Fiscal Policies in a Finite Horizon Model with the Spirit of Capitalism¹ Liutang Gong and Heng-fu Zou²

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Abstract

This paper examines the effects of government debt and deficit, income tax, and public investment on economy in a endogenous growth model with finite-horizon, the spirit of capitalism, and government expenditure. It shows that with the increasing of the spirit of capitalism and life horizon (the probability of death will be decreasing), the growth rate and the public investment (government expenditure-output ratio) will be increasing; but the consumption-capital ratio will be decreasing; The effects of income tax rate and public investment on growth rate and consumption-capital ratio appear Laffer curve style, thus we present the optimal income tax rate and public investment, which is different from Barro (1990), the optimal income tax rate and public investment will not be equal yet. The effects of government debt and social security on economy are included also.

Key Words: Government debt; Taxation; Public investment; The spirit of capitalism.

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1 Introduction

Since Barro (1990) examined the optimal income tax rate and optimal public investment in a simple endogenous growth model with government expenditure, many researchers focused on the topic of government expenditure and economic growth, for instance, Barro and Sala-i-Martin (1992); King and Rebelo (1990), Rebelo (1991), and Turnovsky and Fischer (1995), et al. They have studied the effects of government expenditure and public investment on economy in various infinite-horizon model. For the case of finite horizon, Saint-Paul (1992) presented the effects of fiscal policy in a simple endogenous growth model, and he showed that: 1) Any endogenous growth path is production-efficient. 2) An increase in public debt reduces the growth rate, so there always exists a future generation that will be harmed by such a measure. 3) An unfunded social security system necessarily reduces the growth rate. 4) A reduction in public debt, although it increase the growth rate cannot be Pareto-improving: one current generation must be harmed. and 5) A subsidy to investment or interest income raises the growth rate and can be pareto-improving. Kahn, Jong-Soo Lim, and Rhee Changyong (1997), Mourmouras and Lee (1999) also derive the similar solution in a endogenous growth model with finite-horizon and government expenditure. Recently, Fisher(2001) focus on the fiscal policy in a infinite-horizon model with the spirit of capitalism. Seldom works focus on the government expenditure, government debt, and public investment in a finite-horizon endogenous growth model with the spirit of capitalism.

Following with framework of Barro (1990), Saint-Paul (1992), and Mourmouras and Lee (1999), et al, this paper is to discuss the income tax rate, public investment, and government debt in a endogenous growth model with the finite-horizon, government expenditure, and the spirit of capitalism.

Among the neoclassical growth models, wealth accumulation is often taken to be solely driven by one's desire to increase consumption rewards. The representative agent chooses a consumption path to maximize his discounted utility, which is defined only on consumption. This motive is important for wealth accumulation. It is, however, not the only motive. Because man is a social animal, he also accumulates wealth to gain prestige, social status, and power in the society; see Frank (1985), Cole, Mailath, and Postlewaite (1992, 1995), Fershtman and Weiss (1993), Zou (1994, 1995), Bakshi and Chen (1996), and Fershtman, Murphy, and Weiss (1996). Earlier contributions include Duesenberry (1948), Kurz (1968), and Spence (1974). In these wealth-is-status models, the representative agent accumulates wealth not only for consumption but also for wealth-induced status, mathematically, in light of the new perspective, the utility function can be defined on both consumption, c, and wealth, w: $u(c_t, w_t)$. Another interpretation of these models is in line with the spirit of capitalism in the sense of Weber (1958) and Keynes (1971): capitalists accumulate wealth for the sake of wealth. To cite Weber (1958)³:

Man is dominated by making of money, by acquisition as the ultimate purpose of his life. Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs. This reversal relationship, so irrational from a naive point of view, is evidently a leading principle of capitalism.

Using the wealth-is-status and the-spirit-of-capitalism models, many authors have tried to explain growth, savings, and asset pricing. Cole, Mailath, and Postlewaite (1992) have demonstrated how the presence of social status leads to multiple equilibria in long-run growth. Zou (1994, 1995) has studied the spirit of capitalism and long-run growth and shows that a strong capitalist spirit can lead to unbounded growth of consumption and capital even under the neoclassical assumption of production technology. Bakshi and Chen (1996) have explored empirically the relationship between the spirit of capitalism and stock market pricing and offered an attempt towards the resolution of the equity premium puzzle in Mehra and Prescott (1985). They have shown that when investors care about status they will be more conservative in risk taking and more frugal in consumption spending. Furthermore stock prices tend to be more volatile with the presence of the spirit of capitalism. Gong and Zou (2002) introduce the spirit of capitalism in a stochastic model, and give the effects of fiscal policies and the spirit of capitalism on the economic growth and asset pricing relationship. They also enhance their model to a framework with non-expected utility function.

This paper organizes as followings: In section 2, we extend Barro(1990)'s model to the framework of finite horizons model with the spirit of capitalism and government debt. We derive the effects of government debt, government public investment, and income tax rate on the economy; Also, we examine the effects of the spirit of capitalism and the time horizon on the growth

 $^{^{3}\}mathrm{See}$ Cole, Mailath, and Postlewaite (1992); Zou (1994, 1995); and Bakshi and Chen (1996) for more details.

rate, consumption-capital ratio. In section 3, we extend Saint-Paul (1992)'s framework to the model with government expenditure and the spirit of capitalism. We present the effects of government debt and income tax rate on the growth and consumption-capital ratio. The effects of social security on economy are included. Finally, we summarize the paper in section 4.

2 Extension of Barro(1990)'s model

Following with Barro (1990), Mourmouras and Lee (1999), Blanchard (1985), and Weil (1989,1991), we set up a finite horizon model with the spirit of capitalism, the government expenditure, and government debt. We describe the model briefly.

2.1 The representative agent

At each time, there is a continuum of generations indexed by the date at which they are born, s, people have an infinite horizon but die with a constant probability per unit of time, p. At any instant of time, a large cohort, whose size is normalized to be p, is born⁴.

