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ESSAYS ON A POSITIVE THEORY OF ECONOMIC GROWTH

A thesis presented

by

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to

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Abstract

Departing from the conventional approach to economic growth, these papers collected here tackle the growth problems in capitalist economy, open economy and socialist economy with a novel definition of representative agent's preference: utility is a function of both consumption and capital stock (or wealth). As essays one and two demonstrate, this modelling of preference captures the essential of Max Weber's "spirit of capitalism" and John Maynard Keynes' "psychology of capitalist society". Results derived from this modelling are in striking contrast to the conventional wisdom: different nations with different capitalist spirit tend to have different consumption, capital stock and endogenous growth rates in the long run; inflation affects steady state capital acumulation, and high inflation may lead to high rate of endogeneous growth.

Essays three and four extend this approach to the case of open economy. Foreign asset accumulation, foreign direct investment, and domestic fiscal and monetary policies are studied in the light of the new model.

Essays five and six justify why we should define socialist planners' objective function in both consumption and capital accumulation. The regular phenomena in socialist economy, such as investment hunger, expansion drive, political investment cycles, and endogenous investment cycles, can be more sensibly understood within this positive framework than with the normative Cass model.

Contents

INTRODUCTION

According to Ramsey-Cass-Koopmans aggregate growth model, the maximization of an additive utility function defined on per capita consumption, subject to deminishing returns of per capita capital in production, leads to a unique long-run equilibrium where the marginal productivity of capital equals to the sum of population growth rate and time discount rate. Over time, per capita capital-labor ratio and consumption are expected to equalize across countries. This familiar convergence theorem of economic growth has been facing increasing theoretical and empirical challanges in recent years. Economic growth in the real world tends to diverge instead of converge. To explain the divergent phenomena of growth, new growth models have appeared, and their focus has shifted from standard neoclassical technology to increasing or constant returns in production. In these new models, capital accumulation in the long run depends not only on population growth and time discount rate, but also on the forms of utility functions, which are still defined on per capita consumption.

In this thesis, I will take a novel definition of preference advocated by Modecai Kurz as my starting point. I will endeavor to show that the inclusion of both consumption and capital stock (or wealth) into the utility functions is more realistic for the representative agent in a capitalist economy and social planners in a planned economy. Then I will demonstrate how economic growth across countries and economies can be better understood in the light of the new approach.

Essay one begins with the justification of the model. It is shown that Max Weber's "spirit of capitalism" and John Maynard Keynes' "psychology" of capitalist society can be approached mathematically by defining the representative agent's utility function on both consumption and capital stock. The essence of "the spirit of capitalism" is the continual accumulation of wealth for its own sake, rather than only for the material rewards that it can serve to bring. This approach to economic growth has been taken by many classical economists such as Adam Smith and Karl Marx. The salient feature of this modelling is that long-run capital accumulation depends crucially on the representative agent's preference or "the spirit of capitalism": a result standing in contrast to the modified golden rule of Cass model. Even though, for many simple preferences and technology, there may be multiple equilibria in this model as pointed by M. Kurz, I find that for the popular logarithm utility function and Cobb-Douglas technology, a unique saddle point equilibrium exits. On the saddle equilibrium path, different countries with different spirit of capitalism end up with different equilibria. The country with high spirit of capitalism tends to have high per capita capital stock, high saving rate, and possibly high consumption in the long run. In particular, government spending crowds out both long-run private consumption and capital accumulation. If the technology is assumed to be constant returns, instead of diminishing returns, in per capita capital, then the higher the spirit of capitalism, the higher the endogenous rate of growth. Essay one has also made attempt to demonstrate empirical relevance of this model.

In essay two, I extend the basic model into a monetary economy. The important results are: (1) Money is not super-neutral; on the unique saddle equilibrium path, high inflation leads to high capital accumulation in the long run. (2) It is possible to have positive relationship between inflation and endogenous rate of growth. Both results are different from traditional Sidrauski model and recent endogenous growth models.

Essay three studies foreign asset accumulation and macroeconomic policies in a small open economy. The inclusion of wealth into the representative agent's utility function can be easily justified by looking at the national sentiments of protectionism and new-mercantilism in the world. The effects of macroeconomic policies derived there are substantially different from those existing studies.

Essay four tackles optimal taxation on foreign capital income by open door and close door governments in a growing economy. (Open door government derives nonnegative utility from the presence of foreign capital in the home country, while close door government dislikes foreign capital.) In a two country world with foreign country investing in home country, a close door

government in the home country may be expected to tax more on foreign capital than the open door government does. But the result is surprising, both governments should adopt the same tax rate. Furthermore, the moral is not against close door government. In fact, close door government may do better than the open door government in maintaining higher consumption for its country, if the difference in the technological efficiency between foreign capital and domestic capital is small.

I turn to the growth problems in a typical socialist economy in essays five and six. To be realistic, social planners are not unworldly bureaucrats, and the optimal planning models written in 1960s are more or less mathematical utopia of socialism. In practice, social planners are often investment rate maximizers instead of social welfare maximizers, and their personal interests are more connected to persistent expansion of their organization than to the increase in people's consumption. Therefore, to define social planners' objective function on both consumption and capital stock provide us the thread to the understanding of investment hunger, expansion drive and chronic shortage in socialist economy. Relying on historical evidence in China, I have also shown that political investment cycles can be well illustrated in the new positive model. While political investment cycles can be attributed to the exogenous changes in the politcal regimes, it is also true that, even under the same political leadship, the investment rates often fluctuat cyclically. Is this endogenous cycle rational? I deal with this problem in essay six.

ESSAY ONE

" The Spirit of Capitalism " and Long-run Growth

Traditional optimal growth models such as Phelps (1961), Cass (1965) and Koopmans (1965) have demonstrated that, with typical neoclassical production function and exogeneously given population growth rate, the maximization of an additive utility function defined on per capita consumption by a representative agent or family leads to an unique steady state where the marginal productivity of capital equals to population growth rate plus time discount rate. If two countries have the same technology and population growth rate, it is expected that per capita consumption, per capita capital stock, saving rate and returns on capital are all equalized in steady state. In addition, government spending only crowds out private consumption and does not affect optimal capital stock.

This well-known convergence theorem of economic growth has been facing increasing challenges in recent years. Empirically, convergence theorem can not explain why Japan's saving rate is so apparently high [Hayashi (1986)] and America's national saving rate so low [Summers and Carrol (1987)], why the growth rates in some developing countries with Confucian culture like four East Asian 'miracles' of South Korea, Taiwan, Hongkong and Singapore are so high and the rates in some developing countries so low, why the relative income gap between developed contries and developing countries tends to widen in stead of narrowing, why productivity levels have not converged and why the nations with Protestant religious establishment in 1870 have 1979 per capita income more than one-third higher than the nations with Catholic religion [DeLong (1987)]. To answer these problemes, some new theories have emerged. A typical starting point for these new theories is to "departure from the usual assumption of deminishing returns" [Romer (1986) and Lucas (1988)]. Romer

(1986) has shown that, under increasing returns to scale, the level of per capita income in different countries need not converge.

We hold that culture differences among countries play important role in the determination of economic growth. While this is a century old proposition in sociology, theoretical contributions to economic growth tend to forget this crucial point. Here our alternative growth model is based on "the spirit of capitalism" of Max Weber (1958) and a mathematical model of Mordecai Kurz (1968). In a paper unduly neglected by the economics profession, Kurz (1968) deviates from conventional wisdom of economics and defines utility function on consumption and capital which he calls as wealth effects. As we will show in section I, this novel definition of preference reflects the essence of the spirit of capitalism: the continual accumulation of wealth for its own sake, rather than for the material rewards that it can serve to bring. (We hasten to add that Kurz's original model is a pure technical one and he does not offer any explanation or justification for his inclusion of capital into the preference. We can only hope that our reasonings will not distort Kurz's original ideas.)

As a result of the presence of the so-called wealth effects in the preference, the steady state capital stock is larger than the modified golden rule level, and multiple equilibria stand in contrast to the uniqueness of equilibrium in Cass model. Therefore countries with the same technology, same time discount rate and same population growth rate may have different steady states depending on initial conditions. Even with the same initial condition, difference in the wealth effects or spirit of capitalism may lead to different consumption and capital per capita in different countries.

The advantage of multiple equilibria in Kurz's model is also its disadvantage. For quite simple preference, it is difficult to make comparative studies with the multiple equilibria. In section II, we limit our study to log utility function and Cobb-Douglass technology. We are able to prove that there exists unique saddlepoint equilibrium, and the country with high spirit of capitalism ends up with high per capita capital stock, high saving rate,and possibly high consumption. Unlike the result in Cass model, government spendings in our model crowd out both steady state capital and consumption, hence reduce the saving rate.

In section III, we extend the model to an open economy. Owing to the assumption of perfect capital mobility, the returns on domestic capital and foreign bonds are equalized, but consumption and foreign asset accumulation in different countries may be quite different. Again government spending reduces steady state consumption and foreign asset accumulation.

In section IV, we follow recent literature on endogenous growth and set our model with constant returns to capital input. It is demonstrated that the higher the spirit of capitalism, the higher the growth rate.

Section V applies the model to explain variety of historical experience of economic growth and development in the world --- the success of Japan, the decline of the British industry, the case of Latin America and the four East Asian miracles and the relationship between economic growth and religion. All these have received considerable studies by economists, sociologists and historians. Our contribution is to show that their empirical and historical studies can well be incorporated into an operational mathematical model.

I. Reflections on Kurz Model

Here we present a model essentially the same as in Kurz (1968). A representative family, whose size grows at a natural given rate n, maximizes utility subject to a dynamic constraint of capital accumulation:

$$\text{Max} \int_0^\infty [\ u(c) + v(k)\ ]e^{-\rho t}dt \qquad (1)$$

s.t..

$$\dot{k} = f(k) - c - nk \qquad (2)$$

where c is per capita consumption, k is per capita capital stock, and ρ is the time discount rate which is positive. A dot over a variable denotes time derivative. The utility function and the technology have following standard properties: $u'(c)>0$, $u''(c)<0$, $v'(k)>0$, $v''(k)<0$, $f'(k)>0$, and $f''(k)<0$.

The Hamiltonian is

$$H = [\ u(c) + v(k) + \lambda(\ f(k) - c - nk\ )]e^{-\rho t} \qquad (3)$$

The optimal path of capital accumulation is described by following two dynamic equations:

$$-u''(c)\dot{c} = v'(k) + u'(c)(\ f'(k) - n - \rho\ ) \qquad (4)$$

$$\dot{k} = f(k) - c - nk \quad (5)$$

Let the steady state variables denote as k^* and c^*, then

$$v'(k^*) + u'(c^*)(\ f'(k^*) - n - \rho\ ) = 0 \qquad (6)$$

$$f(k^*) - c^* - nk^* = 0 \qquad (7)$$

Two facts immediately follow from these two equations.

Firstly, the steady state capital stock is higher than modified golden rule level. To see this, we rewrite equation (6) and compare it to the modified golden rule in Cass (1965),

$$f'(k^*) = n + \rho - \frac{v'(k^*)}{u'(c^*)} < n + \rho = f'(k^{mg}) \qquad (8)$$

where k^{mg} denotes the modified golden rule capital. Since $f''(k)<0$ for all k, $K^* > K^{mg}$.

Secondly, there are multiple equilibria in this model, and Kurz has given us both mathematical proof as well as numerical examples. Here we reproduce a figure from his paper.

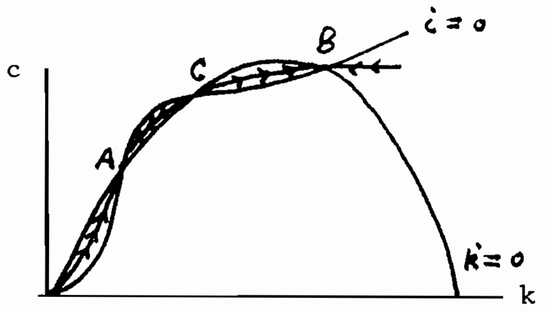

Figure 1

This result stands in contrast to Cass model where the equilibrium is unique.

As a positive approach to understand capitalist growth, in our view, Kurz model provides us a much better tool to tackle with the very complicated phenomena of growth in this world.

The inclusion of wealth or capital into preference reflects "the spirit of capitalism" in the sense of Max Weber (1958). In his famous study, Weber defines capitalism as the rational organization of formally free labour (p.21). The essence of the spirit of capitalism is the continual accumulation of wealth for its own sake, rather than for the material rewards that it can serve to bring (p.4). Even though " at all periods of history, wherever it was possible, there has been ruthless acquisition, bound to no ethical norms whatever" (p.57), only in capitalist economy, "man is dominated by the making of money, by acquisition as the ultimate purpose of his life. *Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs*. This reversal of what we should call the natural relationship, so irrational from a naive point of view, is evidently a *leading principle of capitalism* as it is foreign to all people not under capitalist influence" (p.53, italic added.).

John Maynard Keynes (1971) developes the same idea in his statement of the "psychology" of capitalist society. He says that "Europe was so organised socially and economically as to secure the maximum accumulation of capital. While there was some continuous improvement in the daily conditions of life of the mass of the population, society was so framed as to throw a great part of the increased income into the control of the *class least likely to consume it*. The new rich of the nineteenth century were not brought up to large expenditures, and *preferred the power which investment gave them to the pleasures of immediate consumption*.... Herein lay, in fact, the main justification of the capitalist system. If the rich had spent their new wealth on their own enjoyments, the world would long ago have found such a regime intolerable. But like bees they saved and accumulated, not less to the advantage of the whole community because they themselves held narrow ends in prospect."(p.11, italic added)

Keynes continues to describe the saving behavior of the capitalist classes: they "were allowed to call the best part of the cake theirs and were theoretically free to consume it, on the tacit underlying condition that they consumed very little of it in practice. *The duty of 'saving' became nine-tenths of virtue and the growth of the cake the object of true religion.* ... And so the cake increased; but to what end was not clearly contemplated. Individuals would be exhorted not so much to abstain as to defer, and to cultivate the pleasures of security and anticipation. Saving was for old age or for your children; but this was only in theory --- the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you" (P.12, italic added).

