A Note on Endogenous Timing with Strategic Delegation: Unilateral Externality Case*

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Abstract

We investigated the endogenous choice of roles by managerial firms in the presence of unilateral externality. The choice over timing can be taken either by managers or by owners. It is shown that (i) the choice of the timing by managers entails the same profit that owners would have achieved by specifying the timing in the delegation contract; and (ii) firms move simultaneously if the degree of unilateral externality is small, while sequentially if the degree of unilateral externality is large, with the firm generating unilateral externality as a follower; the owner of the follower firm delegates to restrict output, while his/her counterpart does not delegate it.

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1 Introduction

In their pioneering paper, Hamilton and Slutsky (1990) investigated the issue of endogenous timing by adding a preplay stage at which players decide simultaneously whether to move at the first opportunity or at the second. Based on the framework of Hamilton and Slutsky (1990) with one production period, a variety of strategic settings has emerged.

We introduce unilateral externality into the demand function[1] and investigate the endogenous role choices of managerial firms. In the case of unilateral externality, when one firm is affected

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[1] Theoretical studies that introduce exogenous asymmetry into oligopoly market include Albaek (1990), Mailath (1993), Amir and Grilo (1999), Amir (1995), Robson (1990), Gal-Or (1985), Dowrick (1986) and Boyer and Moreaux (1987), among others.

by the actions of other firms, the former may simply not react because of the positive externality generated by the latter. For instance, many instances of unilateral externality, in which a minor firm is affected by a major one, mark the recent history of corporate interaction. Suppose that two firms, one relatively smaller than the other, produce goods. The major firm has the resources to advertise and expand its market, whereas its minor rival lacks them. For this reason, the smaller of the two competitors has a greater chance of selling its output because of the positive externality generated by the major firm[2]. This situation occurs, for example, in an appliance store where the products of household appliance manufacturers that are not particularly prominent are displayed alongside those of established, prominent brands. As another example, consider a type of DVD player that is already sold by several prominent brands. In this instance, the market channel may be exploited by a fringe firm, since the network size of the famous trademarks is larger than its own. The fringe DVD manufacturer may have either a greater or a lesser chance of selling its output as a result of the positive or negative externality generated by the better known brands. Borrowing the phraseology of Tirole (1988, Chapter 10), positive externality arises when a good is so valuable to consumers that they adopt compatible goods. Thus, a popular automobile is serviced by many dealers, and an unfashionable one, by specific dealers. The size of the relevant good may be industry-wide[3]. However, we focus on unilateral externality.

In our model, the products are perfect substitutes if the unilateral externality is absent.[4] When the unilateral externality is present, the quantity of one firm (say firm 1)'s product has an additional price-raising effect on the other firm (say, firm 2)'s product besides the normal price-reducing effect. Only one firm's product has this price-raising effect and so we call it unilateral externality. When the degree of the unilateral externality is sufficiently large, firm 1's product becomes a complement to firm 2's while it is still a substitute when the degree of the unilateral externality is sufficiently small. For this reason, firms would be concerned about the presence of the externality. Cabral and Majure's (1994) theoretical and empirical study of Portuguese banking presents some relevant evidence. They find that in the case of some banks, the number of branches of rivals is a strategic complement, while for other banks, it is a strategic substitute. This finding is explained by the geographic differences in the expansion patterns of incumbent/public banks and entrant/private banks, stemming from dissimilarities in their relative efficiency (private banks being relatively more efficient in urban areas) and the degrees of customer loyalty (rural customers being more loyal than urban ones).

On the other hand, it is well known that such a situation can arise in standard models without the externality. For example, Bulow *et al.* (1985a, 1985b) suggest several instances in which possible strategic asymmetries exist. One example is the following: the dominant firm in the industry may regard its rivals' outputs as strategic complements, while a fringe firm may regard those of the dominant firm as strategic substitutes. A more recent paper by Tombak (2006) presents further examples and a discussion of strategic asymmetry[5]. In these works on strategic

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[2] Later, we will mention in the Model section why we treat only the positive externality. This is inspired by the work of Nakamura (2006) who just pointed out externality regarding demand functions.

[3] These examples are related to the influence of network externality.

[4] The products can be differentiated. However, the analysis would be same and the results would not change qualitati vely.

