	-0.0002	-0.1054	0.0540	0.0059	-1.1447	37.621	248740^*	-67.54^*	230.38^*

Broadly speaking, a time series is said to be stationary if the properties of one section of the data are much like those of any other section. We used the Augmented Dickey-Fuller (ADF) test to determine whether the series is stationary or non-stationary. The ADF test statistic is given as

$$\Delta y_t = c + (\alpha - 1)y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \epsilon_t, \tag{13}$$

where ϵ_t is a white noise error term. The hypothesis of the form H_0 : non stationarity ($i.e., \alpha = 1$) vs $H_1 : \alpha \neq 1$) is tested using the ADF test given as: $\hat{t}_n = \frac{1-\hat{\alpha}}{\sqrt{\hat{\alpha}}^2 \sum_{t=2}^{n} y_{t-1}^2}$. The ADF test results in Table 1 strongly imply that the null hypothesis of non-stationarity was rejected at 1% level of significance (Dickey & Fuller,1979).

The results of ARCH-LM test provide a strong evidence of ARCH effects in each of the returns series. This confirms that the exchange rate returns series are volatile and need to be modeled using the GARCH class of models.

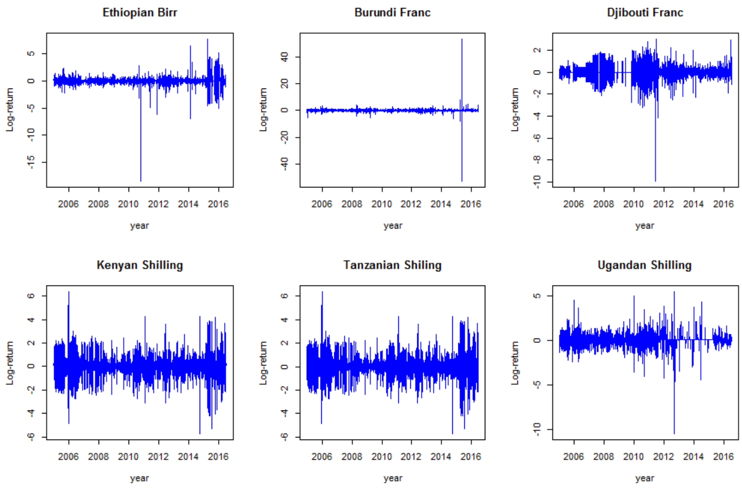

FIG. 2. Time plots of the daily log returns of the exchange rates for selected East African countries (2005 - 2016).

5.2. Specifying a volatility model

In this paper, the Akaike information criterion (AIC) Akaike (1974), Bayesian information criterion (BIC) and log-likelihood function were employed to select optimal GARCH models for the sample of the data available. The formulas for AIC and BIC are

$$\begin{aligned} AIC &= -2 \times LLF + 2k \\ BIC &= -2 \times LLF + k \times \ln(N) \end{aligned} \tag{14}$$

where N is the sample size and k is the number of parameters. LLF is an abbreviation for log-likelihood function. The BIC is consistent but not efficient, and the AIC is inconsistent but efficient. These are the reasons for using both criteria. Table 2 displays the summaries of the AIC, BIC and log-likelihood function of GARCH, EGARCH and GJR-GARCH models, when the errors follow normal (Gaussian), Student's t and generalized error distribution (GED). The minimum value of the criterion indicates that the model which offers the best models among the given models. By looking at these values, EGARCH(1,1) model has the smallest AIC and BIC for Ethiopian, Kenyan and Djiboutian exchange rate returns under GED, GARCH(1,1) for Tanzanian exchange rate return under GED and GARCH(1,1) for Burundi and Ugandan exchange rate returns under

Student's-t error distribution. Hence, these models are the preferred candidate for modelling exchange rate returns data.