If the probability of death is constant, the expected remaining life for an agent of any age is given by $\int_0^\infty tp e^{-pt} dt = p^{-1}$. So, when p goes to zero, p^{-1} goes to infinity, agents have infinite horizons.

Following Blanchard (1985), Saint-Paul (1992), denote by c(s,t), y(s,t), w(s,t), h(s,t) consumption, non-interest income, nonhuman wealth, and human wealth of an agent born at time s, as of time t. Let r(t) be the interest rate at time t. Under the assumption that instantaneous utility is logarithmic, the agent maximizes

$$E_t \int_t^\infty (\log c(s, v) + \beta \log w(s, v)) e^{\theta(t-v)} dv, \qquad (1)$$

where $\theta > 0$ is the discounted rate, and the expectations are taken over the random life length of the individual. β measures the investor's concern with his social status or measures his spirit of capitalism. The larger the parameter β , the stronger the agent's spirit of capitalism or concern for social status.

⁴The same as Balnchard (1985), Saint-Paul (1992), and Barro (1990) et al, we suppose there is no population growth.

Given the constant probability of death p, maximizing the expected utility (1) is equivalent to maximizing

$$\int_{t}^{\infty} (\log c(s,v) + \beta \log w(s,v)) e^{(\theta+p)(t-v)} dv$$
(2)

The effective discount rate is therefore $\theta + p$. Noticing that even if θ is equal to zero, agent will discount the future if p is positive.

If an agent has wealth w(s,t) at time t, he receives r(t)w(s,t) in interest and pw(s,t) from the insurance company. Thus its dynamic budget constraint is

$$\frac{dw(s,t)}{dt} = (r(t) + p)w(s,t) + y(s,t) - c(s,t)$$
(3)

An additional transversality condition is needed to prevent agents from going infinitely into debt and protecting themselves by buying life insurance. Following Blanchard (1985), we impose a condition⁵

$$\lim_{v \to \infty} w(s, v) e^{-\int_t^v (r(\mu) + p)d\mu} = 0 \tag{4}$$

The agent is to choose his consumption path and wealth accumulation path to maximize the discounted utility subject to the dynamic budget constraint (3) and the transversality condition (4). Define the Hamiltonian associated with the optimization problem

$$H = \log c(s, v) + \beta \log w(s, v) + \lambda (r(t) + p)w(s, t) + y(s, t) - c(s, t))$$
(5)

where λ is the costate variable associate with the state variable w(s, t), and it represents the marginal utility of wealth.

The first-order conditions are

$$\frac{1}{c(s,t)} = \lambda \tag{6}$$

$$\frac{d\lambda}{dt} = -\lambda(r(t) - \theta) - \beta \frac{1}{w(s,t)}$$
(7)

and the transversality condition

⁵This condition is presented in Yaari (1965) and Blanchard (1985). This condition requires that the interest rate r is larger than -p.

$$\lim_{v \to \infty} \lambda w(s, v) e^{(\theta + p)(s - v)} = 0.$$

where equation (6) is the formally condition, which presents that the marginal utility of consumption equals the marginal utility of wealth, equation (7) is the familiar Euler equation.

From equations (6) and (7), we have

$$\frac{dc(s,t)}{dt} = x(t)c(s,t) \tag{8}$$

where x(t) is defined as

$$x(t) = r(t) - \theta + \beta \frac{c(s,t)}{w(s,t)}$$
(9)

Now, from equations (8), (9), (3), and condition (4), we can derive the express of consumption as a fraction of income, which is the sum of consumer's asset wealth and his human wealth

$$c(s,t) = \Delta(w(s,t) + h(s,t)) \tag{10}$$

where the human income h(s,t) is defined as

$$h(s,t) = \int_t^\infty y(s,v) e^{-\int_t^v (r(v')+p)dv'} dv$$

and the marginal propensity to consumption is defined as

$$\Delta = \left(\int_{t}^{\infty} e^{\int_{t}^{v} (x(v') - r(v') - p)dv'} dv\right)^{-1}.$$
(11)

Denote aggregate variables by the associated uppercase letters. The relation between any aggregate variable X(t) and an individual counterpart x(s,t) is

$$X(t) = \int_{-\infty}^{t} x(s,t) p e^{p(s-t)} ds$$

Let C(t), Y(t), W(t), and H(t) denote aggregate consumption, noninterest income, nonhuman wealth, and human wealth at time t, respectively. Then, from equation (10), aggregate consumption is given by

$$C = \Delta(W + H) \tag{12}$$

Human wealth is defined as

$$H(t) = \int_{-\infty}^{t} h(s,t) p e^{p(s-t)} ds$$

and it can be written as

$$H(t) = \int_{-\infty}^{t} (\int_{-\infty}^{t} y(s,v) p e^{p(s-v)} ds) e^{-\int_{t}^{v} r(v') dv'} dv$$
$$= \int_{-\infty}^{t} Y(v) e^{-\int_{t}^{v} (r(v')+p) dv'} dv$$

Human wealth is thus the present value of future labor income accruing to those currently alive. The same as Blanchard (1985). we assume for the moment that labor income is equally distributed (All agents work and have the same productivity): y(s, v) = Y(v) for all s. Thus, all agents have the same human wealth and it dynamic behavior can be written as

$$\dot{H} = (r+p)H - Y \tag{13}$$

and

$$\lim_{v \to \infty} H(v) e^{-\int_t^v (r(v')+p)dv'} = 0$$

And from equation (11), the dynamic equation for the marginal propensity to the consumption Δ can be deduced to

$$\frac{d\Delta}{dt} = (x(t) - r(t) - p)\Delta + \Delta^2$$
(14)

The same way, we derive the dynamic equations for the aggregate nonhuman wealth and the aggregate human wealth as

$$W = rW + Y - C \tag{15}$$

where individual wealth accumulates, for those alive, at rate r + p, aggregate wealth accumulates at a rate r. This is because the amount pW is a transfer through life insurance companies, from those who die to those who remain alive, it is not therefore an addition to aggregate wealth. From equation (12), (13), and (14), we find that aggregate consumption evolves over time according to

$$\dot{C} = x(t)C - pW\Delta \tag{16}$$

$$x(t) = r(t) - \theta + \beta \frac{C}{W}$$
(17)

Equations (13), (14), (15), (16), and (17) summarize the dynamic characters of aggregate consumption, nonhuman wealth, human wealth and the marginal propensity to the consume.