[5] Tombak (2006) showed that the recommendation of strategy for an accommodating incumbent differs from that suggested by Fudenberg and Tirole (1984) when one firm regards rival's output as a strategic substitute while its

asymmetry, the leader is determined exogenously by the incumbent firm's investment in the first stage. However, we focus on the endogenous choice of roles in the presence of unilateral externality.

We examine the contract a la Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987), hereafter referred to as VFJS. VFJS showed that when firms delegate the output decision to the manager, the subgame perfect Nash equilibrium results in an output that is higher and in profits that are lower than those that can be observed in the Cournot game[6]. The existing related literature, however, does not discuss the effect of unilateral externality on delegation with firm's moves of order endogenously determined. This study considers the effect on a managerial delegation game when unilateral externality exists across firms. We adopt the generalization of Hamilton and Slutsky's (1990) observable delay game and introduce the descriptions of the delegation game of Vickers (1985) and Lambertini (2000a)[7]. In this regard, Lambertini (2000a) extends the delegation game to model explicitly the timing choices made by managers or firm owners[8]. He demonstrates that when firms compete in quantities, the simultaneous-move game occurs endogenously. If the choice of timing is delegated to the managers, they will play at the first opportunity. On the contrary, if the choice of timing is not delegated, the owners prefer the second opportunity. Observationally, these two cases have the same subgame perfect Nash equilibrium. This feature arises because each owner of the follower firm has the advantage of being able to increase output using delegation while the owner of the leader does not delegate it. This leads to a situation in which all firms first decide to delay and then to delegate control to the managers. Thus, all firms play the simultaneous-move game at the second opportunity.

Following Lambertini (2000a, b), we consider a three-stage game model, which focuses on the issue of endogenous timing with strategic delegation in the presence of unilateral externality. This paper shows that when managers are only delegated the market decision, with the choice of timing remaining with the stockholders, the extended game with observable delay exhibits observationally the same subgame perfect Nash equilibrium as the one in which both decisions are delegated to managers. This result is the same as that of Lambertini (2000a), in the sense that, in his setting, the subgame perfect Nash equilibriums of the two cases are also observationally identical. However, the difference is also worth noting. In Lambertini (2000a), firms move simultaneously in the Cournot setting in equilibrium. This is the same as our result when the degree of unilateral externality is sufficiently small, since the products produced by the two firms are substitutes. However, when the degree of unilateral externality is sufficiently large such that the product of the firm that generates the unilateral externality becomes a complement to the

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rival firm regards the dominant firm's outputs as strategic complement.

[6] Furthermore, Ritz (2008) and Jansen *et al*. (2007) analyzed a case in which the managers' objective function is a weighted sum of the firm's profit and market share. Miller and Pazgal (2002) adopted a case in which the manager's objective function is a weighted sum of the firm's profit and the profits of its rivals. For the extension of this game, see also Witteloostuijn *et al.*(2007), Manasakis *et al*. (2007), Ishibashi (2001) and Theilen (2007).

[7] For the sake of simplicity, our framework of managers' objective functions differs from that analyzed by Jansen *et al.* (2007), because the introduction of market share in the contracts complicated the mathematical methodology in comparison with the sale volume contracts.

[8] On the other hand, Lambertini (2000b) extends the delegation game to explicitly model the timing choices made b y only owners of firms while the output decision is delegated to the manager. It is worth noting that in Lambertini (2 000b) the decision upon the timing of order is up to
the owner, while its implementation at market stage is delegated to the manager.

rival firm, the equilibrium requires sequentiality with the firm generating unilateral externality as a follower; the owner of the firm delegates to *restrict* output, while his/her counterpart in the other firm does not delegate it.

The organization of the paper is as follows. In Section 2, we describe the model. Section 3 presents equilibrium analysis in fixed timing under duopoly. Section 4 determines firms' endogenous choice of timing. Section 5 closes the paper.

