

# A FISCAL FEDERALISM APPROACH TO OPTIMAL TAXATION AND INTERGOVERNMENTAL TRANSFERS IN A DYNAMIC MODEL <sup>1</sup>

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## ABSTRACT

In this paper, we study the optimal choices of the federal income tax, federal transfers, and local taxes in a dynamic model of capital accumulation and with explicit game structures among private agents, the local government, and the federal government. When the federal government is the leader and the local government is the follower in a Stackelberg game with both the consumption tax and property tax available to the local government, the optimal local property tax is zero, and local consumption tax is positive. But federal transfers to the local government are negative, and the federal income tax can be positive or negative. In this case, the local consumption tax is used to finance both local and federal public spending.

*Key Words:* income tax, property tax, consumption tax, intergovernmental transfer, capital accumulation, fiscal federalism.

*JEL Classification #:* E0, H2, H4, H5, H7, O4, R5

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# 1 Introduction

This paper considers optimal choices of the federal income tax, local property tax, local consumption tax, and federal transfers to local governments in an intertemporal model of capital accumulation. There exists an enormous literature on optimal income and commodity taxation. Classical contributions include, for example, Ramsey [26], Mirrlees [20], Diamond and Mirrlees [13], Atkinson and Stiglitz [2, 3], and Samuelson [29]. Comprehensive literature reviews are provided by Atkinson and Stiglitz [4], and Myles [21]. In most of these contributions, the government has often taken to be a single identity without introducing the structure of tax assignments and expenditure assignments among multiple levels of government. But in reality, income tax is mainly collected by central governments in Europe and jointly by the federal government and state governments in the United States, property tax is mainly collected by local governments, and commodity tax is collected by both central governments and local governments in Europe or by local governments in the United States. In most developed countries, each level of government has the power to determine tax rates and tax bases. In addition, intergovernmental transfers in various forms exist among different levels of government in every country of reasonable population size. It is natural to see how the structure of fiscal federalism affects optimal taxation and intergovernmental transfers.

In an earlier contribution to optimal taxation and revenue sharing in the context of fiscal federalism, Gordon [15] has utilized a static model to consider how local governments set the rules of local taxes including tax rates and types of taxes in a decentralized form of decision-making while allowing the central government the role of correcting externalities through grants, revenue sharing, and regulations on local tax bases. Recently, Persson and Tabellini [24, 25] have considered risk sharing and redistribution across local governments in a federation using static models involving risk.

In this paper, on the basis of the contributions by Gordon [15], and Persson and Tabellini [24, 25], we analyze the optimal choices of federal taxes, federal transfer, and local taxes in a dynamic model of capital accumulation and with explicit game structures among private agents, local governments, and the federal government.<sup>2</sup> For ease of the treatment, we

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<sup>2</sup>See Zou [33, 34], Brueckner [7], Devarajan, Swaroop, Zou [11], Davoodi and Zou [10], and Zhang and Zou [32] for related dynamic approaches to multi-level government spending, intergovernmental transfers, federal taxes, and local taxes in a "federation".

focus on federal income tax, local property tax, local consumption tax, and federal matching grant for local public spending. Our dynamic approach is timely because the optimal design of tax assignments, expenditure assignments, and intergovernmental transfers among different levels of government has received considerable attention in the 1990s in the context of fiscal federalism, public sector reforms, and economic growth for both developing and developed countries. One of the most important goals of establishing a sound intergovernmental fiscal relationship is supposed to promote local as well as national economic growth (see Rivlin [28], Bird [6], ; Gramlich [16], ; and Oates [23] ). The paper intends to provide an analytical framework for the ongoing discussion on fiscal federalism and economic growth.

Section 2 presents the optimal choices of taxes and transfer from the dynamic Cournot-Nash game between the federal and local government while assuming the Stackelberg (leader-follower) games between the local government and the private agent and between the federal government and the private agent. Section 3 derives the optimal choices of taxes and transfer by studying the Stackelberg game between the local government and the federal government, while retaining the same Stackelberg games between the two levels of government and the private agent. Section 4 concludes.

## 2 The framework

There are three actors in the economy: a representative agent, a local government, and the federal government.

### 2.1 The agent

Like Arrow and Kurz [1], Barro [5], Turnovsky [30], and Turnovsky and Fisher [31], government expenditures are introduced into the representative agent's utility function. Unlike those studies, public expenditures are divided into the federal and local ones in the model. The agent derives a positive, but diminishing, marginal utility from the expenditures of both the federal and local governments and private consumption. Let  $f$ ,  $s$ , and  $c$  be federal expenditure, local expenditure, and private consumption, respectively. If the utility function  $u(c, f, s)$  is twice differentiable, the assumption is equivalent to:

$$u_c > 0, u_f > 0, u_s > 0, u_{cc} < 0, u_{ff} < 0, u_{ss} < 0. \quad (1)$$

For the cross effects  $u_{cf}$ ,  $u_{cs}$ , and  $u_{fs}$ , they are assumed to be positive in general. In addition,  $u(c, f, s)$  satisfies the Inada condition:

$$\begin{aligned} \lim_{s \rightarrow 0} u_s &= \infty, \quad \lim_{f \rightarrow 0} u_f = \infty, \quad \lim_{c \rightarrow 0} u_c = \infty \\ \lim_{s \rightarrow \infty} u_s &= 0, \quad \lim_{f \rightarrow \infty} u_f = 0, \quad \lim_{c \rightarrow \infty} u_c = 0 \end{aligned} \quad (2)$$

The representative agent's discounted utility is given by

$$U = \int_0^{\infty} u(c, f, s) e^{-\rho t} dt, \quad (3)$$

where  $\rho$  is the positive, constant time preference.

Again following Arrow and Kurz [1], Barro [5], and Turnovsky [30], output  $y$  is produced by a constant-return-to-scale production function with three inputs: private capital stock,  $k$ , federal government expenditure,  $f$ , and local government expenditure,  $s$ , namely

$$y = y(k, f, s), \quad (4)$$

where all variables are in per capita terms. For simplicity, the size of population or the labor force is assumed to be constant.