If the agent has infinite horizons, p = 0 and equation (16) reduces to $\dot{C} = (r(t) - \theta + \beta \frac{C}{W})C$, which is the standard consumption accumulation equation presented by Zou (1994,1995) and Kurz(1968). If we do not consider the spirit of capitalism, $\beta = 0$, then equation (16) reduces to

$$\dot{C} = (r(t) - \theta)C - p(p + \theta)\Delta$$

which appeared in Blanchard (1985).

2.2 Producers

Following Barro (1990), the government purchases a portion of the private output produced in the economy, and then uses these purchases to provide free public services to a single representative firm which stands in for a competitive industry. In other words, such productive services are complementary to private capital, something which raises the long-run growth rate of the economy. Let G be the quantity of productive government services measured in terms of the good produced in the economy. The production function of the representative firm is given by

$$Y = F(K, L, G) = AK^{\alpha}L^{1-\alpha}G^{1-\alpha}$$
(18)

where $0 < \alpha < 1$ is a positive constant, K is the capital stock, G is the government expenditure, and L is the labor input.

Equation (18) presents that the technology of the firm exhibits constant returns to scale in the private capital stock K and the labor input L, also exhibits constant returns to scale in the private capital stock K and the government expenditure G. And the marginal productivity of private capital stock and government expenditure are positive, and diminishing. G is a positive externality for the individual producer, and this is how a positive linkage between government and growth is potentially achieved in this model.

The firm's optimization problem is to choose the capital stock and labor input to maximize its profit

$$\max(1-\tau)F(K,L,G) - rK - wL$$

and we get

$$r = (1 - \tau)\alpha A \left(\frac{K}{G}\right)^{\alpha - 1} L^{1 - \alpha}$$
(19)

$$w = (1 - \tau)(1 - \alpha)AK(\frac{K}{G})^{\alpha - 1}L^{-\alpha}$$
(20)

In the following text, we select L = 1, so the human income can be written as $(1 - \alpha)AK(\frac{K}{C})^{\alpha - 1} = (1 - \alpha)Y$.

Equations (19) and (20) mean that the real interest rate equals the aftertax marginal productivity of capital stock, the wage rate equals the after-tax marginal productivity of labor.

2.3 Government

We introduce a government that spends on goods that do not affect the marginal utility of private consumption and finances spending either by lumpsum taxes or by debt. Its dynamic budget constraint is

$$D = rD - T + G \tag{21}$$

where D is debt, G is government public spending, and T is taxes. T - G represents government surplus, or government deficit. We also require the government spending satisfies the transversality condition⁶

$$\lim_{t \to \infty} D_t e^{-\int_t^v r(v)dv} = 0$$

This condition, together with the government's dynamic budget constraint (21), is equivalent to the statement that the level of debt is equal to the present discounted value of future surpluses.

⁶This condition is given in Blanchard (1985), which means that the interest rate r is nonnegative, at least asymptotically.

2.4 Equilibrium

The macroequilibrium of the economy can be summarized by equations (13)–(17) and (19)-(21). And the total wealth of private can be expressed as the sum of the capital stock and the government debt, W = K + D, So, we get

$$\dot{C} = (r(K) - \theta + \beta \frac{C}{K+D})C - p\Delta(K+D)$$
(22a)

$$\dot{K} = f(K,G) - C - G \tag{22b}$$

$$\frac{d\Delta}{dt} = (-\theta - p + \beta \frac{C}{K+D})\Delta + \Delta^2$$
(22c)

$$\dot{D} = r(K)D + G - T \tag{22d}$$

$$r = (1 - \tau)\alpha A(\frac{K}{G})^{\alpha - 1}$$
(22e)

$$f(K,G) = AK^{\alpha}G^{1-\alpha} \tag{22f}$$

First, we can also derive the same proposition as Saint-Paul, which rules out dynamic inefficiency on the production side of economy.

In any endogenous growth path such that the consumption share is asymptotically bounded away from zero, the growth rate will be strictly less than and bound away from the social marginal product of capital.

From equations (22a)-(22f), we can derive the dynamic characters of capital accumulation, consumption, government debt, and government expenditure. We can analysis the effects of income tax rate on the economy or public investment on economy. In the next section, we derive the growth rate of economy.

2.5 Determination of the growth rate

From equations (22a)-(22f), we have

$$\dot{C} = (\alpha A (\frac{K}{G})^{\alpha - 1} - \theta + \beta \frac{C}{K + D})C - p\Delta(K + D)$$
$$\dot{K} = A K^{\alpha} G^{1 - \alpha} - C - G$$

$$\frac{d\Delta}{dt} = (-\theta - p + \beta \frac{C}{K+D})\Delta + \Delta^2$$
$$\dot{D} = r(K)D + G - T$$

