Fiscal Policies in a Stochastic Model with Hyperbolic Discounting

Liutang Gong* and Heng-fu Zou**

Abstract

In this paper, we study the effects of fiscal policies on economy in a stochastic model with hyperbolic discounting rate. With specific assumptions on the production technology, preferences, and stochastic shocks, we derive the explicit solutions to the growth rates of consumption and savings and equilibrium returns on all assets, and give the effects of fiscal policies and discounting rate on growth.

Keywords: Fiscal policies; Hyperbolic discounting; Stochastic growth.

JEL classification: D91, G11, H0, O0.

* Peking University and Wuhan University, ** Peking University, Wuhan University, and the World Bank, Correspondence may be addressed to Bill Smith, Department of Economics, Fogelman College of Business and Economics, University of Memphis, Memphis, TN 38152, USA. e-mail: wtsmith@memphis.edu. Phone: (901) 678-3675.

1. Introduction

The effects of fiscal policies on economic performance have been widely studied for many years. Among the existing literatures, the stochastic framework becomes more and more important. Eaton (1981), Turnovsky (1993, 2000), Grinols and Turnovsky (1993, 1994), and Obstfeld (1994) have introduced stochastic tax and stochastic government expenditure into the continuous-time growth and asset-pricing models. Under specific assumptions on the production technology, preferences, and stochastic shocks, they have derived explicit solutions to the growth rates of consumption and savings and equilibrium returns on assets. Grinols and Turnovsky (1998) studied the optimal tax and monetary policies in a stochastic monetary growth model, and they found that: the capital income taxes and monetary growth influence the economy through effective risk-adjusted measure, and it can be expressed as a linear function of their mean and variance; money and bonds are neutral are shown in their model; and they presented the optimal tax policies and monetary policies. Gong and Zou (2002) introduce the spirit of capitalism to a stochastic model, and give the effects of fiscal policies and the spirit of capitalism on the economic growth and asset pricing relationship. They also enhance their model to a framework with non-expected utility function.

Among these existing literatures, the basic framework rely on the assumption that households have a constant rate of time preference. Laibson (1994,1997,1998), motivated partly by introspection and by experimental findings, has made compiling observations about ways in which rates of time preference vary. He argues that individuals are highly impatient about consuming between today and tomorrow but are much more patient about choice advanced further in the future, for example, between 365 and 366 days from now. Hence, rate of time preference would be very high in the short run but much lower in the long run, as viewed from today's perspective. Barro (1999) extended the empirical content of Ramsey model with Laibson-style preference to study the consumption and saving behaviors under full commitment and no commitment. Krusell and Smith (2001) studied the consumption-savings problem of an infinitely-lived, rational consumer who has time-inconsistent preference in the form quasi-geometric discounting, they presented a continuum equilibria for the steady state capital stock. Palacios-Huerta (2001) studied the optimal consumption and portfolio rules under hyperbolic discounting. Petersen (2001) studied the equilibrium tax policy with hyperbolic consumers. Few work focus on effects of fiscal polices on the economy in stochastic model with hyperbolic discounting. The important matter in this paper is to understand the effects of fiscal policies on economy with the Laibson's discounting function in a stochastic growth model.

This paper extend Turnovsky (2000) and Gong and Zou (2002)'s model to a stochastic growth model with hyperbolic discounting. With specific assumptions on the production technology, preferences, and stochastic shocks, we derive the explicit solutions to the growth rates of consumption and savings and equilibrium returns on all assets. We further demonstrate how fiscal policies and stochastic shocks affect economic growth, following.

The paper is organized as follows: in section 2, we present a modified growth and asset-pricing framework as in Turnovsky (2000) and Gong and Zou (2002). In section 3, we derive the optimal conditions for macroeconomic equilibrium. In section 4, using a specific utility function, we present explicit solutions to the consumption-wealth ratio, the mean growth rate of the economy, and the expected real return on bonds and capital. In section 5, we discuss the effects of stochastic shocks, fiscal policies, and the varies of discounting rate on the economy. We conclude the paper in section 6.

2. The Model

Along with Eaton (1981) and Turnovsky (2000), we assume output Y and government expenditure G to be proportional to the mean-level output, i.e.

$$dY = \alpha K dt + \alpha K dy \,, \tag{1}$$

$$dG = g\alpha K dt + \alpha K dz \,, \tag{2}$$

where $0 < \alpha < 1$ and $0 < g < 1$ are constants.

Equation (1) asserts that the accumulated flow of output over the period $(t, t+dt)$, given by the right-hand side of this equation, consists of two components. The deterministic component is described as the first term on the right hand, which is the firm's production technology and has been specified as a linear production function. The second part is the stochastic component, which can be viewed as the shock to the production and assumed to be temporally independent, normally distributed, and

$$E(dy) = 0, \mathrm{var}(dy) = \sigma_y^2 dt \,.$$

In equation (2), the deterministic part of government expenditure is expressed in terms of a fraction of mean output, and government expenditure has the stochastic shock dz. It is further assumed that dz is temporally independent, normally distributed, and

$$E(dz) = 0, \mathrm{var}(dz) = \sigma_z^2 dt \,.$$

Following Fischer (1975) and Turnovsky (2000), it is assumed that there are two assets in the economy: government bonds, B, and the capital stock, K. As in Turnovsky (2000), we suppose that the stochastic real rate of return on government bonds, dR_B, over a period dt, is given by

$$dR_B = r_B dt + du_B \,, \tag{3}$$

where r_B and du_B will be determined endogenously in the macroeconomic equilibrium.

Turning to the second asset, capital, and using the production technology in equation (1), the stochastic real rate of return on capital is

$$dR_K = \frac{dY}{dK} = \alpha dt + \alpha dy = r_K dt + du_K \,. \tag{4}$$

Thus, wealth $W(t)$ is the sum of the holdings of B and K, i.e.,

$$W = K + B \,. \tag{5}$$

Let n_B and n_K denote the fractions of wealth invested in government bonds and capital, respectively, i.e.,

$$n_B = \frac{B}{W}, n_k = \frac{K}{W}, \tag{6}$$

and $n_B + n_K = 1$.

Following with Turnovsky (2000) and Gong-Zou (2002), we may assume that, without any loss of generality, taxes are levied on capital income, namely,

$$dT = \tau r_K K dt + \tau' K du_K, \tag{7}$$

where τ and τ' are the tax rates on the deterministic component of capital income and the stochastic capital income, respectively.