2. The Model

We consider an asymmetric duopolistic model with unilateral externality. The products are perfect substitutes if the unilateral externality is absent. When the unilateral externality is present, the quantity of firm 1's product has an additional price-raising effect on firm 2's product besides the normal price-reducing effect. Only firm 1's product has this price-raising effect and so we call it unilateral externality. The inverse demands of firms 1 and 2 are characterized as

$$p_1 = a - q_1 - q_2, \tag{1}$$

$$p_2 = a + (\theta - 1)q_1 - q_2, \tag{2}$$

where p_i is the market price of good i and q_i denotes the output of firm i (= 1, 2), θ measures the degree of unilateral externality, $\theta \in (0,2); \theta \neq 1$.[9] When the degree of the unilateral externality is sufficiently large, i.e., $\theta \in (1,2)$, firm 1's product becomes a complement to firm 2's while it is still a substitute when the degree of the unilateral externality is sufficiently small, i.e., $\theta \in (0,1)$.

Firms' owners may decide whether to delegate control to managers. The protocol described by Vickers (1985) is followed in order to formalize managerial delegation[10]. Each owner can assess the performance of his/her manager in accordance with two readily observable indicators, namely, the profit and output of the firm. The cost of writing a contract is assumed to be fixed, which is normalized to zero with no loss of generality. Let firm i's production cost function be $C(q_i) = f_i + c_i q_i$, where f_i is a fixed cost and c_i is a constant marginal cost. For simplicity, we assume that $f_2 = 0$ and $c_1 = c_2 = 0$ with no loss of generality. As for firm 1's fixed cost f_1, since firm 1 generates unilateral externality, we can assume it is positive. However, as long as it is

[9] We thank referees for suggesting this range of unilateral externality. Moreover, if $\theta = 1$, then the price of firm 2's product is independent of firm 1's quantity. This is not an interesting case.

[10] Fershtman and Judd (1987) defined the manager's objective function as related to a linear combination of profit and revenue. For the sake of simplicity, we adopt Vickers' approach.

sufficiently small, it does not affect our analysis[11]. So we also assume $f_1 = 0$. Thus, we assume that the manager of firm i maximizes the function

$$U_i = \Pi_i + \beta_i q_i = p_i q_i + \beta_i q_i \tag{3}$$

where the parameter β_i identifies the weight attached to the volume of sales and Π_i is the profit of firm i's owner.

We consider the issue of endogenous timing with strategic delegation in the presence of unilateral externality. Two cases are distinguished: (i) Both timing and market decisions are delegated to managers; (ii) Only the market decision is delegated to managers, while the choice of timing remains in the hands of stockholders. Specifically, a three-stage game model is used. The timing of the game is as follows. For case (i), in the first stage, each owner writes his or her manager's incentive contract, in which the manager's objective function is specified, and we call it the contract stage. In the second stage, the managers of the firms announce simultaneously the period in which output is to be produced, knowing each other's incentive contract; this is called the announcement stage. In the third stage, each manager produces outputs in the announced period, with knowledge of each other's incentive contract and production period; this is referred to as the market stage. If both managers decide to produce simultaneously (respectively, in different periods), a Cournot-type (respectively, a Stackelberg-type) game arises. For case (ii), we still have a three-stage game. The first stage is the announcement stage in which it is the owners instead of managers who make the announcement of the production period (or each owner specifies the timing in the contract). The second is the contract stage, and the third is the market stage.

3 Equilibrium Analysis in Fixed Timing

Before presenting the results of equilibrium derived from the model with the observable delay game, we present the equilibrium analysis in fixed timing, namely, (1) when the managers of firms compete simultaneously in the market stage (i.e., Cournot-type), and (2) when they take their output decisions sequentially in the market stage (i.e., Stackelberg-type). We will use superscript “c” to denote the Cournot case, “l” to denote the leader, and “f” to denote the follower.

[**Cournot-type**]: In the market stage, the manager of firm i maximizes his payoff U_i by choosing quantity q_i and both managers choose quantities simultaneously. Thus, both managers’ maximization problems are the following:

$$\max_{q_1} U_1 = \Pi_1 + \beta_1 q_1 = a q_1 - q_1^2 - q_1 q_2 + \beta_1 q_1$$

[11] We thank referees for suggesting this intuitive explanation of the assumption.

$$\max_{q_2} U_2 = \Pi_2 + \beta_2 q_2 = aq_2 + (\theta - 1)q_1 q_2 - q_2^2 + \beta_2 q_2$$