Therefore, defining utility on consumption and capital captures the preference of the representative agent in a capitalist economy. It is a way to model the nature of capitalism mathematically. In this respect, we add two excellent quotations from Gustav Cassel and Alfred Marshall:

"There is a formation of capital for which it is hardly possible to assign any concern about the future as motive. It cannot be said of the leading capitalists who satisfy all their wants of any consequence, and have a capital the returns on which gurantees this satisfaction of wants for all time to them and their families, yet constantly set aside large sum to increase their wealth, that they save out of the concern about the future. In these cases there must be some other motive. It is the economic interest of the capitalist to increase his wealth, and this in time becomes an end in itself. The motives that are at work are numerous. The senseless cupidity that in times finds its sole pleasure in contemplating the growth of its wealth, and may very well be described as an abnormal sluggishness of spirit and a pathological impoverishment of the emotional life, is certainly not the sole explanation. The desire of splendor and of the higher position in the society which the possession of great wealth assures, the stimulation of jealousy of other men, the healthy joy of the strong man in successful work as such, in ruling large masses, in influence especially --- these are all factors that have to be taken into account." Gustav Cassel (1924): The Theory of Social Economy, Pp 228-229.

For Marshall, "there are indeed some who find an intense pleasure in seeing their hoards of wealth grow up under their hands, with scarcely any thought for the happiness that may be got from its use by themselves or by others. They are prompted partly by the instinct of the chase, by the desire to outstrip their rivals; by the ambition to have shown ability in getting the wealth, and to acquire power and social position by its possession. And sometimes the force of habit, started when they were really in need of money, has given them, by a sort of reflex action, an artifial and unreasoning pleasure in amassing wealth for its own sake." Alfred Marshall (1920): Principles of Economics, 8th Edition, P189.

Of this point, many great thinkers in our history have shared the same view. Karl Marx, Before Weber and Keynes, regards the instinct nature of accumulation by capitalists as one of the essential part of capitalism. "Accumulate, accumulate! That is Moses and the prophets! 'Industry furnishes the material which saving accumulates.' Therefore save, save, i.e. reconvert the greatest possible portion of surplus-value or surplus into capital! Accumulation for the sake of accumulation, production for the sake of production: this was the formula in which classical economics expressed the historical mission of the bourgeoisie in the period of its domination." (Marx (1977), P.742) The conventional approach which defines the utility function only on consumption treats ancient slave owners, medieval feudal landlords, modern capitalists and socialist planners all the same. There is no historical and social sense in the basic neoclassical growth model.

To define wealth in the utility function is also an indirect way to express the importance of Schumpeter's entrepreneurial spirit in capitalist development. In our representative family of the capitalist economy, we must admit some elements of Schumpeter's entrepreneurial geniuses and let them play the role of technological innovators. In writing about entrepreneurs, Schumpeter deliberately makes them differ from capitalists. We can make this distinction ex-ante, but they are often coincide ex-post. Unless there is superior way to characterise the entrepreneur's preferece, the inclusion of wealth in the utility function is a first order approximation. Schumpeter's own words offer better explanation: the entrepreneur first " found a private kingdom, ... then there is the will to conquer: the impulse to fight, to prove oneself superior to others, to succeed for the sake, not of the fruits of success, but of success itself. ... *The financial result is a secondary consideration, or, at all events, mainly valued as an index of success and as a symptom of victory, the display of which very often is more important as a motive of large expenditure than the wish for consumers' goods themselves.* Finally, there is the joy of creating, of getting things done, or simply of excercising one's energy." [Schumpeter (1934) P.93] How can the success and the joy of getting things done be separated from their index and symptom - the wealth accumulation? "Pecuniary gain is indeed a very accurate expression of success" (P.94).

If the convention approach treats the man as economic animal wrong, then it is a more serious mistake to totally ignore man as a political animal. Ever since Aristotle, we are taught that "man is by nature an animal of intended to live in a polis".[see Aristotle(1958)]. Wealth or property provides man not only consumption means but also political power and social prestiges.

Possession of wealth is, to a considerable degree, a measure and standard of a person's success in a society. Thus capital and wealth directly enter to the utility function of the representative agent of the capitalist economy. In a recent book on power, Galbraith (1984), following the long tradition of sociology and political science, classifies wealth as one of three resouces of political power. "In past time, so great was the prestige of property that ... it accorded power to its possessor. What the man of wealth said or believed attracted the belief of others as a matter of course." (p49) To this day, "wealth per se no longer gives automatic access to conditional power. The rich man who now seeks such influence hires a public relations firm to win others to his beliefs. Or he contributes to a politician or a political action committee that reflects his views. Or he goes into politics himself and uses his property not to purchase votes but to persuade voters."(p.50) This is just what Lord Acton (1988) puts it:" Power goes with property" (P.572). In a *laisser-faire* capitalist economy, "freedom of accumulation not only carries with it the possibility of cumulative increase in the inequality of economic power,...in addition, economic power confers power in other forms, including the political." [Frank Knight (1942, 1982) P.82]

Seeking high social position and power has been long recognized as one of the most important motivation in capital accumulation. In the discussion related to the spirit of capitalism, Max Weber (1958) explicitly states that "the desire for the power and recognition which the mere fact of wealth brings plays its part" (P.70) in capital accumulation. John Stuart Mill (1909) makes this point even clear in his discussion of capital accumulation in modern history of England. "The earlier decline of feudalism having removed or much weakened invidious distinctions between the originally trading classes and those who have been accustomed to despise them; and a polity having grown up which made wealth the real source of political influence; its acquisition was invested with a factitious value, independent of its intrinsic utility. It becomes synonymous with power; and since power with the common herd of mankind *gives* power, wealth became the chief source of personal consideration, and the measure and stamp of success in life. To get out of one rank in society into the next above it, is the great aim of English middle-class life, and the acquisition of wealth the means" (P.174). The capitalist institutions not only "give a most direct and potent stimulus to the desire of acquiring wealth", but "by the scope they have allowed to individual freedom of action, have encouraged personal activity and self-reliance, while by the liberty they confer of association and combination, they facilitate industrial enterprises on a large scale"(P.174).

Above discussion may sound metaphysical, the real strength of Kurz model lies in its explanation of the real world economic growth. Cass model predicts the convergence of steady state capital stock and consumption for different countries if they have the same technology, same population growth rate and the same time discount rate. Obviously this is not proved by the divergent growth patterns in the world. Delong (1987) has shown, by using historical facts, that in the long run there need not be convergence of per capita income. In Kurz model, even the technology is standard neo-classical, the multiple equilibria clearly demostrate that, depending on the initial conditions of per capita capital stock, different countries reach quite different steady states: some country ends up with both higher consumption and capital per capita. In figure 1, there are two saddle equilibria: A and B with B dominating A both in consumption and capital stock.

Another notable feature of Kurz model is that preference enters to the determination of steady state capital and consumption. Even the initial conditions for two countries are the same, owing to the differences in their preferences, one country may well have higher saving rate, higher per capita consumption and higher per capita capital than another country. These can be simply illustrated in Kurz diagram:

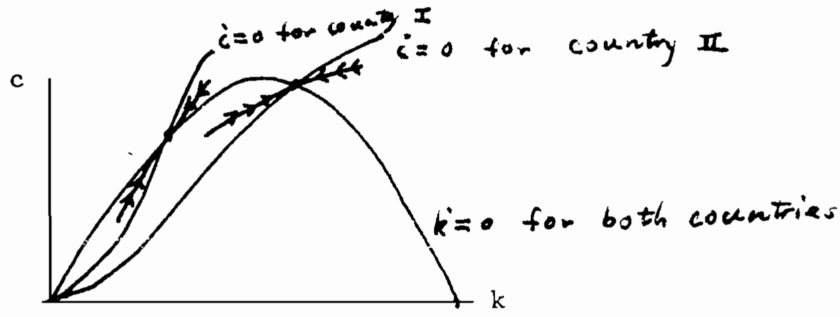

Figure 2

Where country II has higher per capita consumption and capital stock than country I.

II. The model and its implications

For general preferences, multiple equilibria in Kurz's model prevent us from making many interesting comparative studies. In this section, we will focus on a logarithm utility function and Cobb-Douglas technology.

Let the preference of the representative family be given by

$$U(c,k) = \log c + \pi \log k \qquad (9)$$

where π is nonnegative and measures the importance of wealth effects.

The technology is standard Cobb-Douglass

$$f(k) = k^{\alpha} . \qquad 1 > \alpha > 0. \qquad (10)$$

So capital accumulation is characterized by

$$\dot{k} = k^{\alpha} - c - nk \qquad (11)$$

The family maximizes an additive utility function

$$W = \int_0^{\infty} [\log c + \pi \log k] e^{-\rho t} dt \qquad (12)$$

The Hamiltonian is

$$H = [\ \log c + \pi \log k + \lambda(k^{\alpha} - c - nk)] e^{-\rho t} \qquad (13)$$

The first order conditions are

$$\pi c + \alpha k^{\alpha} - (n+\rho)k = \frac{k}{c}\dot{c} \qquad (14)$$

$$k^{\alpha} - nk - c = \dot{k} \tag{15}$$

In equilibrium,

$$f'(k^*) = \alpha(k^*)^{\alpha-1} < n + \rho \ . \tag{16}$$

Proposition one: *the steady state values of capital and consumption are unique, and are given as follows*

$$k^* = \left[\frac{\pi + \alpha}{n\pi + n + \rho} \right]^{\frac{1}{1-\alpha}} \tag{17}$$

$$c^* = k^{*\alpha} - nk^* \tag{18}$$

The proof is straight forward calculation by setting the time derivatives of c and k in equations (14) and (15) equal to zero.

Proposition two: *the equilibrium is a saddlepoint.*

Proof:

Take the first order approximation of the dynamic equations at the equilibrium values:

$$\begin{bmatrix} \dot{c} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} \pi \frac{c^*}{k^*} & \frac{c^*}{k^*} [\alpha^2 (k^*)^{\alpha-1} - (n+\rho)] \\ -1 & \alpha(k^*)^{\alpha-1} - n \end{bmatrix} \begin{bmatrix} c - c^* \\ k - k^* \end{bmatrix}$$

The product of two characteristic values is given by the determinant of the coefficient matrix

$$\Delta = \frac{c^*}{k^*} \{ \pi[(\alpha(k^*)^{\alpha-1} - n] + [\alpha^2(k^*)^{\alpha-1} - (n+\rho)]\} \qquad (19)$$

Since k^* is larger than the modified golden rule capital k^{mg}, the term in the second bracket on the right hand side is negative by eq. (16) and $\alpha < 1$. Hence if k^* is further larger than the golden rule capital k^g , then the term in the first bracket is also negative, and Δ is negative. In this case the equilibrium is a saddlepoint. In fact, as pointed out by Kurz, if the steady state capital is higher than golden rule capital, there exist saddlepoint for all preferences separable in wealth and consumption. We need to show that even if $k^* < k^g$, the system given above is a saddle.

Rewrite Δ as

$$\Delta = \frac{c^*}{k^*} \ [\ (k^*)^{\alpha-1}(\pi\alpha - \alpha^2) - \pi n - n - \rho \ ]$$

As $(k^*)^{\alpha-1} = \dfrac{n\pi + n + \rho}{\pi + \alpha}$, substitute this term into Δ,

$$\Delta = \frac{c^*}{k^*} \left[\frac{(\pi n + n + \rho)(\pi\alpha - \alpha^2) - (\pi n + n + \rho)(\pi + \alpha)}{\pi + \alpha} \right]$$

$$= \frac{c^*}{k^*} \frac{\pi n + n + \rho}{\pi + \alpha} [\ \pi\alpha - \alpha^2 - \pi - \alpha \ ]$$

$$= \frac{c^*}{k^*} \frac{\pi n + n + \rho}{\pi + \alpha} [\ \pi(\alpha-1) - \alpha^2 - \alpha \ ] < 0 \qquad (20)$$

where we have used the fact that $\alpha < 1$, and so the term in the bracket is negative. That is to say, one of the characteristic roots is negative, and one is positive. So the equilibrium is a saddlepoint.

The phase diagram is depicted in figure 3

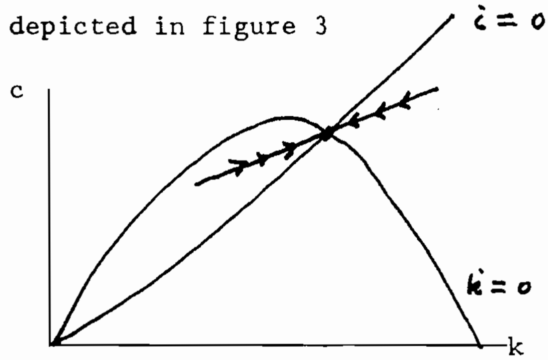

Figure 3

Proposition three: *The greater the importance attributed to wealth accumulation, the greater the steady state capital stock.*

To prove the proposition, just differentiate equation (17) with respect to π,

$$\frac{\partial k^*}{\partial \pi} = (1-\alpha)^{-1}\left[\frac{\pi+\alpha}{\pi n + n + \rho}\right]^{\frac{\alpha}{1-\alpha}} \frac{\rho+(1-\alpha)n}{(\pi n + n + \rho)^2} > 0 \qquad (21)$$

Therefore the country with high spirit of capitalism ends up with high capital stock.

Proposition four : *If capital stock is larger than the golden rule level, consumption is lower in country with high spirit of capitalism; if capital stock is smaller than the golden rule one, consumption is greater in the country with high spirit of capitalism.*

Proof:

$$\frac{\partial c^*}{\partial \pi} = (f'(k^*) - n) \frac{\partial k^*}{\partial \pi} \qquad (22)$$

since $\frac{\partial k^*}{\partial \pi}$ is positive as shown in proposition two, $\frac{\partial c^*}{\partial \pi}$ is positive if $f'(k^*) > n$, and vice versa. But $f'(k) = n$ is just the golden rule at which consumption is maximized.

Proposition five: *The steady state saving rate is positively related to the spirit of capitalism.*

Proof:

The saving rate, s, is given by

$$s = \frac{f(k^*) - c^*}{f(k^*)} = \frac{nk^*}{k^{*\alpha}} = nk^{*(1-\alpha)} \qquad (23)$$

$$\frac{ds}{d\pi} = n(1-\alpha)k^{*(-\alpha)} \frac{dk^*}{d\pi} > 0 \qquad (24)$$

So cultural difference leads to difference in saving rates. That might be a partial answer to the question why Japan's saving rate is so high and America's saving rate is so low [See L. Summers comment on Hayashi (1986)]. In fact , if technology is constant returns to scale in capital and labor, saving rate is always an increasing function of per capita capital.

$\frac{ds}{dk} = \frac{n[f(k)-f'(k)k]}{(f(k))^2} > 0$, since the numerator is just the wages times population growth rate.

