The "Exorbitant Privilege": A Theoretical Exposition

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Abstract: This paper develops a general equilibrium model to study how the "exorbitant advantage" works, whether it is sustainable, and what may be the consequences if it is removed. Its main findings are: (1) the center country that issues the reserve currency enjoys the "exorbitant advantage" in the sense that her current account deficit can be financed by the periphery country's reserve holdings. The "exorbitant privilege" is predicated on the overvaluation of the reserve currency caused by a higher rate of money growth in the center country; (2) the "exorbitant advantage" is not likely to be sustainable in the long run; (3) if the "exorbitant advantage" is removed, the value of the reserve currency will depreciate, the terms of trade will change against the periphery country and sector composition will change in favour of the tradable sector in the center country and in favour of the non-tradable sector in periphery country. These changes will be more pronounced if the center country repays her debt by printing money instead of raising taxes.

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1. Introduction

The term "exorbitant privilege" was coined in 1965 by the then French Minister Valery Giscard d'Estaing to describe the US's unique ability to finance large external deficits (Gourinchas and Rey, 2005). The French economist Jacques Rueff (1971) explains the working of the exorbitant privilege as follows:

> "…when a country with a key currency runs a balance-of-payment deficit - that is to say, the United States, for example - it pays the creditor country dollars, which end up with the latter's central bank. But the dollars are of no use in Bonn or in Tokyo or in Paris. The very same day, they are reloaned to the New York money market, so that they return to the place or origin. Thus the debtor country does not lose what the creditor country has gained. So the key-currency country never feels the effect of a deficit in its balance of payments. And the main consequence is that there is no reason whatever for the deficit to disappear, because it does not appear." (p.78)

Rueff (1971) argues that the American balance-of-payment deficits (over the 1950s) he analyses was not brought about by the United States; rather it was "the outcome of an unbelievable collective mistake" (p.24), namely, the adoption of the Bretton Woods System of gold-exchange standard which accepts the US dollar as a substitute for gold as reserves. In Rueff's words,

> "By accepting this substitution, the banks of issue put themselves in a position of being able to lend immediately to the United States, wholly or in part, the dollars they received in settlement of American balance-of-payments deficits. …

> To the extent that the new system *was* applied, it in fact released the United States from the obligation of settling its foreign debts. Thus the United States was in a position to lend, to give, and even to buy outside its own frontiers without having to worry about its own capacity." (p.65)

More generally, Rueff (1971) contends that the gold-exchange standard "allowed the countries in possession of a currency benefiting from international prestige to give without taking, to lend without borrowing and to acquire without paying." (p.23)

Fifty years on, the Bretton Woods System that Rueff criticizes has long broken down, but the US appears to continue to enjoy the "exorbitant privilege" as evidenced by its apparent ease in acquiring foreign debts to finance its persistent current account and fiscal deficits. This "exorbitant privilege", as Rueff might argue, is attributable to the "international prestige" that the US dollar possesses. Besides being the main vehicle currency in foreign exchange markets, the main invoice currency for international trade, the US dollar is the most important reserve currency and the US Treasury securities are widely held by central banks around the world as official exchange reserves. Given the US dollar's dominant position and the fact that the US dollar is no longer tied to gold, the current international monetary order is referred to as the "dollar standard" (McKinnon, 2001). A notable feature of the dollar standard in recent decades is that some fast growing Asian countries (including those that have officially adopted floating exchange rate systems) choose to fix the value of their local currencies against the US dollar (Calvo and Reinhart, 2002). The outcome of these unilateral choices of exchange rate policies resembles the arrangement under the Bretton Woods system of fixed exchange rates, thus Dooley, Folkerts-Landau, &

Garber (2004) characterise it as the "Revived Bretton Woods system", with the US being the "center country" and Asia in the "periphery".

How does the "exorbitant privilege" work under the "Revived Bretton Woods system"? Is the system sustainable? What might happen to the real economies of the world should the centre country lose its "exorbitant privilege"? These are the questions this paper attempts to tackle. To answer these questions, we develop a simple two-country, three-good trade model with money. The setup of our model is similar to that in Lucas (1982) and Stockman & Dellas (1989), but our model makes four specific assumptions to characterise some stylised features of the "Revived Bretton Woods system": (1) the currency of the centre country (the "dollar") is the medium of exchange used in international trade and is the reserve currency of the periphery country; (2) the periphery country's currency is pegged to the dollar, with the peg maintained by the periphery country's intervention in the foreign exchange market; (3) the periphery country's foreign exchange reserves are invested in country 2's government bonds with no well-defined date of *net* repayment; and (4) the funds raised from the government bond issue are given to the citizens of the center country as transfer payments.

Based on our model, we come to three main conclusions. First, the centre country that issues the reserve currency has the "exorbitant privilege" in the following sense: once the center country's current account deficits translates into increases in reserves of the periphery country, the increased reserve is reinvested in the center country, thus the center country's deficits are more or less automatically funded. However, having access to automatic funding does not mean that the center country will necessarily run a current account deficit. The presence of a current account deficit

is an indication that the currency of the centre country is overvalued. The overvaluation may be caused by a higher rate of money growth in the centre country or by a desire on the part of the periphery country to devalue its currency for mercantilist purposes or to increase its reserves for precautionary considerations. This finding supports Aizenman's (2010) contention that it is a fallacy to regard global imbalance as a "necessary requisite and consequence" (p.23) of the dollar standard. It is also broadly consistent with the casual observation that the increase in global imbalances during the years prior to the global financial crisis of 2008 coincided with both an easy monetary policy environment in the US and a fast rate of official reserve accumulation in Asia.