The marginal productivity of private capital stock, federal government expenditure, and local government expenditure are positive and decreasing:

$$y_k > 0, y_f > 0, y_s > 0, y_{kk} < 0, y_{ff} < 0, y_{ss} < 0. \quad (5)$$

Federal government expenditure,  $f$ , is financed by the income tax on the agent. Local government expenditure,  $s$ , is the sum of the consumption tax<sup>3</sup>,  $\tau_c c$ , the capital or property tax,  $\tau_k k$ , and federal government's transfer,  $gs$ .  $\tau_f$ ,  $\tau_c$ , and  $\tau_k$  are the federal income tax rate, local consumption tax rate, and local capital or property tax rate, respectively, and  $g$  is the rate of federal matching grant for local spending. Hence, the budget constraints for the federal government and local government can be written as follows

<sup>3</sup>The consumption tax has been analyzed recently in growth models with one level of government by King and Rebelo [18], Rebelo [27], and Jones, Manuelli, and Rossi [17], and Turnovsky [30].

$$f = \tau_f y - g s \quad (6)$$

$$s = g s + \tau_k k + \tau_c c \quad (7)$$

respectively, and the budget constraint for the representative agent can be written as

$$\frac{dk}{dt} = (1 - \tau_y)y(k, f, s) - \delta k - \tau_k k - (1 + \tau_c)c \quad (8)$$

where  $\delta$  is the rate of capital depreciation.

The representative agent is assumed to have an infinite planning horizon, to face a perfect capital market, and to have perfect foresight. Given these assumptions, he chooses his consumption path and capital-accumulation path to maximize his discounted utility

$$\max U = \int_0^{\infty} (u(c) + v(f) + w(s))e^{-\rho t} dt \quad (9)$$

subject to (8). His initial capital stock is given by  $k(0) = k_0$ . For simplicity, we have taken the utility function to be separable in  $c$ ,  $f$ , and  $s$  in (9).

The Hamiltonian associated with the optimization problem is defined as

$$H = u(c) + v(f) + w(s) + \lambda((1 - \tau_y)y(k, f, s) - \delta k - \tau_k k - (1 + \tau_c)c) \quad (10)$$

where  $\lambda$  is the costate variable, and it represents the marginal utility of wealth.

The first-order conditions for individual optimization are

$$\frac{dk}{dt} = (1 - \tau_f)y(k, f, s) - (1 + \tau_c)c - (\delta + \tau_k)k \quad (11)$$

$$\frac{d\lambda}{dt} = -\lambda[(1 - \tau_f)\frac{\partial y}{\partial k} - \rho - \delta - \tau_k] \quad (12)$$

$$u_c = (1 + \tau_c)\lambda. \quad (13)$$

And from the last condition (13), we have

$$c = c(\lambda, \tau_c). \quad (14)$$

## 2.2 Local government

The local government and the private agent play the Stackelberg game with the local government as the leader and private agent the follower<sup>4</sup>. At the same time, in this section, we also assume that the local and the federal government react to each other along Cournot-Nash lines. That is to say, given the federal income tax rate, federal matching grant, and federal spending, the local government maximizes the agent's welfare by fully incorporating the agent's first-order conditions in section 2.1 into its own maximization. Specifically, the local government will choose optimal taxes  $\tau_c$  and  $\tau_k$ , public expenditure,  $s$ , private capital stock,  $k(t)$ , and the marginal utility of private wealth,  $\lambda(t)$ , to maximize the agent's welfare

$$\max_{\lambda, k, \tau_c, \tau_k, s} \int_0^{\infty} [u(c(\lambda, \tau_c)) + v(f) + w(s)] e^{-\rho t} dt \quad (15)$$

subject to its own budget constraint

$$s - gs = \tau_c c + \tau_k k \quad (16)$$

and the first-order conditions for private agent's optimization

$$\frac{dk}{dt} = (1 - \tau_f)y(k, f, s) - (1 + \tau_c)c(\lambda, \tau_c) - (\delta + \tau_k)k \quad (17)$$

$$\frac{d\lambda}{dt} = -\lambda[(1 - \tau_f)\frac{\partial y}{\partial k} - \rho - \delta - \tau_k] \quad (18)$$

where we have already used the optimal consumption for the private agent in the objective function:  $c = c(\lambda, \tau_c)$ .

Define the Hamiltonian for local government's optimization problem as

$$\begin{aligned} H = & u(c(\lambda, \tau_c)) + v(f) + w(s) + \beta \{ -\lambda[(1 - \tau_f)\frac{\partial y}{\partial k} - \rho - \delta - \tau_k] \} \\ & + \alpha [(1 - \tau_f)y(k, f, s) - (1 + \tau_c)c(\lambda, \tau_c) - (\delta + \tau_k)k] \\ & + \xi [\tau_c c(\lambda, \tau_c) + \tau_k k + gs - s] + \mu \tau_k + \nu \tau_c \end{aligned}$$

<sup>4</sup>A similar technique is used by Chamley [8, 9], Lucas [19], Devarajan *et al* [11] in the treatment of optimal taxation of capital income with one level of government and a representative agent.

where  $\alpha$  is the "local" costate variable associated with the agent's dynamic budget constraint;  $\beta$  is the "local" costate variable associated with the agent's Euler equation of optimal consumption;  $\xi$  is the multiplier for local government's budget constraint;  $\mu$  is the multiplier for the inequality constraint that  $0 \leq \tau_k < 1$ ,  $v$  is the multiplier for the nonnegative consumption tax constraint  $\tau_c \geq 0$ .