Proposition six: *Increase in government spending reduces both steady state capital and consumption.*

Proof:

After the introduction of government spending g, the equilibrium equations are modified to be

$$\pi c + \alpha k^{\alpha} - (n+\rho)k = 0 \qquad (25)$$

$$k^{\alpha} - nk - c - g = 0 \qquad (26)$$

We will assume that the initial g is equal to zero and the initial equilibrium is given in proposition one.

$$\begin{bmatrix} \pi & \alpha^2 (k^*)^{\alpha-1} - (n+\rho) \\ -1 & \alpha (k^*)^{\alpha-1} - n \end{bmatrix} \begin{bmatrix} \frac{dc}{dg} \\ \frac{dk}{dg} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

As shown in proposition two, the determinant of the coefficient matrix evaluated at the equilibrium value in proposition one is negative. Denote it as Δ'

$$\frac{dc}{dg} = \frac{1}{\Delta'} [(n+\rho) - \alpha^2 (k^*)^{\alpha-1}] < 0 \qquad (27)$$

$$\frac{dk}{dg} = \frac{\pi}{\Delta'} < 0 \qquad (28)$$

Therefore the steady state saving rate is negatively associated with government spending,

$$\frac{ds}{dg} = n(1-\alpha)k^{*(-1)}\frac{dk}{dg} < 0 \qquad (29)$$

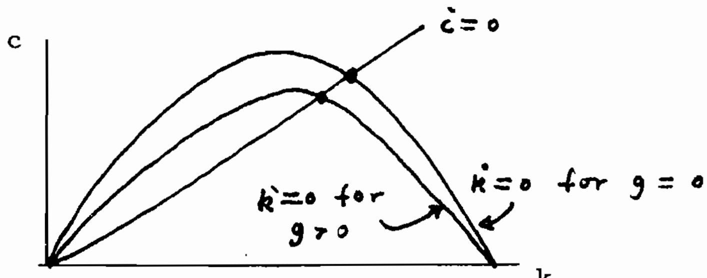

Figure 4

Increase in government spending immediately reduces private consumption, and, for consumers, marginal utility of consumption goes up accordingly. Consumers allocate less output to investment and more output to consumption. Hence the steady state capital stock decreases. Needless to say, we can not obtain the negative relationship between saving rates and government spending in Cass model where government spending only reduces private consumption, and optimal capital is independent of government spending.

Proposition seven: *output tax reduces steady state capital, and may increase or decrease consumption.*

Proof:

The equilibrium conditions with output tax are

$$\pi c + (1-\tau)\alpha k^{\alpha} - (n+\rho)k = 0 \qquad (30)$$

$$(1-\tau)k^{\alpha} - nk - c = 0 \qquad (31)$$

Solve these two equations for k,

$$k = \left[\frac{(1-\tau)(\pi+\alpha)}{n\pi + n + \rho} \right]^{\frac{1}{1-\alpha}} \tag{32}$$

Therefore

$$\frac{dk}{d\tau} = \frac{-1}{1 - \alpha} k^{\alpha} \frac{\pi + \alpha}{n\pi + n + \rho} < 0 . \tag{33}$$

The effect of income taxation on consumption is ambiguous here because increase or decrease in consumption depends on whether the initial capital stock is higher or lower than the golden rule level as shown in proposition four. If the initial capital stock is higher than the golden rule one, then higher tax leads to lower capital stock and higher consumption; and vice versa.

III. The Case of Open Economy

In this section we extend our study to a small open economy. It is assumed that there is perfect capital mobility in the world. The portfolios of representative family consist of domestic capital k and foreign bonds b. The returns on foreign bonds is r, which is exogeneously given to the small economy. The intertemporal optimization for the representative family is modified to be

$$\text{Max}\int_0^\infty [\log c + \pi \log a\,]e^{-\rho t}dt \qquad (34)$$

$$\text{s.t.}\quad \dot{a} = f(k) + (r-n)b - nk - c \qquad (35)$$

$$a = k + b \qquad (36)$$

The current-value Hamiltonian is defined by

$$H = \{\ \log c + \pi \log a + \lambda_1[f(k)+(r-n)b-nk-c] + \lambda_2(a-k-b)\} \qquad (37)$$

The first order conditions for optimization are

$$f'(k) = r \qquad (38)$$

$$\pi c + b(r-n-\rho) + k(r-n-\rho) = \frac{k + b}{c}\ \dot{c} \qquad (39)$$

$$f(k) - nk + (r-n)b - c = \dot{k} + \dot{b} \qquad (40)$$

Equation (38) says that marginal productivity of capital is equal to the returns on foreign bonds in equilibrium. From this equation we can solve k as a function of r with $k'(r) < 0$.

Equation (39) is a rearranged Euler equation. If the term $(r-n-\rho)$ is positive, then the system does not possess equilibrium, consumption will continue to grow to the infinity. we will mainly study the case where $(r-n-\rho)$ is negative.

Another point about equation (39) is worth commenting here. if π equals zero as in Cass model, equation (39) is reduced to be

$$\dot{c} = (r-n-\rho)c \qquad (41)$$

Equilibrium for this differential equation requires that $r = n + \rho$. Which holds only by coincidence. So the introduction of wealth effects into the preference is a nice way to get around this kind of knife-edge problem in the infinite horizon model. Blanchard (1985) has used a finite horizon model to overcome this difficulty.

Since, at optimum, capital stock is a function of world interest rate, it will not change if the interest rate does not change. Therefore the dynamics of equations (39) and (40) can be easily characterized in two dimensional space of consumption and foreign asset holdings. In steady state, (which is garanteed by the assumption of $r < (n+\rho)$.) we have

$$\pi c + b(r - n - \rho) = (n + \rho - r)k(r) \qquad (42)$$

$$c - b(r - n) = f(k(r)) - nk(r) \qquad (43)$$

We divide our study to three cases.

Case 1: $r < n$.

$$\left.\frac{dc}{db}\right|_{\dot{c} = 0} = \frac{n + \rho - r}{\pi} > 0 \qquad (44)$$

$$\left.\frac{dc}{db}\right|_{\dot{a} = 0} = \frac{1}{r - n} < 0 \qquad (45)$$

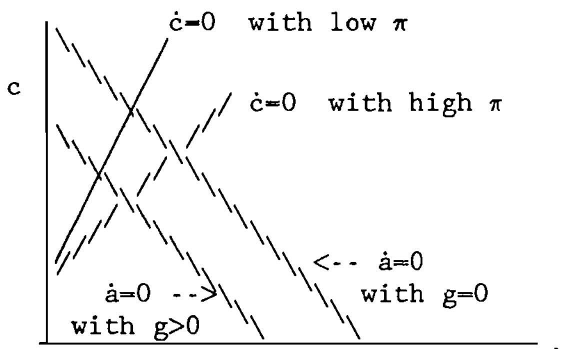

Figure 5

The country with high spirit of capitalism has larger foreign asset holdings and smaller consumption than the country with low spirit of capitalism. The reason is simple. As the returns on foreign bonds are less than population growth rate, the real returns on per capita foreign bonds are negative (r - n < 0). Hence country with high π holds more foreign asset and sacrifices more consumption than the country with low π.

Government spendings shift the line ȧ=0 inward to the left. Both consumption and foreign asset holdings decrease (figure 5).

Case 2: r > n .

$$\left.\frac{dc}{db}\right|_{\dot{a}=0} = \frac{1}{r - n} > 0$$

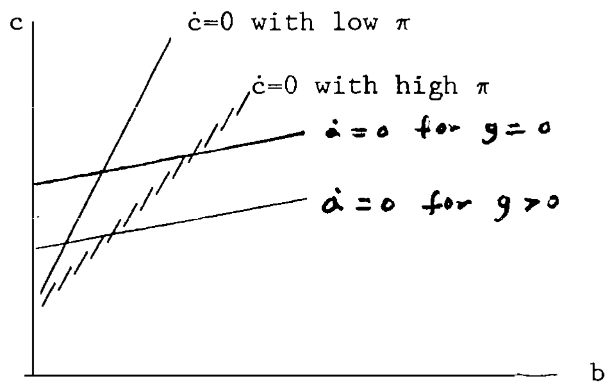

Figure 6

As the returns on foreign asset are higher than the population growth rate, the real returns on per capita foreign asset are positive $(r - n > 0)$. Hence the country with high π holds more foreign bonds, earns more foreign interest income and consumes more than the country with low π in steady state. Again government spendings reduce consumption and foreign asset acquisition.

Case 3: $r = n$.

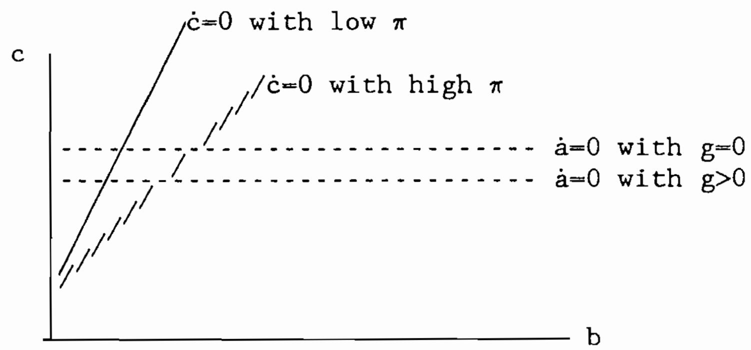

Figure 7

If $r = n$, consumption is the same for countries with different π's. But the country with high spirit of capitalism possesses more foreign assets than the country with low spirit of capitalism. Once more, government spendings in this case reduce steady state consumption and foreign asset holdings.

IV. The Spirit of Capitalism and the Endogenous Rate of Growth

Recent study on endogenous economic growth departs from the standard Cass model by replacing decreasing returns to scale with constant returns or increasing returns. To highlight the effect of the spirit of capitalism on the rate of growth in this setting, we maintain the same utility function used in sections II and III and borrow the story of technology from Rebelo (1987) and Barro (1988).

There is only one input which is called capital in the economy. The production function is constant returns to scale with respect to capital:

$$f(k) = Ak \tag{46}$$

where $A > 0$ is the constant net marginal product of capital, k is the per head capital stock in the representative family. Still the family size is assumed to be growing at constant rate n. According to Barro (1988), "the assumption of constant returns become more plausible when capital is viwed broadly to encompass human and non-human capital" (P.4).

Now the family maximizes

$$\int_0^{\infty} [\log c + \pi \log k] e^{-\rho t} dt$$

s.t.

$$\dot{k} = (A - n)k - c \tag{47}$$

The current value Hamiltonian is given by

$$H = \log c + \pi \log k + \lambda[(A - n)k - c] \tag{48}$$

The optimal conditions are

$$\frac{1}{c} = \lambda \tag{49}$$

$$\frac{\pi}{k\lambda} + (A - n - \rho) = -\frac{\dot{\lambda}}{\lambda} \tag{50}$$

$$\frac{\dot{k}}{k} = A - n - \frac{1}{k\lambda} \tag{51}$$

In writing equation (51), we have used capital accomulation condition (47) and optimal condition (49).

Differentiate condition (49) with respect to time and denote the constant growth rate of consumption as γ,

$$-\frac{\dot{\lambda}}{\lambda} = \frac{\dot{c}}{c} = \gamma \tag{52}$$

Using (52), we rewrite condition (50) as

$$\frac{\pi}{k\lambda} = \gamma + \rho + n - A$$

Or

$$k\lambda = \frac{\pi}{\gamma + \rho + n - A} \tag{53}$$

Differentiate expression (53) with respect to time on both sides,

$$\frac{\dot{k}}{k} = -\frac{\dot{\lambda}}{\lambda} = \gamma \tag{54}$$

That is to say, the growth rate of capital on the balanced growth path is the same as the growth rate of consumption. Substitute (54) into (51), (51) can be written as

$$\gamma = A - n - \frac{1}{k\lambda}$$

Or

$$k\lambda = \frac{1}{A - n - \gamma} \tag{55}$$

Now we use (53) and (55) to solve for the balanced growth rate γ,

$$\frac{\pi}{\gamma + \rho + n - A} = \frac{1}{A - n - \gamma} \tag{56}$$

So the endogenous rate of growth γ is equal to

$$\gamma = (A - n) - \frac{\rho}{1 + \pi} \tag{57}$$

From equation (57), the growth rate is higher if the spirit of capitalism is higher:

$$\frac{\partial \gamma}{\partial \pi} = \frac{\rho}{(1 + \pi)^2} > 0. \tag{58}$$

Even if countries have the same technology, the same population growth rate and the same time preference, they will have different rates of growth due to the difference in the spirit of capitalism.

On the balanced growth path, the saving rate s is

$$s = \frac{\dot{k}}{f(k)} = \frac{\dot{k}}{k}\,\frac{k}{Ak} = \frac{\gamma}{A} = \frac{A - n}{A} - \frac{\rho}{(1+\pi)A} \qquad (59)$$

Again the saving rate is an increasing function of the spirit of capitalism:

$$\frac{ds}{d\pi} = \frac{\rho}{A(1+\pi)^2} > 0. \qquad (60)$$

V. Some Applications

It is interesting to observe that economists tend to explain growth and development by dealing with numerical figures on capital, labor and technology, while historians, political scientists and sociologists pay more attention to the cultural background of growth and development. That might be the division of labor among intellectuals. Our model is an attempt to partly reconcile both stories told by economists and other social scientists. The measurement problem relating to the capitalist spirit may lead many quantitatively trained brains to hesitate in its application. But like many so-called latent variables in social sciences, (see Blalock (1974), Aigner and Goldberger (1977) regarding the problem of using latent variables in social sciences.) the cross country differences in the spirit of capitalism and their effect on economic growth and development can be easily seen. Examples abound.

(A) *Why Has Japan 'Succeeded'?*

Under this sensational title, Michio Morishima (1982),a well-known economic theorist, attibutes the economic success of Japan in Western technology and Japanese Confucianism. His approach is typical Weberian and his comparison between protestant ethic and Japanese ethos is illuminating. For modern capitalism to be established, a religious revolution had to come first. In the Western, "Puritanism's worldly frugality meant opposition to enjoyment and consumption, and luxury consumption especially was completely squeezed out, In this way the formation of capital was carried out through frugality; new capital was then used productively and became a new source of profit. Thus the religious revolution resulting from Protestantism created the modern entrepreneur and capitalism - a new type of person who was the possessor of an earnest faith, and who controlled huge wealth, but nevertheness contended himself with a life of extreme simplicity, striving to accumulate capital" (pp 83 - 84).