Following with Saint-Paul, we assume that the government maintains a constant debt/GDP ratio, is equals to δ . Along with the Balanced growth path, the endogenous variable, capital stock, consumption, human capital will grow at constant growth rates, and we can prove that growth rate are equal, and we denote the common constant ϕ , the efficient discounted rate Δ will convergent to a constant, so we get

$$\phi = (1 - \tau)\alpha A(\frac{K}{G})^{\alpha - 1} - \theta + \beta \frac{C/K}{1 + \delta A(\frac{K}{G})^{\alpha - 1}} - p\Delta \frac{K}{C}(1 + \delta A(\frac{K}{G})^{\alpha - 1})$$
$$\phi = A(\frac{K}{G})^{\alpha - 1} - \frac{C}{K} - \frac{G}{K}$$
$$\Delta = (\theta + p - \beta \frac{C/K}{1 + \delta A(\frac{K}{G})^{\alpha - 1}})$$
$$\phi = (1 - \tau)\alpha A(\frac{K}{G})^{\alpha - 1} + \frac{1}{A(\frac{K}{G})^{\alpha}} \frac{1}{\delta} - \frac{\tau}{\delta}$$

Thus, we have

$$(1-\tau)\alpha A(\frac{K}{G})^{\alpha-1} + \frac{1}{A(\frac{K}{G})^{\alpha}} \frac{1}{\delta} - \frac{\tau}{\delta}$$

$$= ((1-\tau)\alpha A(\frac{K}{G})^{\alpha-1} - \theta + \beta \frac{(1-\alpha(1-\tau))A(\frac{K}{G})^{\alpha-1} - \frac{1}{A(\frac{K}{G})^{\alpha}} \frac{1}{\delta} + \frac{\tau}{\delta} - \frac{G}{K}}{1+\delta A(\frac{K}{G})^{\alpha-1}}) (23a)$$

$$-p(\frac{(\theta+p)(1+\delta A(\frac{K}{G})^{\alpha-1})}{(1-\alpha(1-\tau))A(\frac{K}{G})^{\alpha-1} - \frac{1}{A(\frac{K}{G})^{\alpha}} \frac{1}{\delta} + \frac{\tau}{\delta} - \frac{G}{K}}{-\beta})$$

$$(1-\alpha)A(\frac{K}{G})^{\alpha-1} - \frac{1}{A(\frac{K}{G})^{\alpha}} \frac{1}{\delta} + \frac{\tau}{\delta} - \frac{G}{K} = \frac{C}{K}$$
(23b)

$$\phi = (1 - \tau)\alpha A (\frac{K}{G})^{\alpha - 1} + \frac{1}{A(\frac{K}{G})^{\alpha}} \frac{1}{\delta} - \frac{\tau}{\delta}$$
(23c)

From equation (23a), we can determine the steady-state capital stock to government expenditure, then from equation (23c), we can determine the long-run growth rate; finally, we can determine the steady-state consumption capital ratio from equation (23b).

We cannot derive the explicit solutions from the capital-government ratio, consumption-capital ratio, and the growth rate from equations (23a)-(23c). But we can present the numerical solutions from equations (23a)-(23c):

If we select the parameters: A = 0.3, $\alpha = 0.75$, $\delta = 0.05$, p = 0.05, $\theta = 0.05$, g = 0.25, and $\beta = 1$, we derive the growth rate $\phi = 0.03831$, consumption-capital ratio $\frac{C}{K} = 0.0565$, capital-government expenditure ratio $\frac{K}{G} = 31.9$, and the government public investment: government expenditure-output ratio $\frac{G}{Y} = 0.2484$.

2.6 The comparative solutions

In this subsection, we analysis the effects of the spirit of capitalism, time horizon, income tax rate, and the government debt on the economic growth.

2.6.1 The case without the spirit of capitalism

If we do not including the spirit of capitalism, i.e. we let $\beta = 0$ in the above model, we need change equation (23a) into

$$\phi = (\alpha A(\frac{K}{G})^{\alpha - 1} - \theta) - p(\theta + p) \frac{1 + \delta A(\frac{K}{G})^{\alpha - 1}}{A(\frac{K}{G})^{\alpha - 1} - \phi - \frac{G}{K}}$$
(23a')

other equations remains the same.

If we select the parameters: A = 0.5, $\alpha = 0.75$, $\delta = 0.05$, p = 0.05, $\theta = 0.05$, g = 0.25, and $\beta = 0$, we e derive the growth rate $\phi = 0.052343$, consumption-capital ratio $\frac{C}{K} = 0.135144$, capital-government expenditure ratio $\frac{K}{G} = 16.3809$, and the government public investment: government expenditure-output ratio $\frac{G}{Y} = 0.245627$.

We present the comparative solutions in figure 1e and figure 1f, where we select the parameters as: A = 0.3, $\alpha = 0.75$, $\delta = 0.05$, p = 0.05, $\theta = 0.05$, $\beta = 1$ or $\beta = 5$, and τ varies from 0.05 to 0.40. And from the figures, we know that with the increasing of the spirit of capitalism (β increasing from 1 to 5), the growth rate and the government-output ratio will be increasing; but the consumption-capital ratio will be decreasing. This can be explained as: with the increasing of the spirit of capitalism, the marginal utility of wealth will be increasing, the consumer will decrease the consumption-capital and investment, so the consumption-capital will be decreasing; with the increasing of investment, the growth rate will be

enlarge; and the government expenditure-output ratio (public investment) will be increasing.

2.6.2 The case of infinite horizon

In this subsection, we examine the effects of the horizon on the economy, if we select p = 0, the representative agent will have infinite horizon, thus, we get

$$\phi = (1-\tau)\alpha A(\frac{K}{G})^{\alpha-1} - \theta + \beta \frac{A(\frac{K}{G})^{\alpha-1} - \phi - \frac{G}{K}}{1 + \delta A(\frac{K}{G})^{\alpha-1}}$$
(23a")

Equations (23a"), (23b), and (23c) determine the equilibrium capitalgovernment ratio, consumption-capital ratio, and the growth rate.