Second, if international trade is balanced, reserves in the periphery country do not change, the center country does not utilise its "exorbitant privilege" although it continues to benefit from the existing reserves invested by the periphery country. If the policies of the center country or the periphery country or both make the dollar overvalued, trade imbalance will occur. The imbalance cannot be sustained in the long run because a sustained imbalance requires chronic overvaluation of the reserve currency which undermines the standing of the dollar as a reserve currency. Moreover, a lasting trade imbalance leads to an accumulation of debt in the center country and an accumulation of reserves in the periphery country to increasingly undesirable levels. It also leads to a change in sector composition in the center country in favour of the non-tradable sector.

Third, if the "exorbitant privilege" were to be removed in the sense that the demand for the dollar as a reserve currency falls significantly and the periphery country withdraws (a part of) its reserves invested in the center country, the dollar

would fall in value, the terms of trade would change against the periphery country and the sector composition would change in favour of the tradable sector in country 2 and the non-tradable sector country 1. The dollar depreciation and country 1' terms of trade deterioration would be more drastic, but the changes in sectoral composition would be less pronounced if the centre country chose to repay its debt by "printing money" instead of raising taxes.

Our paper connects with three related strands of literature. The first literature concerns the "exorbitant privilege". Gourinchas and Rey (2005) find that the US enjoys an "exorbitant privilege" in the sense that US's return on its external assets was notably higher than foreigners' return on US liabilities for over half a century since the early 1950s. Gourinchas, Rey and Govillot (2010) further argue that the "exorbitant privilege" comes with an "exorbitant duty" as indicated by a substantial worsening of US foreign asset position during the recent global financial crisis when there was a net selling of US external assets. Gourinchas et al.'s definition of exorbitant privilege is somewhat different from ours since they emphasise the excess return of US external assets whereas we focus on the center country's ability to obtain automatic funding for its external deficits. However our analysis points to an explanation for the excess return of US external assets, namely that, government policies under the dollar standard can create a powerful force that draws large amounts of official reserves from periphery countries to the US. Since safety is the main concern for the owners of official reserves, the reserves are typically invested in low-risk low-yield government securities. In contrast, US investments overseas involve higher risks and enjoy higher returns.

Secondly, our paper relates to the literature concerning the direction of international capital flows. The flow of official reserves from the poor periphery to the rich center country in our model constitutes a significant part of the "global saving glut" which Bernanke (2005, 2007) suggests may be a contributing factor to the global financial crisis. The direction of capital flow in our paper is also consistent with the empirical findings documented by Lucas (1990) and Alfaro et al. (2008).

Thirdly, our paper contributes to the ongoing debate on the causes and implications of the global imbalances. On the optimistic side of the debate, it is argued that the imbalances may be an equilibrium outcome resulting from the fact that different regions of the world differ in their capacity to generate financial assets from real investments (Caballero, Farhi, and Gourinchas, 2008). It is further suggested that the pattern of global capital flow associated with the imbalances may be "optimal" from the emerging economy's perspective of developing a competitive tradable goods sector (Dooley and Garber, 2007). On the pessimistic side, there are concerns that the global imbalances cannot be sustained and that its eventual correction may trigger a dollar crisis and impose substantial adjustment costs on the world economy. For instance, Blanchard, Giavazzi, & Sa (2005) argue that the increases in US demand for foreign goods and in foreign demand for US assets are behind the US current account deficits. They predict that the current account deficit must be reversed in the future accompanied by a substantial depreciation of the US dollar. Similarly, Obstfeld & Rogoff (2005) present a model of an adjustment process through which global balance is returned by a change of demand for tradable versus non-tradable and domestic versus foreign goods. They also predict a large depreciation of the US dollar. The conclusions of our paper are closer to the pessimistic view of the debate. However,

our focus is on the role of the US dollar as the reserve currency and the ramifications of government policies in both the center country and the periphery for the global imbalances.

2. A Model of the "Exorbitant Privilege"

Consider a world with two countries, country 1 (the periphery country) with a population of N_1 and country 2 (the centre country) with a population of N_2. Individuals in both countries derive utility from consuming three goods X, Y and Z over a time horizon T. Good X is produced in country 1, good Y is produced in country 2 and good Z is a non-tradable good produced in both countries. Domestic trade is mediated by each country's domestic currency, the "yuan" in country 1 and the "dollar" in country 2; and international trade is meditated by the dollar. As a result, consumers in country 1 have to purchase "dollars" to pay for imported good Y, so their budget constraint has a domestic currency component and a foreign exchange component. In contrast, consumers in country 2 only use dollars. The yuan-dollar exchange rate is fixed at $\bar{e}$ and country 1's central bank maintains this rate by buying or selling dollars in the foreign exchange market if necessary.

We assume that all of country 1's foreign exchange reserves are in dollars and are invested in one-period bonds issued by the government of country 2 (at an exogenously determined interest rate *i*). Bond repayments are financed by new bond issues. The funds raised by issuing bonds are distributed to country 2's consumers in transfer payments.

The decision problem of a representative consumer in country 1 is:

$$\max_{\{x_{1t},y_{1t},z_{1t},FX_{1t}\}_{t=1}^{T}} \sum_{t=1}^{T} \beta^{t-1}(\alpha_1 \ln x_{1t} + \alpha_2 \ln y_{1t} + \alpha_3 \ln z_{1t}) \qquad (\alpha_1 + \alpha_2 + \alpha_3 = 1), \qquad (1)$$

subject to the budget constraint:

domestic currency: $p_{xt}x_{1t} + p_{z_1t}z_{1t} = w_{1t} - \overline{e}FX_{It}$,

foreign exchange: $p_{yt}y_{1t} = FX_{It}$.

