Product Innovation, Capital Accumulation, and Endogenous Growth

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Introduction

This chapter integrates both product innovation and physical capital accumulation in a simple model of endogenous growth and examines the long–run relationship between product development and capital formation. It also studies the impact of international technology transfers and international trade on long–run capital accumulation.

This work can be regarded as a continuation of the line of research initiated by Romer (1990), Grossman and Helpman (1991), and Helpman (1992). In the Romer model, the innovative products are horizontally differentiated capital goods and are produced from the homogeneous final output. These differentiated capital goods are in turn employed to produce the final output. A different modeling strategy is adopted by Grossman and Helpman. In the Grossman–Helpman model, the innovative products are intermediate inputs into the production of a single, final good. But the final good can be either consumed by households or can be invested in the form of capital accumulation by firms. In both models, a similar, perhaps surprising, conclusion has been drawn: physical capital accumulation plays only a supporting role in the story of long–run growth because the primary sources of growth are a variety of factors such as the rate of time preference, the productivity of product innovation, and the elasticity of substitution across brands, 'while the investment rate adjusts so as to keep the rate of expansion of conventional capital in line with the growth rate of output' (Helpman, 1992). Some related approaches to the dynamics of innovation and long–run growth can be found in Stokey (1988, 1991a, 1991b), Aghion and Howitt (1992), Gort and Klepper (1992), and Stein (1997).

In this chapter, we intend to offer a different perspective on capital accumulation, product innovation, and output growth. In particular, we hope to distinguish the role of the marginal productivity of capital in determining the long–run rates of both product innovation and physical capital accumulation. In our model, all differentiated goods are produced using capital input, and can be consumed, or invested to increase capital stock, or used for product innovation. This modeling option has already been pointed out in Grossman and Helpman (1991), even though they choose to model capital as the homogeneous final good.

We should not argue about the plausibility of treating capital stock as the accumulated differentiated products, because in the real world capital does take many forms such as machinery, buildings, tools, and so on. In modeling capital as differentiated goods, our model agrees with the Romer (1990) model, but it differs from the Romer model in assuming that the final consumption in our model also consists of all differentiated goods instead of a single, homogeneous good as in the Romer model.

In this alternative framework, we will demonstrate how the long-run growth rates of capital accumulation and product innovation are determined. In particular we will show the roles of the productivity of the capital stock and the efficiency of product innovation process in determining the long-run rates. In addition, we extend the basic model to an open economy and show that trade in goods not only improves welfare, but also accelerates capital accumulation. Furthermore, for a developing country receiving technology transfers from a developed country such as in the North–South model, the rate of capital accumulation in the South is shown to be partly determined by the rate of product innovation in the North.

This chapter is organized as follows. The next section will set up the dynamic model with both capital accumulation and product innovation. The growth rates of different variables will be derived. Following this, we consider the effect of technology transfers from the developed country on product innovation and capital accumulation in the developing country. The next section extends the model to the case with international trade and shows the impact of trade on capital accumulation. We then conclude this chapter.

The model

The consumer preference is the standard Dixit–Stiglitz CES utility function, which has been used by Krugman (1979), Judd (1985), and Grossman and Helpman (1991) among many others in studying the dynamic process of product innovation:

$$(4.1)\quad U = \int_0^\infty e^{-\rho t}\left(\int_0^\infty c(n,t)^\theta\, dn\right)dt,$$

where $c(n, t)$ is the rate of consuming good n at time t, ρ is the time discount rate, and $0 < \theta < 1$. Here θ has the usual economic implication that the elasticity of substitution between any two goods is $(1 - \theta)^{-1}$.

At any time t, the available variety of goods in this economy is given by $[0, N(t)]$. New product can be obtained through costly product development:

$$(4.2)\quad \dot{N}(t) = R^\alpha,$$

where R is the spending on product development, and $0 < \alpha < 1$. Obviously, α measures the efficiency level of product innovation as a higher value of α yields more new variety with the same input R than a lower value of α.