The first-order conditions for local government's optimization are

$$\frac{\partial H}{\partial s} = w' + \alpha(1 - \tau_f) \frac{\partial y}{\partial s} - \beta\lambda(1 - \tau_f) \frac{\partial^2 y}{\partial k \partial s} + \xi(g - 1) = 0 \quad (19)$$

$$\frac{\partial H}{\partial \tau_c} = u' c_{\tau_c} - \alpha c - \alpha(1 + \tau_c) c_{\tau_c} + \xi c + \xi \tau_c c_{\tau_c} + v = 0 \quad (20)$$

$$v \tau_c = 0, v \geq 0 \quad (21)$$

$$\frac{\partial H}{\partial \tau_k} = -\alpha k + \beta\lambda + \xi k + \mu = 0 \quad (22)$$

$$\mu \tau_k = 0, \mu \geq 0 \quad (23)$$

$$\begin{aligned} \frac{d\alpha}{dt} &= \rho\alpha - \frac{\partial H}{\partial k} \\ &= \rho\alpha - \alpha[(1 - \tau_f) \frac{\partial y}{\partial k} - \delta - \tau_k] + \beta\lambda(1 - \tau_f) \frac{\partial^2 y}{\partial k^2} - \xi \tau_k \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{d\beta}{dt} &= \rho\beta - \frac{\partial H}{\partial \lambda} \\ &= \rho\beta - u' c_\lambda + \alpha(1 + \tau_c) c_\lambda + \beta[(1 - \tau_f) \frac{\partial y}{\partial k} - \rho - \delta - \tau_k] - \xi \tau_c c_\lambda. \end{aligned} \quad (25)$$

### 2.3 The federal government

We assume that the federal government and the agent play the Stackelberg game with the federal government as the leader and the agent the follower, whereas the federal government and the local government play the Cournot-Nash game. Therefore, taking as given local government's choices of  $\tau_c, \tau_k$ , and  $s$ , the federal government incorporates the private agent's first-order conditions for his optimization into the federal optimization program by choosing



federal income tax,  $\tau_f$ , federal public spending,  $f$ , the rate of federal transfer to the local government,  $g$ , private capital stock,  $k$ , and the marginal utility of private wealth,  $\lambda$ , to maximize the agent's welfare, namely,

$$\max \int_0^{\infty} [u(c(\lambda, \tau_c)) + v(f) + w(s)] e^{-\rho t} dt \quad (26)$$

subject to the agent's optimization conditions:

$$\frac{dk}{dt} = (1 - \tau_f)y(k, f, s) - (1 + \tau_c)c(\lambda, \tau_c) - (\delta + \tau_k)k \quad (27)$$

$$\frac{d\lambda}{dt} = -\lambda[(1 - \tau_f)\frac{\partial y(k, f, s)}{\partial k} - \rho - \delta - \tau_k(k, \lambda, \alpha, \beta)] \quad (28)$$

and the federal budget constraint

$$f + gs = \tau_f y \quad (29)$$

with the initial private capital stock  $k(0)$  given.

Define the Hamiltonian function for the federal government as

$$\begin{aligned} H_f = & u(c(\lambda, \tau_c)) + v(f) + w(s) + \theta_2 \left\{ -\lambda \left[ (1 - \tau_f) \frac{\partial y(k, f, s)}{\partial k} - \rho - \delta - \tau_k \right] \right\} \\ & + \theta_1 [(1 - \tau_f)y(k, f, s) - (1 + \tau_c)c(\lambda, \tau_c) - (\delta + \tau_k)k] \\ & + \eta [\tau_f y - f - gs] + \omega g \end{aligned}$$

where  $\theta_1$  is the "federal" costate variable associated with the agent's dynamic budget constraint;  $\theta_2$  is the "federal" costate variable associated with the agent's Euler equation of optimal consumption;  $\eta$  is the multiplier for the federal budget constraint; and  $\omega$  is the multiplier for the requirement of a non-negative rate of federal transfer, i.e.,  $g \geq 0$ .

The first-order conditions for the federal government's optimization are

$$\frac{\partial H_f}{\partial f} = v' + [\eta \tau_f + \theta_1 (1 - \tau_f)] \frac{\partial y}{\partial f} - \theta_2 \lambda (1 - \tau_f) \frac{\partial^2 y}{\partial k \partial f} - \eta = 0 \quad (30)$$

$$\frac{\partial H_f}{\partial \tau_f} = -\theta_1 y + \theta_2 \lambda \frac{\partial y}{\partial k} + \eta y = 0 \quad (31)$$

$$\begin{aligned} \frac{d\theta_1}{dt} &= \rho \theta_1 - \frac{\partial H_f}{\partial k} \\ &= \rho \theta_1 - \theta_1 [(1 - \tau_f) \frac{\partial y}{\partial k} - \delta - \tau_k] + \theta_2 \lambda [(1 - \tau_f) \frac{\partial^2 y}{\partial k^2} - \eta \tau_f \frac{\partial y}{\partial k}] \end{aligned} \quad (32)$$



$$\frac{d\theta_2}{dt} = \rho\theta_2 - \frac{\partial H_f}{\partial \lambda} \quad (33)$$

$$= \rho\theta_2 - u'c_\lambda + \theta_1(1 + \tau_c)c_\lambda + \theta_2[(1 - \tau_f)\frac{\partial y}{\partial k} - \rho - \delta - \tau_k] - \eta s + \omega = 0, \omega \geq 0. \quad (34)$$

## 2.4 Some results from the Cournot-Nash equilibrium for the federal and local governments

The full dynamic system is extremely complicated. But some results regarding the optimal choices of taxes and federal transfer along the Cournot-Nash lines can be derived from the steady-state or long-run analysis of the full dynamic system. In the steady state,

$$\frac{dk}{dt} = \frac{d\lambda}{dt} = \frac{dc}{dt} = \frac{df}{dt} = \frac{ds}{dt} = \frac{d\tau_f}{dt} = \frac{d\tau_c}{dt} = \frac{d\tau_k}{dt} = \frac{dg}{dt} = 0 \quad (35)$$

and so are various costate variables and multipliers:

$$\frac{d\alpha}{dt} = \frac{d\beta}{dt} = \frac{d\xi}{dt} = \frac{d\mu}{dt} = \frac{d\theta_1}{dt} = \frac{d\theta_2}{dt} = \frac{d\eta}{dt} = \frac{d\omega}{dt} = 0. \quad (36)$$