The decision problem for a representative consumer in country 2 is:

$$\max_{\{x_{2t},y_{2t},z_{2t}\}_{t=1}^{T}} \sum_{t=1}^{T} \beta^{t-1}(\alpha_1 \ln x_{2t} + \alpha_2 \ln y_{2t} + \alpha_3 \ln z_{2t}) \qquad (\alpha_1 + \alpha_2 + \alpha_3 = 1), \tag{2}$$

subject to the budget constraint:

$$\frac{p_{xt}}{\overline{e}} x_{2t} + p_{yt}y_{2t} + p_{z_2t}z_{2t} = w_{2t} + \frac{FX_{Gt}}{N_2}.$$

In the above decision problems, subscript t denotes time t and subscript i (i=1, 2) denotes country i; x_{it}, y_{it}, z_{it} are quantities of goods X, Y and Z consumed; p_{xt}, p_{yt}, p_{z_it} are money prices of goods X, Y and Z denominated in the currency of the country of production; w_{it} is the wage rate; FX_{It} is the foreign exchange (dollars) purchased in the foreign exchange market to pay for imports (good Y); $\overline{e}$ is the fixed exchange rate; FX_{Gt} is the amount of dollars purchased by country 1's central bank in order to maintain the exchange rate at $\overline{e}$. Since FX_{Gt} represents an increase in reserves and the reserves are invested in country 2's government bond, FX_{Gt}/N_2 is the per person transfer payment from the country 2's government funded by its bond issue.

On the supply side, producers in country 1 produce goods X and Z using the following technologies:

$$X_{1t} = a_{1x}L_{1xt}, \quad Z_{1t} = a_{1z}L_{1zt}, \tag{3}$$

where L_{1xt} and L_{1zt} are labor devoted to producing goods X and Z in country 1.

Under the assumption of perfect competition, the following conditions hold:

$$\pi_{z_1 t} = p_{z_1 t} Z_{1t} - w_{1t} L_{1zt} = 0, \tag{4}$$

$$p_{xt} X_{1t} + p_{z_1 t} Z_{1t} - w_{1t}(L_{1xt} + L_{1zt}) = FX_{Gt}\overline{e} \tag{5}$$

Equation (4) is a standard zero-profit condition. Equation (5) stipulates that the difference between country 1's total value of production and its total value of consumption is the value of trade surplus, which is equal to the amount of the foreign exchange purchase by the central bank. If trade is balanced, FX_{Gt} is zero. If country 1 has a trade surplus, country 1's central bank buys FX_{Gt} dollars (with $FX_{Gt}\overline{e}$ yuan), invests the same amount in country 2's government bonds, enabling country 2 to finance its trade deficit.

In country 2, producers make goods Y and Z using the following technologies:

$$Y_{2t} = a_{2y} L_{2yt}, \quad Z_{2t} = a_{2z} L_{2zt}. \tag{6}$$

Perfect competition implies:

$$\pi_{yt} = p_{yt} Z_{2t} - w_{2t} L_{2yt} = 0, \qquad \pi_{z_2 t} = p_{z_2 t} Z_{2t} - w_{2t} L_{2zt} = 0. \tag{7}$$

The market clearing conditions in the economy are summarised below.

(1) The markets for good Z:

$$N_1 z_{1t} = Z_{1t}, \qquad N_2 z_{2t} = Z_{2t}. \tag{8}$$

(2) The market for good X:

$$N_1 x_{1t} + N_2 x_{2t} = X_{1t} = a_{1x} L_{1xt}. \tag{9}$$

(3) The market for good Y:

$$N_1 y_{1t} + N_2 y_{2t} = Y_{2t} = a_{2y} L_{2yt}. \tag{10}$$

(4) Labor markets:

$$L_{1xt} + L_{1zt} = N_1, \qquad L_{2yt} + L_{2zt} = N_2. \tag{11}$$

(5) The foreign exchange market:

$$N_1 FX_{It} + FX_{Gt} = \frac{p_{xt}}{\overline{e}} N_2 x_{2t} . \tag{12}$$

The left-hand side of equation (12) is the sum of private demand for dollars (to pay for imports) and central bank's purchase of dollars; the right hand side is country 1's dollar earnings from exports.

(6) Money markets:

Strictly speaking, there are no separate markets for money – money, as the medium of exchange, features in all markets. In country 1, money supply is affected by government intervention in the foreign exchange market, but the government can still choose a rate of money growth π_1 (although this will affect the amount of foreign exchange intervention required to maintain a given exchange rate). The time path of money supply for country 1 is described by:

$$m_{1t} = (1 + \pi_1) m_{1(t-1)} + FX_{Gt} \overline{e} , \tag{13}$$

where π_{1t} is a measure of money growth rate (on the basis of last period's money supply, excluding growth due to foreign exchange market intervention); $FX_{Gt} \overline{e}$ is the increase in the quantity of money due to foreign exchange market intervention, assuming no sterilisation.

After a foreign exchange market intervention, country 1's reserve holdings are:

$$R_t = FX_{Gt} + (1 + i) R_{t-1} , \tag{14}$$

where R_{t-1} is the reserve holdings in the previous period, and i is the (exogenous) interest rate paid on country 2's government bonds.

The total money supply is equal to the total wage income in each country:

$$m_{1t} = N_1 w_{1t} , \qquad m_{2t} = (1 + \pi_2) m_{2(t-1)} = N_2 w_{2t} , \tag{15}$$

where π_{2t} is the rate of money growth in country 2.

Solving the consumers' decision problems (1) and (2), and applying zero profit conditions and market clearing conditions, we obtain the solutions for all endogenous variables in the model, which are presented in Table 1.[1] Based on the model outlined above and the solutions presented in Table 1, we now attempt to answer the questions set out in the introduction section, namely, how the "exorbitant privilege" works, whether it is sustainable, and what would likely happen to the economies of both the center and the periphery if it were removed.