The production functions for all goods are identical:

$$(4.3) \quad x(n,t) = \beta k(n,t),$$

where $x(n, t)$ is the output of good n at time t, $k(n, t)$ is the capital input to produce good n at time t, and β is the marginal productivity of capital at time t. In the context of endogenous growth, this constant return production function specified in (4.3) has been quite popular, see Barro (1990) and Rebelo (1991) for the arguments.

At time t, the total capital stock is given by $K(t)$:

$$(4.4) \quad K(t) = \int_0^\infty k(n,t)dn = \int_0^N k(n,t)dn .$$

In our model, both physical investment and product development utilize differentiated goods. For simplicity, we assume that all differentiated goods are perfect substitutes for these two purposes, even though they are imperfect substitutes in consumption. Since the utility function is symmetric in the variety of goods and since the marginal utility of each good is diminishing, the optimal consumption of each good at time t is the same: $c(n, t) = C(t)$ for all $n \subseteq [0, N]$. Thus we can write the discounted utility in (4.1) as

$$(4.1') \quad U = \int_0^\infty e^{-\rho t} N(t)C(t)^\theta dt .$$

Furthermore, due to identical consumption for each good and identical production function in (4.3), and due to the perfect substitutability across goods in physical investment and product development, the optimal output of each good at time t is also the same: $X(t) = x(n, t)$ for $n \subseteq [0, N]$ and

$$(4.3') \quad X(t) = \beta K(t)/N(t).$$

Therefore, all products that are not consumed can be either used for investment or for product development:

$$\dot{K}(t) = \int_0^N x(n,t)dn - \int_0^N c(n,t)dn - R - \delta K,$$

here δ is the rate of capital depreciation. Upon substituting $x(n, t) = X(t)$ and $c(n, t) = C(t)$ for all $n \subseteq [0, N]$

(4.5) $\dot{K}(t) = \beta K(t) - N(t)C(t) - R(t) - \delta K(t).$

Equation (4.5) says that the aggregate output is allocated among consumption, product innovation, the replacement of the depreciated capital, and new capital formation.

The optimization problem is to maximize (4.1') subject to the two dynamic constraints (4.5) and (4.2) with the initial values $K(0)$ and $N(0)$ given.

The current value Hamiltonian is:

$$(4.6)\quad \begin{aligned} &H(K,C,N,R,\lambda,\omega) \\ &= N(t)C(t)^{\theta} + \lambda[\beta K(t) - N(t)C(t) - R(t) - \delta K(t)] + \omega R(t)^{\alpha} \end{aligned}$$

where $\lambda(t)$ is the shadow price of capital, and $\omega(t)$ is the shadow price of product variety.

The first–order conditions necessary for optimization are:

(4.7) $\theta C(t)^{\theta-1} = \lambda(t),$

(4.8) $\alpha\omega(t)R(t)^{\alpha-1} = \lambda(t),$

(4.9) $(\beta - \delta - \rho) = -\dot{\lambda}(t)/\lambda(t),$

(4.10) $C(t)^{\theta} - \lambda(t)C(t) = \omega(t)\rho - \dot{\omega}(t),$

(4.11) $\dot{K}(t) = \beta K(t) - N(t)C(t) - R(t) - \delta K(t),$

(4.12) $\dot{N}(t) = R^{\alpha},$

and the transversality conditions:

$$\lim_{t\to\infty} \lambda(t)K(t)e^{-\rho t} = 0, \lim_{t\to\infty} \omega(t)N(t)e^{-\rho t} = 0.$$

Equation (4.7) implies that the marginal utility of consumption for every product and the shadow price of capital are equalized at all time. Equation (4.8) indicates that the allocation of resource for capital formation and product innovation is guided by the equality of their shadow price ratio to their marginal cost ratio: $\lambda(t)/\omega(t) = \alpha R^{1-\alpha}$. Equations (4.9) and (4.10) are the Euler conditions for the shadow prices of capital and innovation, respectively. Equation (4.11) restates the dynamic budget constraint (4.5), and equation (4.12) restates the technology generating new product variety, namely, equation (4.2).