Now, the representative agent chooses the consumption-wealth ratio, c/W, and the portfolio shares, n_B and n_K to maximize his expected utility subject to the budget constraint, i.e.,

$$\max E_t U_t = E_t \left(\int_t^{t+h} e^{-\beta s} u(c_s) ds + \delta \int_{t+h}^{\infty} e^{-\beta s} u(c_s) ds \right)$$

subject to

$$dW = (\rho - c/W) W dt + W dw, \tag{8}$$

$$n_B + n_K = 1, \tag{9}$$

with the initial stocks of nominal bonds and capital are given by $B(0)$ and $K(0)$, respectively.

Where $\beta > 0$, $h \in (0, +\infty)$, and $\delta \in (0,1]$ are constants. In addition, we denote

$$\rho = n_B r_B + n_K (1-\tau) r_K, \tag{10}$$

$$dw = n_B du_B + (1-\tau') n_K du_K. \tag{11}$$

Similar to Palacios-Huerta (2001) and Harris and Laibson (2001), the discount function decays exponentially at a rate β up to time $t+h$, drops discontinuously at time $t+h$ to a fraction δ just prior to $t+h$, and decays exponentially at a rate β thereafter. i.e., the discount function is

$$\begin{array}{ll} e^{-\beta s}, & t \le s \le t+h, \\ \delta e^{-\beta s}, & t+h \le s < \infty. \end{array}$$

Self t takes control of consumption and portfolio decisions from time t to data $t+h$. The future then begins at time $t+h$, where self $t+h$ takes control, and last forever. The ``instantaneous gratification" model in Harris and Laibson (2001) is obtained when $h \to 0$. Harris and Laibson (2001) also studied the stochastic discount function, where they assumed that

h is distributed exponentially with parameter λ. In this paper, we consider the case of deterministic h.

3. Macroeconomic equilibrium

As in Turnovsky (2000), Gong and Zou (2002), the economic system in equilibrium determines the rates of consumption and savings, the value of returns on all assets, and the economic growth rate.

The exogenous variables include the preference parameters, technology parameters, and government fiscal policies including government expenditure g, tax rates τ and τ'. The exogenous stochastic processes consist of government expenditure shocks, dz, productivity shocks, dy are taken to be mutually uncorrelated. The remaining stochastic disturbances---real rates of returns on indexed bonds, du_B, and total wealth, dw, are both endogenous and will be determined by the economic system. The remaining endogenous variables include the following: the consumption-wealth ratio, c/W, the mean growth rate of the economy, the expected real returns on two assets, r_B, and r_K, respectively, and the corresponding portfolio shares n_B and n_K.

To solve the agent's optimization problem, similar to Merton (1971) and Palacios-Huerta (2001), we introduce the value function

$$X(W,t) = \max E_t U_t = E_t \int_t^{t+h} e^{-\beta s} u(c_s) ds + \delta \int_{t+h}^{\infty} e^{-\beta s} u(c_s) ds$$

subject to equations (8)--(11) and the given initial bonds, $B(t)$ and capital, $K(t)$.

We get following proposition

Proposition 1 *The first-order conditions for the optimization problem can be written as follows:*

$$u'(c)e^{-\beta t} = X_W, \tag{12}$$

$$(r_B X_W W - e^{-\beta t}\eta) + \mathrm{cov}(dw, du_B) X_{WW} W^2 = 0, \tag{13}$$

$$((1-\tau) r_K X_W W - e^{-\beta t}\eta) + \mathrm{cov}(dw, (1-\tau') du_K) X_{WW} W^2 = 0, \tag{14}$$

$$n_B + n_K = 1, \tag{9}$$

and the transversality condition (TVC)

$$\lim_{t\to\infty} E_t (X_W W e^{-\beta t}) = 0, \tag{15}$$

where η is the Lagrangian multiplier associated with the portfolio selection constraint (9).

Furthermore, the optimal solutions of the problem must satisfy the Bellman equation

$$X_t - (1-\delta)E_t(u(c(t+h))e^{-\beta(t+h)}) + u(c(t))e^{-\beta t} + W_t(\rho - c/W)X_W + \frac{1}{2}\sigma_w^2 X_{WW}W^2 = 0 \tag{16}$$

with equations (10)--(11).

Condition (12) asserts that in the equilibrium the marginal utility of consumption must equal the marginal utility of wealth; conditions (13) and (14) are the asset pricing relationships; condition (9) is the portfolio selection constraint; and equation (16) is the Bellman equation, from which we will solve the value function $X(W,t)$.

In order to determine the full equilibrium system, we follow Turnovsky (2000) in discussing government behavior. Equations (2) and (7) describe government expenditure policy and tax policies, both of which are proportional to current output. In the absence of lump-sum taxation, government budget constraint can be described as:

$$dB = BdR_B + dG - dT\ . \tag{17}$$

Goods market equilibrium requires that

$$dY = dC + dK + dG\ . \tag{18}$$

Combing with equations (1), (2), and (18), we have

$$\frac{dK}{K} = [\alpha(1-g) - \frac{c}{n_K W}]dt + \alpha(dy - dz) = \phi dt + \alpha(dy - dz)\,, \tag{19}$$

where we denote ϕ the mean growth rate of economy.

Along with the balance growth path, the growth paths of real bonds, capital stocks, and the wealth must follow the same growth rule

$$\frac{dB}{B} = \frac{dK}{K} = \frac{dW}{W} = \phi dt + dw\,. \tag{20}$$

So, we have

$$\phi = \rho - c/W, dw = \alpha(dy - dz)\,. \tag{21}$$

From equations (2) and (7), equation (17) can be written in the form

$$n_B\frac{dB}{B} = (r_B n_B + \alpha(g-\tau)n_K)dt + n_B du_B + \alpha n_K dz - \tau'\alpha n_K dy\,,$$

so, we get

$$dw = n_B du_B + n_K(1-\tau')\alpha dy = \alpha(dy - dz) = \frac{1}{n_B}[n_B du_B + \alpha n_K(dz - \tau' dy)]\,.$$

Summarize the above discussion, we have:

Proposition 2. *The equilibrium system of the economy can be summarized as*:

$$\frac{dK}{K} = [\alpha(1-g) - \frac{c}{n_K W}]dt + \alpha(dy - dz) = \phi dt + \alpha(dy - dz) \tag{22}$$

$$(r_B X_W W - e^{-\beta t}\eta) + \mathrm{cov}(dw, du_B) X_{WW} W^2 = 0 \qquad (23)$$

$$((1-\tau) r_K X_W W - e^{-\beta t}\eta) + \mathrm{cov}(dw, (1-\tau') du_K) X_{WW} W^2 = 0 \qquad (24)$$

$$n_B + n_K = 1 \qquad (9)$$

and the transversality condition (TVC) plus the initial conditions.