TABLE 2.
Information criteria and log-likelihood function for GARCH(1,1), EGARCH(1,1) and GJRGARCH(1,1) models using Normal, Student and GED distributions

Currency	IC	Normal			Student			GED		
		GARCH	EGARCH	GJR	GARCH	EGARCH	GJR	GARCH	EGARCH	GJR
Ethiopia	AIC	−7.984	−7.928	−8.002	−8.812	−8.799	−8.783	−6.396	−10.741	−5.893
	BIC	−7.977	−7.919	−7.994	−8.804	−8.791	−8.774	−6.389	−10.730	−5.882
	log-likelihood	17115	16882	17117	18499	18472	18436	13426	**22547**	12373
Burundi	AIC	−7.646	−7.614	−7.645	−8.312	−8.255	−8.267	−5.609	−6.152	−6.289
	BIC	−7.638	−7.605	−7.636	−8.302	−8.246	−8.258	−5.599	−6.141	−6.279
	log-likelihood	15861	16110	16160	**17448**	17329	17354	11776	12917	13205
Kenya	AIC	−6.913	−6.853	−6.912	−7.314	−7.362	−7.294	−7.636	−8.216	−7.957
	BIC	−6.904	−6.843	−6.903	−7.305	−7.353	−7.285	−7.628	−8.207	−7.946
	log-likelihood	14599	14520	14510	15354	15454	15312	16028	**17247**	16704
Djibouti	AIC	−8.286	−8.276	−8.309	−9.577	−9.177	−8.693	−6.639	−14.740	−6.337
	BIC	−8.279	−8.267	−8.301	−9.568	−9.168	−8.684	−6.631	−14.731	−6.326
	log-likelihood	17373	17252	17070	20104	19263	18249	13939	**30938**	13304
Tanzania	AIC	−7.209	−7.191	−7.210	−7.517	−7.537	−7.514	−8.084	−8.053	−7.749
	BIC	−7.202	−7.181	−7.201	−7.508	−7.528	−7.505	−8.075	−8.044	−7.739
	log-likelihood	15166	15145	15174	15781	15822	15777	**16971**	16906	16273
Uganda	AIC	−7.649	−7.607	−7.666	−9.511	−8.646	−9.673	−6.491	−8.646	−6.075
	BIC	−7.642	−7.598	−7.657	−9.501	−8.637	−9.664	−6.482	−8.637	−6.065
	log-likelihood	16132	16051	16133	**19964**	18149	20304	13628	18149	12756

5.3. Estimation results

The basic estimation model consists of three different univariate GARCH models such as the standard linear GARCH model and the nonlinear GJR-GARCH and EGARCH models. All the six East African exchange rate return volatilities are evaluated by considering the domestic-USD exchange rates. The conditional distribution of the error terms are assumed to be normal, the Student's t-distribution and GED to obtain valid models. The results are presented in Table 3 from which we have several findings. In addition, the SV models are considered, and the estimation results based on the SV model are presented in table 4. We also fitted the multivariate GARCH models such as DCC-GARCH(1,1), DCC-EGARCH(1,1), DCC-GJR-GARCH(1,1), ADCC-GARCH(1,1), ADCC-EGARCH(1,1) and

ADCC-GJR-GARCH(1,1) models. The estimation results based on the DCC-GARCH model are presented in tables 5, 6 and 7.

TABLE 3.

Parameter estimates for univariate GARCH models

Currencies	Model selected	Distribution	α_0	α	β	γ	shape	Log likelihood
Ethiopia	EGARCH(1,1)	GED	-0.3192	0.0135	0.8031	0.3337	0.4785	-659
			(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
Burundi	EGARCH(1,1)	Student	0.0017	0.0500	0.9000	0.0500	2.000	-7655
			(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
Kenya	EGARCH(1,1)	GED	-0.0780	0.1268	0.8292	0.6593	0.5173	-3238
			(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	
Djibouti	GARCH(1,1)	Student	0.000	0.3833	0.6157		2.740	339
			(0.000)	(0.000)	(0.000)		(0.000)	
Tanzania	GARCH(1,1)	GED	0.0664	0.3698	0.6292		0.5962	-3327
			(0.000)	(0.000)	(0.000)		(0.000)	
Uganda	GARCH(1,1)	Student	0.0006	0.4026	0.5964		2.6427	-1080
			(0.000)	(0.000)	(0.000)		(0.000)	

TABLE 4.

Estimation results for the standard SV model.