Denote

$$g = -\dot{\lambda}(t) / \lambda(t).$$

From (4.9),

$$g = \beta - \delta - \rho.$$

For endogenous growth to be possible, g is assumed to be positive as usually done, e.g., Barro (1990) and Rebelo (1991). Then take log–differentiation in (4.7):

(4.13) $\dot{C}(t) / C(t) = g / (1 - \theta)$.

Or

(4.13') $C(t) = C(0) e^{gt/(1-\theta)}$,

where $C(0)$ is the initial consumption of every product, which is discussed in the appendix. Expression (4.13) says that the growth rate is positively related to the marginal productivity of capital β, negatively related to the time preference ρ, and positively related to the elasticity of substitution $(1 - \theta)^{-1}$.

Substituting (4.7) into (4.10):

$$(1 - \theta) C(t)^{\theta} / \omega(t) = \rho - \dot{\omega}(t) / \omega(t).$$

If we focus on a constant growth rate for the shadow price of product variety, the right–hand side of the above equation is constant. Then take log–differentiation on *both sides:*

(4.14) $\dot{\omega}(t) / \omega(t) = \theta \dot{C}(t) / C(t) = \theta g / (1 - \theta)$.

Next, log–differentiate (4.8) and use (4.9) and (4.14):

(4.15) $\dot{R}(t) / R(t) = g / (1 - \theta)(1 - \alpha)$.

Or

(4.15') $R(t) = R(0) e^{gt/(1-\theta)(1-\alpha)}$,

and $R(0)$ is the initial spending on product innovation, and it is determined in the appendix together with the initial consumption $C(0)$. In equation (4.15), the growth rate of the product–development spending is an increasing function of the marginal productivity of capital β, the elasticity of substitution in consumption $(1-\theta)^{-1}$, the efficiency of innovation technology α, but it is a decreasing function of the time preference ρ.

With (4.12) and (4.15'), we can solve the variety of products available at time t given the initial variety $N(0)$:

$$(4.16)\quad \begin{aligned} N(t) &= [R^{\alpha}(0)(1-\theta)(1-\alpha)/\alpha g[e^{\alpha g t/(1-\theta)(1-\alpha)}-1]+N(0) \\ &= R(t)^{\alpha}[(1-\theta)(1-\alpha)/\alpha g]+A(0), \end{aligned}$$

where $A(0) = [N(0) - R^{\alpha}(0)(1-\theta)(1-\alpha)/\alpha g]$. If $R(0)$ is known, $A(0)$ is just a constant because $N(0)$ is given.

The growth rate of the variety of products is given by:

$$\dot{N}(t)/N(t) = \{[(1-\theta)(1-\alpha)/\alpha g]+A(0)R(t)^{-\alpha}\}^{-1}$$

Since $R(t)$ approaches infinity as time t goes to infinity, the long–run growth rate of the variety is:

$$(4.17)\quad \lim_{t\to\infty}\dot{N}(t)/N(t) = \alpha g/(1-\theta)(1-\alpha),$$

which is the product of the efficiency of product innovation, α, and the growth rate of product–development spending,

$$\dot{R}(t)/R(t).$$

With the solutions of consumption and product variety, we can calculate the discounted utility:

$$\begin{aligned} U &= \int_0^{\infty} N(t)C(t)^{\theta}e^{-\rho t}dt \\ &= \int_0^{\infty} R(0)^{\alpha}C(0)^{\theta}[(1-\theta)(1-\alpha)/\alpha g]e^{[-\rho+\theta g/(1-\theta)+\alpha g/(1-\theta)(1-\alpha)]t}dt \\ &\quad + \int_0^{\infty} C(0)^{\theta}A(0)e^{[-\rho+\theta g/(1-\theta)]t}dt. \end{aligned}$$

For the above expression to be bounded, the following condition is required:

$$\theta g/(1-\theta)+\alpha g/(1-\theta)(1-\alpha) < \rho.$$

Since $g = \beta - \delta - \rho > 0$, the condition above is the same as

$$(4.18)\quad (\beta-\delta)(\alpha+\theta-\alpha\theta) < \rho.$$

Condition (18) also implies that

(4.19) $g/(1-\theta)(1-\alpha)<(\beta-\delta)$.