Therefore,

$$(1 - \tau_f)y(k, f, s) - (1 + \tau_c)c(\lambda, \tau_c) - (\delta + \tau_k)k = 0 \quad (37)$$

$$-\lambda[(1 - \tau_f)\frac{\partial y}{\partial k} - \rho - \delta - \tau_k] = 0 \quad (38)$$

$$u_c = (1 + \tau_c)\lambda \quad (39)$$

$$s - gs = \tau_c c + \tau_k k \quad (40)$$

$$w' + \alpha(1 - \tau_f)\frac{\partial y}{\partial s} - \beta\lambda(1 - \tau_f)\frac{\partial^2 y}{\partial k \partial s} + \xi(g - 1) = 0 \quad (41)$$

$$u'c_{\tau_c} - \alpha c - \alpha(1 + \tau_c)c_{\tau_c} + \xi c + \xi\tau_c c_{\tau_c} + v = 0 \quad (42)$$

$$v\tau_c = 0, v \geq 0. \quad (43)$$

$$-\alpha k + \beta\lambda + \xi k + \mu = 0 \quad (44)$$

$$\mu\tau_k = 0, \mu \geq 0 \quad (45)$$

$$\beta\lambda(1 - \tau_f)\frac{\partial^2 y}{\partial k^2} - \xi\tau_k = 0 \quad (46)$$

$$\rho\beta - u'c_\lambda + \alpha(1 + \tau_c)c_\lambda - \xi\tau_cc_\lambda = 0 \quad (47)$$

$$f + gs = \tau_f y \quad (48)$$

$$v' + [\eta\tau_f + \theta_1(1 - \tau_f)]\frac{\partial y}{\partial f} - \theta_2\lambda(1 - \tau_f)\frac{\partial^2 y}{\partial k\partial f} - \eta = 0 \quad (49)$$

$$-\theta_1 y + \theta_2\lambda\frac{\partial y}{\partial k} + \eta y = 0 \quad (50)$$

$$\theta_2\lambda(1 - \tau_f)\frac{\partial^2 y}{\partial k^2} - \eta\tau_f\frac{\partial y}{\partial k} = 0 \quad (51)$$

$$\rho\theta_2 - u'c_\lambda + \theta_1(1 + \tau_c)c_\lambda = 0 \quad (52)$$

$$-\eta s + \omega = 0, \omega g = 0, \omega \geq 0. \quad (53)$$

**Proposition 1** *The steady-state optimal property tax rate is zero, but the steady-state consumption tax is positive.*

*Proof:* First from equation (46), we have

$$\beta\lambda(1 - \tau_f)\frac{\partial^2 y}{\partial k^2} = \xi\tau_k \geq 0. \quad (54)$$

Hence,  $\beta \leq 0$ .

Suppose the optimal property tax rate is strictly positive:  $\tau_k > 0$ . From equations (44) and (47), we have

$$(\xi - \alpha)k + \beta\lambda = 0, \quad (55)$$

$$u' - \alpha(1 + \tau_c) + \xi\tau_c = \frac{\rho\beta}{c_\lambda}. \quad (56)$$

Substituting equations (55) and (56) into equation (42), we obtain

$$\frac{\rho\beta}{c_\lambda}c_{\tau_c} - \beta\lambda\frac{c}{k} + v = 0. \quad (57)$$

From equation (39), we have

$$c_\lambda = \frac{1 + \tau_c}{u_{cc}}, \quad c_{\tau_c} = \frac{\lambda}{u_{cc}}. \quad (58)$$

Substituting equation (58) into equation (57), we get

$$\frac{\beta\lambda}{1 + \tau_c} \left( \rho - \frac{(1 + \tau_c)c}{k} \right) + v = 0. \quad (59)$$

Substituting equations (37) and (38) into equation (59), we have

$$\frac{\beta\lambda}{1 + \tau_c} (1 - \tau_f) \left( \frac{\partial y}{\partial k} - \frac{y}{k} \right) + v = 0. \quad (60)$$

Because of the assumption on the production function, we have

$$\frac{\partial y}{\partial k} k < y. \quad (61)$$

If  $\tau_c \geq 0$ , we have  $v \geq 0$ . Now, equation (60) implies

$$\beta = 0. \quad (62)$$

Hence, from equations (44) and (46), we have

$$\xi = \alpha = 0. \quad (63)$$

Then, from equation (41), we obtain

$$w'(s) = 0, \quad (64)$$

which is impossible because  $w'(s)$  is strictly positive by our assumptions. Therefore, we must have  $\tau_k = 0$ .

*Q.E.D.*

This result is rather intuitive. For the local government, consumption tax has no distortionary effect on private production and private capital accumulation, whereas local property tax directly reduces private capital accumulation. It is always welfare maximizing for the local government to finance local public spending through the less distortionary consumption tax instead of capital or property tax.

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**Proposition 2** *The steady-state federal transfer is zero:  $g = 0$ .*

*Proof:* Suppose that  $g$  is not equal to zero. Then, from equation (53), we have

$$\omega = 0, \eta = 0. \quad (65)$$

That is to say, from equation (51),

$$\theta_2 = 0. \quad (66)$$

Then, from equation (50),

$$\theta_1 = 0. \quad (67)$$

Now from equation (49) we must have

$$v'(f) = 0. \quad (68)$$

This contradicts our assumption that  $v'(f) > 0$ . Therefore, we must have

$$g = 0. \quad (69)$$

*Q.E.D.*

This is also intuitively convincing. The federal government and the local government decide their optimal choices along the Cournot-Nash lines without taking into consideration the interactions of their choices. In this case it is always in the federal government's interest to provide zero subsidy to the local government. In the next section this picture will dramatically change when the two governments play a Stackelberg game with the federal government as the leader and the local government as the follower.

Before we conclude this section, please note the following three points. First, the steady-state consumption tax and property tax cannot be zero at the same time. This is true because, from proposition 2, we know that the steady-state government matching grant is zero. From equation (40), if  $\tau_k = \tau_c = 0$ , we have  $s = 0$ , which cannot be optimal in view of the Inada conditions (2) on the utility function. Second, if the local consumption tax is set to zero, i.e.,  $\tau_c = 0$ , then local spending must be financed by a positive capital or property tax. Still, the Cournot-Nash game between the federal and local governments will result in a zero federal transfer to locality:  $g = 0$ . The proof is similar to the one in proposition 2. Finally, along the Cournot-Nash lines, the optimal federal income tax must be positive because of the Inada condition for federal spending.