Country	Para.	mean	Std Dev.	MC error	5%	Median	95%	Log-likelihood
Ethiopia	α	-11.878	0.085	0.060	-12.019	-11.879	-11.740	10460
	ϕ	0.812	0.031	0.031	0.758	0.814	0.860	
	τ	0.612	0.053	0.048	0.528	0.610	0.701	
Kenya	α	9.700	1.000	0.085	-11.000	-9.700	-8.060	15445
	ϕ	1.000	0.000	0.061	1.00	1.000	1.000	
	τ	0.006	0.003	0.128	0.003	0.006	0.011	
Tanzania	α	-6.944	5.545	0.065	-11.000	-9.379	4.042	15805
	ϕ	0.977	0.060	0.033	0.830	1.000	1.000	
	τ	0.015	0.011	0.063	0.005	0.012	0.029	
Uganda	α	-9.725	0.990	0.126	-11.000	-9.724	-8.075	30077
	ϕ	1.000	0.000	0.016	1.000	1.000	1.000	
	τ	0.013	0.005	0.049	0.006	0.012	0.022	
Djibouti	α	3.274	0.033	0.140	3.200	3.270	3.330	38790
	ϕ	0.806	0.121	0.020	0.590	0.821	0.971	
	τ	0.015	0.013	0.077	0.000	0.011	0.040	
Burundi	α	3.955	0.033	0.064	3.900	3.960	4.010	17461
	ϕ	0.832	0.110	0.036	0.610	0.853	0.973	
	τ	0.013	0.011	0.069	0.001	0.011	0.036	

TABLE 5.

Information criterions and log likelihood functions for DCC and ADCC-GARCH models under the multivariate normal (mvnorm), multivariate Laplace (mvaplace) and multivariate t (mvt) distributions

	DCC(1,1)								
	GARCH			EGARCH			GJR-GARCH		
	mvnorm	mvaplace	mvt	mvnorm	mvlaplace	mvt	mvnorm	mvlaplace	mvt
AIC	5.7180	−1.4816	−3.6341	8.1222	1.4916	−3.6347	7.8435	0.79967	−3.64
BIC	5.7800	−1.4197	−3.5435	8.1932	1.5626	−3.5440	7.9146	0.87068	−3.54
log-likelihood	−11961	3151	7688	−17001	−3084	7689	−16417	−1632	**7691**
	ADCC(1,1)								
AIC	5.7185	−1.4811	−1.8306	8.1226	1.4921	1.0881	7.8440	0.80015	0.42955
BIC	5.7820	−1.4177	−1.7656	8.1952	1.5646	1.1621	7.9165	0.87267	0.50358
log-likelihood	−11961	3151	3885	−17001	−3084	−2235	−16417	−1632	−853

5.3.1. Univariate GARCH models based exchange rate volatilities: analysis of results

The estimation results of the GARCH(1,1) and EGARCH(1,1) models are presented in Table 3. The regression coefficients α_0 (constant), ARCH term α (short-run persistency of shocks) and GARCH term β (long-run persistence of shocks) for GARCH (1, 1) are statistically significant at 1%. The estimated values of the GARCH term is significant and show a positive sign, implying that last period news can still have a significant impact on volatility. Moreover, the impacts of shocks on exchange rate volatility are clearly observed since the estimated values of α and β are highly significant (Khan & Azim, 2013).

From the estimation results, it is quite important to note that $\beta > \alpha$ imply that the shocks to conditional variance take a long time to die out, hence volatility is 'persistent'. In other words, the large magnitude of β can also be an evidence of long memory in variances (Khan & Azim, 2013). Furthermore, the sum $(\alpha + \beta)$ capturing the persistent in volatility for all exchange rate returns is very close to 1, indicating the presence of ARCH and GARCH effects in exchange rate data and also a required to have a mean reverting variance process, indicating that volatility shocks are quite persistent (Abdalla, 2012). The GARCH models cannot capture the asymmetric impact of negative and positive shocks on the conditional volatility of subsequent observations. When the asymmetric effect of shocks on volatility γ is positive and significant implies the presence of leverage effect (Ahmed & Suliman, 2011). The results presented in table 3 show that the coefficient γ for EGARCH models are significant implying that

TABLE 6.
DCC-GJR-GARCH(1,1) model estimation results under the multivariate t-distribution