The maximization problem of each manager yields the *managerial reaction function*:

$$q_1 = \frac{a + \beta_1 - q_2}{2}, \tag{4}$$

$$q_2 = \frac{a + \beta_2 + (\theta - 1)q_1}{2}, \tag{5}$$

which yields

$$q_1 = \frac{a + 2\beta_1 - \beta_2}{3 + \theta}, q_2 = \frac{(\theta + 1)a + (\theta - 1)\beta_1 + 2\beta_2}{3 + \theta}. \tag{6}$$

It follows that

$$\Pi_1 = \frac{(a + 2\beta_1 - \beta_2)(a - \beta_2 - \beta_1 - \beta_1\theta)}{(3 + \theta)^2}, \tag{7}$$

$$\Pi_2 = \frac{[a(1+\theta) - \beta_1(1-\theta) + 2\beta_2][a(1+\theta) - \beta_1(1-\theta) - \beta_2(1+\theta)]}{(3+\theta)^2}, \tag{8}$$

$$U_1^c = \frac{(a + 2\beta_1 - \beta_2)^2}{(3+\theta)^2}, \tag{9}$$

$$U_2^c = \frac{[a(1+\theta) + \beta_1(\theta - 1) + 2\beta_2]^2}{(3+\theta)^2}. \tag{10}$$

Given the market stage, the owners of both firms simultaneously select their delegation parameters in order to maximize their objectives during the contract stage. Then each firm's profit maximization problem in the contract stage is to maximize Π_i by choosing β_i.

The first-order conditions for firm i are:

$$\beta_1 = \frac{(a - \beta_2)(1-\theta)}{4(1+\theta)}, \quad \beta_2 = \frac{a(1-\theta)(1+\theta) - \beta_1(1-\theta)^2}{4(1+\theta)}, \tag{11}$$

which yields each owner's reaction function

$$\beta_1^c = \frac{a(1+\theta)(1-\theta)}{\theta^2 + 10\theta + 5}, \quad \beta_2^c = \frac{a(1-\theta)(1+3\theta)}{\theta^2 + 10\theta + 5}. \tag{12}$$

When firms move simultaneously, each β_i^c can be either a positive or a negative value depending on the value of θ. Thus, in contrast with the results reported by Vickers (1985) and Fershtman and Judd (1987), delegation can be utilized to restrict output (respectively, expand) output if $\theta \in (1,2)$ (respectively, $\theta \in (0,1)$). Therefore, if $\theta \in (1,2)$ (respectively, $\theta \in (0,1)$), owners are able to commit themselves and ensure defensive (respectively, offensive) behavior through an incentive contract. Straightforward computation gives us the equilibrium quantities and profits:

$$q_1^c = \frac{2a(1+\theta)}{\theta^2 + 10\theta + 5},\ q_2^c = \frac{2a(1+3\theta)}{\theta^2 + 10\theta + 5}, \tag{13}$$

$$\Pi_1^c = \frac{2a^2(1+\theta)^3}{(\theta^2 + 10\theta + 5)^2}, \quad \Pi_2^c = \frac{2a^2(1+\theta)(1+3\theta)^2}{(\theta^2 + 10\theta + 5)^2}. \tag{14}$$

[Stackelberg-type 1]: Let us consider the Stackelberg game in which the manager of firm 1 is the leader. To solve for the backwards-induction quantity of this game, we use the manager of firm 2's managerial reaction function (5) as in the simultaneous-move games. Hence, the manager of firm 1 selects the output to maximize

$$\max_{q_1} U_1 = \left(a - q_1 - \frac{a + \beta_2 + (\theta - 1)q_1}{2} \right) q_1 + \beta_1 q_1.$$

Straightforward computation yields each firm's equilibrium quantity:

$$q_1 = \frac{a + 2\beta_1 - \beta_2}{2(1+\theta)}, q_2 = \frac{2\beta_1(\theta - 1) + \beta_2(3+\theta) + a(1+3\theta)}{4(1+\theta)}. \tag{15}$$

It follows that

$$\Pi_1 = \frac{(a + 2\beta_1 - \beta_2)\left(a - 2\beta_1 - \beta_2\right)}{8(1+\theta)}, \tag{16}$$

$$\Pi_2 = \frac{[a(1+3\theta) + \beta_2(3+\theta) + 2\beta_1(\theta-1)][a(1+3\theta) + 2\beta_1(\theta-1) - \beta_2(1+3\theta)]}{16(1+\theta)^2}, \tag{17}$$

$$U_1^l = \frac{(a + 2\beta_1 - \beta_2)^2}{8(1+\theta)}, \tag{18}$$

$$U_2^f = \frac{[a(1+3\theta) + 2\beta_1(\theta-1) + \beta_2\left(3+\theta\right)]^2}{16(1+\theta)^2}. \tag{19}$$

Each firm's profit maximization problem in the contract stage is to maximize Π_i by choosing β_i.

