Foreign Asset Accumulation and Macroeconomic Policies

in a Model of Mercantilism

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I. Introduction

This paper examines the effects of macroeconomic policies on foreign asset accumulation in a small open economy. It obtains policy implications that are very different from many existing studies such as Turnovsky (1985, 1987) and, in particular, Obstfeld (1981).

In an often-cited paper, Obstfeld (1981) presents three interesting results regarding the effects of government policies on foreign asset holdings: (1) foreign exchange intervention is found to have no real effects when official foreign reserves earn interest that is distributed to the public; (2) inflation leads to higher long-run consumption and foreign claims; (3) an increase in government consumption induces a surplus on current account in the short run and larger foreign asset accumulation in the long run. The intertemporal optimization framework used by Obstfeld in this study and some related studies Obstfeld, 1982, 1990) has also influenced the open economy macroeconomics in the past decade.

In this paper, we are going to show that the policy implications of Obstfeld's model hinge on the special assumption of Uzawa's (1968) time preference and they are totally reversed and substantially changed in a dynamic optimization model with the wealth effect. The wealth effect approach developed in our paper is adapted from the models of Bardhan (1967), Kurz (1968), Calvo (1980) and Blanchard (1983) and defines the representative agent's utility function on foreign asset in addition to consumption and real balances. The main results derived from our model stand in striking contrast to the ones in Obstfeld paper: (1) foreign exchange intervention leads to more foreign asset holdings and more consumption in the long run; (2) if the utility function is separable in consumption and real balances as in Obstfeld (1981), inflation has no effect on the real variables in both short run and long run; if the utility function is nonseparable, inflation results in more foreign asset accumulation when the cross derivative of consumption and real balances is positive; (3) government spending always reduces foreign asset accumulation and crowds out private consumption.
Our paper is organized as follows. Section II sets up a simple wealth effect model with money and discusses the stability and some policy implications of the model. Section III makes detailed comparative study on the effects of macroeconomic policies. We conclude our paper in Section IV.

II. A Simple Model

We consider a small economy in a competitive world market. The economy is populated with many identical people. We follow Bardhan (1967), Kurz (1968), Calvo (1980) and Blanchard (1983) and define a representative agent's instantaneous utility as

\[ U(c, m, b) = u(c) + v(m) + \alpha w(b) \]

where \( c \) is consumption, \( m \) is real balance holdings, \( b \) is the foreign asset holdings, and \( \alpha (\alpha > 0) \) measures the wealth effect. A negative \( b \) is foreign debt, and \( \alpha w(b) \) is the disutility of debt as in Bardhan (1967) and Blanchard (1983). For a positive \( b \), \( \alpha w(b) \) reflects the wealth effects introduced by Kurz (1968). It is assumed that \( u'(c) > 0, w'(b) > 0, v'(m) > 0, u''(c) < 0, w''(b) < 0, v''(m) < 0 \).

The representative agent maximizes a discounted utility over an infinite horizon:

\[ \int_0^\infty [u(c) + v(m) + \alpha w(b)]e^{-\rho t} dt \]  

where \( \rho \) is the time discount rate and \( 0 < \rho < 1 \).

The budget constraint is

\[ \dot{a} = y + rb + x - c - \pi m \]  
\[ a = b + m \]

and the initial asset is given by \( b(0) \). Where where a dot over a variable is the time derivative, \( y \) is output, \( g \) is government spending, \( x \) is the government transfer, \( a \) is total wealth of the representative agent, \( \pi \) is the expected inflation rate and \( r \) is the returns on foreign bonds, which is given in the world
capital market. Except for the preference and time discount rate, the set up of our model is identical to the one in Obstfeld (1982).

The home price of the goods is $p$, and the corresponding world price is $p^*$. Assuming purchasing power parity, we have

$$p = E p^*$$  \hspace{1cm} (4)

where $E$ is the exchange rate. With proper normalization, $p^*$ can be set to one. The Hamiltonian function is

$$H = u(c) + v(m) + \alpha \omega(b) + \lambda_1(y + rb + x - c - \pi m) + \lambda_2(a - b - m)$$

The necessary conditions for optimization are

$$v'(m) - \pi u'(c) = \alpha \omega'(b) + ru'(c)$$  \hspace{1cm} (5)

$$\alpha \omega'(b) + (r - \rho)u'(c) = -u''(c)c$$  \hspace{1cm} (6)

plus the budget constraints and transversality condition.