Currencies	Parameter	Estimate	Std. Error	t value	p-value
Ethiopia	μ	0.000	0.003	0.004	0.997
	α	0.248	0.049	5.082	0.000
	β	0.489	0.096	5.097	0.000
	γ	0.118	0.072	1.641	0.101
Burundi	μ	0.000	0.003	0.004	0.996
	α	0.505	0.070	7.242	0.000
	β	0.381	0.078	4.900	0.000
	γ	−0.159	0.079	−2.031	0.042
Djibouti	μ	0.000	0.000	0.048	0.962
	α	0.351	0.038	9.191	0.000
	β	0.625	0.023	27.16	0.000
	γ	0.047	0.048	0.961	0.336
Kenya	μ	−0.000	0.006	−0.025	0.980
	α	0.402	0.069	6.834	0.000
	β	0.596	0.058	10.25	0.000
	γ	−0.270	0.053	−5.054	0.000
Tanzania	μ	0.000	0.006	0.005	0.996
	α	0.411	0.057	7.900	0.000
	β	0.581	0.048	12.48	0.000
	γ	−0.249	0.052	−4.801	0.000
Uganda	μ	0.018	0.000	180.0	0.000
	α	0.035	0.055	10.62	0.000
	β	0.959	0.042	62.65	0.000
	γ	0.334	0.173	1.928	0.054
	θ_1	0.051	0.000	5.177e+05	0.000
	θ_2	0.949	0.000	1.918e+07	0.000
	mshape	4.000	1.043	3.836	0.000

the series is not symmetric and leverage effects are present. Therefore, EGARCH models are better models to model the Ethiopian, Burundian and Kenyan exchange rate volatilities.

5.3.2. The SV models based exchange rate volatilities: analysis of results

We estimated the SV model as an alternative to the GARCH model. The SV model estimation results are summarized in Table 4 along with the median, MC error and the 95% confidence intervals of the estimated values. The standard deviation and MC error of the parameter estimates

TABLE 7.

DCC-GJR-GARCH model estimation results

Currencies	Ethiopia		Burundi		Djibouti		Kenya		Tanzania		Uganda	
	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2
Ethiopia												
Burundi	0.1203 (0.000)	0.8789 (0.000)										
Djibouti	0.0674 (0.000)	0.9119 (0.000)	0.1479 (0.1826)	0.8521 (0.000)								
Kenya	0.1263 (0.000)	0.8630 (0.000)	0.0162 (0.259)	0.9838 (0.000)	0.0798 (0.050)	0.9200 (0.000)						
Tanzania	0.1251 (0.000)	0.8648 (0.000)	0.0163 (0.288)	0.9837 (0.000)	0.0818 (0.060)	0.9181 (0.000)	0.1754 (0.000)	0.7882 (0.000)				
Uganda	0.0110 (0.001)	0.8199 (0.000)	0.0000 (0.910)	0.9186 (0.291)	0.0118 (0.000)	0.9799 (0.000)	0.0771 (0.000)	0.9072 (0.000)	0.0810 (0.000)	0.9021 (0.000)		

in most cases are quite small indicating that the estimates are quite precise. Furthermore, the estimated persistence parameters from the SV model are close to 1 for Kenyan (1.000), Tanzanian (0.9769) and Ugandan (1.000) exchange rate returns, which is a sign of a high persistence of volatility. It is also clear that the 95% confidence interval contains the estimated values.

Figure 4 compares the estimates of the conditional standard deviations of the log returns by the EGARCH and stochastic volatility models. The green, blue and red lines are for the stochvol, EGARCH and return square, respectively. The Kenyan, Tanzanian and Ugandan Shillings seem highly volatile when compared to other East African currencies considered in this paper. The Burundi Franc and the Ethiopian birr seem less volatile. The volatility based on the SV model is higher than the volatility based on the EGARCH model.