Now to find the optimal path of capital accumulation, we substitute (4.13'), (4.15'), and (4.16) into (4.11) and solve:

$$
\begin{aligned}
K(t) = &-[R(0)+C(0)R(0)^{\alpha}(1-\theta)(1-\alpha)/\alpha g] \\
(4.20)\qquad &\cdot[g/(1-\theta)(1-\alpha)-(\beta-\delta)]^{-1}e^{gt/(1-\theta)(1-\alpha)} \\
&-A(0)C(0)[g/(1-\theta)-(\beta-\delta)]^{-1}e^{gt/(1-\theta)}+B(0)e^{(\beta-\delta)t}
\end{aligned}
$$

where

$$
\begin{aligned}
B(0) = K(0)&+[R(0)+C(0)R(0)^{\alpha}(1-\theta)(1-\alpha)/\alpha g] \\
&\cdot[g/(1-\theta)(1-\alpha)-(\beta-\delta)]^{-1} \\
&+A(0)C(0)[g/(1-\theta)-(\beta-\delta)]^{-1}.
\end{aligned}
$$

For capital accumulation and product innovation, we want to make sure that the transversality conditions are satisfied. Using condition (4.19), we can easily show that

$$\lim_{t\to\infty}\omega(t)N(t)e^{-\rho t}=0.$$

But for the capital stock,

$$\lim_{t\to\infty}\lambda(t)K(t)e^{-\rho t}=\lim_{t\to\infty}\theta C(0)^{\theta-1}K(t)e^{-(\beta-\delta)t}=\theta C(0)^{\theta-1}B(0).$$

Hence the transversality condition requires that

(4.21) $B(0)=0$.

Substituting (4.21) into the capital-accumulation equation (4.20):

$$
\begin{aligned}
K(t) = &-[R(0)+C(0)R(0)^{\alpha}(1-\theta)(1-\alpha)/\alpha g] \\
(4.22)\qquad &\cdot[g/(1-\theta)(1-\alpha)-(\beta-\delta)]^{-1}e^{gt/(1-\theta)(1-\alpha)} \\
&-A(0)C(0)[g/(1-\theta)-(\beta-\delta)]^{-1}e^{gt/(1-\theta)}.
\end{aligned}
$$

Since $g/(1-\theta)(1-\alpha)<(\beta-\delta)$ the first term on the right hand side of (22) is positive. Furthermore, since $g/(1-\theta)(1-\alpha)>g/(1-\theta)$ the first term will dominate the second term as time t goes to infinity, and the long–run growth rate of the capital stock is:

$$(4.23)\quad \lim_{t\to\infty} \dot{K}(t)/K(t) = g/(1-\theta)(1-\alpha).$$

Thus the long–run growth rate of capital is the same as the growth rate of product development spending. In particular, equation (23) implies that in the long run the preference, the innovation technology, and the productivity of capital jointly determine the growth rate of capital. This result is very different from the ones obtained by Romer (1990) and Helpman (1992) because the marginal productivity of capital plays no role in the determination of the long–run growth rate in their models.

In concluding this section, we make a general observation based on the expressions for the growth rates of the endogenous variables. Even though the growth rates for the endogenous variables are eventually constant, they differ in their magnitudes. In fact, the long–run growth rate of capital accumulation and the growth rate of product development spending are higher than the consumption growth rate:

$$(4.24)\quad g/(1-\theta)(1-\alpha) > g/(1-\theta)$$

for $0 < \alpha < 1$. The long–run growth rate of product variety is smaller than the long–run growth rate of the capital stock, but it may be higher or lower than the rate of consumption growth depending on whether $\alpha/(1-\alpha)$ is larger or smaller than one.