Proposition 3. *The stochastic component of real rate of return on government bonds,* du_B *and total wealth,* dw *, are determined by:*

$$dw = \alpha(dy - dz), \qquad (25)$$

$$du_B = \frac{\alpha}{n_B}[(1 - n_K(1-\tau'))dy - dz]. \qquad (26)$$

These two equations enable us to compute all the necessary covariances and variances in the full equilibrium system. Equation (25) implies that the stochastic shocks of government expenditure and production determine the stochastic rate of return on government bonds.

4. An explicit example

In order to find explicit solutions, we specify the utility function as in Turnovsky (2000) and Gong and Zou (2002)

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \qquad (27)$$

where $\gamma > 0$ is a constant, represents the inverse of substituting elasticity of intertemporal consumption.

Under the form of the utility function in (27), we have

Proposition 4 *The first-order optimal conditions are*

$$\frac{c}{W} = (b(1-\gamma))^{-\frac{1}{\gamma}} \equiv x, \qquad (12')$$

$$(r_B - \frac{\eta}{b(1-\gamma)W^{1-\gamma}}) = \gamma\, \mathrm{cov}(dw, du_B), \qquad (13')$$

$$((1-\tau) r_K - \frac{\eta}{b(1-\gamma)W^{1-\gamma}}) = \gamma\, \mathrm{cov}(dw, (1-\tau') du_K), \qquad (14')$$

where η *is the Lagrangian multiplier associated with constraint* (9),

$$\rho = n_B r_B + n_K (1-\tau) r_K,$$

$$dw = n_B du_B + (1-\tau')n_K du_K ,$$

$$\sigma_w^2 = n_B^2 \sigma_B^2 + n_K^2 (1-\tau')^2 \sigma_K^2 + 2n_B n_K (1-\tau')\sigma_{BK} ,$$

and b *is determined by*

$$-\beta + (\gamma - (1-\delta)e^{-\beta h} E_0 (\frac{W_h}{W_0})^{1-\gamma})[b(1-\gamma)]^{-\frac{1}{\gamma}} + \rho(1-\gamma) - \frac{1}{2}\sigma_w^2 (1-\gamma)\gamma = 0 , \tag{28}$$

$$E_0 (\frac{W_h}{W_0})^{1-\gamma} = \exp((1-\gamma)(\rho - [b(1-\gamma)]^{-\frac{1}{\gamma}} - \frac{1}{2}\gamma\sigma_w^2)h) . \tag{29}$$

Equations (28) and (29) determine the constant b, and from equation (12') we can determine c/W. If $\delta = 1$, we have

$$\frac{c}{W} = \frac{\beta - \rho(1-\gamma) + \frac{1}{2}\sigma_w^2 (1-\gamma)\gamma}{\gamma} ,$$

which presented by Turnovsky (2000).

If $\gamma = 1$, we have

$$\frac{c}{W} = \frac{\beta}{1-(1-\delta)e^{-\beta h}} .$$

Therefore, for a logarithmic utility function in consumption, the consumption-wealth ratio is always equal to constant.

If $\gamma \neq 1$, then the effect of an increase in the expected net-of-tax return on the consumption-wealth ratio will be

$$\frac{\partial(c/W)}{\partial\rho} = \frac{(\gamma-1)(1-(1-\delta)hxe^{-\beta h} E_0 (W_h/W_0)^{1-\gamma})}{\gamma + [(1-\gamma)xh - 1](1-\delta)e^{-\beta h} E_0 (W_h/W_0)^{1-\gamma}} .$$

For the small h, we have $\frac{\partial(c/W)}{\partial\rho} > 0$ when $\gamma > 1$, and $\frac{\partial(c/W)}{\partial\rho} < 0$ when $\gamma < 1$.

Therefore, an increase in the expected net-of-tax return ρ will raise the consumption-wealth ratio if $\gamma > 1$, and lower it otherwise. This can be explained as follows. When $\gamma < 1$, the elasticity of intertemporal substitution, $\frac{1}{\gamma}$, is relatively small. The representative agent will increase current consumption more than investment and wealth. On the other hand, when $\gamma < 1$, the elasticity of intertemporal substitution is relatively large, and the agent will increase wealth holding more than consumption.

Similar analysis holds for the effect of the variance of wealth, σ_w^2, on c/W:

$$\frac{\partial(c/W)}{\partial\sigma_w^2}=\frac{1}{2}\frac{\gamma(1-\gamma)(1-(1-\delta)hxe^{-\beta h}E_0(W_h/W_0)^{1-\gamma})}{\gamma+[(1-\gamma)xh-1](1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}}.$$

Therefore, an increase in the variance of wealth reduces the consumption-wealth ratio when $\gamma<1$, and increases the ratio when $\gamma>1$.

Equations (13') and (14') illustrates the asset pricing relationships. The term of $\frac{\eta}{b(1-\gamma)W^{1-\gamma}}$ in equation (13') is equal to the real rate of return on nominal bonds, which is riskless and is uncorrelated with the stochastic term dw. But equation (13') implies that the return on government bonds is equal to the riskless return plus a risk premium, which is proportional to the covariance between total wealth and government bonds. Similarly, in equation (14'), for the net return on the risky capital, it is also equal to the riskless return plus a risk premium, which is also proportional to the covariance between total wealth and risky capital. In the absence of risk, these two equations imply that the net returns on two assets are all equal.

Since ρ is still endogenous in terms of holding shares for various assets, we now use the full equilibrium system to derive explicit solutions to c/W, n_B, n_K, r_B, and ϕ. With proposition 3, and from the optimal conditions (13') and (14') plus equation (9), we have:

$$\sigma_w^2=\alpha^2(\sigma_y^2+\sigma_z^2),$$

$$\text{cov}(dw,du_B)=\frac{\alpha^2}{n_B}[(1-n_K(1-\tau'))\sigma_y^2+\sigma_z^2]dt,$$

$$\text{cov}(dw,(1-\tau')du_K)=\alpha^2(1-\tau')\sigma_y^2dt,$$

and

Proposition 5 *The mean return on government bonds and the stochastic growth rate of the economy are*

$$r_B=\alpha(1-\tau)+\frac{\gamma}{n_B}\alpha^2(\sigma_y^2-(1-\tau')^2n_K\sigma_y^2-(1-\tau')^2n_B\sigma_y^2+\sigma_z^2), \qquad (30)$$

$$\phi=\frac{r_Bn_B+(g-\tau)\alpha n_K}{n_B}=\rho-\frac{c}{W}. \qquad (31)$$

The first term on the right-hand side of equation (30) is the net (after-tax) return on capital, which is the same as in Turnovsky (2000); the second term on the right-hand side is the stochastic component of the return on government bonds.