3. Analysis

3.1. How does the "exorbitant privilege" work?

In our model, the "exorbitant privilege" is characterised as the central country's ability to obtain automatic funding for its trade deficit because the dollar is the periphery country's reserve currency and the reserves are invested in the center country's government bonds. By this characterisation, the "exorbitant privilege" is only utilised when country 2 runs a trade deficit which is funded by country 1's increase in reserves, that is, when $FX_{Gt} > 0$. From Table 1, we have:

$$FX_{Gt} = \frac{\alpha_1(1+\pi_2)m_{2(t-1)}\overline{e} - \alpha_2(1+\pi_1)m_{1(t-1)}}{(1-\alpha_1+\alpha_2)\overline{e}}, \tag{16}$$

which implies,

$$FX_{Gt} \geq 0 \Leftrightarrow \overline{e} \geq \frac{\alpha_2(1+\pi_1)m_{1(t-1)}}{\alpha_1(1+\pi_2)m_{2(t-1)}}. \tag{17}$$

[1] These solutions describe a temporary "state of rest" in the sense that consumers behave optimally and markets clear; they are not real equilibrium solutions as external trade is not balanced. There is a market tendency to regain balance as explained in section 3.

Since the fixed exchange rate ($\overline{e}$) coincides with the free market exchange rate when $FX_{Gt} = 0$, equation (17) suggests that the "exorbitant privilege" is utilised when $\overline{e}$ is higher than the free market rate, or in other words, when the dollar is overvalued. Suppose $\overline{e}$ is equal to the free market rate at time *t-1*, an overvaluation of the dollar may occur at time *t* if country 1 devalues its currency ($\overline{e}$ becomes larger), and/or if the rate of money growth in country 2 is higher than that in country 1 ($\pi_2 > \pi_1$).

Once the exorbitant privilege is utilised at time *t (i.e.,* $FX_{Gt} > 0$), the money supply in country 1 increases by $FX_{Gt}\overline{e}$ (assuming no sterilisation). Other things equal, the increase in country 1's money supply lowers the market value of country 1's currency relative to the dollar, thereby driving the fixed exchange rate towards the free market rate in the next period. Therefore, in the absence of continued devaluation by country 1 or excess money growth in country 2, there is a tendency for external imbalances to correct themselves. This suggests that if external imbalances persist, it is likely that the policies in the periphery country or the centre country or both have kept the dollar over-valued.

3.2. Is the "exorbitant privilege" sustainable?

While government policies in the center country and/or in the periphery can lead to persistent external imbalances, it does not mean that "exorbitant privilege" is sustainable. A necessary condition for the "exorbitant privilege" is that the periphery country is willing to use the dollar as its reserve currency. An important reason why the dollar is chosen as the reserve currency is that it is perceived to be a stable currency based on past experiences. If, as shown above, the utilisation of the "exorbitant privilege" requires a continued over-valuation of the dollar, then the

process of utilising the "exorbitant privilege" weakens the very foundation which supports the dollar as a reserve currency, therefore the system cannot be sustainable in the long run.

The use of the "exorbitant privilege" also has other consequences that lower the sustainability of the system. From the periphery country's perspective, the "exorbitant advantage" leads to a continued increase in its foreign exchange reserves which over time tend to go above its desired level. As the increase in reserves is lent back to the central country on an indefinite basis, it leads to the peculiar direction of long-term capital flow from the relatively poor periphery country to the relatively rich centre country, and since the capital flow is the outcome of a net flow of goods in the opposite direction, it has the effect of transferring real resources from the periphery to the centre.

While the centre country benefits from the real resource transfer in the short run, there are long run costs. First of all, the flip side of the increase in reserves by the periphery country is the centre country's accumulation of debt. Although the debt is denominated in dollars and has no well-defined time frame for net repayment, sovereign debt problems can escalate and result in financial crises as the history of the last eight centuries amply testifies (Reinhart and Rogoff, 2009). Secondly, the use of the "exorbitant privilege" also leads to a change in sectoral competition in country 2. From Table 1, we have:

$$\frac{L_{1xt}}{L_{1zt}} = \frac{\alpha_1 + \alpha_2}{\alpha_3} \tag{18}$$

$$\frac{L_{2yt}}{L_{2zt}} = \frac{(2-\alpha_3)\alpha_2\bar{e}(1+\pi_2)m_{2(t-1)} + \alpha_2\alpha_3(1+\pi_1)m_{1(t-1)}}{\alpha_3(1+\alpha_2)\bar{e}(1+\pi_2)m_{2(t-1)} - \alpha_2\alpha_3(1+\pi_1)m_{1(t-1)}} \tag{19}$$

It is easy to derive from equations (18) and (19) that

$$\frac{L_{2yt}}{L_{2zt}} < \frac{L_{1xt}}{L_{1zt}} \Leftrightarrow \bar{e} > \frac{\alpha_2(1+\pi_1)m_{1t}}{\alpha_1(1+\pi_2)m_{2t}}, \tag{20}$$

which implies that if the "exorbitant privilege" is at work, then country 2 has a lower proportion of resources devoted to the tradable sector relative to country 1. In other words, the use of the "exorbitant privilege" results in a change in the sectoral composition in the centre country in favour of the non-tradable sector at the expense of the tradeable sector. This is politically undesirable as it raises concerns about the "hollowing out" of the manufacturing sector in the centre country. The sectoral composition bias is also economically inefficient as it is driven by debts other than comparative advantage, and entails future adjustment costs to the extent that the bias has to be corrected when debts are eventually paid.

3.3. What would happen if the "exorbitant privilege" were removed?

If, as argued above, the "exorbitant privilege" cannot be sustained in the long run, the logical next question is what would happen if it were removed. By the removal of the "exorbitant privilege", we mean that the standing of the dollar as a reserve currency is in question and the periphery country not only stops accumulating dollar reserves, but also gradually reduces its dollar reserves over time. As the dollar reserves are held in the form of the centre country's government bonds, a reduction of reserves amounts to demanding the centre country's government to repay its debt. The centre country's government may raise taxes to repay the debt (case 1), or it may simply print money to meet its debt obligations since its debt is denominated in dollars (case 2). In the following we present two simple models to illustrate what would likely happen to the economies of the centre and the periphery countries in these two cases.