"If the Japanese had not adopted the belief of frugality, which was another of the prerequisites of capitalism, then modern capitalism could certainly not have been achieved in Japan. In Japan in those days Buddhism and Shinto, the traditional religions, did not have that great influence on the everyday life of the Japanese people. However, ... as a result of the Tokugawa Bakufu's cultural policy, Confucianism had spread widely and deeply among the Japanese people. Confucianism was understood in Japan as an ethical system rather than a religion, and it directly taught the Japanese people that frugal behavior was noble behavior. Therefore Japan, at the end of the Meiji Revolution, had already fulfilled the second prerequisite for capitalism" (p.86).

The impact of Confucianism on Japanese economy analyzed by Morishima may well manifest today in the high rate of saving in Japan. As our theoretical model predicts that there are positive relationship between high spirit of capitalism and high saving rate, why should we stick to the conventional models and expect the saving rates in the U.S. and Japan to be converged? (see Hayashi (1986) and many comments on his paper.)

Japan's story is not exceptional. Now South Korea, Taiwain, Singapore and Hongkong are following the Japanese suit. As observed by Roderick MacFarquhare (1985), for all these countries, "the significant coincidence is culture, the shared heritage of centuries of inculcation with Confucianism. That ideology is as important to the rise of the east Asian hyper-growth economies as the conjunction of Protestantism and the rise of capitalism in the west." "Post-Confucian economic man works hard and plays hard, buys much, but saves more".

(B) *English Culture and the Decline of the industrial Spirit, 1850 - 1980*

This is the title of the book by historian Martin Wiener. Controversial as it may be, (see M. Olson (1985) for different opinion.) it does reflect an important theme in our model: with the decline of the capitalist spirit, the economy ends up in a new steady state with lower per capita capital and consumption.

For Britain, Wiener argues that, after the Great Exhibition of 1851, "social and psychological currents began to flow in a different direction" (p157). "The emerging culture of industrialism ... was itself transformed. The thrust of new values borne along by the revolution in industry was contained in the later nineteenth century; the social and intellectual revolution implicit in industrialism was muted, perhaps even aborted" (p157). "For a century and a half the industrialist was an essential part of English society, yet he was never quite sure of his place. The educated public's suspicions of business and industry inevitably colored the self-image and goals of the business community. Industrialists responded to their mental environment, sometimes by seeking to leave the world of production for more acceptable realms of gentility, and sometimes by striving to adapt their way of life to the canons of gentility. ... As a rule, leaders of commerce and industry in England over the past century have accomodated themselves to an elite culture blended of preindustrial aristocratic and religious values and more recent bureauratic values that inhibited their quest for expansion, productivity, and profit" (p.127). The gentrification of the industrialists discouraged "commitment to a wholehearted pursuit of economic growth" (P.127) and led to "the waning of the industrial spirit" (P.159) or the capitalist spirit.

The attitude of Englishmen in the early of this century towards American was quite similiar to the one adopted today by the American and European towards Japanese. "Americans in particular (and more recently Japanese) were perceived by many leaders of industry ... as narrow people obsessed with the economic side of life, paying for their material success with the quality of personal life" (P.142). The confession by Samuel Courtauld, cited in Wiener's book, is typical: "I view the so-called 'Americanization of Europe' with the utmost dislike. I doubt whether American ideals of living - purely materialistic as they are - will finally lead to a contended working nation anywhere when the excitement of constant expansion has come to an end." (P.142)

History is really ironic. Let us listen to Jean-Jacques Rousseau's criticism of his contemporaries: "Ancient politicians incessantly talked about morals and virtue, those of our time talk only of business and money". [Rousseau (1964) P.51]

(C) *Underdevelopment Is a State of Mind*

Lawrence Harrison may have drawn a surprising conclusion from his twenty year experience with Latin America. His definition of "state of mind" is much broader than the spirit of capitalism; but the latter is a very important part of the former in his argument. We cite his work here not because we fully agree with his conclusion, but we regard his work as an empirical application of our theoretical model.

The basic message of Harrison (1985) is that culture is the principal determinant of development (P.166). "In the case of Latin America, we see a cultural pattern, derivative of traditional Hispanic culture, that is ... anti-entrepreneurial, and at least among the elite, anti-work" (P.165). " Unlike the traditional attitude of predominantely Protestant societies, work is not thought to be a positive value; it is regarded as a necessary evil, something people must do to live, but something to be avoided" (P.147). "The low value that attaches to work and practical achievement has probably suppressed the entrepreneurial instinct and performance ... thereby also contributing importantly to reduced rates of economic growth" (P.148).

(D) *Have Productivity Levels Converged?*

Our model can find comfortable support from a recent empirical study by De Long (1987). In this short paper, De Long finds that , contrary to the claim by William Baumol (1986), productivity levels among once-rich twenty-two countries in 1870 have not converged in 1979. In fact, "holding constant 1870 per capita income, nation that has Protestant religious establishments in 1870 have 1979 per capita income more than one-third higher than do nations that had Catholic establishments". And he shows that "there is one striking ex-ante association between growth over 1870-1979 and an exogeneous variable: a nation's dominant religious establishment. ... A religious establishment variable that is one for Protestant, one-half for mixed, and zero for Catholic nations is significantly correlated with growth as long as measurement error variance is not too high."(P.8) Indeed, "it does serve as an example of how culture may be associated with substantial divergence in growth performance".(p.8)

De Long's findings are no surprise at all to many eminent sociologists, historians and economists. Max Weber(1958) and Werner Sombart (1913, 1915) directly attribute the origin of the capitalist spirit to religious beliefs, even though they have different opinions about which religion gives birth to the spirit of capitalism. R. H. Tawney (1926) has made an important contribution to the study of relation between the rise of capitalism and religion. And economist Kenneth Boulding (1973) titles one of his interesting papers as "religious foundations of economic progress", in which he argues that Protestant ethic has not only influenced the development of capitalism, but "the Protestant ethic has contributed to the *success* of capitalist institutions, particularly in regard to their fostering a high rate of economic progress."(p.45)

VI. Conclusion

As far as we know, preferential or cultural difference among nations have not been seriously modelled in theoretical studies of economic growth. When traditional optimal growth model can not explain real world economic growth, the alternatives are often found in the modifications of technology, e.g., Romer (1986) and Lucas (1988). We must admit that technology innovations in nations, and technology transfers among nations have contributed most outstandingly to the productivity progress in the world. But technology can not explain why nations with different religions tend to have different per capita income; it can not expain why Japan and the US have so different saving rates; it can not explain why economic miracles are only made in four Asian countries with Confucian culture, but not achieved in Latin America; it can not explain why Great Britain with the highest potential for increasing returns to scale in last century has suffered from economic stagnation while West Germany has created economic supremacy on the ruins of World War.

In some very simple models we have shown that differences in the spirit of capitalism can explain varieties of phenomena in economic growth and development. These pure subjective and cultural elements may be one of the most important factors in the determination of the differences in growth rates, saving rates, per capita consumption and per capita capital stock among nations.

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ESSAY TWO

"The Spirit of Capitalism" and Monetary Growth

I. Introduction

This chapter simply extends the "spirit of capitalism" model into a monetary economy. In its mathematical form, it is a hybrid model of Sidrauski (1967a) and Kurz (1968): the representative family's utility function is defined on consumption, liquidity services and *wealth*. As we argued in essay one, appearance of wealth in the preference is an explicit way to capture the nature and spirit of capitalism, and distinguish the representative agent in a capitalist economy from other societies. Human beings are different among different societies not because people are born differently, but because people try to adapt to their different social and economic enviornments, and form their different values and behaviors.

In section II, the basic model is set up. It is shown that money is not superneutral and the growth rate of money affects capital accumulation. In particular, along a saddle equilibrium path, with separable preference, higher inflation leads to higher capital stock in steady state. This stands in contrast to Sidrauski (1967) where the growth rate of money have no effects on capital accumulation in both the long run and the short run if the preference is separable in consumption and liquidity services. (The case of nonseparable preferences has been analysed in Fischer (1979).) In addition, fiscal policy such as government spending crowds out consumption and may reduces or increases capital accumulation in steady state, which has no effect on steady state capital other than consumption in Sidrauski's model.

Section III takes up the study of the effect of monetary growth rate on the endogeneous rate of economic growth. With logarithm utility function and the same assumption about technology as in Rebelo (1987) and Barro (1988), inflation is shown to increase the balanced rate of growth. If the capitalist spirit is absent in the model, the balanced rate of growth is independent of the growth rate of money.

II. The Model

A representative family maximizes an additive utility function

$$\int_0^\infty (U(c, m) + \pi v(a))e^{-\rho t}dt \qquad (1)$$

where c stands for consumption, and m for real balance, and a for wealth. All these variables are in per capita terms. π measures the capitalist spirit and is positive. ρ is the positive time discount rate.

There is a standard neoclassical production function available for the family, f(k). k is per head capital stock, and $f'(k) > 0$, $f''(k) < 0$.

Let i be the expected inflation rate and x the government transfers to each member of the family. The dynamic budget constraint for the family is

$$\dot{a} = f(k) + x - c - nk - (n + i)m - g \qquad (2)$$

where "dot" means the time derivative of the variable, and n is the constant growth rate of the family size, and g is the government spending.

The wealth can be hold in the forms of capital and real balances:

$$a = k + m \tag{3}$$

Therefore

$$\dot{a} = \dot{k} + \dot{m} \tag{4}$$

Let λ_1 be the costate variable attached to budget constraint (2), and λ_2 the costate variable attached to the wealth equation (3), the Hamiltonian function for the family's maximization is given as follows,

$$H = \{ U(c,m) + \pi v(a) + \lambda_1[f(k) + x - c - nk - (i+n)m - g] + \lambda_2(a - k - m)\}e^{-\rho t} \tag{5}$$

For simplicity, it is assumed that U(c,m) is separable in c and m, and

$$U(c,m) = u(c) + l(m) \tag{6}$$

The conditions for a maximum are

$$u'(c) = \lambda_1 \tag{7}$$

$$l'(m) = u'(c)(f'(k) + i) \tag{8}$$

$$\pi v'(a) + u'(c)(f'(k) - n - \rho) = - u''(c)\dot{c} \tag{9}$$

$$\lim_{t \to \infty} a\lambda_1 e^{-\rho t} = 0 \tag{10}$$

Plus the dynamic budget constraint

$$\dot{k} + \dot{m} = f(k) + x - c - nk - (i+n)m - g \tag{2'}$$

By definition,

$$\dot{m} = \left(\theta - \frac{\dot{p}}{p} - n \right)m \tag{11}$$

where θ is the constant growth rate of money, and p is the price level. On perfect foresight paths, the expected inflation rate is equal to the actual one:

$$\frac{\dot{p}}{p} = i \qquad (12)$$

We also note that the government transfer, x, is equal the net increase of money supply:

$$x = \theta m \qquad (13)$$

Substitute (11), (12) and (13) into conditions (8) and (2'), and rewrite condition (9), we have

$$\pi v'(a) + u'(c)\,(f'(k) - n - \rho) = -u''(c)\dot{c} \qquad (9)$$

$$\left[\, f'(k) + \theta - n - \frac{l'(m)}{u'(c)} \,\right] m = \dot{m} \qquad (14)$$

$$f'(k) - nk - c - g = \dot{k} \qquad (15)$$

In steady state $\dot{c} = \dot{k} = \dot{m} = 0$. Two points about steady state equation (9) are worth commenting here. First, as v'() is positive, (f'(k) - n - ρ) has to be negative in steady state, namely, the steady state capital is larger than modified golden rule level. Second, if π is zero in (9), we are back to Sidrauski's case, and capital accumulation in steady state is independent of monetary growth; for π is positive in our model, capital accumulation will in general depends on monetary factor.

There may be multiple equilibria in our model; later we will focus on the steady state with only one negative characteristic root. That is to say, in the neighborhood of this steady state, there exists an unique perfect foresight path leading to the equilibrium. Denote any steady state values of variables as k^*, c^*, and m^*, and linearize around them, we have

$$\begin{bmatrix} \dot{c} \\ \dot{m} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} -(f'(k^*) - n - \rho) & \dfrac{\pi v''()}{-u''(c^*)} & \dfrac{\pi v''()+u'(c^*)f''(k^*)}{-u''(c^*)} \\ \dfrac{l'(m^*)u''(c^*)m^*}{u'(c^*)^2} & \dfrac{l''(m^*)m^*}{-u'(c^*)} & f''(k^*)m^* \\ -1 & 0 & f'(k^*) - n \end{bmatrix} \begin{bmatrix} c - c^* \\ m - m^* \\ k - k^* \end{bmatrix} \tag{16}$$

Call the 3x3 matrix as T. The trace of the matrix T is equal to the sum of three characteristic roots, which is positive here

$$\text{trace of } T = \rho - \frac{l''(m^*)m^*}{u'(c^*)} > 0 \tag{17}$$

So at least one root is positive.

The determinant of the matrix T, which we denote as Δ, does not possess definite sign,

$$\Delta = (f'(k^*)-n-\rho)(f'(k^*)-n)\frac{l''(m^*)m^*}{u'(c^*)} + f''(k^*)\frac{\pi v''()m^*}{u''(c^*)} +$$

$$+ \frac{\pi v''()+u'(c^*)f''(k^*)}{u''(c^*)}\frac{l''(m^*)m^*}{u'(c^*)} + \frac{\pi v''()}{u''(c^*)}\frac{l'(m^*)u''(c^*)m^*}{u'(c^*)^2}(f'(k^*)-n) \tag{17}$$

where the second and the third terms are always negative, the first and the fourth terms have opposite sign depending on whether $(f'(k^*)-n)$ is larger or less than zero, i.e., whether the capital is larger or less than golden rule level.

Proposition One: *If preference and technology are such that the golden rule capital is an equilibrium, then, in the neigborhood of the equilibrium point, there is a unique perfect foresight path converges to the steady state.*

We know that the determinant of matrix T is equal to the product of the three characteristic roots. If golden rule capital is an equilibrium point, then $f'(k^*) = n$, and the first and the fourth terms of the determinant of matrix T vanish. Hence Δ is negative. That is to say, there are either three negative roots or one. Since the trace of matrix T is positive as shown above, at least one of the roots is positive. Therefore the dynamic systems have one negative root and two positive roots, and there is a unique perfect foresight path converging to the golden rule steady state. Proposition one can also be viewed as an approximate result for any equilibrium point near the golden rule one.