5.3.3. Estimation of CCC, DCC and ADCC multivariate GARCH Models

Figure 3 shows a scatterplot matrix of exchange rate returns for six East African countries. Tail dependence can be observed from this figure. This suggests that the multivariate t-distribution is a promising model for them. The baseline models considered in this paper are DCC-GARCH(1,1), DCC-EGARCH(1,1), DCC-GJR-GARCH(1,1), ADCC-GARCH(1,1), ADCC-EGARCH(1,1) and ADCC-GJR-GARCH(1,1) models. We should note that among the baseline models considered in this paper, DCC-GJR-

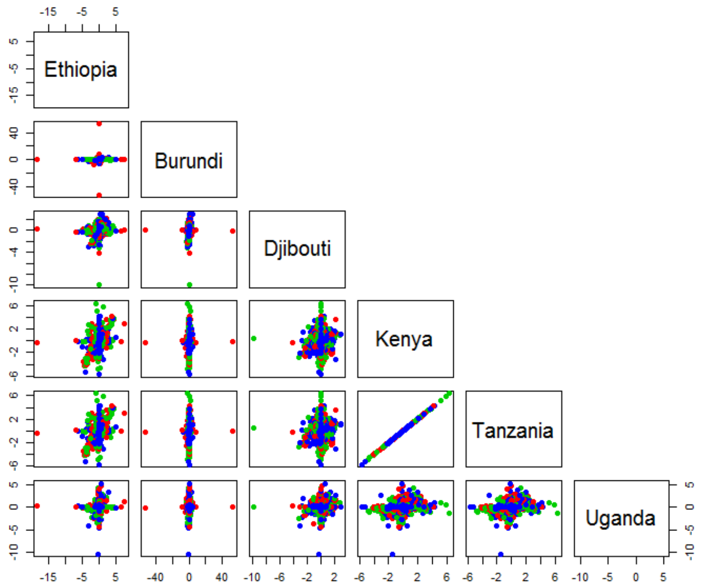

FIG. 3. Scatterplot matrix of daily returns of six East African currencies

GARCH(1,1) has been used to model the asymmetry in variance, ADCC-GARCH(1,1) has been used to model the asymmetry in correlation and ADCC-GJR-GARCH(1,1) has been used to model the asymmetry in both variance and correlation. The candidate models are compared by using the AIC, BIC and log-likelihood functions under the multivariate normal, multivariate Laplace and multivariate t distributions. Multivariate Laplace and multivariate t distributions are considered in order to assess the robustness of the estimates to possible deviations from the normality assumptions.

The DCC-GJR-GARCH model under the multivariate t-distribution seems better with respect to the other models in terms of information criteria and log-likelihood function. All the estimates of the GARCH parameters α and β are positive and significant. The sum of the estimates $(\hat{\alpha} + \hat{\beta})$ in a descending order are given as follows: Kenya $(0.402182 + 0.595236 = 0.997418 < 1)$, Uganda $(0.034962 + 0.959193 = 0.994155 < 1)$, Tanzania $(0.411081 + 0.581116 = 0.992197 < 1)$, Djibouti $(0.351244 + 0.624516 = 0.97576 < 1)$, Burundi $(0.505199 + 0.381335 = 0.886534 < 1)$ and Ethiopia $(0.247653 + 0.489045 = 0.736698 < 1)$. All the sums except Burundi and Ethiopia are close to 1, indicating that high persistence in the conditional