First-order conditions imply

$$\beta_1^l = 0, \beta_2^f = \frac{a(1-\theta)}{3+\theta}. \tag{20}$$

Firm 1 functions as the leader and decides that it is optimal not to allow for any output expansion and restriction, whereas firm 2 functions as the follower, and uses the delegation to restrict (respectively, expand) output if $\theta \in (1,2)$ (respectively, $\theta \in (0,1)$). Both firms' quantities and payoffs are

$$q_1^l = \frac{a}{3+\theta},\ q_2^f = \frac{a}{2},\ \Pi_1^l = \frac{a^2(1+\theta)}{2(3+\theta)^2},\ \ \Pi_2^f = \frac{a^2(1+3\theta)}{4(3+\theta)}. \tag{21}$$

[Stackelberg-type 2]: Finally, consider the game in which the manager of firm 2 functions as the leader. Similar to the previous Stackelberg game, we utilize the manager of firm 1's managerial reaction function (4) as in the simultaneous-move games. Hence, the manager of firm 2 selects the output to maximize

$$\max_{q_1} U_2 = \left(a - q_2 - \frac{a+\beta_1 - q_2}{2} + \theta\left(\frac{a+\beta_1 - q_2}{2}\right)\right)q_2 + \beta_2 q_2.$$

Straightforward computation then gives us

$$q_1^f = \frac{a(1+\theta)+\beta_1(3+\theta)-2\beta_2}{4(1+\theta)}, q_2^l = \frac{2\beta_2 - \beta_1(1-\theta)+a(1+\theta)}{2(1+\theta)}. \tag{22}$$

It follows that

$$\Pi_1 = \frac{[a(1+\theta)+\beta_1(3+\theta)-2\beta_2][a(1+\theta)-2\beta_2-\beta_1-3\beta_1\theta]}{16(1+\theta)^2}, \tag{23}$$

$$\Pi_2 = \frac{[a(1+\theta)-\beta_1(1-\theta)+2\beta_2][a(1+\theta)-2\beta_2-\beta_1(1-\theta)]}{8(1+\theta)}, \tag{24}$$

$$U_1^f = \frac{[a(1+\theta)+\beta_1(3+\theta)-2\beta_2]^2}{16(1+\theta)^2}, \tag{25}$$

$$U_2^l = \frac{[a(1+\theta)+\beta_1(\theta-1)+2\beta_2]^2}{8(1+\theta)}. \tag{26}$$

Each firm's profit maximization problem in the contract stage is to maximize Π_i by choosing β_i.

First-order conditions imply

$$\beta_1^f = \frac{a(1-\theta)(1+\theta)}{(3+\theta)(1+3\theta)}, \beta_2^l = 0. \quad (27)$$

Firm 1 acts as the follower and uses the delegation to restrict (respectively, expand) output if $\theta \in (1,2)$ (respectively, $\theta \in (0,1)$), whereas firm 2 functions as the leader, and decides that it is optimal not to allow for any output expansion and restriction. Both firms' quantities and payoffs are then

$$q_1^f = \frac{a(1+\theta)}{2(1+3\theta)}, q_2^l = \frac{a(1+6\theta+\theta^2)}{(1+\theta)(1+3\theta)}, \quad (28)$$

$$\Pi_1^f = \frac{a^2(1+\theta)^2}{4(3+\theta)(1+3\theta)},\ \Pi_2^l = \frac{a^2(1+\theta)(1+6\theta+\theta^2)}{2[(1+3\theta)(3+\theta)]^2}. \quad (29)$$

4. The Choice of Timing

4.1 Delegation of Both Decisions

Having derived the equilibrium for three fixed timing games in the previous section, we will investigate the subgame perfect Nash equilibrium (SPNE) of the entire game for case (i), namely, when both timing and market decisions are delegated to managers. The reduced form of endogenous timing game can be represented by payoff table 1. In the table, "F" and "S" represent the first period and second period with regard to quantity choice, respectively.