The unique perfect foresight path is crucial for the assumption of rational expectation in a monetary economy as noted by Brock (1974), Calvo (1979) and Fischer (1979). In our model, this amounts to a negative Δ. For the rest part of the section, we focus on this unique path.

Proposition Two: *Along the unique perfect foresight path, high growth rate of money leads to high capital accumulation in steady state.*

Totally differentiate equations (9), (14) and (15) in a saddlepoint equilibrium,

$$T \begin{bmatrix} dc \\ dm \\ dk \end{bmatrix} = \begin{bmatrix} \dfrac{v'()}{u''(c^*)} d\pi \\ -m^* d\theta \\ dg \end{bmatrix} \tag{18}$$

By Cramer's rule, (and note that the determinant of T, Δ, is negative)

$$\frac{dk}{d\theta} = \frac{-\pi v''()m^*}{\Delta u''(c^*)} > 0 \tag{19}$$

The reason is straightforward. As the growth rate of money rises, inflation goes up, and the opportunity cost of money holdings increases. The representative family stores its wealth more in the form of capital. This is similiar to the standard explanation in Tobin's ad hoc monetary growth model. (see Tobin (1965), also Sidrauski (1967b)) Of course, the result here is derived from the "rational choice" of the representative family.

It is a basic fact that consumption is maximized at golden rule steady state. If capital stock is less than that level, increase in inflation results in higher capital and higher consumption; if capital stock is already above golden rule one, inflation leads to higher capital but lower consumption. Mathematically,

$$\frac{dc}{d\theta} = (f'(k^*) - n)\frac{dk}{d\theta} \tag{20}$$

As $(dk/d\theta)$ is positive, $(dc/d\theta)$ depends on whether $(f'(k^*) - n)$ is positive or negative.

The effects of inflation on money demand are not clear-cut. To see this, use Cramer's rule on linear system (18),

$$\frac{dm}{d\theta} = \frac{1}{\Delta}(f'(k^*)-n-\rho)m^*(f'(k^*)-n) + \frac{\pi v''()+u'(c^*)f''(k^*)m^*}{\Delta\, u''(c^*)} \tag{21}$$

where the second term on the right hand side is always negative, which represents the substitution effect of money demand. If capital stock is larger than the golden rule, then the first term is also negative. In this case, inflation reduces real balances. If steady state capital is less than the golden rule, the first term is positive, and the effect of inflation on money demand is ambiguous. This is the income effect on money demand: as capital stock is less than golden rule level, increase in inflation will raise net output, $f(k^*) - nK^*$, which will in turn raise demand for both consumption and real balances.

Proposition Three: *The higher the spirit of capitalism, the higher the steady state capital; The higher the government spending, the lower the steady state consumption and real balances. The effect of government spending on steady state capital is ambiguous.*

Again use (18),

$$\frac{dk}{d\pi} = \frac{-l''(m^*)m^*v'()}{\Delta u'(c^*)u''(c^*)} > 0 \tag{22}$$

$$\frac{dk}{dg} = \frac{(f'(k^*)-n-\rho)l''(m^*)m^*}{\Delta u'(c^*)} + \frac{\pi v''()l'(m^*)m^*u''(c^*)}{\Delta u''(c^*)u'(c^*)^2} \gtrless 0? \tag{23}$$

$$\frac{dm}{dg} = \frac{-l'(m^*)u''(c^*)m^*[\pi v''()+u'(c^*)f''(k^*)]}{\Delta u''(c^*)u'(c^*)^2} +$$

$$+ \frac{1}{\Delta}(f'(k^*) - n - \rho)f''(k^*)m^* < 0 \tag{24}$$

$$\frac{dc}{dg} = \left\{ \frac{-\pi v''()f''(k^*)m^*}{u''(c^*)} + \frac{-[\pi v''()+u'(c^*)f''(k^*)]l''(m^*)m^*}{u'(c^*)u''(c^*)} \right\} \frac{1}{\Delta} < 0 \quad (25)$$

To see the effect of government spending on capital accumulation, we first look at the magnitude of its effect on consumption. From equation (25), the two terms in the parentheses are just the two negative terms in Δ. Unless capital stock is in the golden rule state, (dc/dg) will not equal to minus one. In this case, the change on capital stock resulted from government spending can be clearly seen. Differentiate equilibrium condition (15) with respect to g,

$$\frac{dk}{dg} = \left(1 + \frac{dc}{dg} \right) \frac{1}{f'(k^*) - n} \quad (26)$$

Suppose that $(f'(k^*) - n)$ is positive. (dk/dg) will be positive if (dc/dg) is larger than minus one, and negative if (dc/dg) is smaller than minus one. The case of negative $(f'(k^*) - n)$ is just the opposite.

In Sidrauski's rational choice model, government spending fully crowds out consumption and exerts no effect on capital accumulation in the steady state. In our model, government spending may increase or decrease capital stock in the long run. This also differs from the result in chapter one where, along saddle equilibrium path, government spending always reduces capital accumulation.

III. Inflation and the Endogenous Rate of Growth

Even though recent study on endogenous growth has reformulated many growth models, e.g., Romer's (1986) and Lucas' (1988) revisions of traditional Solow model, Barro's (1988) extension of the new model into government spending, and Rebelo's (1987) study of multi-sector and cash-in-advance models, the Sidrauski's money-in-utility model has received little attention as far as we know. This is not strange as we will show that, with separable preference on money and consumption, the balanced rate of growth is independent of the rate of monetary growth in Sidrauski's model.

Here we continue the study of our model under new assumptions about technology and utility function. Technology is assumed to be constant returns to scale in capital, which is the only input here as in Barro (1988). The size of the family is assumed to be fixed. Using the same notations as in last section, we write the new family production function and budget constraint as follows,

$$f(k) = Ak \tag{27}$$

where $A > 0$ is the constant net marginal product of capital, and

$$\dot{a} = Ak + x - c - im \tag{28}$$

$$a = k + m \tag{29}$$

The family maximizes a discounted logarithm utility function defined on consumption, real balances and wealth separably subject to constraints (28) and (29)

$$\int_0^\infty (\log c + \log m + \pi \log a)e^{-\rho t}dt \tag{30}$$

The optimal conditions are

$$\lambda = \frac{1}{c} \tag{31}$$

$$\lambda\ (A + i) = \frac{1}{m} \tag{32}$$

$$\frac{\pi}{a} + \lambda\ (A - \rho) = -\dot{\lambda} \tag{33}$$

$$\dot{k} + \dot{m} = Ak + x - c - im \tag{34}$$

$$\lim_{t \to \infty} e^{-\rho t}\lambda a = 0 \tag{35}$$

Substitute equations (11), (12) and (13) into optimal conditions (32) and (34), we obtain (with a little bit manipulation)

$$\lambda = \frac{1}{c} \tag{31}$$

$$\frac{m}{c} = \left[A + \theta - \frac{\dot{m}}{m} \right]^{-1} \tag{32'}$$

$$\frac{m + k}{c} = \pi \left[\rho - A - \frac{\dot{\lambda}}{\lambda} \right]^{-1} \tag{33'}$$

$$\dot{k} = Ak - c \tag{34'}$$

where λ is the Lagrangian multiplier associated with budget constraint (28) and reflects the shadow price of wealth.

In the rest of this section, we focus on a particular solution to the dynamic systems: the balanced growth path. Along this path, all variables, i.e. per head consumption, real balance and capital, grow at constant rate (or rates). This steady state differs from last section where all per capita variables are constant in steady state in stead of their growth rates.

Let γ denote the growth rate of consumption on the balanced growth path,

$$\gamma = \frac{\dot{c}}{c} \qquad (36)$$

By condition (31),

$$\frac{\dot{\lambda}}{\lambda} = - \frac{\dot{c}}{c} = - \gamma \qquad (37)$$

Using equation (36), (37), and the constancy of growth rates of all variables on the balanced growth path, we first to show

Proposition Four: *On the balanced growth path, consumption, real balances and capital grow at the same rate,* γ.

To show that real balances grow at the rate γ, we take logarithm on both sides of equation (32′) and differentiate all terms with respect to time. By assumption, the growth rate of real balance is constant, so the right hand side of (32′) is a constant. Hence

$$\frac{\dot{m}}{m} - \frac{\dot{c}}{c} = 0$$

and

$$\frac{\dot{m}}{m} = \gamma \qquad (38)$$

Similarly, divide both sides of equation (34′) by k and re-arrange terms,

$$\frac{k}{c} = [A - \frac{\dot{k}}{k}]^{-1} \qquad (39)$$

Again by definition of balanced path, the growth rate of k is constant, so the right hand side of (39) is constant. Take log differentiation on both sides of (39),

$$\frac{\dot{k}}{k} - \frac{\dot{c}}{c} = 0$$

and

$$\frac{\dot{k}}{k} = \gamma \qquad (40)$$

Therefore all variables grow at the same constant rate γ.

Next we want to solve the balanced growth rate, γ, in terms of technology, preference, and the growth rate of money.

Substitute equations (38), (37) and (40) into (32′), (33′) and (39), respectively. We have

$$\frac{m}{c} = [A + \theta - \gamma]^{-1} \qquad (41)$$

$$\frac{k}{c} = [A - \gamma]^{-1} \qquad (42)$$

$$\frac{m + k}{c} = \pi [\rho - (A - \gamma)]^{-1} \qquad (43)$$

The balanced growth rate, γ, can be solved from equations (41), (42) and (43). In fact, (41) plus (42) is equal to (43)

$$[\theta + A - \gamma]^{-1} + [A - \gamma]^{-1} = \pi [\rho - (A - \gamma)]^{-1} \qquad (44)$$

Simple Algebra leads to

$$A - \gamma = \frac{-[(\pi+1)\theta - 2\rho] \pm \sqrt{[(\pi+1)\theta-2\rho]^2 + 4(2+\pi)\theta\rho}}{2(2 + \pi)} \qquad (45)$$

From (42), ($A - \gamma$) must be positive. So

Proposition Five: *the balanced growth rate* γ *is given by*

$$\gamma = A + \frac{[(\pi+1)\theta-2\rho] - \sqrt{[(\pi+1)\theta-2\rho]^2 + 4(2+\pi)\theta\rho}}{2(2 + \pi)} \qquad (46)$$

In passing we note that, if the capitalist spirit is not present in the model, i.e. $\pi = 0$, the balanced growth rate is directly given by equations (33) and (37)

$$\gamma = A - \rho \qquad (47)$$

which is independent of inflation, and is exactly the same as the case of real economy analysed by, for example, Barro (1988).

Proposition Six: *The higher the inflation, the higher the balanced rate of growth.*

To show this, differentiate γ with respect to θ in equation (46)

$$\frac{\partial\gamma}{\partial\theta} = \frac{1}{2(2+\pi)} \left\{ (\pi+1) - \frac{[(\pi+1)\theta-2\rho](\pi+1) + 2(2+\pi)\rho}{\sqrt{[(\pi+1)\theta-2\rho]^2 + 4(2+\pi)\theta\rho}} \right\} \tag{48}$$

which is shown to be positive with some tedious but very simple algebra in the appendix. The possible reason for this result might be following: with higher rate of money supply and higher inflation, the representative family tends to substitute real balance holdings with capital in its portfolios. That will stimulate the rate of investment and capital accumulation, which in turn raises the balanced growth rate in the economy. In the end, as the balanced growth rate goes up, the rise in the rate of money growth does not bring about a full proportional rise in inflation rate. To see this, just look at the following identity (which comes from equations (11), (12) and (13).)

$$\frac{\dot{m}}{m} = \theta - i \tag{49}$$

On the balanced growth path, equation (49) is the same as

$$\gamma = \theta - i \tag{50}$$

Differentiate equation (50) with respect to the rate of money growth,

$$\frac{di}{d\theta} = 1 - \frac{d\gamma}{d\theta} < 1 \tag{51}$$

Therefore, inflation falls short of the rate of money growth.

Here we may pose to speculate that proposition six depends on the specific preference given in this section. We have not tried our hands on other general models, and we admit the complexity of the more general excercises.

Proposition Seven: *The higher the spirit of capitalism, the higher the balanced rate of growth.*

In equation (46) differentiate γ with respect to π, and re-arrange terms,

$$\frac{d\gamma}{d\pi} = \frac{1}{2(2+\pi)^2} \{(\theta+2\rho) + \sqrt{[(\pi+1)\theta-2\rho]^2+4(2+\pi)\theta\rho} - \frac{(2+\pi)(\pi+1)\theta^2}{\sqrt{[(\pi+1)\theta-2\rho]^2+4(2+\pi)\theta\rho}}\} \tag{52}$$

again which is shown to be positive in the appendix. Thus the positive association between the spirit of capitalism and economic growth holds in both real economy and monetary economy. We have elaborated on this point theoretically and empirically in chapter one, and omit our further effort here.

In addition, it is straightforward to verify that the higher the net marginal productivity, the higher the growth rate; the higher the subjective time discount rate, the lower the growth rate.

IV. Conclusion

In a more realistic setting of capitalist economic growth, we have demonstrated that money is not superneutral both in the steady state in the sense of Sidrauski (1967a) and on the balanced growth path of Lucas (1988). The rate of money growth exerts positive effects on both steady state capital accumulation and balanced growth rate.

The limitations of our presentation are obvious. This is especially true of section II where we only treat the case of a unique perfect forsight path. In a place where the specter of multiple equilibria haunts, our narrow focus may be justified.

Appendix

Here we prove propositions six and seven:

For proposition six, we only need to show that the terms in the parentheses of equation (48) are positive. Suppose not, then

$$(\pi + 1) < \frac{[(\pi+1)\theta - 2\rho](\pi+1) + 2(2+\pi)\rho}{\sqrt{[(\pi+1)\theta-2\rho]^2 + 4(2+\pi)\theta\rho}} \qquad \text{(A1)}$$

For $\theta > 0$, the numerator on the right hand side of (A1) is positive, so both sides are positive. Take square on both sides, and cross multiply

$$(\pi+1)^2\{ [(\pi+1)\theta-2\rho]^2+4(2+\pi)\theta\rho \} < \{ [(\pi+1)\theta-2\rho](\pi+1)+2(2+\pi)\rho \}^2 \qquad \text{(A2)}$$

Expand the expression

$$(\pi+1)^2[(\pi+1)\theta-2\rho]^2+(\pi+1)^2(2+\pi)4\theta\rho < (\pi+1)^2[(\pi+1)\theta-2\rho]^2+(\pi+1)^2(2+\pi)4\theta\rho -$$

$$- 8\rho^2(\pi+1)(2+\pi) + 4\rho^2(2+\pi)^2 \qquad \text{(A3)}$$

Namely,

$$0 < - 8\rho^2(\pi+1)(2+\pi) + 4\rho^2(2+\pi)^2 \qquad \text{(A4)}$$

or

$$0 < 4\rho^2(2+\pi)(2+\pi-2\pi-2) \qquad \text{(A5)}$$

$$0 < - 4\pi\rho^2(2+\pi) \qquad \text{(A6)}$$

inequality (A6) is a contradiction for both π and ρ are positive, and the right hand side is negative. Hence the right hand side of equation (48) is positive.