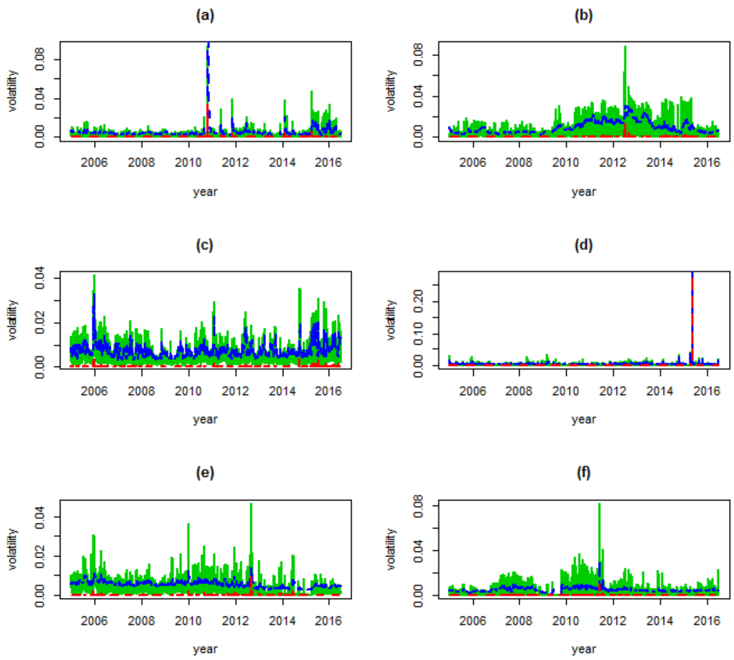

FIG. 4. Time plots of fitted volatility for the daily log returns of the exchange rates for (a) Ethiopian Birr, (b) Kenyan Shilling, (c) Tanzanian Shilling, (d) Burundi Franc, (e) Ugandan Shilling and Djibouti Franc (f). The green, blue and red lines are for the stochvol, EGARCH and return square, respectively.

variances. Kenya, Uganda and Tanzania have the highest volatility persistence among the East African exchange rate returns. Furthermore, all the sums of the estimates of the GARCH parameters are less than 1, implying that conditional variance is finite and the series is strictly stationary. One can test $\theta_1 = \theta_2 = 0$ in order to check whether the assumption of the conditional correlations is empirically relevant. As shown in Tables 6 & 7, the DCC correlation parameters show significant variations over time, implying that the correlation between the East African currency exchange rate returns is dynamic. The correlation parameter estimates fulfil the necessary condition of $\theta_1 + \theta_2 = 0.9999 < 1$, suggesting that the DCC model is adequate in measuring the time-varying conditional correlations.

Figure 5 presents the covariance terms based on the CCC (black) and DCC (red) models. This figure presents a sample of covariance between exchange rate returns. It shows that the estimated covariance of CCC-GARCH is quite stable when compared to the DCC-GARCH model.

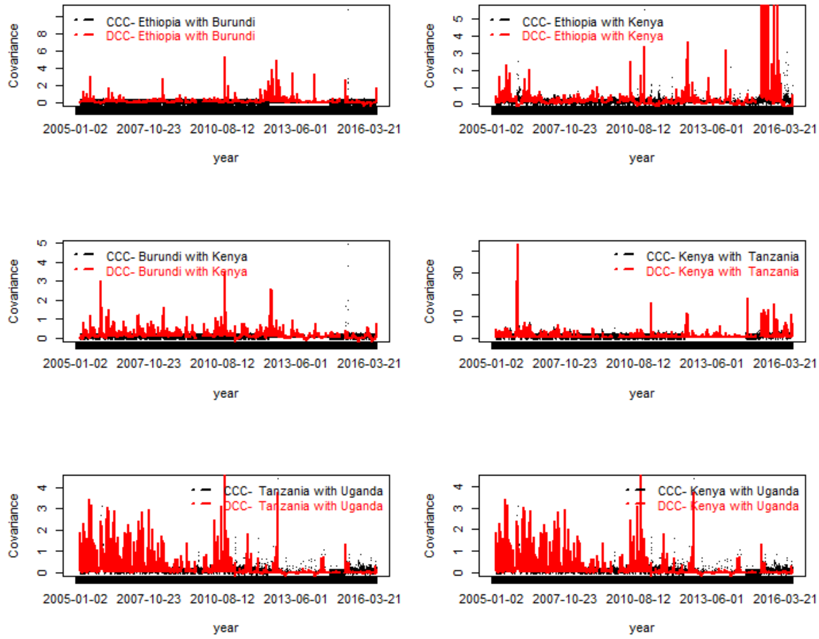

FIG. 5. Sample estimation covariance between exchange rate returns of East African countries.