Effects of technology transfers

Technology transfers have recently received considerable attention since Krugman (1979) has formally modeled them in a North–South product–cycle model; see Dollar (1986, 1987). Here we extend our model to the case of exogenous technology transfers from the North to the South. Think the country in our model as the South. As in Krugman (1979), the South receives technology transfers from the North in the following way: at any time t, it obtains part of the know–how about how to produce the product variety in the developed world without incurring any cost:

$$(4.25)\quad \dot{N}(t) = R(t)^{\alpha} + \pi N^{*}(t).$$

In (4.25), $N^{*}(t)$ is the product variety known in the North, and $\pi(>0)$ is the rate of technology transfers.

With equation (4.25) replacing equation (4.2), the optimal conditions for consumption and product development spending are not altered. In particular, the growth rates are the same:

$$\dot{C}(t)/C(t) = g/(1-\theta),$$

and

$$\dot{R}(t)/R(t) = g/(1-\theta)(1-\alpha).$$

The important change is the dynamic equation for product innovation. Substitute $R(t)$ into $N(t)$ and solve for $N(t)$:

$$(4.26)\quad N(t) = A'(0) + [R^{\alpha}(0)(1-\theta)(1-\alpha)/\alpha g]e^{\alpha g t/(1-\theta)(1-\alpha)} + \int \pi N^{*}(t)dt.$$

If the growth rate of product variety in the North is given by an exogenous rate γ: $\dot{N}^{*}/N^{*} = \gamma$, as in Krugman (1978), then

$$N(t) = A'(0) + [R^{\alpha}(0)(1-\theta)(1-\alpha)/\alpha g]e^{\alpha g t/(1-\theta)(1-\alpha)} + \pi N^{*}(0)e^{\gamma t}/\gamma.$$

where $A'(0) = N(0) - [R^{\alpha}(0)(1-\theta)(1-\alpha)] - \pi N^{*}(0)/\gamma$. If $\gamma > \alpha g/(1-\theta)(1-\alpha)$, then $N(t)$ will grow eventually at the rate of γ. If we imagine that over time the South can catch up with the efficiency levels of both capital and innovation in the North, then $\gamma > \alpha g/(1-\theta)(1-\alpha)$. In general the long-run growth rate is:

$$(4.27)\quad \lim_{t\to\infty} \dot{N}(t)/N(t) = \max[\alpha g/(1-\theta)(1-\alpha,\gamma].$$

Again we want to make sure that the discounted utility is bounded. Substitute $C(t)$ and $N(t)$ into the objective function:

$$\begin{aligned} U = & \int_0^{\infty} C(0)^{\theta} A'(0) e^{(-\rho+\theta g/(1-\theta))t} dt \\ & + \int_0^{\infty} R(0)^{\alpha} C(0)^{\theta} [(1-\theta)(1-\alpha)/\alpha g] e^{[-\rho+\theta g/(1-\theta)+\alpha g/(1-\theta)(1-\alpha)]t} dt \\ & + \int_0^{\infty} [C(0)^{\theta} \pi/\gamma] e^{[-\rho+\theta g/(1-\theta)+\gamma]t} dt. \end{aligned}$$

For the expression above to be bounded, the following condition is required:

$$(4.28)\quad \rho > \max[\gamma + \theta g/(1-\theta), \alpha g/(1-\theta)(1-\alpha) + \theta g/(1-\theta)].$$