### 3 The Stackelberg game between the federal government and local government

In section 2, we find that if the federal and local governments play the Cournot-Nash game in choosing their individually optimal taxes and transfer, respectively, then federal transfer to the local government is zero. Here we suppose that the federal and the local governments play a Stackelberg game while retaining the same game structures of the private agent versus the two levels of government. In the new setting, it is natural to let the federal government be the leader, and the local government be the follower. In order to by-pass the complexity of the general solutions of these complicated, multi-stage Stackelberg games, and provide some explicit solutions to the optimal choices of taxes and federal transfer, we use specific utility function and production technology.

#### 3.1 The agent

The production function of the agent is assumed to take the following form

$$y = k^\alpha f^\beta s^\gamma, \quad (70)$$

where  $\alpha > 0$ ,  $\beta > 0$ ,  $\gamma > 0$  and  $\alpha + \beta + \gamma < 1$ . In equation (70), the output and inputs are all measured in terms of the representative agent's labor input. This is why  $\alpha + \beta + \gamma < 1$ . For simplicity, the agent's labor input is assumed to be constant.

His utility function is logarithmic:

$$u(c, f, s) = \ln c + \vartheta_1 \ln f + \vartheta_2 \ln s \quad (71)$$

where  $\vartheta_1$  and  $\vartheta_2$  are constant and positive.

With these choices of preferences and technology, it is simple to show that steady-state capital, output, and consumption are the functions of federal income tax, federal spending, local property tax, local consumption tax, local spending, and various technology and preference parameters:

$$\begin{aligned} k &= \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{1}{\alpha-1}} f^{\frac{\beta}{1-\alpha}} s^{\frac{\gamma}{1-\alpha}} \\ y &= \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{\alpha}{\alpha-1}} f^{\frac{\beta}{1-\alpha}} s^{\frac{\gamma}{1-\alpha}} \end{aligned} \quad (72)$$

$$c = \frac{\rho + (1 - \alpha)(\delta + \tau_k)}{\alpha(1 + \tau_c)} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{1}{\alpha-1}} f^{\frac{\alpha}{1-\alpha}} s^{\frac{\gamma}{1-\alpha}}.$$

### 3.2 The local government

The local government maximizes the steady-state agent's welfare

$$\max_{\tau_c, \tau_k, s} \ln c + \vartheta_1 \ln f + \vartheta_2 \ln s \quad (73)$$

subject to the individual's optimal choices of consumption and capital stock given in equation (72), and its own budget constraint:

$$s - gs = \tau_c c + \tau_k k. \quad (74)$$

Substituting equation (72) into equation (16) yields

$$\begin{aligned} s &= \frac{\tau_c}{1-g} \frac{\rho + (1 - \alpha)(\delta + \tau_k)}{\alpha(1 + \tau_c)} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{1}{\alpha-1}} f^{\frac{\alpha}{1-\alpha}} s^{\frac{\gamma}{1-\alpha}} \\ &\quad + \frac{\tau_k}{1-g} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{1}{\alpha-1}} f^{\frac{\alpha}{1-\alpha}} s^{\frac{\gamma}{1-\alpha}} \\ &= \frac{\tau_c(\rho + (1 - \alpha)\delta) + (\tau_c + \alpha)\tau_k}{\alpha(1 + \tau_c)} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{1}{\alpha-1}} f^{\frac{\alpha}{1-\alpha}} s^{\frac{\gamma}{1-\alpha}} \frac{1}{1-g}. \end{aligned}$$

Therefore, we have

$$s = \left( \frac{\tau_c(\rho + (1 - \alpha)\delta) + (\tau_c + \alpha)\tau_k}{\alpha(1 + \tau_c)} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \left( \frac{1}{1-g} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{-1}{1-\alpha-\gamma}} f^{\frac{\alpha}{1-\alpha-\gamma}} \quad (75)$$

Now, the local government's objective function upon substitution becomes:

$$\begin{aligned} \ln c + \vartheta_2 \ln s &= \ln(\rho + (1 - \alpha)(\delta + \tau_k)) - \ln(1 + \tau_c) + \frac{1}{\alpha - 1} \ln(\rho + \delta + \tau_k) \\ &\quad + \left( \frac{\gamma}{1 - \alpha} + \vartheta_2 \right) \ln s + \text{constant} \\ &= \ln(\rho + (1 - \alpha)(\delta + \tau_k)) - \ln(1 + \tau_c) + \frac{1}{\alpha - 1} \ln(\rho + \delta + \tau_k) \\ &\quad + \left( \frac{\gamma}{1 - \alpha} + \vartheta_2 \right) \frac{1 - \alpha}{1 - \alpha - \gamma} [\ln(\tau_c(\rho + (1 - \alpha)\delta) + (\tau_c + \alpha)\tau_k) - \ln(1 + \tau_c)] \\ &\quad - \left( \frac{\gamma}{1 - \alpha} + \vartheta_2 \right) \frac{1}{1 - \alpha - \gamma} \ln(\rho + \delta + \tau_k) + \text{constant} \end{aligned}$$

(76)

Hence, the local government's optimization problem is equivalent to maximizing equation (76) by determining  $\tau_c$  and  $\tau_k$ . The first-order conditions for the optimal choices of taxes of the local government are:

$$\begin{aligned} & \frac{1-\alpha}{\rho+(1-\alpha)(\delta+\tau_k)} + \frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma} \frac{\tau_c+\alpha}{\tau_c(\rho+(1-\alpha)\delta)+(\tau_c+\alpha)\tau_k} \\ & - \frac{\vartheta_2(1-\alpha)+\gamma}{(1-\alpha)(1-\alpha-\gamma)} \frac{1}{\rho+\delta+\tau_k} + \frac{1}{\alpha-1} \frac{1}{\rho+\delta+\tau_k} = 0, \end{aligned} \quad (77)$$

$$-\frac{1}{1+\tau_c} + \frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma} \left[ \frac{\rho+(1-\alpha)\delta+\tau_k}{\tau_c(\rho+(1-\alpha)\delta)+(\tau_c+\alpha)\tau_k} - \frac{1}{1+\tau_c} \right] = 0. \quad (78)$$

To simplify the calculations, the rate of capital depreciation is set to zero:  $\delta = 0$ .