If we select the parameters: A = 0.3, $\alpha = 0.75$, $\delta = 0.05$, p = 0, $\theta = 0.05$, $\tau = 0.25$, and $\beta = 1$, we e derive the growth rate $\phi = 0.052559$, consumption-capital ratio $\frac{C}{K} = 0.0370083$, capital-government expenditure ratio $\frac{K}{G} = 31.7289$, and the government public investment: government expenditure-output ratio $\frac{G}{Y} = 0.249339$.

Compare with the finite horizon, p = 0.05, we find that with the increasing of horizon (the small of the parameter p), the growth rate and the government expenditure-output (public investment) will be increasing, but the consumption-capital stock will be decreasing. This is because that with the increasing of the horizon of the representative agent, he will smooth his consumption among longer life time, (with the decreasing of the probability of death, the consumer will leave more consumption later), thus the consumption-capital will be decreasing; with the decreasing of consumption, the savings will be increasing, so the private investment, thus the growth rate will be increasing; firgure1c-figure 1d show the comparative solution.

Furthermore, if we let $\delta = 0$ and $\beta = 0$, we will deduce to the model of Barro (1990). Which we can find the explicit growth rate

$$\phi = (1 - \tau)\alpha (\frac{1}{\tau A})^{\alpha - 1} - \theta \tag{24}$$

Which we can compute the consumption-capital ratio and capital-government expenditure ratio explicit.

2.6.3 The effects of government debt

Figure 7-figure 9 present the effects of government debt on the growth rate, consumption-capital ratio, and the government expenditure-output ratio (public investment). Where we compare the effects while government-output ratio increase from 0.05 to 0.5, the growth rate will be decreasing, the consumption-capital will be decreasing, and government expenditure-output ratio will be decreasing.

The reason can be explained as: with the increasing of government debt, private capital will be decreasing, this leads the reducing of consumptioncapital ratio and the output,

2.6.4 The optimal income tax

Barro (1990) present the optimal income tax rate from the maximizing the social welfare, because of the equivalency, he maximized the growth rate, i.e. maximize the growth rate in equation (24) with respect to the income tax rate τ , and easy calculated

$$\tau = 1 - \alpha \tag{25}$$

In this paper, we cannot derive the explicit solution, we present the effects of income tax rate on the growth rate in figure 1a, figure 1c, and figure 1e, we find that the effects of income tax rate on the growth appears the Laffer's curve, namely, there exist a critical income tax rate, τ^* , the growth rate will be increasing when $\tau < \tau^*$; the growth rate will be decreasing when $\tau > \tau^*$. Thus, the critical point is the optimal tax rate, from the figure 1, when $\beta = 1$, we find that when tax rate increasing from 0.05 to 0.25, the growth rate will increasing from 0.015 to 0.038; while tax rate increasing from 0.25 to 0.4, the growth rate will be decreasing from 0.38 to 0.33, thus the optimal income tax rate $\tau = 0.25$. Conside the case $\beta = 5$ in figure 1, p = 0 or p = 0.05 in figure 2, and $\delta = 0.05$ or $\delta = 0.5$ in figure 3, we get the same results. This is also consistence with the result present in Barro (1990), the optimal tax rate $\tau = 1 - \alpha$.

Figure 1b, 1d, and 1f present the effects of income tax rate on the consumption-capital ratio. The are also appears Laffer curve's effects, this is because that with the increasing of income tax rate, the cost of investment will be increasing, this leads the decreasing of capital stock, thus the consumption-capital ratio is increasing.

Figure 2a–2c shows the effects of income tax rate on the government expenditure-output ratio, we find that with the increasing of income tax rate, the government-output ratio will be increasing, this is obviously. With the increasing of income tax rate, government income increase, then the government-output ratio increase.

2.7 The case of nonseparable utility function

In the above section, we discuss fiscal policy with the separable utility function. In this subsection, we select the nonseparable utility function

$$u(c,w) = \frac{c^{1-\sigma}w^{-\beta}}{1-\sigma}$$

where $\sigma > 0$, and $\beta \ge 0$ when $\sigma \ge 1$, and $\beta < 0$ otherwise; $|\beta|$ measures the investor's concern with his social status or measures his spirit of capitalism. The larger the parameter $|\beta|$, the stronger the agent's spirit of capitalism or concern for social status.

The same technology, we can derive the dynamics of aggregate consumption, C, aggregate capital stock, K, and the marginal propensity to the consumption Δ

$$\dot{C} = \left(\frac{1}{\sigma}\left((1-\tau)\alpha A\left(\frac{K}{G}\right)^{\alpha-1} - \theta - \frac{\beta}{1-\sigma}\frac{C}{K+D} - \beta\left(\frac{W}{W} + p\right)\right)C - p\Delta(K+D)$$
$$\dot{K} = AK^{\alpha}G^{1-\alpha} - C - G$$
$$\frac{d\Delta}{dt} = \left(-\theta - p + \beta\frac{C}{K+D}\right)\Delta + \Delta^{2}$$
$$\dot{D} = r(K)D + G - T$$

Thus, we derive the growth rate, consumption-capital ratio, and the government expenditure-output ratio satisfy that

$$\phi = \frac{1}{\sigma} \left((1-\tau)\alpha A \left(\frac{K}{G}\right)^{\alpha-1} - \theta - \frac{\beta}{1-\sigma} \frac{C}{K} - \beta \left(\frac{K}{G}\right)^{\alpha-1} - \beta \left(\phi + p\right) \right)$$
(26a)
$$-p(\theta+p) \left(1+\delta A \left(\frac{K}{G}\right)^{\alpha-1}\right) \frac{K}{C} - \beta \right)$$
$$(1-\alpha)A \left(\frac{K}{G}\right)^{\alpha-1} - \frac{1}{A \left(\frac{K}{G}\right)^{\alpha}} \frac{1}{\delta} + \frac{\tau}{\delta} - \frac{G}{K} = \frac{C}{K}$$
(26b)

$$\phi = (1 - \tau)\alpha A \left(\frac{K}{G}\right)^{\alpha - 1} + \frac{1}{A(\frac{K}{G})^{\alpha}} \frac{1}{\delta} - \frac{\tau}{\delta}$$
(26c)

If we select parameters: A = 0.3, $\alpha = 0.75$, $\delta = 0.05$, p = 0.05, $\theta = 0.05$, $\tau = 0.25$, $\sigma = 2$, and $\beta = 1$, we e derive the growth rate $\phi = 0.0112347$, consumption-capital ratio $\frac{C}{K} = 0.0836485$, capital-government expenditure ratio $\frac{K}{G} = 32.1268$, and the government public investment: government expenditure-output ratio $\frac{G}{Y} = 0.247018$.