For simplicity of illustration, our models assume: (1) the government of the periphery country an equal amount of principal in each period; and (2) the debt repayments are distributed equally to citizens of the periphery country in transfer payments.

Consider case 1 where debt repayments by country 2 are financed by taxes. The model for case 1 is similar to that presented in section 2 except for 3 differences.

First, an individual consumer in country 1 now receives a transfer payment which changes the foreign exchange component of the budget constraint. The decision problem of the representative consumer is:

$$\max_{\{x_{1t},y_{1t},z_{1t},FX_{It}\}_{t=1}^{T}} \sum_{t=1}^{T} \beta^{t-1}(\alpha_1 \ln x_{1t} + \alpha_2 \ln y_{1t} + \alpha_3 \ln z_{1t}), \qquad (\alpha_1 + \alpha_2 + \alpha_3 = 1), \tag{21}$$

subject to the budget constraint:

domestic currency: $p_{xt} x_{1t} + p_{z_1 t} z_{1t} = w_{1t} - \overline{e} FX_{It}$,

foreign exchange: $p_{yt} y_{1t} = FX_{It} + \dfrac{\Delta R_t}{N_1}$,

where $\Delta R_t / N_1$ is the per capita transfer payment.

Second, an individual consumer in country 2 now has to pay a tax to fund the debt repayment. The decision problem of the representative consumer is:

$$\max_{\{x_{2t},y_{2t},z_{2t}\}_{t=1}^{T}} \sum_{t=1}^{T} \beta^{t-1}(\alpha_1 \ln x_{2t} + \alpha_2 \ln y_{2t} + \alpha_3 \ln z_{2t}), \qquad (\alpha_1 + \alpha_2 + \alpha_3 = 1), \tag{22}$$

subject to the budget constraint:

$$\frac{p_{xt}}{\overline{e}} x_{2t} + p_{yt} y_{2t} + p_{z_2 t} z_{2t} = w_{2t} - \frac{\Delta R_t}{N_2},$$

where $\Delta R_t / N_2$ is the per capita tax imposed for the purpose of debt repayment.

Third, the periphery country no longer intervenes in the foreign exchange market. As a result, the clearance condition for the foreign exchange market becomes:

$$N_1 FX_{It} = \frac{p_{xt}}{e_t} N_2 x_{2t}, \tag{23}$$

which determines the free market exchange rate e_t.

Correspondingly, country 1's money supply equation becomes:

$$m_{1t} = (1+\pi_1) m_{1(t-1)} \tag{24}$$

Now consider case 2 where country 2's government simply discharges its debt obligations with "printed" dollars. The model for case 2 differs from that for case 1 in two aspects. First, consumers in country 2 are no longer required to pay tax to fund the debt repayments, thus the decision problem for the representative consumer in country 2 is:

$$\max_{\{x_{2t}, y_{2t}, z_{2t}\}_{t=1}^{T}} \sum_{t=1}^{T} \beta^{t-1} (\alpha_1 \ln x_{2t} + \alpha_2 \ln y_{2t} + \alpha_3 \ln z_{2t}) \qquad (\alpha_1 + \alpha_2 + \alpha_3 = 1), \tag{25}$$

subject to budget constraint:

$$\frac{p_{xt}}{\overline{e}} x_{2t} + p_{yt} y_{2t} + p_{z_2 t} z_{2t} = w_{2t}$$

Secondly, the dollar "printing" changes country 2's money supply, which becomes:

$$m_{2t} = (1+\pi_2) m_{2(t-1)} + \Delta R_t . \tag{26}$$

We solve for the equilibrium for each of the models outlined above and present the solutions in Table 2. Based on the results in Table 2, we investigate how the economies would respond to the removal of the "exorbitant privilege", focusing on the exchange rate, the terms of trade, and sectoral composition in each country.

Intuitively, the removal of the "exorbitant privilege" means that the demand for the dollar would fall, therefore the value of the dollar would fall as well. The value

of the dollar would fall by more if country 2 also increases its supply of dollar to discharge its debts. As shown in Table 2, the exchange rates under the two cases are:

Case 1: $$(e_t)_1 = \frac{\alpha_2(1+\pi_1)m_{1(t-1)}}{\alpha_1[(1+\pi_2)m_{2(t-1)}] + \alpha_3\Delta R_t}, \tag{27}$$

Case 2: $$(e_t)_2 = \frac{\alpha_2(1+\pi_1)m_{1(t-1)}}{\alpha_1[(1+\pi_2)m_{2(t-1)} + \Delta R_t] + \alpha_3\Delta R_t}. \tag{28}$$

From equation (17), we know that for the "exorbitant privilege" to work, the dollar needs to be overvalued, that is,

$$\overline{e} > \frac{\alpha_2(1+\pi_1)m_{1(t-1)}}{\alpha_1(1+\pi_2)m_{2(t-1)}}. \tag{29}$$

It is easy to show that

$$\overline{e} > (e_t)_1 > (e_t)_2. \tag{30}$$

Inequality (30) implies that if the "exorbitant privilege" were removed, the dollar would depreciate in value; and that the dollar would depreciate more in the case 2 (where country 2 resorts to money printing to meet its debt obligations) than in case 1 (where debt repayment is financed by tax revenue).

Now consider the terms of trade. From Table 1, we have the prices for tradable goods in the model of the "exorbitant privilege":

$$(p_{xt})_{ep} = \frac{\alpha_1(1-\alpha_1-\alpha_2)(1+\pi_1)m_{1(t-1)} + \alpha_1(1+\alpha_1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e}}{(1-\alpha_1+\alpha_2)(\alpha_1+\alpha_2)a_{1x}N_1}, \tag{31}$$

$$(p_{yt})_{ep} = \frac{(1+\pi_2)m_{2(t-1)}}{a_{2y}N_2}, \tag{32}$$

where subscript *ep* indicates the "exorbitant privilege".