The only new thing about these conditions is equation (5), which says that the marginal benefit of holding money ($v'(m) - \pi u'(c)$) is equal to the marginal benefit of holding foreign asset ($\alpha \omega'(b) + ru'(c)$) at optimum.

To fully spell out the dynamics, we need to specify the government sector. Government revenue comes from money creation and interest earnings from the central bank's reserves, i.e., $(\dot{M}/p) + rR$, and $R$ denotes the amount of reserves. Government also consumes goods, $g$, makes transfer, $x$, to the representative agent. So its budget is given by

$$g + x = \dot{M}/p + rR$$

or

$$g + x = (\dot{M}/M)m + rR$$  \hspace{1cm} (7)

Let the money growth rate be a positive constant:
\[ M/M = \theta \]  \hspace{1cm} (8)

Then we can write (7) as
\[ x = \theta n + rR - g \]  \hspace{1cm} (9)

By definition,
\[ \dot{m} = [(\dot{M}/M) - \dot{p}/p]m \]  \hspace{1cm} (10)

On the perfect foresight path, the expected inflation rate is equal to the actual inflation rate:
\[ \dot{p}/p = \dot{E}/E = e = \pi \]  \hspace{1cm} (11)

where \( e \) is expected rate of exchange rate depreciation. Therefore
\[ \dot{m} = (\theta - \pi)m \]  \hspace{1cm} (12)

Substitute (12) into (5), and substitute (9) and (12) into (3),
\[ \dot{c} = [\alpha v'(b) + (r - \rho)u'(c)][(-1/u''(c))] \]  \hspace{1cm} (13)
\[ \dot{m} = [\alpha v'(b) + (r + \theta)u'(c) - v'(m)](m/u'(c)) \]  \hspace{1cm} (14)
\[ \dot{b} = y + rb + rR - c - g \]  \hspace{1cm} (15)

To understand the stability of the system, we linearize (13), (14) and (15) around the steady state values \( \tilde{c}, m \) and \( \tilde{b} \):
\[
\begin{bmatrix}
\dot{c} \\
\dot{m} \\
\dot{b}
\end{bmatrix} =
\begin{bmatrix}
p - r \\
(r + \theta)mu''(\tilde{c})/u'(\tilde{c}) - mv''(\tilde{m})/u'(\tilde{c}) \\
-1
\end{bmatrix}
\begin{bmatrix}
0 \\
-\alpha w(b)/u''(c) \\
0
\end{bmatrix}
\begin{bmatrix}
c - \tilde{c} \\
m - \tilde{m} \\
b - \tilde{b}
\end{bmatrix}
\]  \hspace{1cm} (16)

The determinant of the 3x3 matrix is given by
\[
\frac{\nu''(\tilde{m})m}{u'(\tilde{c})u''(c)} \Delta
\]
which is negative if $\Delta$ is negative. In this case, the dynamic system has one negative and two positive characteristic roots because the product of the three roots is negative and the sum of the three roots is also positive and is given by the trace of the 3x3 matrix, $(p - \nu''(\bar{m})\bar{m} / u'(\bar{c}))$. Therefore, the dynamic system has a unique perfect foresight path near the steady state.

### III. Policy Analysis

In this section, we extend the basic model into a monetary economy and study the effects of inflation, government spending and foreign exchange intervention.

We note that equations (13) and (15) are independent of real balances and inflation rate in both short run and long run:

Proposition Four: With consumption and real balances separable in the preference, an increase in inflation (or exchange rate depreciation) has no effects on both the short-run and the long-run consumption and foreign asset holdings.