Now we have the following results

**Proposition 3** *If it is required that  $\tau_k \geq 0$ , the optimal property tax and consumption tax are*

$$\tau_k = 0 \quad (79)$$

$$\tau_c = \frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma}. \quad (80)$$

Therefore, the constrained optimal property tax is always zero as shown in proposition 1. With the specific example in this section, we can obtain explicit solutions to optimal local tax rates

**Proposition 4** *If  $\tau_k$  can take any value, we have*

$$\begin{aligned} \tau_k &= -\rho \frac{\vartheta_2}{1-\alpha-\gamma} - \rho \frac{\gamma}{(1-\alpha)(1-\alpha-\gamma)}, \\ \tau_c &= \frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma} + \frac{\rho(\vartheta_2(1-\alpha)+\gamma)}{(1-\alpha)(1-\alpha-\gamma)-\vartheta_2(1-\alpha)-\gamma}. \end{aligned} \quad (81)$$

In this case, the optimal  $\tau_k$  for the local government is in fact negative, whereas  $\tau_c$  is strictly positive. This result conforms to our intuition. The local government taxes consumption to subsidize capital investment. This tax subsidy scheme leads to more welfare for the agent in the long run.



### 3.3 The federal government

Unlike the Cournot-Nash game between the federal and local governments, in the Stackelberg game the federal government takes into consideration the optimal choices of both the agent and the local government when it maximizes the agent's steady-state welfare

$$\max \ln c + \vartheta_1 \ln f + \vartheta_2 \ln s \quad (82)$$

by choosing  $\tau_f, g, \tau_c, \tau_k, f$ , and  $s$ . The federal budget constraint is still

$$f + gs = \tau_f y. \quad (83)$$

Substituting equations (72) and (75) into equation (83), we have

$$\begin{aligned} f &= \tau_f \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{\tau_c(\rho + (1 - \alpha)\delta) + (\tau_c + \alpha)\tau_k}{\alpha(1 + \tau_c)} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \\ &\quad \times \left( \frac{1}{1 - g} \right)^{\frac{\gamma}{1-\alpha-\gamma}} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{-1}{1-\alpha-\gamma}} f^{\frac{\theta}{1-\alpha-\gamma}} \\ &\quad - g \left( \frac{\tau_c(\rho + (1 - \alpha)\delta) + (\tau_c + \alpha)\tau_k}{\alpha(1 + \tau_c)} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \left( \frac{1}{1 - g} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{-1}{1-\alpha-\gamma}} f^{\frac{\theta}{1-\alpha-\gamma}} \\ &= f^{\frac{\theta}{1-\alpha-\gamma}} \left\{ \tau_f \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{\tau_c(\rho + (1 - \alpha)\delta) + (\tau_c + \alpha)\tau_k}{\alpha(1 + \tau_c)} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \right. \\ &\quad \times \left( \frac{1}{1 - g} \right)^{\frac{\gamma}{1-\alpha-\gamma}} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{-1}{1-\alpha-\gamma}} \\ &\quad \left. - g \left( \frac{\tau_c(\rho + (1 - \alpha)\delta) + (\tau_c + \alpha)\tau_k}{\alpha(1 + \tau_c)} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \left( \frac{1}{1 - g} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{-1}{1-\alpha-\gamma}} \right\}. \end{aligned}$$

Therefore, we have

$$f = \left\{ \tau_f \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} A^{\gamma - \frac{g}{1 - g}} A^{1-\alpha} \right\}^{\frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma}} \left( \frac{\rho + \delta + \tau_k}{\alpha(1 - \tau_f)} \right)^{\frac{-1}{1-\alpha-\beta-\gamma}} \left( \frac{1}{1 - g} \right)^{\frac{\gamma}{1-\alpha-\beta-\gamma}}, \quad (84)$$

where  $A = \left( \frac{\tau_c(\rho + (1 - \alpha)\delta) + (\tau_c + \alpha)\tau_k}{\alpha(1 + \tau_c)} \right)^{\frac{1-\alpha}{1-\alpha-\gamma}}$ .

Substituting equations (72), (75), and (84) into the federal government's objective function yields

$$\begin{aligned}
& \ln c + \vartheta_1 \ln f + \vartheta_2 \ln s \\
&= \frac{1}{1-\alpha} \ln(1-\tau_f) + \left( \frac{\gamma}{1-\alpha} + \vartheta_2 \right) \left[ -\frac{1-\alpha}{1-\alpha-\gamma} \ln(1-g) + \frac{1}{1-\alpha-\gamma} \ln(1-\tau_f) \right] \\
& \quad + \left[ \left( \frac{\gamma}{1-\alpha} + \vartheta_2 \right) \frac{\beta}{1-\alpha-\gamma} + \frac{\beta}{1-\alpha} + \vartheta_1 \right] \ln f + \text{constant} \\
&= \frac{1}{1-\alpha} \ln(1-\tau_f) + \left( \frac{\gamma}{1-\alpha} + \vartheta_2 \right) \left[ -\frac{1-\alpha}{1-\alpha-\gamma} \ln(1-g) + \frac{1}{1-\alpha-\gamma} \ln(1-\tau_f) \right] \\
& \quad + \left[ \left( \frac{\gamma}{1-\alpha} + \vartheta_2 \right) \frac{\beta}{1-\alpha-\gamma} + \frac{\beta}{1-\alpha} + \vartheta_1 \right] \left\{ \right. \\
& \quad \left. \frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma} \ln \left[ \tau_f \frac{\rho+\delta+\tau_k}{\alpha(1-\tau_f)} A^\gamma - \frac{g}{1-g} A^{1-\alpha} \right] \right. \\
& \quad \left. + \frac{1}{1-\alpha-\beta-\gamma} \ln(1-\tau_f) - \frac{\gamma}{1-\alpha-\beta-\gamma} \ln(1-g) \right\} + \text{constant}.
\end{aligned} \tag{85}$$