From Table 2, we have the prices for tradable goods in the two models with the removal of the "exorbitant privilege":

Case 1: $(p_{xt})_1 = \frac{(1+\pi_1)m_{1(t-1)}}{a_{1x}N_1}$, $(p_{yt})_1 = \frac{(1+\pi_2)m_{2(t-1)}}{a_{2y}N_2}$, (33)

Case 2: $(p_{xt})_2 = \frac{(1+\pi_1)m_{1(t-1)}}{a_{1x}N_1}$, $(p_{yt})_2 = \frac{(1+\pi_2)m_{2(t-1)}}{a_{2y}N_2} + \frac{\Delta R_t}{a_{2y}(1-\alpha_3)N_2}$, (34)

where subscripts 1 and 2 indicate case 1 and case 2, respectively.

From equations (31)-(34), it is easy to show that

$$(\frac{p_{xt}}{p_{yt}})_{ep} > (\frac{p_{xt}}{p_{yt}})_1 \quad \text{if} \quad \overline{e} > \frac{\alpha_2(1+\pi_1)m_{1(t-1)}}{\alpha_1(1+\pi_2)m_{2(t-1)}}, \tag{35}$$

$$(\frac{p_{xt}}{p_{yt}})_1 > (\frac{p_{xt}}{p_{yt}})_2. \tag{36}$$

Equations (35) and (36) suggest that the removal of the "exorbitant privilege" would leads to a fall in country 1's terms of trade; and the fall would be more pronounced in case 2 than in case 1. The fall in country 1' terms of trade is driven by a change in relative demand for goods X and Y due to income changes in country 1 and country 2. The relative price change is larger in case 2 because the increase in money supply in country 2 further drives up the price of good Y.

Finally we consider how each country's sector composition (as described by the ratio of labor employed in the tradable sector relative to that in the non-tradable sector) would respond to a removal of the "exorbitant advantage". From Table 1, we have each country's sector composition in the model of the "exorbitant privilege":

Country 1: $$(\frac{L_{1xt}}{L_{1zt}})_{ep} = \frac{\alpha_1+\alpha_2}{\alpha_3}, \tag{37}$$

Country 2: $$(\frac{L_{2yt}}{L_{2zt}})_{ep} = \frac{(2-\alpha_3)\alpha_2\overline{e}(1+\pi_2)m_{2(t-1)} + \alpha_2\alpha_3(1+\pi_1)m_{1(t-1)}}{\alpha_3(1+\alpha_2)\overline{e}(1+\pi_2)m_{2(t-1)} - \alpha_2\alpha_3(1+\pi_1)m_{1(t-1)}}. \tag{38}$$

From Table 2, we have each country's sector composition with the removal of the "exorbitant privilege" in case 1 and case 2:

Case 1: $$(\frac{L_{1xt}}{L_{1zt}})_1 = \frac{(\alpha_1 + \alpha_2 \frac{\alpha_2 m_{2t}}{\alpha_2 m_{2t} + \alpha_3 \Delta R_t})}{(1-\alpha_1 - \alpha_2 \frac{\alpha_2 m_{2t}}{\alpha_2 m_{2t} + \alpha_3 \Delta R_t})} \quad (39)$$

$$(\frac{L_{2yt}}{L_{2zt}})_1 = \frac{(\alpha_1 + \alpha_2 + \alpha_3 \frac{\Delta R_t}{m_{2t}})}{(1-\alpha_1 - \alpha_2 - \alpha_3 \frac{\Delta R_t}{m_{2t}})} \quad (40)$$

Case 2: $$(\frac{L_{1xt}}{L_{1zt}})_2 = \frac{(1-\alpha_3 - \alpha_3 \frac{\alpha_2 \Delta R_t}{\alpha_1 m_{2t} + \alpha_3 \Delta R_t})}{(\alpha_3 + \alpha_3 \frac{\alpha_2 \Delta R_t}{\alpha_1 m_{2t} + \alpha_3 \Delta R_t})} \quad (41)$$

$$(\frac{L_{2yt}}{L_{2zt}})_2 = \frac{1-\alpha_3}{\alpha_3} \quad (42)$$

From equations (37)-(42), we derive

$$(\frac{L_{1xt}}{L_{1zt}})_{ep} > (\frac{L_{1xt}}{L_{1zt}})_2 > (\frac{L_{1xt}}{L_{1zt}})_1 \quad (43)$$

$$(\frac{L_{2yt}}{L_{2zt}})_{ep} < (\frac{L_{2yt}}{L_{2zt}})_2 < (\frac{L_{2yt}}{L_{2zt}})_1 \quad \text{if} \quad \bar{e} > \frac{\alpha_2(1+\pi_1)m_{1(t-1)}}{\alpha_1(1+\pi_2)m_{2(t-1)}} \quad (44)$$

Inequalities (43) and (44) suggest that with the removal of the "exorbitant privilege", labour resources would move away from the tradable sector (producing good X) in country 1 and move towards the tradable sector (producing good Y) in country 2. The changes in sectoral composition are driven by changes in relative demand for good X and good Y. The extent of change in sector composition would be smaller in case 2 than in case 1 because in case 2, debt repayment financed by dollar "printing" drives

up of the price of good Y, therefore the relative demand increase for good Y is smaller.

4. Concluding remarks

In this paper, we have presented a set of three simple models to study how the "exorbitant advantage" works, whether it is sustainable, and what would be the likely consequences if it were removed. We have shown that the presence of the "exorbitant privilege" is an indication of the over-valuation of the dollar which may be caused by policies in the periphery and/or the center country. We have argued that the "exorbitant privilege" is unsustainable in the long run, and that its removal would likely lead to a dollar depreciation, a change in the terms of trade against the periphery, and a change in sector composition in favour of the tradable sector in the center country and in favour of the non-tradable sector in the periphery.