Table 1: Endogenous Timing Game (Delegation of Both Decisions)

Firm 2

		F	S
Firm 1	F	U_1^c, U_2^c	U_1^l, U_2^f
	S	U_1^f, U_2^l	U_1^c, U_2^c

Straightforward computation shows that

$$U_1^f / U_1^c = \left\{ \frac{(3+\theta)[a(1+\theta)+\beta_1(3+\theta)-2\beta_2]}{4(1+\theta)(a+2\beta_1-\beta_2)} \right\}^2 = \left\{ 1 + \frac{(\theta-1)[a(1+\theta)+\beta_1(\theta-1)+2\beta_2]}{4(1+\theta)(a+2\beta_1-\beta_2)} \right\}^2$$

$$\Rightarrow U_1^f / U_1^c \begin{cases} >1 & \text{when } \theta \in (1,2) \\ <1 & \text{when } \theta \in (0,1) \end{cases}, \text{[12]} \tag{30}$$

$$U_1^c / U_1^l = \frac{8(1+\theta)}{(3+\theta)^2} < 1. \tag{31}$$

These inequalities tell us that firm 1 prefers producing sequentially (respectively, in the first opportunity) if $\theta \in (1,2)$ (respectively, $\theta \in (0,1)$). We then find that

$$\begin{aligned} U_2^f / U_2^c &= \left\{ \frac{(3+\theta)[a)(1+3\theta)+2\beta_1(\theta-1)+\beta_2(3+\theta)]}{4(1+\theta)[a(1+\theta)+\beta_1(\theta-1)+2\beta_2]} \right\}^2 \\ &= \left\{ 1 - \frac{(\theta-1)^2[a+2\beta_1-\beta_2]}{4(1+\theta)[a(1+\theta)+\beta_1(\theta-1)+2\beta_2]} \right\}^2 < 1, \end{aligned} \tag{32}$$

$$U_2^c / U_2^l = \frac{8(1+\theta)}{(3+\theta)^2} < 1. \tag{33}$$

These inequalities tell us that firm 2 prefers producing in the first opportunity. So there is a unique subgame perfect Nash equilibrium in the observable delay game of managerial delegation: Firm 1 acts as a follower, i.e., (S, F) if $\theta \in (1,2)$ while both firms move in the first opportunity, i.e., (F, F) if $\theta \in (0,1)$. Thus, we have the following proposition:

Proposition 1: *When both decisions are delegated to managers, under the assumption of unilateral externality with $\theta \in (0,2)$, there is a unique subgame perfect Nash equilibrium. If $\theta \in (0,1)$, the equilibrium involves both firms moving in the first opportunity; on the contrary, if $\theta \in (1,2)$, it involves firm 2 producing as a leader while the firm generating externality, i.e., firm 1, as a follower.*

The fact that producing in the first opportunity is the manager of firm 2's strictly dominant strategy plays an important role in the derivation of the result. Since it is the managers who are making decisions of timing, clearly, being a leader is always better than moving simultaneously.

[12] From (6), we know that $a+2\beta_1-\beta_2>0$ and that $[a(1+\theta)+\beta_1(\theta-1)+2\beta_2>0$.

Hence, $U_2^l > U_2^c$ and $U_1^l > U_1^c$. If the manager of firm 1 chooses to produce in the first period, the manager of firm 2 prefers moving simultaneous to being a follower. Suppose the manager of firm 2 produces as a follower. Anticipating this, if $\theta \in (1,2)$, firm 1 produces few outputs so that firm 2, as a follower, will not produce too many outputs (recall that firm 2's reaction function is upward if $\theta \in (1,2)$) and the price of firm 1's product will not be too low; if $\theta \in (0,1)$, firm 1 produces many outputs so that firm 2, as a follower, will produce few outputs (recall that firm 2's reaction function is downward if $\theta \in (0,1)$). On the contrary, if firm 2 produces simultaneously with firm 1 in period 1, firm 2 produces more, yielding more payoff to the manager of firm 2. Hence, $U_2^c > U_2^f$. We find that firm 2 wants to produce in the first opportunity regardless of in which period firm 1 produces.