We also can derive the comparative static solution of various parameters, we find the similar effects of government debt, the spirit of capitalism, the horizon, and the optimal tax rate as before.

2.8 The effects of an increase in public investment

Barro(1990) presented the effects of government public investment on the economy. Without government debt, the budget constraints of government can be simplified as

$$G = \tau Y$$

Thus, if we define government expenditure-output ratio as g,i.e. G/Y = g, then we have

$$g = \tau$$

thus the optimal income tax rate and the government expenditure-output ratio are equal. But, in this model, we cannot get this simple relationship between the income and the government expenditure-output ratio. Suppose the government expenditure-output ratio is constant, we get

$$\phi = \frac{1}{\sigma} \left((1-\tau)\alpha A\left(\frac{K}{G}\right)^{\alpha-1} - \theta - \frac{\beta}{1-\sigma} \frac{C/K}{1+\delta A\left(\frac{K}{G}\right)^{\alpha-1}} - \beta(\phi+p) \right) (27a)$$
$$-p\left(\frac{(\theta+p)\left(1+\delta A\left(\frac{K}{G}\right)^{\alpha-1}\right)}{C/K} - \beta\right)$$
$$\phi = A\left(\frac{K}{G}\right)^{\alpha-1} - \frac{C}{K} - \frac{G}{K}$$
(27b)

where $\frac{K}{G}$ and τ can be derived from equations

$$g = A(\frac{K}{G})^{\alpha} \tag{27c}$$

$$\phi = (1 - \tau)\alpha A \left(\frac{K}{G}\right)^{\alpha - 1} + \frac{1}{A\left(\frac{K}{G}\right)^{\alpha}} \frac{1}{\delta} - \frac{\tau}{\delta}$$
(27d)

We cannot find the explicit solution, we present the numerical solution: If we select parameters: A = 0.3, $\alpha = 0.75$, $\delta = 0.05$, p = 0.05, $\theta = 0.05$, g = 0.25, $\sigma = 2$, and $\beta = 1$, we derive the growth rate $\phi = 0.0115233$, and consumption-capital ratio $\frac{C}{K} = 0.0833629$. If we increase government debt-output ratio to $\delta = 0.5$, we derive the growth rate $\phi = 0.00863026$, and consumption-capital ratio $\frac{C}{K} = 0.0862$.

We present the effects of debt and public investment (government expenditureoutput ratio) on the growth rate, consumption-capital stock ratio in figure 13-18.

Figures 4a and 4b present the effects of public investment on the growth rate, we get the Laffer curve of effects of public investment on the growth rate. There exists a critical point, we denote g^* , the growth rate will be increasing when $g < g^*$; the growth rate will decreasing while $g > g^*$. the critical point g^* is called the optimal public investment level. In figure 4a, when $\delta = 0.5$, we find that, when public investment increasing from 0.05 to 0.24, the growth rate will increase from 0 to 0.8%, when δ increasing from 0.24 to 0.4, the growth rate will decreasing from 0.8% to 0.6%, the optimal public investment =0.24; The solid curve present the similar effects of public investment on growth when $\delta = 0$, but the optimal public investment will be 0.25. Figure 4b present the effects of public investment on growth rate under p = 0 or p = 0.05. They show the similar Laffer's curve's effects, and the optimal investment will be around 0.25. Compare with the effects of public investment on growth rate presented by Barro (1990) and (2000), the effects are similar, but the optimal public investment will not satisfy $\tau = g$ yet.

Figures 4c and 4d present the effects of public investment on the consumptioncapital ratio, we also derive the Laffer's curve effects of public investment on consumption-capital ratio. Different from Barro (1990) and Mourmouras and Lee (1999), the critical point will not be 0.25 when government debt is included.

The above effects can be explained as: when the public investment is small, the marginal productivity of government expenditure will be larger, with the increase of government expenditure, the public investment will be increase, the output will be increasing, and the growth rate will be increasing, but with the increasing of output, the consumption will be increasing, thus the consumption-capital stock will be increasing. With the increasing of government expenditure, the marginal productivity of government expenditure will be decreasing, thus there exists a critical point, on the right of this point, the government expenditure crowd out the private investment, thus decreasing the growth rate. The optimal public investment equals the marginal productivity of government expenditure without government debt. With the increasing of government debt, the optimal public investment will decreasing.

Figure 4e-4f present the effects of government debt on the growth rate and consumption-capital ratio, we find that, with the increasing of government debt, the growth rate will be decreasing, and the consumption-capital ratio will be increasing. These are consistence with the results of Saint-Paul (1992) presented.

3 Government expenditure in Saint-Paul's model

In this section, we fellow Saint-Paul (1992)'s discussion, extend this model to a model with government expenditure and the spirit of capitalism, discuss the effects of public investment and government debt on the economic growth and the consumption.