With proposition 5, we now have our main theorem of this section:

Theorem 6 *The explicit solutions of the economy system are:*

$$\frac{c}{W} = (b(1-\gamma))^{-\frac{1}{\gamma}}, \tag{12'}$$

$$\phi = \alpha(1-\tau) + \alpha^2\gamma(\tau'\sigma_y^2 + \sigma_z^2) - \frac{c}{W}, \tag{32}$$

$$n_K = \frac{c/W}{\alpha(\tau - g) + c/W - \alpha^2\gamma(\tau'\sigma_y^2 + \sigma_z^2)}, \tag{33}$$

and the TVC

$$\lim_{t\to\infty} E(bW^{1-\gamma}e^{-\beta t}) = 0, \tag{34}$$

and b *is determined by*

$$\begin{aligned} &-\beta + (\gamma - (1-\delta)e^{-\beta h}E_0(\frac{W_h}{W_0})^{1-\gamma})[b(1-\gamma)]^{-\frac{1}{\gamma}} + \alpha(1-\tau)(1-\gamma) \\ &+\frac{1}{2}\alpha^2((2\tau'-1)\sigma_y^2 + \sigma_z^2)(1-\gamma)\gamma = 0, \end{aligned} \tag{28'}$$

$$E_0(\frac{W_h}{W_0})^{1-\gamma} = \exp((1-\gamma)(\alpha(1-\tau) - [b(1-\gamma)]^{-\frac{1}{\gamma}} + \frac{1}{2}\alpha^2\gamma((2\tau'-1)\sigma_y^2 + \sigma_z^2))h). \tag{29'}$$

Proof: Notice the conditions

$$n_B + n_K = 1,$$

$$\rho = n_B r_B + n_K(1-\tau)r_K,$$

$$r_B = \alpha(1-\tau) + \frac{\gamma}{n_B}\alpha^2(\sigma_y^2 - (1-\tau')^2 n_K\sigma_y^2 - (1-\tau')^2 n_B\sigma_y^2 + \sigma_z^2).$$

We obtain

$$\rho = \alpha(1-\tau) + \alpha^2\gamma(\tau'\sigma_y^2 + \sigma_z^2).$$

Thus, we have equations (28'), (29'), and (32). With equation (19), we have

$$\phi = \alpha(1-g) - \frac{c}{n_K W},$$

and equation (33).

Using equations (30), (31), and the portfolio-selection constraint $n_B + n_K = 1$, we have equation (33). *Q.E.D.*

With equation (31), the portfolio shares of government bonds are determined as a residual from the portfolio-selection constraint $n_B + n_K = 1$.

Please also notice that, from transversality condition (34) can be shown to be equivalent to $c/W > 0$. In fact, since

$$dW = \phi W dt + W dw,$$

we have

$$W(t) = W(0)\exp\{(\phi - \frac{1}{2}\sigma_w^2)t + w(t) - w(0)\}.$$

The TVC will be met if and only if

$$(1-\gamma)(\phi - \frac{\gamma}{2}\sigma_w^2) - \beta < 0.$$

By equation (32), we have:

$$(1-\gamma)(\alpha(1-\tau) + \alpha^2\gamma(\tau'\sigma_y^2 + \sigma_z^2) - \frac{c}{W} - \frac{\gamma}{2}\sigma_w^2) < \beta. \qquad (35)$$

From equation (35), we have

$$e^{\beta h}\exp((1-\gamma)(\alpha(1-\tau) - x + \frac{1}{2}\alpha^2\gamma((2\tau'-1)\sigma_y^2 + \sigma_z^2))h) < 1.$$

From equations (28') and (29'), we have

$$\begin{aligned}&(\gamma - (1-\delta)e^{-\beta h}\exp((1-\gamma)(\alpha(1-\tau) - x + \frac{1}{2}\alpha^2\gamma((2\tau'-1)\sigma_y^2 + \sigma_z^2))h)x \\ &= \beta - \alpha(1-\tau)(1-\gamma) - \frac{1}{2}\alpha^2\gamma(1-\gamma)((2\tau'-1)\sigma_y^2 + \sigma_z^2)).\end{aligned}$$

From equation (35) and simple calculation we know that the right-hand side (we denote it RHS) of above equation is a increasing function of x, and

$$RHS|_{x=0} = 0; \lim_{x\to+\infty} RHS = +\infty$$

Thus, if we impose the condition[1] $\beta - \alpha(1-\tau)(1-\gamma) - \frac{1}{2}\alpha^2\gamma(1-\gamma)((2\tau'-1)\sigma_y^2 + \sigma_z^2)) > 0$, we can get unique x from the above equation. Therefore, equations (28') and (29') determine unique b.

5. Comparative dynamics

Theorem 6 determines mean growth rate, consumption-wealth ratio, real return on assets, and the holding shares of assets. From which, we can discuss how stochastic shocks (in production and government spending) and government fiscal policies affect the equilibrium. First, from Theorem 6, we find that the mean government expenditure has no effects on the growth and consumption-wealth ratio, it affects the portfolio selection. Next, we focus on the effects of stochastic shocks and government taxations on economy.\

Effects of stochastic shocks

Differentiating with respect to σ_z^2 and σ_y^2, respectively, in equation (29), we have,

$$\frac{\partial(c/W)}{\partial\sigma_z^2} = -\frac{1}{2}\frac{\alpha^2\gamma(1-\gamma)(1-(1-\delta)hxe^{-\beta h}E_0(W_h/W_0)^{1-\gamma})}{\gamma + [(1-\gamma)xh - 1](1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}} > 0,$$

[1] This condition is just the condition for a positive consumption-wealth ratio, which is similar to Turnovsky (2000).

$$\frac{\partial(c/W)}{\partial\sigma_y^2} = -\frac{1}{2}\frac{\alpha^2\gamma(2\tau'-1)(1-\gamma)(1-(1-\delta)hxe^{-\beta h}E_0(W_h/W_0)^{1-\gamma})}{\gamma+[(1-\gamma)xh-1](1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}} < 0.$$

for $\gamma > 1$ and $\tau' < 50\%$. Where we use the conditions[2] $1-hxe^{-\beta h}E_0(W_h/W_0)^{1-\gamma} > 0$ and $\gamma+[(1-\gamma)xh-1](1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma} > 0$.

Therefore, when the intertemporal elasticity of substitution is relatively small, a higher variance in government expenditure increases the consumption-wealth ratio, whereas the stochastic shock in production lowers the consumption-wealth ratio.