Next we show that the terms in the parentheses of equation (52) is positive. Which is the same as

$$(\theta+2\rho) + \sqrt{[(\pi+1)\theta-2\rho]^2+4(2+\pi)\theta\rho} > \frac{(2+\pi)(\pi+1)\theta^2}{\sqrt{[(\pi+1)\theta-2\rho]^2+4(2+\pi)\theta\rho}} \tag{A7}$$

Multiply both sides by the positive number $\sqrt{[(\pi+1)\theta-2\rho]^2+4(2+\pi)\theta\rho}$, and simplify

$$(\theta+2\rho)\sqrt{[(\pi+1)\theta-2\rho]^2+4(2+\pi)\theta\rho} + 4\rho^2 + 4\theta\rho > (\pi + 1)\theta^2 \tag{A8}$$

But the left hand side of inequality (A8) is the same as

$$(\theta+2\rho)\sqrt{(\pi+1)^2\theta^2+4\rho^2+4\theta\rho} + 4\rho^2 + 4\theta\rho > (\theta+2\rho)(\pi+1)\theta + 4\rho^2 + 4\theta\rho >$$

$$> (\pi + 1)\theta^2 \tag{A9}$$

which is just what we need to prove.

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ESSAY THREE

Optimal Foreign Asset Accumulation and Wealth Effects

I. Introduction

In a world with flexible exchange rate, purchasing power parity and perfect capital mobility, there is no shortage of models studying the optimal monetary policy and optimal foreign asset accumulation for a small open economy; the recent examples are Obstfeld (1981) and Turnovsky (1985, 1987) among others. To derive interesting results about the effects of monetary policy on asset accumulation, deviations from the standard Sidrauski's model are crucial. Otherwise, in steady state, monetary factor does not matter as far as asset accumulation and consumption are concerned. Turnovsky (1985, 1987) introduces leisure or disutility of labor into the representative agent's preference in addition to consumption and real balances. Obstfeld (1981) allows agent's subjective rate of time preference to be endogeneously determined in a manner suggested by Uzawa (1968). In Turnovsky's model (1985), a change in the monetary growth rate causes the economy to jump to a new steady state where all real variables such as employment, output, consumption, and the real balance remain unchanged. A fiscal expansion leads to higher output, lower private consumption and unchanged foreign bonds holdings. The findings by Obstfeld (1981) are just the opposite. An increase in the monetary growth rate raises long-run foreign asset holdings and long-run consumption; and an increase in government consumption excerts no effect on long-run private consumption, it raises short-run current account surplus and long-run bond holdings.

In this paper we intend to set up a new model and offer some new answers to those old questions. In section II, the representative agent's preference is defined on consumption, real balances and *wealth*. In its form, it is a hybrid utility function of Sidrauski (1967) and Kurz (1968). We will argue that this extension is a more realistic definiton of the representative agent's preference in a capitalist world. As for the technology and world bond market, we will maintain all the assumptions in Obstfeld (1981).

In section III, we use the model to analyze the effects of different government's macroeconomic policies on private consumption, foreign asset accumulation, and current account. The results are substantially different from Obstfeld's. The excercise teaches us a lesson: unless there is a satisfactory model characterizing the real world, many conclusions derived from existing models are partial and one-sided; and policy prescriptions based on these models should be avoided.

II. Justifications and Set-Up of the Model.

Except for the preference and time discount rate, the basic assumptions about the small open economy are borrowed from Obstfeld (1981). There is only one consumption good in the world, whose price, p^*, is given in the world market. As the economy is small, it cannot influence this price. Let E denote the exchange rate, i.e. the domestic money price of foreign exchange. Then the price of consumption good in domestic market, p, and the one in world market are related as follows: $p = Ep^*$. The exchange rate is assumed to be flexible, and it adjusts to maintain the equilibrium of real money demand and real money supply at each instant. The price of consumption good in the world market is normalized to be one, $p^* = 1$.

For simplicity, the population of the small economy is assumed to be constant and equal to one; and the per capita real output, y, is fixed at each time period t. The income can be divided between consumption, c, and wealth accumulation. The wealth takes the forms of real balances, m ($m = M/p$, and M is the nominal money.), and foreign bonds, b, the latter has a fixed rate of return, r, in the world bond market. If we denote the representative agent's wealth as a, his dynamic budget constraint is

$$\dot{a} = y + rb + x - c - im \qquad (1)$$

$$a = b + m \qquad (2)$$

where x is the government transfer, and i is the expected inflation rate, and a dot above a variable is the time derivative.

The instantaneous utility of the representative agent takes the form of $U(c,m) + \pi v(a)$. π is a positive measure of wealth effects defined by Kurz (1968). As usual, U(.) and v(.) are nonnegative, strictly concave, and twice continuously differentiable.

In a closed economy, we have argued in essay one that this novel definition is a way to mathematically model the "spirit of capitalism" stated by Max Weber (1956), John Maynard Keynes (1971) and Karl Marx (1977), among others. Accoding to them, wealth accumulation is not viewed narrowly as just the means to enhance and smooth consumption in a capitalist economy, it is an end in itself. To avoid repetition, here we only offer some justifications of this modelling in the setting of world economy.

At first glance, to define utility on foreign asset and wealth may sound mercantilistic, which, we have been taught for more than two hundred years, are wrong and silly. But even today the sentiments of common people have not been deeply affected by economists' preach, and "new mercantilism" and protectionism are very popular in both developing and developed countries. Recently economists have worked out many imperfect competition models which may favor protectionist policies. In our opinion, all classical and new trade theories have made a fatal mistake: defining the representative agent's preference only in consumption. Human beings are divided into different countries, nationalism and patriotism are the foundation of the existence of nations. Citizens of nations take great pride and enjoyment (or enjoy utility in vulgar economics terminology) in both political independence and economic independence. No citizen of a nation will be happy under political domination of foreign country; similarly no citizen of a nation will be happy under economic domination of foreign country. In broad sense, mercantilism is right just because it actually reflects the sentiments of citizens. Utility is not only derived from consumption good, but also from the economic power and political power of a nation in the world --- the base of power is wealth.

The connection of power and economic wealth was clearly conceived by mercantilists. For them, "economic well-being and betterment were not defined in terms of or measured by the satisfying of revealed community consumption preferences." [Allen (1987)] In 17th century, "accumulated wealth was vital, as well as the result of, power. And wealth was intimately associated with specie. ... Wealth and specie were closely associated, however, and rising accumulation of gold and silver was taken as reflection of , even if it did not literally constitute, increasing wealth". Indeed, "money is the sinews of war". Mercantilists were not narrow minded people, and "mercantilism was oriented towards geopolitics as well as economics". "It appears most reasonable to describe the basic mercantilistic position as one of considering wealth and power each to be vital, with the two being mutually dependent and harmonious in the longrun. As concluded by Child, 'Foreign trade produces riches, riches power, power preserves our trade and religion'". [quotations from Allen (1987)]

Today's international politics is different from that of 17th century, but economic power and wealth possessed by a nation is still the foundation of political power exercised by a nation in the world. When developing countries voice the danger of multinational corporations from developed countries in influencing the host country's politics, social institutions and daily life, developed countries may have the right to deride the "complaining" as psychological over-sensitivity. Now with the rise of Japanese economic power and crusade of Japanese firms into developed countries, even Wall Street begins to worry about the Japanese financial magnitude, to say nothing about the common people's response to the Japanese in the western world.

Of course, many economists do not share the sentiments of common people. This is because, firstly, as Joan Robinson points out, when "we were being received into the fraternity of economists", the benefit of free trade "was imposed upon our young minds as a dogma" [Robinson (1966)]; then economists impose an imaginative utility function defined only on consumption upon the common people and desperately preach against the "wrong-doing" of the people. How about if the people also derive negative utility from the presence of foreign capital in their home country and positive utility from their own capital's presence in the foreign land? In this case, mercantilism may be right. It is a simple excercise to undermine many classical results in trade theory if we define a more realistic preference of the representative agent: utility derived from both consumption and wealth.

In the discussion of foreign borrowing and debt problems, some economists have explicitly introduced disutility of debt in the utility functions, e.g. Bardhan (1967) and Blanchard (1983). But this is not commonly accepted practice and is suspected by the conventional wisdom of economics. Without further justifing the components of representative agent's preference, we are going to demonstrate the working of the model.

The agent maximizes the welfare over an infinite horizon subject to budget constraints (1) and (2):

$$\text{Max} \int_0^\infty [U(c,m) + \pi v(a)]e^{-\rho t}dt \tag{3}$$

$$\text{s.t. } \dot{a} = y + rb + x - c - im$$

$$a = b + m$$

where ρ is the positive and constant time discount rate.

To simplify calculation, we assume that U(.) is separable in c and m as in Obstfeld (1981): $U(c,m) = u(c) + l(m)$. The first-order conditions are

$$\dot{c} = \frac{1}{-u''(c)} [\, \pi v'(a) + u'(c)(r - \rho)] \tag{4}$$

$$l'(m) - u'(c)(r + i) = 0 \quad (5)$$

$$\dot{m} + \dot{b} = y + rb + x - c - im \quad (6)$$

and the transversality condition

$$\lim_{t \to \infty} u'(c)ae^{-\rho t}dt = 0 \quad (7)$$

Next we consider the government budget constraint. Government revenue comes from money creation and interest earnings from the central bank's reserves, i.e. $\dot{M}/p + rR$, and R denotes the amount of reserves. Government also consumes goods, g, and makes transfer, x. So its budget is given by

$$g + x = \dot{M}/p + rR$$

$$g + x = (\dot{M}/M)m + rR \quad (8)$$

As in Obstfeld (1981), it is assumed that the central bank varies the transfer x such that the money growth rate is constant:

$$\dot{M}/M = \theta \quad (9)$$

Using (9), equation (8) can be written as

$$x = \theta m + rR - g \quad (10)$$

By definition,

$$\dot{m} = (\dot{M}/M - \dot{p}/p)m = (\theta - \dot{p}/p)m \quad (11)$$

On the perfect foresight path, the expected inflation rate is equal to the actual inflation rate:

$$\frac{\dot{p}}{p} = i \quad (12)$$

Therefore

$$\dot{m} = (\theta - i)m \qquad (13)$$

Substitute equation (13) into optimal condition (5), and equations (10) and (13) into condition (6), and rewrite condition (4),

$$\dot{c} = [\ \pi v'(a) + u'(c)(r - \rho)\ ] \frac{1}{-u''(c)} \qquad (14)$$

$$\dot{m} = [\ (r + \theta) - \frac{l'(m)}{u'(c)}\ ]m \qquad (15)$$

$$\dot{b} = y + r(b + R) - c - g \qquad (16)$$

Suppose that r is less than ρ, and $(r + \theta)$ is positive. Then the existence of steady state for these dynamic systems is guranteed. Set the time derivatives of c, m and b in equations (14), (15) and (16) equal to zero, and implicitly solve the three equations for the steady state values c^*, m^* and b^* (there may be multiple steady states.).

To study the dynamics of the systems in the neighborhood of the steady state, we linearize (14), (15) and (16) around steady state values c^*, m^* and b^*, and obtain:

$$\begin{bmatrix} \dot{c} \\ \dot{m} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} \rho - r & \dfrac{-\pi v''(a^*)}{u''(c^*)} & \dfrac{-\pi v''(a^*)}{u''(c^*)} \\ \dfrac{m^* l'(m^*) u''(c^*)}{u'(c^*)^2} & \dfrac{-m^* l''(m^*)}{u'(c^*)} & 0 \\ -1 & 0 & r \end{bmatrix} \begin{bmatrix} c - c^* \\ m - m^* \\ b - b^* \end{bmatrix} \qquad (17)$$

Denote the 3x3 matrix as T. To discuss the stability of the equilibrium point, we first note that the trace of matrix T is positive:

$$\text{Trace of } T = \rho - \frac{m^* l''(m^*)}{u'(c^*)} > 0 \qquad (18)$$

For the trace is the sum of three characteristic roots of the dynamic systems, positive trace implies that at least one of the roots is positive, and the equilibrium is not stable.

The determinant of the matrix T, which is also the product of the three characteristic roots, is

$$\Delta = -(\rho - r) r \frac{l''(m^*) m^*}{u'(c^*)} + \frac{\pi v''(a^*) l''(m^*) m^*}{u''(c^*) u'(c^*)} + \frac{\pi v''(a^*) l'(m^*) u''(c^*) m^* r}{u''(c^*)(u'(c^*))^2} \qquad (19)$$

where the first term on the right hand side of the expression is positive, and the second and the third ones are negative. As in original Kurz (1968) model, there may be multiple equilibria in this model. So, for general preferences, we can not say that the determinant, Δ, always takes negative or positive sign. But if $u(c) = \log c$, $l(m) = \log m$, and $v(a) = \log a$, we can easily show that there is only one equilibrium for the dynamic systems, and the equilibrium is a saddle point. Because in this case, the determinant is

$$\Delta = \frac{\pi c^{*2}}{a^{*2} m^*} (r b^* - c^*) \qquad (20)$$

To get this result we have used that, in steady state,

$$\rho - r = \frac{\pi v'(a^*)}{u'(c^*)} \quad .$$

If output is larger than government spending (a reasonable fact), then $(rb^* - c^*) = g - y - rR < 0$ in steady state. So $\Delta < 0$. That is to say, there is either three negative characteristic roots or one; but from (18), there is at least one positive root; hence the systems have only one negative root and there exists a unique perfect foresight path converging to the equilibrium point. Our later analysis will focus on this unique path (which is the same as assuming that Δ is negative).