Given producing in period 1 is the manager of firm 2's strictly dominant strategy, the manager of firm 1 chooses to produce in period 2 as a follower if $\theta \in (1,2)$, i.e., if the degree of unilateral externality from firm 1 to firm 2 is sufficiently large so that firm 1's product becomes a complement to firm 2's product. Suppose firm 1 produces in period 2. Anticipating this firm 2 produces few outputs so that firm 1 can produce many outputs (recall that firm 1's reaction function is downward) and the price of firm 2's product will be high. If firm 1 produces simultaneously with firm 2 in period 1, firm 1 produces less, yielding lower payoff to the manager of firm 1. Hence, $U_1^f > U_1^c$. Thus, the unique subgame perfect Nash equilibrium stated in Proposition 1 obtains. If $\theta \in (0,1)$, i.e., if the degree of unilateral externality from firm 1 to firm 2 is sufficiently small so that firm 1's product is still a substitute to firm 2's product, the standard result remains hold, i.e., producing in the first opportunity is a strictly dominant strategy for both firms and thus both firms producing in the first period is a strictly dominant strategy equilibrium, the same as in Lambertini (2000a).

4.2 Delegation of Only the Market Decision

Similar to previous subsection, we will investigate the subgame perfect Nash equilibrium of case (ii), namely, when only the market decision is delegated to managers, while the choice of timing remains in the hands of stockholders. The reduced form of endogenous timing game can be represented by payoff table 2. In the table, "F" and "S" represent the first period and second period with regard to quantity choice, respectively.

Table 2: Endogenous Timing Game (Delegation of Only the Market Decision)

Firm 2

	F	S

Firm 1	F	Π_1^c, Π_2^c	Π_1^l, Π_2^f
	S	Π_1^f, Π_2^l	Π_1^c, Π_2^c

To find subgame perfect Nash equilibrium, we need to compare profits. Straightforward computation shows that

$$\Pi_1^f - \Pi_1^c = \frac{a^2(1+\theta)^3(\theta-1)^4}{4(3+\theta)(1+3\theta)(\theta^2+10\theta+5)^2} > 0, \tag{34}$$

$$\Pi_1^c - \Pi_1^l = \frac{a^2(1+\theta)\left(3\theta^2+18\theta+11\right)\left(\theta-1\right)^2}{2(3+\theta)^2(\theta^2+10\theta+5)^2} > 0\,. \tag{35}$$

These inequalities tell us that firm 1 prefers producing in the second opportunity. We then find that

$$\Pi_2^f - \Pi_2^c = \frac{a^2(3\theta+1)(\theta-1)^4}{4(3+\theta)(\theta^2+10\theta+5)^2} > 0\,, \tag{36}$$

$$\Pi_2^l - \Pi_2^c = \frac{a^2(1+\theta)^2\left(\theta-1\right)^3(11+78\theta+132\theta^2+34\theta^3+\theta^4)}{2[(1+3\theta)(3+\theta)]^2(\theta^2+10\theta+5)^2}\begin{cases} >0 & \text{when } \theta\in\left(1,2\right) \\ <0 & \text{when } \theta\in\left(0,1\right) \end{cases}. \tag{37}$$

So there is a unique subgame perfect Nash equilibrium in the observable delay game of managerial delegation: Firm 1 functions as a follower, i.e., (S,F) if $\theta \in (1,2)$, while both firms move in the second opportunity, i.e., (S, S) if $\theta \in (0,1)$. Thus, we have the following proposition:

Proposition 2: *When managers are being delegated only the market decision, under the assumption of unilateral externality with $\theta \in (0,2)$, there is a unique subgame perfect Nash equilibrium. If $\theta \in (0,1)$, the equilibrium involves both firms moving in the second opportunity; on the contrary, if $\theta \in (1,2)$, it involves firm 2 producing as a leader while the firm generating externality, i.e., firm 1, as a follower.*