Compared to Obstfeld (1981), this result is very strong. Following Uzawa's definition of time preference, Obstfeld has shown that inflation increases foreign asset accumulation in the long run and causes current account surplus in the short run when the utility is separable in consumption and real balances. If the severability assumption is retained in our wealth effect model, money is supernuetral in both the short run and the long run.

In steady state, equation (37) is

$$\alpha v'(\bar{b}) + ru'(\bar{c}) = v'(\bar{m}) - \theta t'(\bar{c}) .$$

As $\bar{c}$ and $\bar{b}$ are not affected by inflation rate $\theta$, a higher $\theta$ means a lower $\bar{m}$ in the new equilibrium:

$$dm / d\theta = u'(\bar{c}) / v''(\bar{m}) < 0 .$$
In the wealth effect model, inflation will have real effects on consumption and foreign asset accumulation if the preference is not separable in $c$ and $m$. We will focus on this case in the rest of this section. With non-separable preference in $c$ and $m$, the representative agent maximizes

$$\int_{\alpha}^{\infty} [u(c, m) + \alpha v(b)] e^{-\rho t} dt$$

subject to the dynamic budget constraints (25) and (26).

It is further assumed that assume $u_{cm}(c, m) > 0$, and $u(c, m)$ is concave in $c$ and $m$:

$$u_{cc}u_{mm} - (u_{cm})^2 > 0$$

The necessary conditions for maximization are

$$u_m(c, m) - \pi u_c(c, m) = \alpha v'(b) + ru_c(c, m)$$

$$\alpha v'(b) + u_c(c, m)(r - \rho) = -u_{cc}(c, m)\dot{c} - u_{cm}(c, m)\dot{m}$$

plus the budget constraints and transversality condition.

Again substitute (34) into (42), and substitute (31) and (34) into (25), and assume steady state:

$$u_c(\bar{c}, \bar{m})(\theta + r) + \alpha v'(\bar{b}) - u_m(\bar{c}, \bar{m}) = 0$$

$$aw'(\bar{b}) + u_c(\bar{c}, \bar{m})(r - \rho) = 0$$

$$y + r\bar{b} + rR - \bar{c} - g = 0$$

To find the effects of various macroeconomic policies on foreign asset accumulation, we totally differentiate (44), (45) and (46):

$$\frac{d}{dt} = \begin{bmatrix} \alpha v''(\bar{b}) & (\theta + r)u_{cm} - u_{mm} \\ (r - \rho)u_{cc} & \alpha v''(\bar{b}) \\ -1 & r \end{bmatrix} \begin{bmatrix} dc \\ db \\ dm \end{bmatrix} = \begin{bmatrix} -u_{c}d\theta - w'(\bar{b})d\alpha \\ -w'(\bar{b})d\alpha \\ dg + rdR \end{bmatrix}$$

Denote the determinant of the 3x3 matrix as $\Delta''$:

$$\Delta'' = (\rho - r)\alpha v''(\bar{b})u_{cm} + [(r - \rho)r + \alpha v''(\bar{b})][(\theta + r)u_{cm} - u_{mm}] +$$
$$\Delta''\text{ is negative because } (\rho - r)\text{ is positive, } u_{cm} > 0, u_{cc} < 0, u_{mm} < 0, \text{ and } w''(\bar{b}) < 0.$$

With preference nonseparable in consumption and real balances, we have

**Proposition Five:** If $u_{cm} > 0$, inflation increases foreign asset accumulation and consumption.

From (47),

$$db / d\theta = (r - \rho)u_c u_{cm} / \Delta'' > 0 \quad (49)$$

and

$$db / d\theta = r(r - \rho)u_c u_{cm} / \Delta'' = r(db / d\theta) > 0 \quad (50)$$

To provide some economic intuition to this result, we can rely on equation (45):

$$\alpha v'(\bar{b}) + u_c(\bar{c}, \bar{m})(r - \rho) = 0 \quad (45)$$

Since the real balances and consumption is nonseparable in the utility function, an increase in the cost of real balances as a result of higher inflation rate leads people to lower money holdings and, in (45), the marginal utility of consumption will be smaller as $u_{cm}$ is positive. In this case, the marginal utility of foreign asset is too high compared to marginal utility of consumption and people will invest more in foreign asset and consume less in the short run. That will lead to a short-run surplus in the current account. In the long run, there will be more foreign asset, more interest income and hence more consumption. As for the real balances, higher cost of inflation tends to reduce them and higher income tends to raise them; and the sign is ambiguous.