On the other hand, when $\gamma < 1$, we have just the opposite results, namely,

$$\frac{\partial(c/W)}{\partial\sigma_z^2} < 0, \quad \frac{\partial(c/W)}{\partial\sigma_y^2} > 0.$$

From equation (30), the equilibrium growth rate, ϕ, varies with the stochastic shocks of government spending as follows. For all values of γ,

$$\frac{\partial\phi}{\partial\sigma_z^2} = \alpha^2\gamma\frac{\gamma+((1-\gamma)xh-1)(1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}+1-(1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma})}{2[\gamma+((1-\gamma)xh-1)(1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}]} > 0$$

because $\gamma+((1-\gamma)xh-1)(1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma} > 0$ and $1-(1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}) > 0$.

Therefore, more volatility in government spending always increases the rate of economic growth. This is true because an increase in σ_z^2 raises the risk of government bonds. The agent reduces his holding of government bonds and invests more in capital, which in turn leads to more output growth.

But for the shocks to the productivity, we have

---

[2] Because the transversality condition (35), we have

$$e^{-\beta h}E_0(\frac{W_h}{W_0})^{1-\gamma} < 1,$$

and for small h, we have $hx < 1$.

Thus, we have

$$1-(1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}) > 0,$$

and

$$\gamma+((1-\gamma)xh-1)(1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma} > 0.$$

These conditions are used throughout this paper.

$$\frac{\partial \phi}{\partial \sigma_y^2} = \alpha^2 \gamma \frac{2\tau'(1 - hx(1-\delta)e^{-\beta h} E_0 (W_h / W_0)^{1-\gamma}) - (1-\gamma)(1 - hx(1-\delta)e^{-\beta h} E_0 (W_h / W_0)^{1-\gamma})}{2[\gamma + ((1-\gamma)xh - 1)(1-\delta)e^{-\beta h} E_0 (W_h / W_0)^{1-\gamma}]}$$

$$= \alpha^2 \gamma \frac{(2\tau' - 1)(1 - (1-\delta)e^{-\beta h} E_0 (W_h / W_0)^{1-\gamma}) + \gamma + (hx(1-\gamma) - 1)(1-\delta)e^{-\beta h} E_0 (W_h / W_0)^{1-\gamma})}{2[\gamma + ((1-\gamma)xh - 1)(1-\delta)e^{-\beta h} E_0 (W_h / W_0)^{1-\gamma}]}.$$

Therefore, the mean growth rate of the economy can increase or decrease depending on the values of γ and other parameters. For example, $\frac{\partial \phi}{\partial \sigma_y^2} > 0$ when $\gamma > 1$, or $\gamma < 1$ and $\tau' > 50\%$. In this case, an increase in the variance of the productivity shocks lowers the growth rate. But when $\gamma < 1$ and $\tau' < 50\%$, $\frac{\partial \phi}{\partial \sigma_y^2}$ has an ambiguous sign. Our results confirm the complicated pictures of the effects of stochastic shocks on output growth in Obstfeld (1994), Turnovsky (2000), and Gong and Zou (2002).

The dependence of the shares of asset holding on the stochastic shocks can be derived from equation (33):

$$\frac{\partial n_K}{\partial \sigma_z^2} = n_K \{ \frac{(1 - n_K) \partial (c/W) / \partial \sigma_z^2}{c/W} + \alpha^2 \gamma \frac{n_K}{c/W} \},$$

$$\frac{\partial n_K}{\partial \sigma_y^2} = n_K \{ \frac{(1 - n_K) \partial (c/W) / \partial \sigma_y^2}{c/W} + \alpha^2 \gamma \tau' \frac{n_K}{c/W} \}.$$

The first equation above tells us that the stochastic shock in government expenditure will enhance the holding of risky capital for $\gamma > 1$, and the second equation shows the positive effect of the stochastic shock in production on the holding of risky capital for $\gamma < 1$. The effects of these two shocks on holding shares are ambiguous for rest cases. As for the holding share of government bonds n_B, we can use the portfolio-selection condition and derive their responses to various shocks and fiscal policies.

We have derived the value function $X(W, t)$ in appendix B. Let $W(0)$ denote the initial stock of wealth. We have the following welfare function:

$$X(W(0), 0) = bW(0)^{1-\gamma},$$

where

$$b = \frac{1}{1-\gamma} (c/W)^{-\gamma}.$$

However, $W(0)$ is itself endogenously determined by

$$W(0) = \frac{K(0)}{n_K}.$$

Therefore, with some simple manipulations, welfare is given by:

$$X(W(0)) = n_K^{\gamma-1} \frac{1}{1-\gamma} (c/W)^{-\gamma} K_0^{1-\gamma}, \tag{36}$$

where c/W and n_K are determined as in Theorem 6. Taking differentiation in equation (36), we get

$$\frac{dX}{X} = (\gamma-1)\frac{dn_K}{n_K} - \gamma \frac{d(c/W)}{c/W}.$$

Now we have

$$\frac{\partial X}{X\partial\sigma_z^2} = -(1+(\gamma-1)n_K)\frac{\partial(c/W)/\partial\sigma_z^2}{c/W} + \alpha^2\gamma\frac{n_K}{c/W},$$

$$\frac{\partial X}{X\partial\sigma_y^2} = -(1+(\gamma-1)n_K)\frac{\partial(c/W)/\partial\sigma_y^2}{c/W} + \alpha^2\gamma\tau'\frac{n_K}{c/W}.$$

Thus, the stochastic shock in government expenditure will improve social welfare for $\gamma > 1$, and the effect of the stochastic shock in production on the welfare is positive when $\gamma < 1$. The effects of these two shocks on social welfare are ambiguous for rest cases.

Effects of fiscal policies

Now we turn to how taxes on capital income impact on the equilibrium. First, differentiating all endogenous variables with respect to the tax on the deterministic part of capital income, τ, in equations (29), (30), and (31), we have

$$\frac{\partial(c/W)}{\partial\tau} = \frac{\alpha(1-\gamma)(1-(1-\delta)hxe^{-\beta h}E_0(W_h/W_0)^{1-\gamma})}{\gamma+[(1-\gamma)xh-1](1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}},$$

$$\frac{\partial\phi}{\partial\tau} = -\frac{\alpha(1-(1-\delta)hxe^{-\beta h}E_0(W_h/W_0)^{1-\gamma})}{\gamma+[(1-\gamma)xh-1](1-\delta)e^{-\beta h}E_0(W_h/W_0)^{1-\gamma}} < 0,$$

$$\frac{\partial n_K}{\partial\tau} = n_K\{\frac{(1-n_K)\partial(c/W)/\partial\tau}{c/W} - \alpha\frac{n_K}{c/W}\},$$

$$\frac{\partial X}{X\partial\tau} = -(1+(\gamma-1)n_K)\frac{\partial(c/W)/\partial\tau}{c/W} - \alpha\frac{n_K}{c/W}.$$