Figures 6 and 7 present the estimated correlations using the DCC and ADCC models under the multivariate normal (black), multivariate Laplace (red) and multivariate t (green) distributions. First of all, the correlations between exchange rate returns are within the range [-1,1]. As shown in figures 6 and 7, the estimates of pairwise correlation by both DCC and ADCC models are more positive than negative, which means that most of the time the exchange rates of East African currencies are positively correlated. It means that the correlations between currency exchange rate returns decrease more during the 2008-2009 crisis than they increase when the market performs well. The correlations seem to be trending upwards during the first five years, but trending downwards in the last five years except for Ethiopia & Burundi and Burundi & Kenya. The pairwise correlations between Kenya & Tanzania, Tanzania & Uganda and Kenya & Uganda have a similar pattern for the DCC and ADCC GARCH models under the three different distributions. Furthermore, the correlations es-

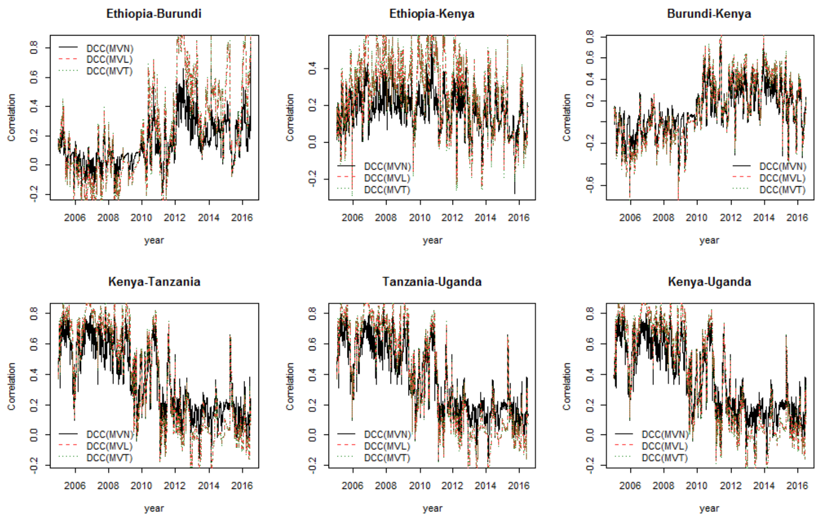

FIG. 6. The DCC between exchange rate returns of East African countries under the multivariate normal (black), multivariate Laplace (red) and multivariate t (green) distributions

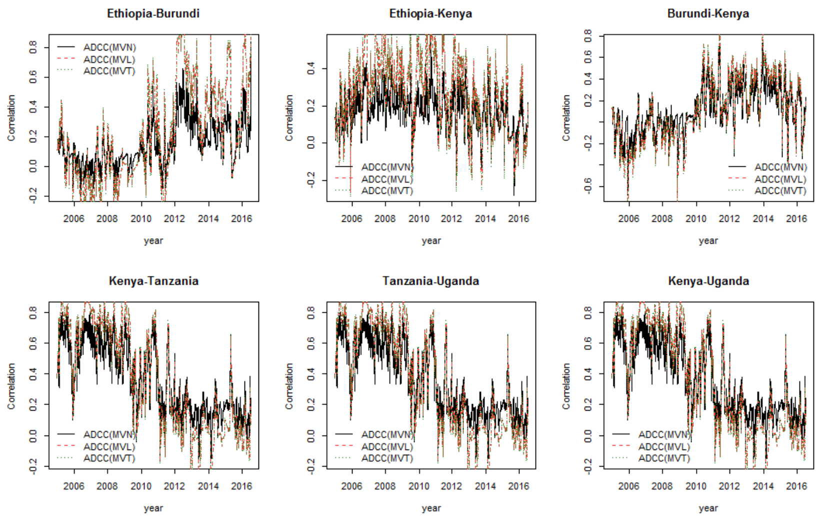

FIG. 7. The ADDC between exchange rate returns of East African countries under the multivariate normal (black), multivariate Laplace (red) and multivariate t (green) distributions

timated by the DCC and ADCC GARCH models are very volatile. The correlation estimate between Kenya & Ethiopia is exceptionally volatile.