Given substitutions above,  $\tau_c$ ,  $\tau_k$ ,  $f$ , and  $s$  are all functions of  $\tau_f$  and  $g$ . Now the federal government's optimization is equivalent to maximize the agent's welfare in equation (85) by choosing  $\tau_f$  and  $g$ . The first-order conditions for maximization are

$$\begin{aligned}
& \frac{(1+\vartheta_2)}{1-\alpha-\gamma} + \left[ \left( \frac{\gamma}{1-\alpha} + \vartheta_2 \right) \frac{\beta}{1-\alpha-\gamma} + \frac{\beta}{1-\alpha} + \vartheta_1 \right] \\
& \quad \left\{ \frac{1}{1-\alpha-\beta-\gamma} + \frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma} \frac{-\frac{\rho+\delta+\tau_k}{\alpha(1-\tau_f)} A^\gamma (1-g)}{\tau_f \frac{\rho+\delta+\tau_k}{\alpha(1-\tau_f)} (1-g) A^\gamma - g A^{1-\alpha}} \right\} \\
&= 0,
\end{aligned} \tag{86}$$

$$\begin{aligned}
& \frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma} + \left[ \left( \frac{\gamma}{1-\alpha} + \vartheta_2 \right) \frac{\beta}{1-\alpha-\gamma} + \frac{\beta}{1-\alpha} + \vartheta_1 \right] \\
& \quad \left\{ \frac{\gamma}{1-\alpha-\beta-\gamma} + \frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma} \frac{-A^{1-\alpha}}{\tau_f \frac{\rho+\delta+\tau_k}{\alpha(1-\tau_f)} (1-g) A^\gamma - g A^{1-\alpha}} \right\} \\
&= 0.
\end{aligned} \tag{87}$$

From these two first-order conditions, we have

**Proposition 5** *The optimal federal income tax and federal transfer to the local government are*

$$\begin{aligned}\tau_f &= 1 - \frac{\frac{C\alpha}{1-\alpha-\beta-\gamma} + \frac{1+\alpha\vartheta_2-\gamma}{1-\alpha-\gamma}}{\frac{1+\vartheta_2}{1-\alpha-\gamma} + \frac{C}{1-\alpha-\beta-\gamma}} \frac{1}{1 - \frac{\alpha}{\rho} A^{1-\alpha-\gamma}} \\ g &= 1 - \frac{\frac{C\alpha}{1-\alpha-\beta-\gamma} + \frac{1+\alpha\vartheta_2-\gamma}{1-\alpha-\gamma}}{\frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma} + \frac{C\gamma}{1-\alpha-\beta-\gamma}} \frac{\frac{\alpha}{\rho} A^{1-\alpha-\gamma}}{1 - \frac{\alpha}{\rho} A^{1-\alpha-\gamma}}\end{aligned}\quad (88)$$

where

$$\begin{aligned}C &= \left(\frac{\gamma}{1-\alpha} + \vartheta_2\right) \frac{\beta}{1-\alpha-\gamma} + \frac{\beta}{1-\alpha} + \vartheta_1 \\ A &= \left(\frac{\tau_c(\rho + (1-\alpha)\delta) + (\tau_c + \alpha)\tau_k}{\alpha(1+\tau_c)}\right)^{\frac{1}{1-\alpha-\gamma}}.\end{aligned}\quad (89)$$

It is interesting that when the consumption tax is available to the local government, and when the federal government acts as the leader in the Stackelberg game with the local government, federal transfer to the local government can be negative. At the same time, it is unclear whether federal income tax must be positive. The reason is now obvious enough: with a less distortionary consumption tax at the local level, and given the Stackelberg game between the federal and local governments, the federal government can impose a negative transfer to locality and at the same time ask the local government to levy a high rate of consumption tax. Hence, the local consumption tax can be used to finance both federal and local spending, and subsidize private investment.

To see how the signs of federal income tax and federal transfer are determined, we make some numerical calculations based on propositions 3 and 5. In this case,  $\tau_k = 0$  and  $\tau_c = \frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma}$ . We let the marginal utility of local public spending,  $\vartheta_2$ , take different values. Other parameters are fixed as follows:  $\alpha = 0.3$ ,  $\beta = 0.2$ ,  $\gamma = 0.1$ ,  $\vartheta_1 = 0.1$ , and  $\rho = 0.05$ . From Table 1, as  $\vartheta_2$  rises from 0.05 to 0.30, (i.e., the marginal utility of local public spending rises),  $\tau_c$  increases from 18.5% to 92.5%. Without the constraint

that  $g \geq 0$ ,  $g$  is negative all the time, and the "reverse" transfer rate from locality to the federal government rises from 18.8% to 191.7%. At the same time, the federal income tax,  $\tau_f$ , decreases and eventually becomes negative. Thus the local consumption tax finances local public spending, federal public spending through a negative federal transfer, and federal subsidies to private production through a negative income tax.

Table 1: Optimal Tax Rates and Federal Transfer

$\vartheta_2$	0.05	0.10	0.15	0.20	0.25	0.30
$\tau_c$	0.184615	0.283333	0.40	0.54	0.711111	0.925
$\tau_f$	0.232575	0.187222	0.132	0.0641538	-0.0203292	-0.1275
$g$	-0.0188034	-0.345833	-0.675676	-1.03143	-1.43593	-1.91731

If we impose the condition that federal transfer to the local government must be positive in the federal government's optimization problem, it is easy to show the next proposition.

**Proposition 6** *If  $g \geq 0$ , the optimal federal income tax is*

$$\tau_f = \frac{\frac{C(1-\alpha-\gamma)}{1-\alpha-\beta-\gamma}}{\frac{1+\vartheta_2}{1-\alpha-\gamma} + \frac{C}{1-\alpha-\beta-\gamma}}$$

$$g = 0.$$

In this case, since it is not feasible for the federal government to collect any revenues from the local government, federal income tax is strictly positive with the Inada conditions on the utility function. As an illustration, we still choose  $\tau_k = 0$  and  $\tau_c = \frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma}$  from local government's optimal choices of tax rates in proposition 3. We also let  $\vartheta_2$  vary and let other parameters be fixed at  $\alpha = 0.3$ ,  $\beta = 0.2$ ,  $\gamma = 0.1$ ,  $\vartheta_1 = 0.1$ , and  $\rho = 0.05$ . The optimal federal transfer is always zero, and the optimal federal income tax and optimal local consumption tax are calculated in Table 2.