If $\gamma = 1$, c/W is independent of the tax rate, because in this case $\frac{c}{W} = \frac{\beta}{1-(1-\delta)e^{-\beta h}}$, which is independent of τ. When $0 < \gamma < 1$, we notice that a rise in the taxation on the deterministic component of capital income has an ambiguous effect on welfare. But, it is clear that

$$\frac{\partial(c/W)}{\partial\tau} > 0, \frac{\partial\phi}{\partial\tau} < 0, \frac{\partial X}{\partial\tau} < 0.$$

Therefore, a higher tax on the deterministic component of capital income will increase the consumption-wealth ratio and decrease the economic growth rate and social welfare. This can be explained as follows: a higher tax on capital income will lower the return on capital. As the agent switches away from capital to bonds and consumption, this reduces capital accumulation, lowers the growth rate, and increases the consumption-wealth ratio.

When $\gamma > 1$, we still find that capital income taxation reduces the holding share of risky capital and lowers the growth rate:

$$\frac{\partial \phi}{\partial \tau} < 0, \frac{\partial n_K}{\partial \tau} < 0.$$

But it reduces the consumption-wealth ratio: $\frac{\partial (c/W)}{\partial \tau} < 0$.

Second, we look at the effects on the equilibrium of the tax on the stochastic component of capital income:

$$\frac{\partial (c/W)}{\partial \tau'} = -\frac{\alpha \sigma_y^2 (1-\gamma)(1-(1-\delta)hxe^{-\beta h} E_0 (W_h / W_0)^{1-\gamma})}{\gamma + [(1-\gamma)xh - 1](1-\delta)e^{-\beta h} E_0 (W_h / W_0)^{1-\gamma}},$$

$$\frac{\partial \phi}{\partial \tau'} = \frac{\alpha^2 \gamma \sigma_y^2 (1-(1-\delta)hxe^{-\beta h} E_0 (W_h / W_0)^{1-\gamma})}{\gamma + [(1-\gamma)xh - 1](1-\delta)e^{-\beta h} E_0 (W_h / W_0)^{1-\gamma}} < 0,$$

$$\frac{\partial n_K}{\partial \tau'} = n_K \{ \frac{(1-n_K)\partial (c/W)/\partial \tau'}{c/W} + \alpha^2 \gamma \sigma_y^2 \frac{n_K}{c/W} \},$$

$$\frac{\partial X}{X \partial \tau'} = -(1+(\gamma-1)n_K) \frac{\partial (c/W)/\partial \tau'}{c/W} + \alpha^2 \gamma \sigma_y^2 \frac{n_K}{c/W}.$$

These results are just opposite to the ones for the tax on the deterministic component of capital income. Still,

$$\frac{\partial (c/W)}{\partial \tau'} < 0, \frac{\partial \phi}{\partial \tau'} > 0, \frac{\partial X}{\partial \tau'} > 0,$$

when $0 < \gamma < 1$; and

$$\frac{\partial (c/W)}{\partial \tau'} > 0, \frac{\partial \phi}{\partial \tau'} > 0, \frac{\partial n_K}{\partial \tau'} > 0,$$

when $\gamma > 1$.

Effects of discounting rate

Finally, we examine the effects of discounting rate on the economy, in our model the discounting rate is

$$\begin{aligned} &e^{-\beta s}, \quad t \le s \le t+h, \\ &\delta e^{-\beta s}, \ t+h \le s < \infty. \end{aligned}$$

Thus, the parameters h, β, and δ determine the discounting rate. Compare with the exponential discounting rate, the parameters h and δ determine the difference between the exponential discounting rate and the hyperbolic discounting rate presented here, with the increasing of h and δ, the difference between the exponential discounting rate and the hyperbolic discounting rate is decreasing, please figure 1. We will analyze the effects of these two parameters on the economy below.

(Please insert figure 1 about here!!)}

First, take differentiation on equations (28), (29), (32)-(34) with respect to δ, we have

$$\frac{\partial(c/W)}{\partial\delta}=\frac{xe^{-\beta h}E_0\left(W_h/W_0\right)^{1-\gamma}}{\gamma+[(1-\gamma)xh-1](1-\delta)e^{-\beta h}E_0\left(W_h/W_0\right)^{1-\gamma}}>0,$$

$$\frac{\partial\phi}{\partial\delta}=-\frac{\partial(c/W)}{\partial\delta}<0,$$

$$\frac{\partial n_K}{\partial\delta}=n_K\frac{(1-n_K)\partial(c/W)/\partial\delta}{c/W}>0,$$

$$\frac{\partial X}{X\partial\delta}=-(1+(\gamma-1)n_K)\frac{\partial(c/W)/\partial\delta}{c/W}<0.$$

Thus, with the increasing of δ, the consumption-wealth ratio and the holding share of capital stock will be increasing, the mean growth rate and the social welfare will be decreasing.

Take differentiation on equations (28), (29), (32)-(34) with respect to h, we also have

$$\frac{\partial(c/W)}{\partial h}=\frac{-(1-\delta)xe^{-\beta h}E_0\left(W_h/W_0\right)^{1-\gamma}((1-\gamma)(\alpha(1-\tau)-x+\frac{1}{2}\alpha^2\gamma((2\tau'-1)\sigma_y^2+\sigma_z^2))-\beta)}{\gamma+[(1-\gamma)xh-1](1-\delta)e^{-\beta h}E_0\left(W_h/W_0\right)^{1-\gamma}}>0,$$

$$\frac{\partial\phi}{\partial h}=-\frac{\partial(c/W)}{\partial h}<0,$$

$$\frac{\partial n_K}{\partial h}=n_K\frac{(1-n_K)\partial(c/W)/\partial h}{c/W}>0,$$

$$\frac{\partial X}{X\partial h}=-(1+(\gamma-1)n_K)\frac{\partial(c/W)/\partial h}{c/W}<0.$$

because of equation (35).

Therefore, we get the similar effects of h on the economy, with the increasing of h, the consumption-wealth ratio and the holding share of capital stock will be increasing, the mean growth rate and the social welfare will be decreasing.

With the increasing of parameters h and δ, the discounting rate will be increasing, and the consumer will increase his consumption level today, thus the consumption-wealth ratio will be increasing, with the increasing of consumption, the investment will be decreasing, this leads to the lower mean growth rate and the social welfare.