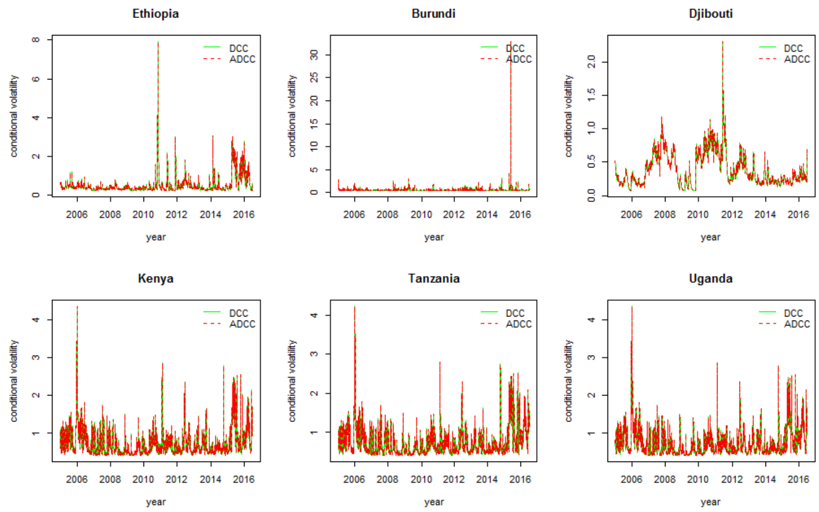

FIG. 8. Time plots of fitted conditional volatility for the daily log returns of the exchange rates under the multivariate t distribution.

Figure 8 shows the time plots of fitted conditional volatility for the daily log returns of the exchange rates under the multivariate t distribution for both DCC and ADCC models. The Burundi exchange rate return is less volatile when compared to the rest of the currency exchange rate returns. The Ethiopian currency exchange rate return has been less volatile before the crisis. However, the Djibouti, Kenyan, Tanzanian and Ugandan currency exchange rate returns have been highly volatile over the sample periods.

6. CONCLUSIONS

This paper examines the dynamic linkages among East African currency exchange rate data. To this effect, we used daily exchange rate returns observed over the January 1^{st} 2005 - June 30^{th} 2016 G.C. and has a total of 4199 observations. We estimated both univariate and multivariate time series models in terms of their ability to model East African currency exchange rate dynamics.

We estimate the East African currency exchange rate returns using three univariate GARCH models (the linear GARCH model and the nonlinear GJR-GARCH and EGARCH models) and the univariate SV model. The univariate GARCH models are estimated assuming both gaussian innovations and fat-tailed distributions, such as the Student's t and the GED. In terms of the log-likelihood function, the SV model is strongly favoured for Ugandan, Djibouti and Burundi exchange rate returns.

We also considered multivariate GARCH framework to model the time-varying correlations between exchange rate returns for both Gaussian returns and returns with heavy tails and skewness. These models are DCC-GARCH, DCC-EGARCH, DCC-GJR-GARCH, ADCC-GARCH, ADCC-EGARCH and ADCC-GJR-GARCH under the multivariate normal, multivariate Laplace and multivariate t distributions. We obtained the AIC, BIC and log-likelihood function values of the models in order to identify whether the DCC or ADCC fits the data better. According to the maximum value of the likelihood function and the minimum value of the information criteria, we conclude that DCC-GIR-GARCH model under the multivariate t distribution is the most appropriate model for modelling the correlation dynamics of exchange rate returns.

The paper reports positive pairwise correlations between the currency exchange rates of Ethiopia & Kenya, Kenya & Tanzania, Tanzania & Uganda, and Kenya & Uganda before the crisis. However, the pairwise correlations between the currency exchange rates of Ethiopia & Burundi and Burundi & Kenya are positive after the crisis. Moreover, there appears to have been a structural shift in the correlation between the currency exchange rates after the crisis.

The DCC-GIR-GARCH(1,1) parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ are significant implying that the conditional correlation between East African currency exchange rates is dynamic. Moreover, the pairwise conditional covariances between currency exchange rate returns are highly persistent since the sums of $\hat{\theta}_1$ and $\hat{\theta}_2$ are very close to 1.

Furthermore, this paper provides a better understanding of the correlation dynamics between East African countries exchange rate returns, which can be applied by policymakers and researchers. Understanding volatility and correlation dynamics between currency exchange rates/markets are important because it aids in decision making, asset pricing, risk management and portfolio allocation.

Lastly, there are a few points that can be improved in the future. For instance, we can compare the multivariate GARCH, Bayesian multivariate GARCH and multivariate stochastic volatility models. Since there appears to be a structural shift after the financial crisis, multivariate Markov switching dynamic conditional correlation GARCH models might be appropriate to model the correlation dynamics between the currency of East African exchange rate data.

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