As we mentioned earlier, Obstfeld (1981) has shown a surprising result that an increase in government spending leads to higher asset accumulation in the new equilibrium. In our wealth effect model, we have
Proposition Six: Government spending always reduces long-run foreign asset accumulation and consumption.

By Cramer's rule, (47) gives us

\[
\frac{db}{dg} = \frac{[(\rho - r)(c_u c_{mm} - c_{rm}^2)]}{\Delta''} < 0
\]

(51)

The whole expression is negative because \((\rho - r) > 0, u(c,m)\) is concave (see (41)), and \(\Delta'' < 0\).

The economic reason for this result is the same as in Proposition One of the last section. Again the short run implications of government spending are current account deficit and lower private saving. As for the long-run consumption, from (46),

\[
dc/dg = r(db/dg) < 0,
\]

That is to say, government spending reduces foreign asset accumulation, which in turn reduces interest income and consumption.

Another interesting comparison between Obstfeld's model and ours is the result of the central bank's foreign exchange intervention. In Obstfeld's model, if the central bank intervenes in the foreign exchange market by purchasing foreign bonds from the public with domestic currency, the total real asset in the economy is not affected, and, as the central bank's reserves also earn interest income which is distributed to the public in lump-sum form, the representative agent's real income and wealth remain the same. Therefore the central bank's intervention does not have real effects on foreign asset holdings, consumption and real balances. It only occasions a rise in the price level exactly proportional to the increase in money supply. In our wealth-effects model, the budget constraint does not change as the interest income earned by the central bank's reserves is still redistributed to the public, but, as foreign bonds are directly valued in the utility function, the symmetry of foreign bonds and the central bank's reserves in Obstfeld's model disappears. Shortly after the intervention of the central bank, the reduction of foreign bonds held by the private sector results in higher marginal utility of foreign asset, and the
equilibrium conditions (44) and (45) no longer hold. In fact, when the initial equilibrium foreign asset is
reduced by $dR$ and real balances increased by $\Delta R$, equilibrium condition (44) becomes

$$u_c(\tilde{c}, \tilde{m} + dR)(\theta + r) + \alpha'w'(\tilde{b} - dR) - u_m(\tilde{c}, \tilde{m} + dR) > 0.$$ 

To restore equilibrium, the representative agent will in the short run cut consumption, reduce real
balances and buy more foreign bonds. In the new equilibrium, as total asset (the sum of private asset
and the central bank's asset) have gone up, private consumption and real balances also go up.

Therefore we have

**Proposition Seven:** The central bank's purchase of foreign claims from the public with domestic
currency will lead to more foreign asset accumulation (the sum of central bank's reserve and
private holdings), more consumption and more real balances.

We also mention in passing that the wealth effects exert positive effect on foreign asset
accumulation in this monetary model as in Proposition Three of the last section. From (47),

$$\frac{db}{d\alpha} = \{(r - \rho)w'(\tilde{b})u_{cm} - [(\theta + r)u_{cm} - u_{mm}]\}/\Delta'' > 0$$
IV. Conclusion

In this paper, we have studies the effects of macroeconomic policies on foreign asset accumulation in a wealth effect model used by Bardhan (1967), Kurz (1968), Calvo (1980) and Blanchard (1983). Our results differ dramatically from the ones in Obstfeld (1981). In particular, we have shown that government spending always reduces foreign asset accumulation (or increases foreign borrowing). While Obstfeld's model turned the conventional Mundell-Fleming model on its head, our wealth effect approach has restored its validity.

Evaluating the consequences of macroeconomic policies is complicated; and the results are often very sensitive to the optimization framework we have utilized. Our wealth effect model only provides a different perspective to the problems and it should be taken as complementary to many existing models.
References