6. Conclusion

In this paper, we study optimal fiscal policies in a stochastic model of growth with hyperbolic discounting. With specific assumptions on the production technology, preferences, and stochastic shocks, we derive the explicit solutions to the growth rates of consumption and savings and equilibrium returns on all assets. Finally, we give the effects of fiscal policies, the spirit of capitalism, and stochastic shocks on growth, asset pricing, and welfare.

Similar to the conclusions presented by Turnovsky (2000), Gong and Zou (2002), we show that the real growth rate ϕ is independent of the mean government expenditure. The comparative static analysis shows that with increasing of the capital income tax rate, the mean growth rate will be decreasing. With the increasing of the capital income tax rate, the consumption-wealth ratio will be increase when the intertemporal elasticity of consumption is greater than 1, i.e. $0 < \gamma < 1$; while the consumption-wealth ratio will be decrease when the intertemporal elasticity of consumption is smaller than 1, i.e. $\gamma > 1$.

The effects of stochastic shocks on government expenditure and production on economy have been investigated also, we find that with the increasing of stochastic shock of government expenditure, the mean growth rate of economy will be increasing, but the effects of stochastic shocks of government expenditure on consumption-wealth,portfolio holding shares, and social welfare depends on the intertemporal elasticity of consumption. The effects of production shocks on growth is ambiguous, it depends on the intertemporal elasticity of consumption and the tax on the stochastic component of capital income.

The direct effects of discounting rate has been also explicitly considered in this paper, we find that with the increasing of h and δ, the consumption-wealth ratio and the holding share of capital stock will be increasing, the mean growth rate and the social welfare will be decreasing.

Further research would extend this paper to the framework with stochastic discount rate, and study the effects of fiscal policies and discounting rate on economy. Another important extension is to introduce the hyperbolic discounting function to Grinols and Turnovsky (1998) 's stochastic monetary model to discuss the optimal monetary policy and the effects of monetary policies on economy.

Appendix A

Consider the optimization problem

$$\max E_t U_t = E_t \int_t^{t+h} e^{-\beta s} u(c_s) ds + \delta \int_{t+h}^{\infty} e^{-\beta s} u(c_s) ds.$$

subject to

$$dW = (\rho - c/W) W dt + W dw, \tag{A1}$$

$$n_B + n_K = 1, \tag{A2}$$

the initial stocks of nominal bonds and capital are given by $B(0)$ and $K(0)$ respectively.

Where $\beta > 0$, $h \in (0, +\infty)$, and $\delta \in (0,1]$ are constants. In addition, we denote

$$\rho = n_B r_B + n_K (1-\tau) r_K, \tag{A3}$$

$$dw = n_B du_B + (1-\tau') n_K du_K. \tag{A4}$$

From equation (A4), we have

$$\sigma_w^2 = n_B^2 \sigma_B^2 + n_K^2 (1-\tau')^2 \sigma_K^2 + 2 n_B n_K (1-\tau') \sigma_{BK}.$$

To solve the problem, similar to Merton (1971) and Palacios-Huerta (2001), we define the value function $X(W,t)$

$$X(W,t) = \max E_t U_t = E_t \int_t^{t+h} e^{-\beta s} u(c_s) ds + \delta \int_{t+h}^{\infty} e^{-\beta s} u(c_s) ds$$

subject to equation (A1)-(A4).

From Palacios-Huerta (2001), we get the Bellman equation associated with the above problem

$$X_t - (1-\delta) E_t (u(c_{t+h}) e^{-\beta(t+h)}) + \max\{u(c_t) e^{-\beta t} + W_t (\rho - c/W) X_W + \frac{1}{2} \sigma_w^2 X_{WW} W^2\} = 0 \tag{A5}$$

subject to the portfolio selection constraint (A2).

The Lagrangian associated with the above problem is

$$u(c_t) e^{-\beta t} + W_t (\rho - c/W) X_W + \frac{1}{2} \sigma_w^2 X_{WW} W^2 + e^{\beta t} \eta (1 - n_K - n_B),$$

where η is the Lagrangian multiplier associated with the portfolio selection constraint (A2).

So, we get the first-order conditions:

$$u'(c) e^{-\beta t} = X_W, \tag{A6}$$

$$(r_B X_W W - e^{-\beta t} \eta) + \text{cov}(dw, du_B) X_{WW} W^2 = 0, \tag{A7}$$

$$((1-\tau)r_K X_W W - e^{-\beta t}\eta) + \operatorname{cov}(dw,(1-\tau')du_K)X_{WW}W^2 = 0, \qquad \text{(A8)}$$

$$n_B + n_K = 1, \qquad \text{(A2)}$$

These equations determine the optimal choices of n_B, n_K, and η as the functions of X_W and X_{WW}.

Furthermore, the optimal solutions of the problem must satisfy the Bellman equation

$$X_t - (1-\delta)E_t(u(c_{t+h})e^{-\beta(t+h)}) + u(c_t)e^{-\beta t} + W_t(\rho - c/W)X_W + \frac{1}{2}\sigma_w^2 X_{WW}W^2 = 0. \qquad \text{(A9)}$$

Now, we have completed the proof of proposition 3.

Appendix B

To show the proposition 4, we rewrite the utility function here:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{B1}$$

where $\gamma > 0$ is a positive constant, represents the substituting elasticity of intertemporal consumption.

The form of the value function is postulated as:

$$X(W,t) = bW^{1-\gamma}e^{-\beta t}, \tag{B2}$$

where b is to be determined.

Differentiating with respect to W yields

$$X_W = b(1-\gamma)W^{-\gamma}e^{-\beta t}, X_{WW} = -b(1-\gamma)\gamma W^{-\gamma-1}e^{-\beta t}.$$

Now the corresponding first-order conditions are:

$$\frac{c}{W} = (b(1-\gamma))^{-\frac{1}{\gamma}}, \tag{B3}$$

$$(r_B - \frac{\eta}{b(1-\gamma)W^{1-\gamma}}) = \gamma \operatorname{cov}(dw, du_B), \tag{B4}$$

$$((1-\tau)r_K - \frac{\eta}{b(1-\gamma)W^{1-\gamma}}) = \gamma \operatorname{cov}(dw, (1-\tau')du_K), \tag{B5}$$

And, we have

$$E_t(u(c_{t+h})e^{-\beta(t+h)}) = e^{-\beta(t+h)}[b(1-\gamma)]^{\frac{\gamma-1}{\gamma}} \frac{W_t^{1-\gamma}}{1-\gamma} E_t(\frac{W_{t+h}}{W_t})^{1-\gamma},$$

with

$$dW_t = W_t(\rho - c/W_t)dt + W_t dw,$$

we obtain

$$E_t(\frac{W_{t+h}}{W_t})^{1-\gamma} = E_0(\frac{W_h}{W_0})^{1-\gamma} \tag{B6}$$