It is obvious that technology transfers can lead to more rapid product innovation in the developing country. But how do these transfers affect capital accumulation in the developing South? To answer this question, we solve for $K(t)$ in the budget constraint (11) with substitution for $C(t)$, $N(t)$, and $R(t)$:

$$
(4.29)\quad \begin{aligned} K(t) = &-[R(0) + C(0)R(0)^{\alpha}(1-\theta)(1-\alpha)/\alpha g] \\ &\cdot [g/(1-\theta)(1-\alpha) - (\beta-\delta)]^{-1} e^{gt/(1-\theta)(1-\alpha)} \\ &-[C(0)N^{*}(0)\pi/\gamma][\gamma + g/(1-\theta) - (\beta-\delta)]^{-1} e^{[\gamma+g/(1-\theta)]t} \\ &- A^{*}(0)C(0)[g/(1-\theta) - (\beta-\delta)]^{-1} e^{gt/(1-\theta)}. \end{aligned}
$$

As before, in deriving (4.29) we have used the transversality condition to impose the requirement:

$$
(4.30)\quad \begin{aligned} B^{*}(0) = &K(0) \\ &+[R(0) + C(0)R(0)^{\alpha}(1-\theta)(1-\alpha)/\alpha g][g/(1-\theta)(1-\alpha) - (\beta-\delta)]^{-1} \\ &+[C(0)N^{*}(0)\pi/\gamma][\gamma + g/(1-\theta) - (\beta-\delta)]^{-1} \\ &+ A^{*}(0)C(0)[g/(1-\theta) - (\beta-\delta)]^{-1} = 0. \end{aligned}
$$

In examining (4.29), we note that the coefficient for $e^{[\gamma + g/(1-\theta)]t}$ is always positive under the condition (28) because $[\gamma + g/(1-\theta)] < \rho < (\beta - \delta)$ (recall that $g = \beta - \delta - \rho > 0$ or $\beta - \delta > \rho$). Therefore, the rate of product innovation in the developed North, γ, stimulates capital accumulation in the developing South. Not only this, the long run growth rate of capital also depends on the rate of product innovation in the developed North:

$$
(4.31)\quad \lim_{t\to\infty} \dot{K}(t)/K(t) = \max[\gamma + g/(1-\theta), g/(1-\theta)(1-\alpha)]
$$

that is to say, if $\gamma > \alpha g/(1-\theta)(1-\alpha)$, i.e., the rate of product innovation in the North is larger than the rate of product innovation in the South without technology transfers, then the long–run rate of capital accumulation in the South, not only the level of capital accumulation, is partly determined by the rate of product innovation in the North. In this case, the higher the rate of product innovation in the developed North, the higher the product innovation in the developing South, and the higher the long–run equilibrium growth rate of the capital stock in the developing South. Thus the link between capital accumulation in the developing country and technology transfers from the developed world is established in our model.

The economic intuition of this link is as follows. As technology transfers from the North accelerate product innovation in the South, more product variety will become available in the South, and more consumption demand for variety will be generated. To meet the rising consumption demand, the South will expand its capital stock and raise the production capacity. Thus technology transfers from the North lead to faster capital accumulation in the South.

Finally, to complete our solution, we need to determine the initial values $C(0)$ and $R(0)$. Schematically, we can just follow what we have done in the appendix and we omit this part here.

Effects of international trade

In this section, we want to show that foreign trade, even without technology transfers, can stimulate the rate of capital accumulation. This result applies to the developing country as well as to the developed country.

Assume that there are two countries in the world: the home country and the foreign country. The model we consider here is for, say, the home country. With foreign goods introduced into the model symmetrically as in Judd (1985), and Grossman and Helpman (1991), the objective function of the home country is modified to be:

$$(4.32)\quad \max U = \int_0^\infty e^{-\rho t}[N(t)+N^*(t)]C(t)^\theta \, dt,$$

subject to

$$(4.33)\quad \dot{K}(t) = \beta K(t) - [N(t)+N^*(t)]C(t) - R(t) - \delta K(t),$$

$$(4.34)\quad \dot{N}(t) = R(t)^\alpha,$$

where $N^*(t)$ is the number of product variety in the foreign country. In writing (4.32) and (4.33), we have assumed that all foreign goods prices are equal to one in terms of home goods. Since all goods are symmetric in the utility function, the consumption level of each good will be the same, namely, $C(t)$.