Before we analyze the impact of different government policies, we note that the more importance the representative agent attributes to wealth accumulation, the higher consumption he enjoys in the steady state. To show this, we first totally differentiate steady state equations of (14), (15) and (16) and obtain:

$$T \begin{bmatrix} dc \\ dm \\ db \end{bmatrix} = \begin{bmatrix} \frac{v'(a^*)}{u''(c^*)} d\pi \\ -m^* d\theta \\ dg \end{bmatrix} \qquad (21)$$

From above, it is understood that the determinant of matrix T, Δ, is negative. Simple calculations lead to

$$\frac{dc}{d\pi} = \frac{-rl''(m^*)m^*v'(a^*)}{\Delta\, u''(c^*)u'(c^*)} > 0 \qquad (22)$$

$$\frac{dm}{d\pi} = \frac{-l'(m^*)m^* v'(a^*)u''(c^*)}{\Delta\, u''(c^*)(u'(c^*))^2} > 0 \tag{23}$$

$$\frac{db}{d\pi} = \frac{-l''(m^*)m^* v'(a^*)}{\Delta u''(c^*)u'(c^*)} > 0 \tag{24}$$

This fact may highlight the importance of wealth effects in understanding the differences in foreign asset accumulation and consumption among nations.

III. The Effects of Macroeconomic Policies

Using (21), we can study how monetary growth rate and government spending affect consumption and asset accumulation in the long-run. The short-run dynamics can be made clear by comparing the changes of relevant variables in different steady states. As usual, before any macroeconomic disturbance happens, the economy is assumed to be in steady state; the disturbance is expected to continue for ever.

(A) *An Increase in Monetary Growth rate*

A rise in growth rate of money leads to higher inflation in the steady state, and which in turn raises the cost of money holdings. The representative agent lowers the holdings of real balances and purchases more foreign bonds immediately following the permanent rise of the inflation. And the newly purchased assets bring about more interest income and higher consumption. As for the real balances in the new steady state, the cost effect --- higher inflation --- tends to reduces them, but the income effect --- more asset returns --- tends to increase them. So the net effect is ambiguous. Mathematically we have:

$$\frac{db}{d\theta} = - \frac{m^* v''(a^*)\pi}{\Delta u''(c^*)} > 0 \qquad (25)$$

$$\frac{dc}{d\theta} = \frac{1}{\Delta} \left[\frac{\pi l''(m^*) m^* v''(a^*)}{u''(c^*) u'(c^*)} - \frac{\pi v''(a^*) m^* r}{u''(c^*)} \right] > 0 \qquad (26)$$

$$\frac{dm}{d\theta} = \frac{(r-\rho) r m^*}{\Delta} + \frac{\pi v''(a^*) m^*}{\Delta u''(c^*)} \qquad (27)$$

In the short run, exchange rate depreciation overshoots its long run equlilibrium level as high growth rate of money leads to excess supply of real money. With time going, higher income raises money demand and appreciates the exchange rate. The change of exchange rate in new equilibrium is just given by the the rate of monetary growth rate.

From above, the effects exerted by inflation on consumption, asset accumulation and real balance holdings are qualitatively the same as in Obstfeld (1981), even though the optimal conditions are different.

(B) *Increase in Government Spending*

The surprising result in Obstfeld (1981) is that an increase in government consumption only leads to higher asset accumulation in the new steady state, and does not change long run private consumption and real money holdings. That result owes its existence to the special maximizing model in Obstfeld's paper. In our model, higher government spending always reduces long run private consumption and real balances. Furthermore foreign asset holdings may be reduced as a result of higher government spending. The short run implication is more Keynesian: higher government spending may lead to lower private saving and current account deficit. To show this, we set $d\pi$ and $d\theta$ equal to zero on the right hand side of (21) and solve the responses of c, m and b to the change in g:

$$\frac{dc}{dg} = - \frac{\pi v''(a^*)l''(m^*)m^*}{\Delta\, u''(c^*)u'(c^*)} < 0 \tag{28}$$

$$\frac{db}{dg} = \frac{1}{\Delta} \left[(r-\rho) \frac{l''(m^*)m^*}{u'(c^*)} + \frac{\pi v''(a^*)l'(m^*)m^*}{u'(c^*)^2} \right] \tag{29}$$

As for the real balances, we note that, in steady state, $(r + \theta) = [l'(m^*)/u'(c^*)]$. Differentiate both sides with respect to government consumption,

$$\underset{(-)}{\frac{l'(m^*)u''(c^*)}{u'(c^*)^2}} \frac{dc}{dg} = \underset{(-)}{\frac{l''(c^*)}{u'(c^*)}} \frac{dm}{dg} \tag{30}$$

namely, consumption and real balances moves in the same direction. From (28), dc/dg is negative. So is dm/dg.

(C) *Foreign Exchange Intervention*

Another significant difference between our model and Obstfeld's is the result

about foreign exchange intervention. In Obstfeld's model, the distribution of the ownership of foreign asset between the public and the central bank is irrelevant, and the equilibrium private consumption and real balances depend only on the aggregate amount of foreign asset owned by the small economy. When the central bank intervenes in the foreign exchange market by purchasing foreign bonds from the private sector with domestic money, the transaction does not affect the aggregate foreign assets in the economy, hence does not change private consumption and real money holdings.

In our model, as foreign bonds are also valued directly in the utility function, the symmmetry of foreign bonds and central bank's reserves in Obstfeld's model disappears. Shortly after the intervention of the central bank, the reduction in private bonds results in higher marginal utility of bonds. People will cut their consumption and save more in the short run, and acquire more foreign assets. In new equilibrium, as total assets (the sum of private assets and the central bank's reserves) have gone up, private consumption and real balances should also go up.

As for the exchange rate, it need not depreciate exactly proportionally to the increase in the nominal money supply at new equilibrium. In fact, its short run level again overshoots its long run one.

IV. Conclusion

In this short paper, we have shown that, in our opinion, a more realistic intertemporal optimization model has quite different predictions from the existing maximizing models, in particular, Obstfeld's model. In broad scope, our model can tell more richer stories about the connection between international politics and economic wealth, the connection between cultural differences among nations and their foreign asset accumulation.

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ESSAY FOUR

Optimal Taxation of Foreign Capital Income in A Growing Economy with Different Governments

I. Introduction

Foreign direct investment and taxation have been comprehensively studied in static settings by many people, e.g., MacDougall [1960], Hamada [1966], Jones [1967], Feldstein and Hartman [1979]. The dynamic counterparts of these models are relatively few. In this short essay, by integrating the dynamics of both domestic capital accumulation and foreign direct investment into an intertemporal growth model, we will make some effort to study the optimal taxation of foreign capital income from the viewpoint of the host country. Throughout this paper, it is assumed that there are two countries: home country and foreign country; only foreign country undertakes direct investment in home country, and home country levies tax on foreign capital earnings.

In section II, with some *ad hoc* assumptions on home country's saving rate and tax rate, we are going to set up the basic dynamic model describing domestic capital accumulation and foreign direct investment in the home country. The stability of the equilibrium and some comparative statics will also be studied. Section III introduces two types of governments: open door government and close door government(to be defined later). In the framework of optimal choice on the part of governments, it is shown that the optimal aggregate capital (i.e. the sum of domestic and foreign capital) is still determined by the modified golden rule as in Cass [1965], and the optimal tax rate on foreign capital income is the same for both kinds of governments. While the optimal saving rate under close door government is higher than under open door government, the corresponding equilibrium per capita consumption may be higher or lower than under open door government. The moral here is not against close door government. In fact, close door government may do better than the open door government in maintaining higher consumption for its country if the population growth rate is low, and the difference in the technological efficiency between foreign capital and domestic capital is small.

II. A Basic Model

As a convention, the home country and foreign country produce only one good which can either be consumed or invested. At any given time, the home country's total capital stock, K, is divided into domestically owned capital, K_d, and foreign owned capital, δK_f. Here the parameter δ takes values which are larger than or equal to one; that is to say, foreign capital (or multinational corporations' investment) is more efficient than, or as equally efficient as, its home country's domestic capital. This can be justified if we take into account of the technological advantage possessed by multinational corporations (see Caves [1982]). The parameter δ is definitely larger than one if we consider developed countries' foreign investment in developing countries. As for another input of production, the home country's labor force at any time is L, which is growing at exogenously given rate n. Write the capital stock in per capita term,

$$k = k_d + \delta k_f \qquad \delta \geq 1 \qquad (1)$$

where $k = K/L$, $k_d = K_d/L$, and $k_f = K_f/L$.

The production function, $f(k)$, is the standard neoclassical one with continuously differentiable first and second order derivatives, and $f'(k) > 0$, $f''(k) < 0$, $f(0) = 0$, $f'(0) = \infty$.

If the home country's tax rate on foreign capital income is τ, $(1-\tau)f'(k)k_f$ would be repatriated to foreign country by foreign investors (or retained by foreign investors). Thus in the home country the per capita gross national product, y, is

$$y = f(k) - (1 - \tau)f'(k)k_f = f(k) + (\tau - 1)f'(k)k_f \qquad (2)$$

Suppose that home country's saving rate is s, domestic capital accumulation by home country is described by following dynamics:

$$\dot{k}_d = sy - nk_d = s[f(k)+(\tau-1)f'(k)k_f] - nk_d \qquad (3)$$

Foreign investors maximize expected profits and adjust their investment according to the return differential between the world interest rate and host country's (here home country's) capital returns:

$$\dot{k}_f = \beta[(1-\tau)f'(k) - r^*] \qquad (4)$$

where the constant β is larger than zero, $(1-\tau)f'(k)$ is net return (after tax return) on foreign capital, and r^* is the alternative net return on foreign capital in the world capital market.

Giving the parameters s and τ, equations (3) and (4) constitute a pair of dynamic systems in k_d and k_f. In equilibrium, we have

$$s\bar{y} - n\bar{k}_d = 0 \qquad (5)$$

$$(1-\tau)f'(\bar{k}) - r^* = 0 \qquad (6)$$

where a bar above a variable denotes the equilibrium value of the variable. To say the stability of the equilibrium, we linearize equations (3) and (4) around the equilibrium values of k_d and k_f:

$$\begin{bmatrix} \dot{k}_d \\ \dot{k}_f \end{bmatrix} = \begin{bmatrix} s\partial y/\partial k_d - n & s\partial y/\partial k_f \\ \beta(1-\tau)f''(\bar{k}) & \beta(1-\tau)f''(\bar{k})\delta \end{bmatrix} \begin{bmatrix} k_d-\bar{k}_d \\ k_f-\bar{k}_f \end{bmatrix} \qquad (7)$$

It is easy to see that both $\partial y/\partial k_d$ and $\partial y/\partial k_f$ in (7) are positive, by differentiating (2),

$$\partial y/\partial k_d = f'(k) + (\tau-1)f''(k)k_f > 0$$

$$\partial y/\partial k_f = f'(k)(\delta-1+\tau) + (\tau-1)f''(k)\delta k_f^2 > 0.$$

which is simply to say, increase in capital stock always brings about more national income. Knowing the signs of $\partial y/\partial k_d$ and $\partial y/\partial k_f$, we are ready to analyze the properties of the equilibrium. Firstly, we will show that there exists a stable equilibrium if the growth rate of labor force is larger than the product of saving rate s and marginal contribution of domestic capital to national product $\partial y/\partial k_d$, i.e.

$$n > s\partial y/\partial k_d \qquad (8)$$

Suppose that inequality (8) holds, the trace of the matrix in (7) is negative as $[s\partial y/\partial k_d - n] + [\beta(1-\tau)f''(k)\delta] < 0$; and the determinant of the matrix is positive as $[s\partial y/\partial k_d - n][\beta(1-\tau)f''(k)\delta] - s\partial y/\delta k_f\beta(1-\tau)f''(k) > 0$. So we obtain two negative eigenvalues for the linearized dynamic systems, and the systems are stable. The phase diagram is in figure 1 below.

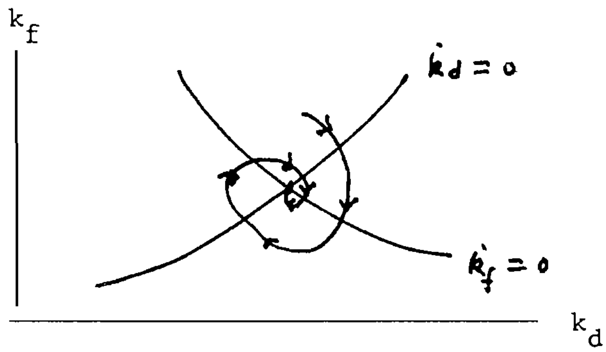

figure 1

The curve for equation $dk_f/dt = 0$ is downward slopping, its slop is given by $(-1/\delta)$. With the assumption that $n > s\partial y/\partial k_d$, the curve for $dk_d/dt = 0$ is upward slopping, and its slope is $(n - s\partial y/\partial k_d)/s\partial y/\partial k_f$. Below $dk_f/dt = 0$, for any given amount of k_f, k_d is relatively small, and $(1-\tau)f'(k)$ is higher than r^*, so k_f will increase. The converse argument goes for capital stock above the line of $dk_f/dt = 0$. Similarly, below $dk_d/dt = 0$; for any given k_f, k_d is relatively small. To maintain equilibrium, k_d has to increase. And vice versa. Therefore the dynamic path spirals inward clockwise in (k_d, k_f) space of figure 1.

In this model, an increase in tax rate on foreign capital income will always reduce foreign direct investment in the home country, but its effect on home country's capital stock is ambiguous. This can be easily seen from figure 2. As a result of higher tax τ, $dk_f/dt = 0$ curve shifts inward because, for any initial equilibrium values of k_d and k_f, $[(1-\tau)f'(k) - r^*]$ is less than zero; foreign investors will reduce their investment in host country. For $dk_d/dt = 0$ curve, higher tax rate raises more tax revenue and leads to higher national income initially, the curve shifts downward accordingly. In new equilibrium, k_f is reduced, and k_d may go up or down. In the diagram, we depict a situation where both k_f and k_d decrease as a result of higher tax on foreign capital.

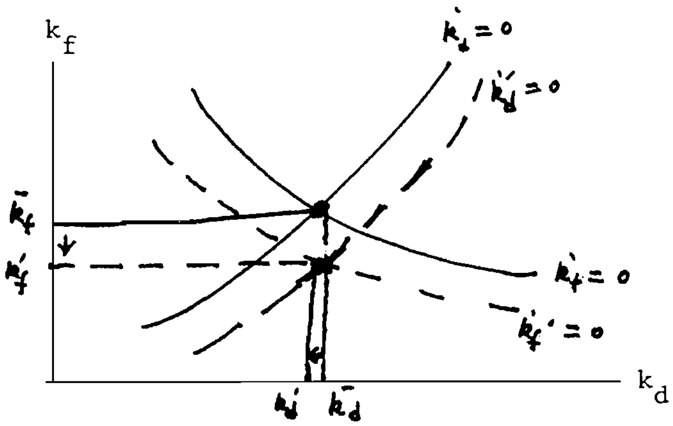

figure 2: tax increase

The economic explanation for above results is simple: High rate of capital tax reduces foreign capital stock. The total tax revenue may increase or decrease depending on the elasticity of foreign investment with respect to tax (a case of familiar Laffer curve of taxation). If high tax raises more total tax revenue, k_d is expected to go up; if high tax lowers tax revenue, k_d will go down. This naturally leads to the optimal taxation by the home country's government, which we will deal with in next section.