Substituting equation (B3) and (B6) in the Bellman equation (A9) leads to

$$-\beta + (\gamma - (1-\delta)e^{-\beta h}E_0(\frac{W_h}{W_0})^{1-\gamma})[b(1-\gamma)]^{-\frac{1}{\gamma}} + \rho(1-\gamma) - \frac{1}{2}\sigma_w^2(1-\gamma)\gamma = 0 \tag{B7}$$

Because

$$dW_t = W_t(\rho - c/W_t)dt + W_t dw,$$

we have

$$W_h = W_0 \exp((\rho - c/W - \frac{1}{2}\sigma_w^2)t + w(t) - w(0)),$$

then

$$\begin{aligned} E_0(\frac{W_h}{W_0})^{1-\gamma} &= E_0(\exp((1-\gamma)(\rho - c/W - \frac{1}{2}\sigma_w^2)h + (1-\gamma)(w(t) - w(0))) \\ &= \exp((1-\gamma)(\rho - [b(1-\gamma)]^{-\frac{1}{\gamma}} - \frac{1}{2}\gamma\sigma_w^2)h). \end{aligned} \tag{B8}$$

Equations (B7) and (B8) determine the constant b .

References:

Barro, R. (1999). Ramsey meets Laibson in the neoclassical growth model. *Quarterly Journal of Economics* November,1125-1152.

Eaton, J. (1981). Fiscal policy, inflation, and the accumulation of risk capital. *Review of Economic Studies* 48, 435-445.

Fischer, S. (1975). The demand for index bonds. *Journal of Political Economy* 83, 509-534.

Gong, Liutang and Heng-fu Zou (2002). Direct wealth effects, fiscal policies, asset pricing, and stochastic growth. *Journal of Economic Dynamics and Control* 26(2), 243-270

Grinols, E. and S. Turnovsky (1993). Risk, the financial market, and macroeconomic equilibrium. *Journal of Economic Dynamics and Control* 17, 1-36

Grinols, E. and S. Turnovsky (1994). Exchange rate determination and asset prices in a stochastic small open economy. *Journal of International Economics* 36, 75-97.

Grinols, E. and S. Turnovsky (1998). Risk, optimal government finance and monetary policies in a growth economy. *Economica* 65,401-427.

Harris, C. and D. I. Laibson (2001). Instantaneous gratification. *Mimeo*. Department of Economics, Harvard University.

Krusell, P. and A. A. Smith (2001). Consumption-saving decisions with quasi-geometric discounting. *Working Paper*. University of Rochester.

Laibson, David, Rae Repetto, and Jeremy Tobacman (1998). Self-control and saving for retirement. *Brookings Papers on Economic Activity* 1, 191-196.

Laibson, David (1994). Essays in hyperbolic discounting. Ph.D. dissertation, MIT.

Laibson, David (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics* 112, 443-77.

Laibson, David (1998). Life-cycle consumption and hyperbolic discount functions. *European Economic Review* 42, 861-71.

Merton, R.C. (1971). Optimal consumption and portfolio rules in a continuous-time model, *Journal of Economic Theory* 3, 373-413.

Obstfeld, M. (1994). Risk-taking, global diversification, and growth. *The American Economic Review* 84, 1310-29.

Palacios-Huerta, I. (2001). Consumption and portfolio rules under hyperbolic discounting. *Working Paper*. Brown University.

Petersen, T. K. (2001). General equilibrium tax policy with hyperbolic consumers. W*orking Paper*. Yale University.

Turnovsky, S. (1990). The effects of taxes and dividend policy on capital accumulation and macroeconomic behavior. *Journal of Economic Dynamics and Control* 14, 491-521.

Turnovsky, S. (1993). Macroeconomic policies, growth, and welfare in a stochastic economy. *International Economic Review* 35, 953-981.

Turnovsky, S. (2000). *Methods of Macroeconomic Dynamics*. 2nd ed. MIT Press.

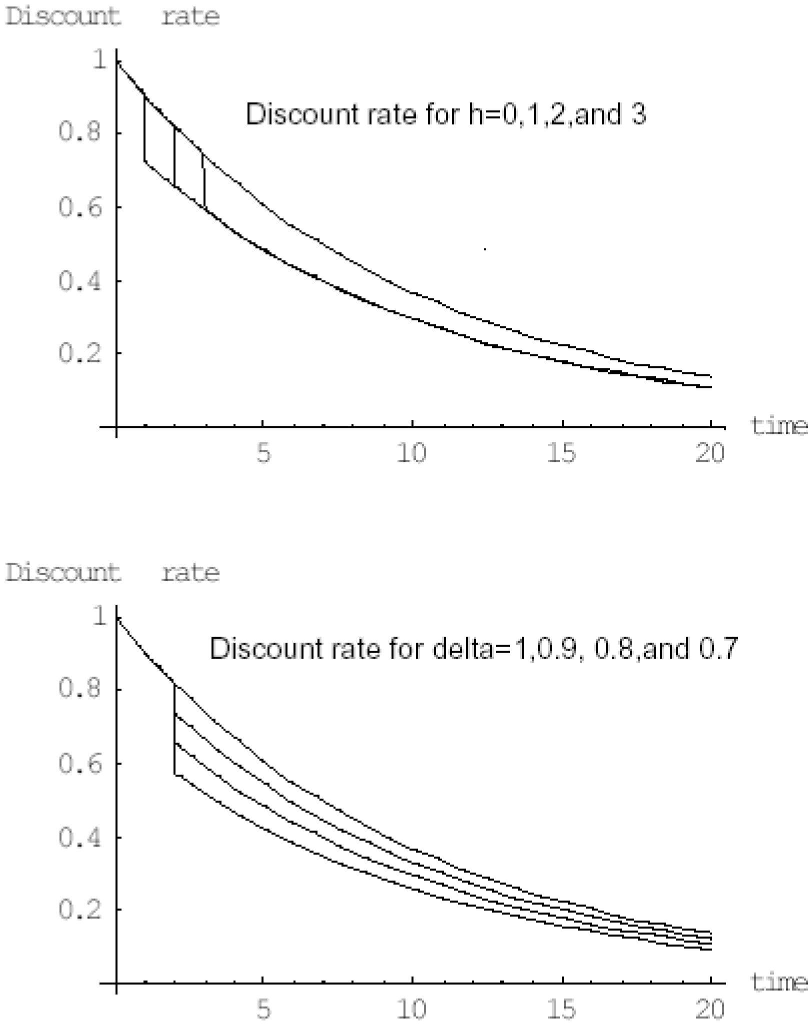

Figure 1: The effects of h and δ on the discounting rate.