With these modifications, no change has been made on equations (4.7), (4.8), (4.9), and (4.12). Therefore, the growth rates for the consumption level of each good, the product-development spending, and the product variety remain the same as in section 2. Therefore we still have

$$(4.13)\quad \dot{C}(t)/C(t) = g/(1-\theta),$$

$$(4.15)\quad \dot{R}(t)/R(t) = g/(1-\theta)(1-\alpha),$$

$$(4.17)\quad \lim_{t\to\infty} \dot{N}(t)/N(t) = \alpha g/(1-\theta)(1-\alpha).$$

Thus, unlike the case of technology transfers, the growth rate of product variety is not affected by foreign trade even though the number of variety consumed in the home country increases by $N^*(t)$ at time t.

We still assume that the product variety in the foreign country grows at an exogenous rate ($\gamma = \dot{N}^*(t)/N^*(t)$) and the initial variety in the foreign country is $N^*(0)$. Then the discounted utility in the home country is

$$U = \int_0^\infty C(0)^\theta A(0) e^{[-\rho+\theta g/(1-\theta)]t} dt$$
$$+ \int_0^\infty R(0)^\alpha C(0)^\theta [(1-\theta)(1-\alpha)/\alpha g] e^{[-\rho+\theta g/(1-\theta)+\alpha g/(1-\theta)(1-\alpha)]t} dt$$
$$+ \int_0^\infty [C(0)^\theta N^*(0)] e^{[-\rho+\theta g/(1-\theta)+\gamma]t} dt.$$

The last term in this expression represents the welfare gain from consuming the foreign variety of products. Again, for this discounted utility to be bounded, the following condition is required:

$$(4.28)\quad \rho > \max[\gamma + \theta g/(1-\theta), \alpha g/(1-\theta)(1-\alpha) + \theta g/(1-\theta)].$$

To see the impact of trade on capital accumulation in the home country, we solve (4.33):

$$(4.35)\quad \begin{aligned} K(t) = &-[R(0) + C(0)R(0)^\alpha (1-\theta)(1-\alpha)/\alpha g] \\ &\cdot \{g/(1-\theta)(1-\alpha) - (\beta-\delta)\}^{-1} e^{gt/(1-\theta)(1-\alpha)} \\ &- [C(0)N^*(0)][\gamma + g/(1-\theta) - (\beta-\delta)]^{-1} e^{[\gamma+g/(1-\theta)]t} \\ &- A(0)C(0)[g/(1-\theta) - (\beta-\delta)]^{-1} e^{gt/(1-\theta)}. \end{aligned}$$

The imposition of the transversality condition on the capital stock gives rise to:

$$(4.36)\quad \begin{aligned} B^*(0) = &K(0) \\ &+ [R(0) + C(0)R(0)^\alpha (1-\theta)(1-\alpha)/\alpha g][g/(1-\theta)(1-\alpha) - (\beta-\delta)]^{-1} \\ &+ [C(0)N^*(0)][\gamma + g/(1-\theta) - (\beta-\delta)]^{-1} \\ &+ A(0)C(0)[g/(1-\theta) - (\beta-\delta)]^{-1} = 0. \end{aligned}$$

In equation (4.35), we note that the coefficient for $e^{[\gamma + g/(1-\theta)]t}$ is again positive under the condition (4.28) because $[\gamma + g/(1-\theta)] < \rho < (\beta - \delta)$. Therefore, trade with the foreign country brings about more capital accumulation in the home country. Essentially, trade plays the role of technology transfers in stimulating capital accumulation in the home country. In the long run,

$$(4.37)\quad \begin{aligned} \lim_{t\to\infty} \dot{K}(t)/K(t) &= \max[\gamma + g/(1-\theta), g/(1-\theta)(1-\alpha)] \\ &= \max[(\dot{N}^*/N^*) + g/(1-\theta), g/(1-\theta)(1-\alpha)]. \end{aligned}$$

Thus, the growth rate of the capital stock in the home country is increasing with the growth rate of product variety available from the foreign country. To provide the economic intuition for this result, we note that the availability of foreign goods is always welfare–enhancing for the home country given the Dixit–Stiglitz consumer preference. But the rising consumption of the foreign goods needs to be financed through the exports of the home goods, which in turn call for the expansion of the capital stock in the home country in order to produce more home goods in exchange for more foreign goods.