3.1 The model

Following Saint-Paul, each individual has a labor endowment that is equal to v + p when he is born and declines at a rate v, so that the total labor endowment in the economy is one. Suppose the human wealth can be defined as the present discounted value of future gross wage income hs(s,t) minus the PDV of future taxes ht(s,t)

$$h(s,t) = hs(s,t) - ht(s,t)$$
(28)

and

$$hs(s,t) = \int_{t}^{\infty} (p+\nu)\omega_{\nu}e^{-\nu(\nu-s)}e^{-\int_{t}^{\nu}(r(\mu)+p)d\mu}dv$$
 (29a)

$$ht(s,t) = \int_{t}^{\infty} \tau(v,s) e^{-v(v-s)} e^{-\int_{t}^{v} (r(\mu)+p)d\mu} dv$$
 (29b)

where $\tau(v, s)$ is the taxes in time v, ω_v is the wage rate at time v.

After aggregation, we have

$$H = HS - HT \tag{30}$$

$$\dot{HS} = (r+p+v)HS - \omega_t \tag{31}$$

The same assumption for the production function

$$Y = F(K, L, G) = AK^{\alpha}L^{1-\alpha}G^{1-\alpha}$$

and from the firm's optimization problem we get the first-order conditions

$$r = \alpha A \left(\frac{K}{G}\right)^{\alpha - 1} L^{1 - \alpha} \tag{32}$$

$$w = (1 - \alpha)AK(\frac{K}{G})^{\alpha - 1}L^{-\alpha}$$
(33)

Now, the wealth W = K+D, and the dynamic characters of the economy can be summarized as equations (11), (22b), (22c), (22d), (30), (31), (32), and (33). We will determine the growth rate from these equations

3.2 Determination of the growth rate

In this section, we solve for a balanced growth path of the model in the case where the tax rate is constant τ . We suppose that $\tau(s,t) = \tau \omega_t (p+v) e^{-v(s-t)}$, $T_t = \tau \omega_t$, and $HT = \tau HS$.

Following with Saint-Paul, we assume that the government maintains a constant debt/GDP ratio, is equals to δ . Public investment is defined as the government expenditure-output ratio, $\frac{G}{Y} = g$. Along with the Balanced growth path, the endogenous variable, capital stock, consumption, human capital will grow at constant growth rates, and we can prove that growth rate

are equal, and we denote the common constant ϕ , the efficient discounted rate Δ will convergent to a constant,

First, from equation (22c), we know that

$$\Delta = \theta + p - \beta \frac{C}{K+D}$$

and from equations (30) and (31), we have

$$\frac{HS}{Y} = \frac{1-\alpha}{r+p+\upsilon-\phi}$$

and from equation (11), we have

$$\frac{C}{K} = (\theta + p - \beta \frac{C/K}{1 + \delta A(\frac{K}{G})^{\alpha - 1}})(1 + \delta A(\frac{K}{G})^{\alpha - 1} + A(\frac{K}{G})^{\alpha - 1} \frac{(1 - \tau)(1 - \alpha)}{r + p + \upsilon - \phi})$$

So, we get

$$\frac{C}{K} = \frac{(\theta+p)(1+\delta A(\frac{K}{G})^{\alpha-1} + A(\frac{K}{G})^{\alpha-1}\frac{(1-\tau)(1-\alpha)}{r+p+\nu-\phi})}{1+\beta\frac{1}{1+\delta A}(1+\delta A(\frac{K}{G})^{\alpha-1} + A(\frac{K}{G})^{\alpha-1}\frac{(1-\tau)(1-\alpha)}{r+p+\nu-\phi})}$$

Thus, from equation (22b), we derive

$$\frac{\dot{K}}{K} = A(\frac{K}{G})^{\alpha-1} - \frac{C}{K} - \frac{G}{K}$$

and the express of growth rate can be determined by the equation

$$\phi = A \left(\frac{K}{G}\right)^{\alpha - 1} - \frac{(\theta + p)(1 + \delta A(\frac{K}{G})^{\alpha - 1} + A(\frac{K}{G})^{\alpha - 1}\frac{(1 - \tau)(1 - \alpha)}{r + p + v - \phi})}{1 + \beta \frac{1}{1 + \delta A(\frac{K}{G})^{\alpha - 1}}(1 + \delta A(\frac{K}{G})^{\alpha - 1} + A(\frac{K}{G})^{\alpha - 1}\frac{(1 - \tau)(1 - \alpha)}{r + p + v - \phi})} - \frac{G}{K}$$
(34)

Also, from equation (22b), we can express taxation as the function of debetoutput ratio.

$$\phi = \alpha A \left(\frac{K}{G}\right)^{\alpha - 1} - \tau \left(1 - \alpha\right) \frac{1}{\delta} + \frac{1}{A\left(\frac{K}{G}\right)^{\alpha}} \frac{1}{\delta}$$
(35)

where $\frac{K}{G} = (\frac{1}{Ag})^{\frac{1}{\alpha}}$ and $\tau = \frac{\delta \alpha A(\frac{K}{G})^{\alpha-1} - \delta \phi + g}{(1-\alpha)}$. If we select parameters: $A = 0.3, \ \alpha = 0.75, \ \delta = 0.05, \ p = 0.05, \ \theta = 0.05$,

If we select parameters: A = 0.3, $\alpha = 0.75$, $\delta = 0.05$, p = 0.05, $\theta = 0.05$, g = 0.25, v = 0.03, and $\beta = 1$, we derive the growth rate $\phi = 0.044631$, and consumption-capital ratio $\frac{C}{K} = 0.0502552$. If we increase government

debt-output ratio to $\delta = 0.5$, we derive the growth rate $\phi = 0.0423576$, and consumption-capital ratio $\frac{C}{K} = 0.0525286$.

We present the effects of government debt and public investment (government expenditure-output ratio) on the growth rate, consumption-capital stock ratio in figure 5.

Figure 5a and 5b present the effects of public investment on the growth rate, we get the Laffer curve of effects of public investment on the growth rate. There exists a critical point, we denote g^* , the growth rate will be increasing when $g < g^*$; the growth rate will decreasing while $g > g^*$. the critical point g^* is called the optimal public investment level. we find when $\delta = 0.5$, the optimal public investment =0.29, and when $\delta = 0$, the optimal public investment will be 0.30. Thus, with the increasing of the government debt, the optimal public investment will be decreasing.