Table 2: Optimal Tax Rates

$\vartheta_2$	0.05	0.10	0.15	0.20	0.25	0.30
$\tau_c$	0.184615	0.283333	0.40	0.54	0.711111	0.925
$\tau_f$	0.248529	0.233333	0.218766	0.204918	0.191972	0.180282

In Table 2, as  $\vartheta_2$  rises from 0.05 to 0.30, the marginal utility of local public services rises sharply, and so does the local consumption tax, which increases from 18.5% to 92.5%. At the same time, as  $\vartheta_1 = 0.1$ , the marginal utility of federal public spending falls relative to local public spending. Therefore, it is less socially desirable to finance as much of federal public spending as before. Hence, federal income tax falls from 24.9% to 18%.

### 3.4 The case of zero consumption tax

In our optimal-tax framework with a multiple levels of government the optimal property tax is always zero or negative given the availability of consumption tax for the local government. In reality, of course, local governments in most developed countries rely on property tax to finance their local public services. While we do not want to argue whether the reality deviates from the theoretical optimality, we can allow some role of a positive, optimal property tax if we set local consumption tax to zero. Then, letting  $\tau_c = 0$  in equation (77), we have

$$\begin{aligned} & \frac{1-\alpha}{\rho + (1-\alpha)(\delta + \tau_k)} + \frac{\vartheta_2(1-\alpha) + \gamma}{1-\alpha-\gamma} \frac{1}{\tau_k} \\ & - \frac{\vartheta_2(1-\alpha) + \gamma}{(1-\alpha)(1-\alpha-\gamma)} \frac{1}{\rho + \delta + \tau_k} + \frac{1}{\alpha-1} \frac{1}{\rho + \delta + \tau_k} \\ & = 0. \end{aligned} \quad (90)$$

Now from equation (90) we have the optimal local property tax  $\tau_k$ . From equations (86) and (87), we have the optimal federal income tax and federal transfer,  $\tau_f$  and  $g$ , respectively.

**Proposition 7** *The optimal property tax, optimal federal income tax, and optimal federal transfer are*

$$\begin{aligned} \tau_k = & \frac{\sqrt{[4\alpha(\gamma + (1-\alpha)\vartheta_2)(1-\alpha)(1+\vartheta_2)\rho^2 + [\alpha^2(1+\vartheta_2) + \gamma + \vartheta_2 - (3\vartheta_2 + 2)\alpha]^2\rho^2}}{2\alpha(1-\alpha)(1+\vartheta_2)} \\ & + \frac{[\alpha^2(1+\vartheta_2) + \gamma + \vartheta_2 - (3\vartheta_2 + 2)\alpha]\rho}{2\alpha(1-\alpha)(1+\vartheta_2)} \end{aligned} \quad (91)$$

$$\tau_f = 1 - \frac{\frac{C\alpha}{1-\alpha-\beta-\gamma} + \frac{1+\alpha\vartheta_2-\gamma}{1-\alpha-\gamma}}{\frac{1+\vartheta_2}{1-\alpha-\gamma} + \frac{C}{1-\alpha-\beta-\gamma}} \frac{1}{1-\frac{\alpha}{\rho}\tau_k} \quad (92)$$

$$g = 1 - \frac{\frac{C\alpha}{1-\alpha-\beta-\gamma} + \frac{1+\alpha\vartheta_2-\gamma}{1-\alpha-\gamma}}{\frac{\vartheta_2(1-\alpha)+\gamma}{1-\alpha-\gamma} + \frac{C\gamma}{1-\alpha-\beta-\gamma}} \frac{\frac{\alpha}{\rho}\tau_k}{1-\frac{\alpha}{\rho}\tau_k} \quad (93)$$

where

$$C = \left(\frac{\gamma}{1-\alpha} + \vartheta_2\right) \frac{\beta}{1-\alpha-\gamma} + \frac{\beta}{1-\alpha} + \vartheta_1. \quad (94)$$

To provide some intuition on how the optimal tax and transfer rates are determined, we compute the three optimal rates in Table 3 for different values of  $\alpha$ , which measures the productivity of private capital stock. For all other parameters, their values are fixed at  $\beta = 0.2$ ,  $\gamma = 0.1$ ,  $\vartheta_1 = 0.1$ ,  $\vartheta_2 = 0.1$ , and  $\rho = 0.05$ .

Table 3: Optimal Property Tax, Income Tax, and Transfer

$\alpha$	0.2	0.25	0.3	0.35	0.40
$\tau_k$	0.0446251	0.035477	0.0232356	0.0187604	0.0157411
$\tau_f$	0.24934	0.262329	0.264068	0.261329	0.256353
$g$	0.0541665	0.180578	0.230509	0.249001	0.250828

From Table 3 it is clear that, because local property tax is highly distortionary, the optimal property tax declines steadily from 4.46% to 1.57% as the productivity of private capital stock rises from .2 to .4. At the same time, the federal government raises its transfer rate to the local government sharply from 5.4% to 25%, without significantly altering the rate of federal income tax.

## 4 Summary

In this paper, we have studied the optimal choices of federal income tax, federal transfer, and local taxes in a dynamic model of capital accumulation and with explicit game structures among the representative agent, the local

government, and the federal government. We summarize our main findings as follows.

When the federal and the local governments choose their optimal tax rates and transfer scheme along the Cournot-Nash lines, optimal local property tax is zero, optimal local consumption tax is strictly positive, optimal federal income tax is strictly positive, and optimal federal transfer is zero.

When the federal government is the leader and the local government is the follower in a Stackelberg game with both consumption tax and property tax available to the local government, again, the optimal local property tax is zero, and local consumption tax is positive. But federal transfer to the local government is negative, and federal income tax can be positive or negative. In this case, the local consumption tax can be used to finance both local and federal public spending. This "reverse" transfer from the local government to the federal government is optimal from the perspective of welfare maximization because the local consumption tax is less distortionary than both local property tax and federal income tax. When the local consumption tax is set to zero, optimal local property tax can be positive, as are the federal income tax and federal transfer to the local government.

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