Conclusion

This chapter has extended the Romer model and the Grossman–Helpman model to the case here in which consumption, investment, and product development use differentiated goods, while all goods are produced with capital. This simple framework has shown that the interaction among the productivity of capital, the efficiency in innovation and consumer preferences determine the long–run rates of both product innovation and capital accumulation. Thus the one–way causality from product innovation to capital accumulation in the Romer model and the Grossman–Helpman model has been revised to the two–way interaction in our model.

This simple framework has also been utilized to examine the effects of technology transfers and international trade on product innovation and capital accumulation in an open economy. Even though the stimulating impact of trade and technology transfers on capital accumulation and growth has observed empirically in many developing countries, our theoretical model has provided a strong argument in establishing the causality from technology transfers and trade to rapid capital accumulation and product innovation.

The simple model can also be extended to deal with other issues related to economic openness and growth in developing countries. In particular, we can consider how exports and technology imports in developing countries affect capital accumulation, consumption, and innovation in a two–gap model with both domestic technology (domestic capital) and foreign technology (foreign capital) (Zou, 1998a). In addition, in this model, we have assumed perfect competition in the world product market. If a developing country's exports have certain market power, intertemporal pricing of exports becomes an important issue for a developing country in determining the optimal paths of accumulation and innovation, and we have studied part of this issue in Zou (1998b).

Appendix: *determination of the initial values*

To complete our solutions to the dynamic paths of consumption, product development spending, innovation, and capital accumulation in section 2, we must determine the initial values of consumption $C(0)$ and product–development spending $R(0)$, given the initial values of two state variables $K(0)$ and $N(0)$. That can be done as follows.

We first note that condition (21), $B(0) = 0$, provides us a nonlinear relationship between $C(0)$ and $R(0)$ with $K(0)$ and $N(0)$ given.

Now differentiate the dynamic equation of capital accumulation (4.22) with respect to t and evaluate the derivative at $t = 0$:

$$\begin{aligned}\dot{K}(0) = &-[R(0) + C(0)R(0)^{\alpha}(1-\theta)(1-\alpha)/\alpha g] \\ &\cdot [g/(1-\theta)(1-\alpha) - (\beta-\delta)]^{-1} g/(1-\theta)(1-\alpha) \\ &- A(0)C(0)[g/(1-\theta) - (\beta-\delta)]^{-1} g/(1-\theta).\end{aligned}$$

Combining this initial investment equation with the dynamic budget constraint in (4.11) evaluated at $t = 0$:

$$\dot{K}(0) = \beta K(0) - N(0)C(0) - R(0) - \delta K(0),$$

we obtain another relationship between $C(0)$ and $R(0)$:

$$\begin{aligned}&-[R(0) + C(0)R(0)^{\alpha}(1-\theta)(1-\alpha)/\alpha g] \\ &\cdot[g/(1-\theta)(1-\alpha) - (\beta-\delta)]^{-1} g/(1-\theta)(1-\alpha) \\ &-A(0)C(0)[g/(1-\theta) - (\beta-\delta)]^{-1} g/(1-\theta) \\ &= \beta K(0) - N(0)C(0) - R(0) - \delta K(0).\end{aligned} \tag{4.38}$$

With $C(0)$ and $R(0)$ determined from equations (4.22) and (4.38), the optimal initial invest is given by the dynamic budget constraint (4.11) and the initial increase in product variety is given by

$$\dot{N}(0) = R(0)^{\alpha}.$$

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