Figure 5c and 5d present the effects of public investment on the consumptioncapital ratio, we also derive the Laffer's curve effects. And we derive the different effects of public investment on the consumption-capital ratio from Barro (1990) and Mourmouras and Lee (1999). The critical point will not be 0.25 when government debt is included.

The above effects can be explained as: when the public investment is small, the marginal productivity of government expenditure will be larger, with the increase of government expenditure, the public investment will be increase, the output will be increasing, and the growth rate will be increasing, but with the increasing of output, the consumption will be increasing, thus the consumption-capital stock will be increasing. With the increasing of government expenditure, the marginal productivity of government expenditure will be decreasing, thus there exists a critical point, on the right of this point, the government expenditure crowd out the private investment, thus decreasing the growth rate. The optimal public investment equals the marginal productivity of government expenditure without government debt. With the increasing of government debt, the optimal public investment will decreasing.

Figures 5e-5f present the effects of government debt on the growth rate and consumption-capital ratio, we find that, with the increasing of government debt, the growth rate will be decreasing, and the consumption-capital ratio will be increasing. These are consistence with the conclusion of Saint-Paul (1992) presented.

The same as in Saint-Paul (1992), we consider the impact of a social security system. Fellow Saint-Paul (1992), we assume that the income pro-

file is now equivalent to what would be obtained if labor endowment was declining at a rate v > v, instead of v and if the wage rate was $\varepsilon \omega$ instead of ω . The model is described by the same equations, except that v is replaced by v in equation (34), we present the effects of v on the economic growth, we find the growth rate will be decreasing with the social security system, the consumption-capital stock will be increasing, this is consistence with the finds as Saint-Paul (1992), where he absent the government expenditure and the spirit of capitalism. Figures 6a and 6b present these effects.

4 Conclusion remarks

This paper present two models to discuss the effects of government debt, government public investment, and income tax on economy, one model extend Barro (1990)'s work to the a finite-horizon model with the spirit of capitalism; another model extend Saint-Paul (1992)'s work, including government expenditure and the spirit of capitalism.

In the first model, we cannot derive the explicit solution for the growth rate. From the numerical solution, we find that with the increasing of the spirit of capitalism, the growth rate and the government-output ratio will be increasing; but the consumption-capital ratio will be decreasing. With the increasing of horizon (the small of the parameter p), the growth rate and the government expenditure-output (public investment) will be increasing, but the consumption-capital stock will be decreasing.

As for the effects of income tax rate on economy, we present the Laffer curve's effects on the growth rate and the consumption-capital stock ratio: there exists a critical point τ^* , with the increasing of income tax rate, the growth rate and the consumption -capital ratio will be increasing when $\tau < \tau^*$; the growth rate and consumption-capital ratio will be decreasing when $\tau > \tau^*$, thus the income tax rate τ^* reaches the maximal growth rate, it is the optimal income tax rate. From the numerical solution, we find the optimal income tax rate still satisfy $\tau = 1 - \alpha$, which is the same as Barro (1990) presented, it shows that the optimal income tax rate satisfy that the marginal productivity of public expenditure equals the income tax rate.

The effects of public investment on the growth rate and consumptioncapital ratio also be discussed in the first model. Similar to Barro (1990), we still get the Laffer curve's effects of public investment on the growth rate and the consumption-capital ratio. there exists a critical point g^* , with the increasing of public investment rate, the growth rate and the consumption -capital ratio will be increasing when $g < g^*$; the growth rate and consumption-capital ratio will be decreasing when $g > g^*$, thus the public investment rate g^* reaches the maximal growth rate, it is the optimal public investment rate. Different from Barro (1990), the optimal public investment rate will not satisfy that $g = \tau$ when government debt is included.

We also present the effects of government debt on the growth rate and consumption-capital ratio, we find that with the increasing of government debt, the growth rate will be decreasing, and the consumption-capital ratio will be increasing. These are consistence with the conclusion of Saint-Paul (1992) presented.

In the second model, we extend Saint-Paul (1992) model to the model including government expenditure and the spirit of capitalism, we find similar effects of public investment on the growth rate and the consumption-capital ratio, they are also appears Laffer curve style effects, but the optimal public investment will not satisfy $g = \tau$ yet.

For the effects of social security on the economic, we find that with the increasing of social security, the growth rate will be decreasing, but the consumption- capital ratio will be increasing.

Further research should focus on the empiric test of public investment, government debt, and the time horizon on economy. Also, we should extend these models to the reference with the habit formation, catch up with Jones et al.

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Figure 1: (a) income tax rate versus growth rate;(b) income tax rate versus consumption-capital ratio;(c)income tax rate versus growth rate; (d) income tax rate versus consumption-capital ratio;(e)income tax rate versus growth rate;(f) income tax rate versus consumption-capital ratio.



Figure 2: (a) income tax rate versus government expenditure-output ratio;(b) income tax rate versus government expenditure-output ratio;(c)income tax rate versus government expenditure-output ratio.



Figure 3: (a) income tax rate versus growth rate; (b) income tax rate versus consumption-capital ratio; (c) income tax rate versus government expenditure-output ratio.



Figure 4: (a) public investment versus growth rate; (b) public investment versus growth rate; (c) public investment versus consumption-capital ratio; (d) public investment versus consumption-capital ratio; (e) government debt versus growth rate; (f) government debt versus consumption-capital ratio.



Figure 5: (a) public investment versus growth rate; (b) public investment versus growth rate; (c) public investment versus consumption-capital ratio; (d) public investment versus consumption-capital ratio; (e) government debt versus growth rate; (f) government debt versus consumption-capital ratio.



Figure 6: (a) social security versus growth rate; (b) social security versus consumption-capital ratio.