Our model has some simplifying assumptions which we hope to relax in future research. Although relaxing these assumptions does not qualitatively change the main conclusions of the paper, it will give us a better representation of the economic phenomena we want to study so that we can gain a deeper understanding of them. For example, in this paper we assume no private lending or borrowing. This greatly simplifies the model as the consumers' decisions become time independent. With the inclusion of private lending and borrowing in the model, the extent of the "exorbitant privilege" may be greater because the capital flowing into the center country can be used to finance both the center country's current account deficit and its external asset acquisition, enabling the center country to become the world's "venture capitalist" (Gourinchas and Rey, 2005).

Another simplifying assumption in this paper is that there is no private demand for money therefore money does not enter the consumers' utility function. Under the dollar standard, there is a demand for dollars in the periphery but virtually no demand for the periphery countries' currencies in the center country. If this asymmetry in money demand is included in the model, a higher monetary growth rate in the center country (which results in the utilisation of the "exorbitant privilege") can lead to a greater resource transfer from the periphery to the center country. This is because higher money growth leads to higher prices which increase the transaction demand for dollars in the periphery, and this increase in demand for dollar holdings needs to be met by additional exports earnings (Cheng and Zhang, forthcoming).

Finally, we assume no sterilisation following the periphery country's purchase of dollars in the foreign exchange market. With the introduction of a partial (but not complete) sterilisation, it will slow down but not stop the process through which the fixed exchange rate approaches the free market level, consequently the exorbitant privilege can persist for longer but not forever in the absence of continued policy interventions.

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Table 1. Solutions for the model of "Exorbitant Privilege"

	Equilibrium solutions
Purchase of "dollars" by country 1's central bank	$FX_{Gt} = \dfrac{\alpha_1(1+\pi_2)m_{2(t-1)}\overline{e} - \alpha_2(1+\pi_1)m_{1(t-1)}}{(1-\alpha_1+\alpha_2)\overline{e}}$
Wages	$w_{1t} = \dfrac{(1-\alpha_1)(1+\pi_1)m_{1(t-1)} + \alpha_1(1+\pi_2)m_{2(t-1)}\overline{e}}{(1-\alpha_1+\alpha_2)N_1}$ $w_{2t} = \dfrac{(1+\pi_2)m_{2(t-1)}}{N_2}$
Prices	$p_{xt} = \dfrac{\alpha_1(1-\alpha_1-\alpha_2)(1+\pi_1)m_{1(t-1)} + \alpha_1(1+\alpha_1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e}}{(1-\alpha_1+\alpha_2)(\alpha_1+\alpha_2)a_{1x}N_1}$, $p_{z_1t} = \dfrac{w_{1t}}{a_{1z}} = \dfrac{(1-\alpha_1)(1+\pi_1)m_{1(t-1)} + \alpha_1(1+\pi_2)m_{2(t-1)}\overline{e}}{(1-\alpha_1+\alpha_2)a_{1z}N_1}$ $p_{yt} = \dfrac{w_{2t}}{a_{2y}} = \dfrac{(1+\pi_2)m_{2(t-1)}}{a_{2y}N_2}$ $p_{z_2t} = \dfrac{w_{2t}}{a_{2z}} = \dfrac{(1+\pi_2)m_{2(t-1)}}{a_{2z}N_2}$
Consumption	$x_{1t} = a_{1x}(\alpha_1+\alpha_2)\dfrac{(1-\alpha_1)(1+\pi_1)m_{1(t-1)} + \alpha_1(1+\pi_2)m_{2(t-1)}\overline{e}}{(1-\alpha_1-\alpha_2)(1+\pi_1)m_{1(t-1)} + (1+\alpha_1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e}}$ $y_{1t} = \alpha_2 a_{2y}\dfrac{N_2}{N_1}\dfrac{(1-\alpha_1)(1+\pi_1)m_{1(t-1)} + \alpha_1(1+\pi_2)m_{2(t-1)}\overline{e}}{(1-\alpha_1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e}}$ $z_{1t} = \alpha_3 a_{1z}$ $x_{2t} = (\alpha_1+\alpha_2)a_{1x}\dfrac{N_1}{N_2}\dfrac{(1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e} - \alpha_2(1+\pi_1)m_{1(t-1)}}{(1-\alpha_1-\alpha_2)(1+\pi_1)m_{1(t-1)} + (1+\alpha_1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e}}$ $y_{2t} = \alpha_2 a_{2y}\dfrac{(1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e} - \alpha_2(1+\pi_1)m_{1(t-1)}}{(1-\alpha_1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e}}$, $z_{2t} = \alpha_3 a_{2z}\dfrac{(1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e} - \alpha_2(1+\pi_1)m_{1(t-1)}}{(1-\alpha_1+\alpha_2)(1+\pi_2)m_{2(t-1)}\overline{e}}$
Labor ratio: tradable vs. non-tradable	$\dfrac{L_{1xt}}{L_{1zt}} = \dfrac{\alpha_1+\alpha_2}{\alpha_3}$ $\dfrac{L_{2yt}}{L_{2zt}} = \dfrac{(2-\alpha_3)\alpha_2\overline{e}(1+\pi_2)m_{2(t-1)} + \alpha_2\alpha_3(1+\pi_1)m_{1(t-1)}}{\alpha_3(1+\alpha_2)\overline{e}(1+\pi_2)m_{2(t-1)} - \alpha_2\alpha_3(1+\pi_1)m_{1(t-1)}}$