The fact that producing in the second opportunity is the owner of firm 1's strictly dominant strategy plays an important role in the derivation of the result. It is well known that the owner of the leading firm prefers not to delegate (i.e., $\beta^l = 0$). Lambertini (2000a) finds that the owner of each firm prefers moving in the second opportunity in the Cournot setting when products are substitutes since it is profitable for the owner to use the delegation device to shift his/her own reaction function outwards ($\beta^f > 0$). Hence, in our model, if $\theta \in (0,1)$, i.e., if the degree of unilateral externality from firm 1 to firm 2 is sufficiently small so that firm 1's product is still a substitute to firm 2's product, we find the unique subgame perfect Nash equilibrium in which both firms move in the second opportunity. The new finding is that even if $\theta \in (1,2)$, i.e., if the degree of unilateral externality from firm 1 to firm 2 is sufficiently large so that firm 1's product becomes a complement to firm 2's product, firm 1 still prefers moving in the second opportunity. The reason is the following: if $\theta \in (1,2)$, then firm 1's product becomes a complement to firm 2's product, and the owner of firm 1 wants to restrict its output so that firm 2 does not produce too many outputs (recall that firm 2's reaction function is upward). Hence, the owner of firm 1 wants to use the delegation device to shift his/her own reaction function inwards ($\beta^f < 0$).

Given producing in period 2 is the owner of firm 1's strictly dominant strategy, the owner of firm 2 chooses to produce in period 1 as a leader if $\theta \in (1,2)$. Since firm 1 chooses to restrict its output in the second opportunity if $\theta \in (1,2)$, firm 2 chooses to be a leader so that it can produce more output and earn more profit. Thus, we get the unique subgame perfect Nash equilibrium stated in Proposition 2.

From Propositions 1 and 2, we immediately have the following corollary.

Corollary 1: *When managers are being delegated only the market decision, while the choice of timing remains in the stockholders' hands, the extended game with observable delay observationally exhibits the same subgame perfect Nash equilibrium as the one when both decisions are delegated to managers. Namely, firms move simultaneously if* $\theta \in (0,1)$ *while sequentially if* $\theta \in (1,2)$ *with firm 1 generating unilateral externality as a follower.*

The corollary seems the same as Proposition 1 in Lambertini (2000a), who analyzed the case of substitutes, in the sense that the subgame perfect Nash equilibrium is observationally the same when managers are being delegated only the market decision as when both decisions are delegated to managers. However, the difference is also worth noting. In Lambertini (2000a), firms move simultaneously in the Cournot setting in equilibrium. This is the same as our result when $\theta \in (0,1)$, since the products produced by the two firms are substitutes. However, when $\theta \in (1,2)$, equilibrium requires sequentiality with firm 1 generating unilateral externality as a follower.

5. Concluding Remarks

This study investigated the endogenous choice of roles by managerial firms, under the assumption that there exists an asymmetric circumstance of unilateral externality. In our setting, the products produced by two firms are originally perfect substitutes. However, besides the normal price-reducing effect, the quantity of firm 1's product has an additional price-raising effect on the price of firm 2's product. This is what we called unilateral externality. The main finding is that, when managers are being delegated only the market decision, while the choice of timing remains in the stockholders' hands, the extended game with observable delay observationally exhibits the same subgame perfect Nash equilibrium as the one when both decisions are delegated to managers. Namely, firms move simultaneously if the degree of unilateral externality is small, i.e., $\theta \in (0,1)$, while sequentially if the degree of unilateral externality is large, i.e., $\theta \in (1,2)$, with firm 1 generating unilateral externality as a follower. This finding seems the same as Proposition 1 in Lambertini (2000a), who analyzed the case of substitutes, in the sense that the subgame perfect Nash equilibrium is observationally the same when managers are being delegated only the market decision as when both decisions are delegated to managers. However, we should note that only when $\theta \in (0,1)$, the equilibrium outcome is the same as in Lambertini (2000a): firms move simultaneously in equilibrium. On the contrary, when $\theta \in (1,2)$, equilibrium requires sequentiality. So the degree of unilateral externality plays an important role in firms' choice of roles.

However, this study has its limitations. For example, with the externality across firms, our findings differ from those observed when the managers' objective function is a weighted sum of the firm's profit and market share, as in the study of Jansen *et al*. (2007). Future studies in this field will require a comprehensive analysis of strategic variables, including managers' objective functions[13]. The precise details of the managerial delegation structure and the choice of strategic variables require further research.

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[13] Barcena-Ruiz and Casdo-Izaga (2005) analyze whether owners of firms have incentives to delegate firms' location decisions to managers or not. They show that the owners have incentives to keep their long-run decision (the location of the firms) for themselves. Other related studies, dealing with long-run plans of firms, include Zhang and Zhang (1997) and Mitrokostas and Petrakis (2008).

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