Table 2. Equilibrium Solutions for the models of debt repayment

	Case 1	Case 2
Exchange rate	$e_t = \frac{\alpha_2(1+\pi_1)m_{1(t-1)}}{\alpha_1[(1+\pi_2)m_{2(t-1)}]+\alpha_3\Delta R_t}$	$e_t = \frac{\alpha_2(1+\pi_1)m_{1(t-1)}}{\alpha_1[(1+\pi_2)m_{2(t-1)}+\Delta R_t]+\alpha_3\Delta R_t}$
Wages	$w_{1t} = \frac{(1+\pi_1)m_{1(t-1)}}{N_1}$ $w_{2t} = \frac{(1+\pi_2)m_{2(t-1)}}{N_2}$	$w_{1t} = \frac{(1+\pi_1)m_{1(t-1)}}{N_1}$ $w_{2t} = \frac{(1+\pi_2)m_{2(t-1)}}{N_2}$
Prices	$p_{xt} = \frac{(1+\pi_1)m_{1(t-1)}}{a_{1x}N_1}$ $p_{z_1t} = \frac{(1+\pi_1)m_{1(t-1)}}{a_{1z}N_1}$ $p_{yt} = \frac{(1+\pi_2)m_{2(t-1)}}{a_{2y}N_2}$ $p_{z_2t} = \frac{(1+\pi_2)m_{2(t-1)}}{a_{2z}N_2}$	$p_{xt} = \frac{(1+\pi_1)m_{1(t-1)}}{a_{1x}N_1}$ $p_{z_1t} = \frac{(1+\pi_1)m_{1(t-1)}}{a_{1z}N_1}$ $p_{yt} = \frac{(1+\pi_2)m_{2(t-1)}}{a_{2y}N_2} + \frac{\Delta R_t}{a_{2y}(1-\alpha_3)N_2}$ $p_{z_2t} = \frac{(1+\pi_2)m_{2(t-1)}}{a_{2z}N_2}$
Consumption	$x_{1t} = a_{1x}\alpha_1(1+\frac{\alpha_2\Delta R_t}{\alpha_1[(1+\pi_2)m_{2(t-1)}]+\alpha_3\Delta R_t})$ $y_{1t} = a_{2y}\frac{N_2}{N_1}(\frac{\alpha_1(1+\pi_2)m_{2(t-1)}+(\alpha_2+\alpha_3)\Delta R_t}{(1+\pi_2)m_{2(t-1)}})$ $z_{1t} = \alpha_3 a_{1z}\{1+\frac{\alpha_2\Delta R_t}{\alpha_1[(1+\pi_2)m_{2(t-1)}]+\alpha_3\Delta R_t}\}$ $x_{2t} = \alpha_1\alpha_2 a_{1x}\frac{N_1}{N_2}\frac{[(1+\pi_2)m_{2(t-1)}]-\Delta R_t}{\alpha_1[(1+\pi_2)m_{2(t-1)}]+\alpha_3\Delta R_t}$ $y_{2t} = \alpha_2 a_{2y}\{1-\frac{\Delta R_t}{[(1+\pi_2)m_{2(t-1)}]}\}$ $z_{2t} = \alpha_3 a_{2z}\{1-\frac{\Delta R_t}{[(1+\pi_2)m_{2(t-1)}]}\}$	$x_{1t} = a_{1x}\alpha_1\{1+\frac{\alpha_2\Delta R_t}{\alpha_1[(1+\pi_2)m_{2(t-1)}+\Delta R_t]+\alpha_3\Delta R_t}\}$ $y_{1t} = (1-\alpha_3)a_{2y}\frac{N_2}{N_1}[\frac{\alpha_1(1+\pi_2)m_{2(t-1)}+\Delta R_t}{(1-\alpha_3)(1+\pi_2)m_{2(t-1)}+\Delta R_t}]$ $z_{1t} = \alpha_3 a_{1z}\{1+\frac{\alpha_2\Delta R_t}{\alpha_1[(1+\pi_2)m_{2(t-1)}+\Delta R_t]+\alpha_3\Delta R_t}\}$ $x_{2t} = \alpha_1\alpha_2 a_{1x}\frac{N_1}{N_2}\frac{[(1+\pi_2)m_{2(t-1)}+\Delta R_t]-\Delta R_t}{\alpha_1[(1+\pi_2)m_{2(t-1)}+\Delta R_t]+\alpha_3\Delta R_t}$ $y_{2t} = \alpha_2(1-\alpha_3)a_{2y}[\frac{(1+\pi_2)m_{2(t-1)}}{(1-\alpha_3)(1+\pi_2)m_{2(t-1)}+\Delta R_t}]$ $z_{2t} = \alpha_3 a_{2z}$
Labor ratio: tradable vs. non-tradable	$\frac{L_{1xt}}{L_{1zt}} = \frac{\alpha_1+\alpha_2\frac{\alpha_2[(1+\pi_2)m_{2(t-1)}]}{\alpha_2[(1+\pi_2)m_{2(t-1)}]+\alpha_3\Delta R_t}}{1-\alpha_1-\alpha_2\frac{\alpha_2[(1+\pi_2)m_{2(t-1)}]}{\alpha_2[(1+\pi_2)m_{2(t-1)}]+\alpha_3\Delta R_t}}$ $\frac{L_{2yt}}{L_{2zt}} = \frac{\alpha_1+\alpha_2+\alpha_3\frac{\Delta R_t}{[(1+\pi_2)m_{2(t-1)}]}}{1-\alpha_1-\alpha_2-\alpha_3\frac{\Delta R_t}{[(1+\pi_2)m_{2(t-1)}]}}$	$\frac{L_{1xt}}{L_{1zt}} = \frac{1-\alpha_3-\alpha_3\frac{\alpha_2\Delta R_t}{\alpha_1[(1+\pi_2)m_{2(t-1)}+\Delta R_t]+\alpha_3\Delta R_t}}{\alpha_3+\alpha_3\frac{\alpha_2\Delta R_t}{\alpha_1[(1+\pi_2)m_{2(t-1)}+\Delta R_t]+\alpha_3\Delta R_t}}$ $\frac{L_{2yt}}{L_{2zt}} = \frac{1-\alpha_3}{\alpha_3}$