Another interesting case is the effects of change of saving rate on foreign and domestic capital. Suppose that saving rate s is up. The curve $dk_f/dt = 0$ is unchanged, but $dk_d/dt = 0$ shifts downward. In the new equilibrium, k_d is higher, and k_f is lower. See figure 3.

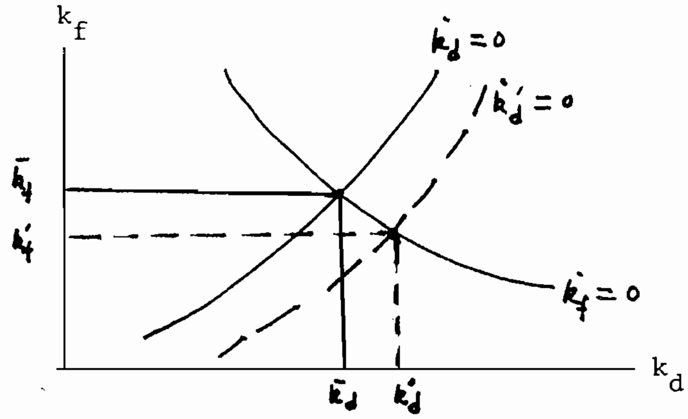

figure 3: saving rate increase

This approach to study foreign direct investment and domestic capital accumulation in the home country is much in the spirit of Findlay [1978]. In reality, many countries, especially many developing countries, do make delibrate choices about the tax on foreign capital income and saving for domestic investment. Thus a more elaborate model is needed to describe the behaviors of governments in many countries. That will be our next task.

III. Optimal Choices of Taxation and Saving by the Governments

Following Bardhan (1967), we define governments' preferences on per capita consumption and foreign capital stock in home country. The preferrences are separable in consumption and capital:

$$U(c, k_f) = u(c) + v(k_f) \qquad (9)$$

An open door government is defined to be one deriving nonnegative utility from foreign capital's presence in home country, that is to say, $v'(k_f) \geq 0$

for open door government. On the contrary, a close door government dislikes foreign capital and derives disutility from foreign capital, i.e., $v'(k_f) < 0$. Of course, for both governments, $u'(c) > 0$. In practice, we should have no difficulty in identifying these two kinds of governments.

Let both kinds of governments maximize their long-run welfare by choosing tax rate on foreign capital income and domestic saving rate subject to the two dynamic constraints: equations (3) and (4). Write out the optimal problem fully:

$$\text{Max} \int_0^\infty [u(c) + v(k_f)]\, e^{-\rho t} dt \qquad (10)$$

s.t.

$$c = (i - s)y = (1-s)[f(k) + (\tau-1)f'(k)k_f] \qquad (11)$$

$$\dot{k}_d = s[f(k) + (\tau-1)f'(k)k_f] - nk_d \qquad (3)$$

$$\dot{k}_f = \beta[(1-\tau)f'(k) - r^*] \qquad (4)$$

where ρ is the time discount rate, and $0 < \rho < 1$.

To avoid boundary solution $s = 1$, and to have sufficiency of the first order conditions, the following assumptions are made: $u'(0) = \infty$, $u''(c) < 0$ and $v''(k_f) < 0$. By substituting expression (11) into the objective function, we have the Hamiltonian function H,

$$H = e^{-\rho t}\{u[(1-s)(f(k) + (\tau-1)f'(k)k_f)] + v(k_f) + \lambda_1[s(f(k) + (\tau-1)f'(k)k_f) - nk_d] + \lambda_2[\beta(1-\tau)f'(k) - r^*]\} \qquad (12)$$

The first order conditions are

$$\partial H/\partial s = e^{-\rho t}[-u'(c)y + \lambda_1 y] = 0 \qquad (13)$$

$$\partial H/\partial \tau = e^{-\rho t}[u'(c)(1-s)f'(k)k_f + \lambda_1 sf'(k)k_f - \lambda_2 \beta f'(k)] = 0 \qquad (14)$$

$$\partial H/\partial k_d = - d(e^{-\rho t}\lambda_1)/dt \qquad (15)$$

$$\partial H/\partial k_f = -d(e^{-\rho t}\lambda_2)/dt \qquad (16)$$

and dynamic constraints (3) and (4), plus transversality conditions.

By equation (13) and (14), we get the shadow prices of domestic capital and foreign capital:

$$\lambda_1 = u'(c) \qquad (17)$$

$$\lambda_2 = u'(c)k_f/\beta \qquad (18)$$

With (17) and (18), optimal conditions (15) and (17) can be written as

$$\frac{u''(c)}{u'(c)}\frac{dc}{dt} = n + \rho - f'(k) \qquad (19)$$

$$d\lambda_2/dt = -[u'(c)(\delta-1+\tau)f'(k) + v'(k_f)] + \rho u'(c)k_f/\beta \qquad (20)$$

In steady state, we obtain

$$\beta[(1-\tau)f'(k) - r^*] = 0 \qquad (21)$$

$$s[f(k) + (\tau-1)f'(k)k_f] - nk_d=0 \qquad (22)$$

$$n + \rho - f'(k) = 0 \qquad (23)$$

$$-[(\delta - 1 + \tau)f'(k)u'(c) + v'(k_f)] + u'(c)k_f\rho/\beta = 0 \qquad (24)$$

Given specific forms of production function and utility function, we can solve these equations and find out steady state levels of τ, s, k_d, and k_f. Without going to special technology and preference, following interesting results can be easily derived.

Result 1: Optimal aggregate capital stock is the same under both kinds of governments, and its level is determined by the modified golden rule.

This is just a statement of equation (23), which is exactly the modified golden rule as in Cass (1965), and , in particular, independent of government prefrences. This result may be a little bit surprising for our model is so different from the original Cass model.

Result 2: The optimal tax rate on foreign capital income is the same for both kinds of governments:

$$\tau = \frac{n + \rho - r^*}{n + \rho} \qquad (25).$$

To see this, we substitute the modified golden rule into equation (21). The result follows immediatly. From intuition, the close door government should tax more on foreign capital income than the open door government. That is possible in the present model only if the close door government has higher discount rate than the open door government. Another way to obtain different tax rates for different governments is to follow Kurz (1968) and introduce both domestic capital and foreign capital into utility functions --- so called wealth effects. We will discuss this possible extension at the end of this paper.

From (25), we note that the higher the time discount rate, the higher the tax rate:

$$\partial\tau/\partial\rho = r^{*}/(n + \rho)^{2} > 0 \qquad (26).$$

Similarly, the faster the population growth, the more the governments need to provide people with the same consumption and capital goods. So governments tax more to meet the growing demand:

$$\partial\tau/\partial n = r^{*}/(n + \rho)^{2} > 0 \qquad (27)$$

But obviously enough, the higher the alternative return on foreign capital, the less the tax rate:

$$\partial\tau/\partial r^{*} = - 1/(n + \rho) < 0 \qquad (28).$$

Result 3: At optimum, the marginal benefit from foreign capital is equal to the marginal cost.

This comes from equation (24),

$$u'(c)[(\delta-1)f'(k)] + u'(c)\tau f'(k) + v'(k_f) = u'(c)k_f\rho/\beta$$

On the left hand side of above equation, the first term is the gain in technological efficiency contributed by foreign capital (recall $\delta \geq 1$), the second term is the tax revenue from foreign capital, and the third term is the direct utility gain (loss) if $v'(k_f) > 0$ ($v'(k_f) < 0$). So the left hand side is the net marginal benefit resulting from foreign capital. The right hand side, $u'(c)k_f\rho/\beta$, is just λ_2, the imputed price of foreign capital.

Result 4: Foreign capital per capita under open door government is higher than under close door government.

This is what we expect. To show this result, we denote foreign capital per capita under open door and close door governments as k_f^{o} and k_f^{c}, respectively, consumption per capita as c^o and c^c, and saving rates as s^o and s^c. We write equation (24) for open door government and close door government separately (note $f'(k) = n + \rho$) in steady state):

$$u'(c^o)[(\delta - 1 + \tau)(n + \rho) - k_f{}^o\rho/\beta] = -v'(k_f{}^o)$$

and

$$u'(c^c)[(\delta - 1 + \tau)(n + \rho) - k_f{}^c\rho/\beta] = -v'(k_f^c)$$

For open door government, $v'(k_f{}^c) > 0$. That requires

$$[(\delta - 1 + \tau)(n + \rho) - k_f{}^o\rho/\beta] < 0.$$

For close door government, $v'(k_f{}^c) < 0$. That requires

$$[(\delta - 1 + \tau)(n + \rho) - k_f{}^c\rho/\beta] > 0.$$

Therefore,

$$k_f^o > (\delta - 1 + \tau)(n + \rho)\beta/\rho > k_f{}^c \qquad (29)$$

which is what we want to demonstrate.

Result 5: If $(r^*/\delta) > n$, consumption per capita under open door government is lower than under close door government; if $(r^*/\delta) < n$, consumption per capita under open door government is higher than under close door government. They are equal if $(r^*/\delta) = n$.

Proof: In steady state,

$$c= (1 - s)y = y - nk_d = f(k) + (\tau - 1) f'(k)k_f - nk_d \qquad (30)$$

Substitute $f'(k) = n + \rho$, $\tau = (n + \rho - r^*)/(n + \rho)$, and $k_f = (k - k_d)/\delta$ into equation (30), we get

$$c = f(k) - (r^*/\delta)k - (n - r^*/\delta)k_d \qquad (31)$$

Since k and f(k) are the same for both governments in steady state,

$$c^o = f(k) - (r^*/\delta)k - (n - r^*/\delta)k_d{}^o$$

and

$$c^c = f(k) - (r^*/\delta)k - (n - r^*)k_d^c$$

So

$$c^c - c^o = (n - r^*/\delta)(k_d^o - k_d^c).$$

We know $(k_d^o - k_d^c)$ is negative as $k_f^o > k_f^c$ (from equation (29)) and k is the same for both governments in steady state. Hence

$$c^c > c^o \text{ if } n < r^*/\delta;$$

$$c^c < c^o \text{ if } n > r^*/\delta;$$

$$c^c = c^o \text{ if } n = r^*/\delta.$$

That is to say, for given world capital return r^* and home country's population growth rate n, if foreign capital is much more technologically efficient than domestic capital in home country, the open door government provides more consumption than the close door government. This is because larger foreign capital under open door government brings about higher tax revenue and greater efficiency gain. It is of some interest to note a special case when $\delta = 1$, i.e. foreign capital and domestic capital have the same efficiency level. In this case, the comparison between domestic population growth rate and world capital return provides a simple criterion to assess the welfare enjoyed by the people. If population growth rate is larger than world interest rate, open door government will perform better by the criterion of consumption per capita. If population growth rate is smaller than the world interest rate, close door government will be favored by the people. This line of arguments by comparing population growth with capital return definitely reminds us of assessing dynamic efficiency in a close economy.

Result 6: Steady state saving rate under open door government is lower than *under close door government.*

By substituting the same tax rate τ and modified golden rule into (22),

$$s\,[f(k) - r^* k_f] - nk_d = 0$$

Namely,

$$s = \frac{nk_d}{f(k) - r^* k_f}$$

For the two governments, the saving rates are

$$s^c = \frac{nk_d^c}{f(k) - r^* k_f^c} \qquad s^o = \frac{nk_d^o}{f(k) - r^* k_f^o}$$

Therefore

$$s^c - s^o = \frac{n(k_f^o - k_f^c)[\delta f(k) - (1-\tau)\, f'(k)k]}{(f(k) - r^* k_f^c)(f(k) - r^* k_f^o)} \tag{32}$$

which is positive since $(k_f^o - k_f^c) > 0$, and $[\delta f(k) - (1 - \tau)f'(k)k) > f(k) - f'(k)k$ = marginal returns on labor input > 0. (Recall $\delta \geq 1$)

IV. A Possible Extension of the Optimal Choice Model

A straight forward extension of the model in last section is to define governments' preferences on both foreign capital and domestic capital --- a hybrid preference of Bardhan (1967) and Kurz (1968). The justification may be borrowed from essay three. For example, the governments' utility function may be separable in consumption, domestic capital and foreign capital:

$$U(c,\ k_d,\ k_f) = u(c) + v(k_f) + w(k_d).$$

Of course, for both governments, $w'(k_d) > 0$.

While this approach is interesting, we can hardly get any simple result from it owing to the existence of multiple equilibria. It will be true that optimal choices of capital stock and tax rate are different for different governments. It may be possible to find out a specific example as in the first three essays of this thesis to illustrate the differences in optimal choices between open door government and close door government. But we find that, even relying on a conventional approach in international economics, the results in section three are interesting and surprising.

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Essay Five

A Model of Socialist Economic Growth and Political Investment Cycles

I. Introduction

In traditional optimal growth models for a centrally planned economy, e. g. Cass (1965) and Koopmans (1965), social planners maximize an intertemporal social welfare function defined on per capita consumption, subject to the dynamic constraint of capital accumulation. The results from these models have become the folklore of modern economics: there exists a unique optimum path converging asymptotically to the unique equilibrium; the optimum capital stock in long-run is determined by the famous modified golden rule, i.e., marginal productivity of capital is equal to the natural growth rate of population plus the time discount rate of social planners. In these models, social planners all act in the interest of the society, they do not have any objective function other than the welfare of the people, and their personal images only reflect in the time discount rate. Cass (1965) provides a typical picture of the central planners: "The central planning authority's concept of social welfare is related to the ability of the economy to provide consumption goods over time. In particular, welfare at any point of time is measured by a utility index of current consumption per capita .... The central planning authority recognizes that consumption tomorrow is not the same thing as consumption today. For this reason, it takes the political pragmatic view that its planning obligation is stronger to present and near future generation than to far removed future generations. This view is implemented in practice by discounting future welfare at a positive rate".

This approach to socialist growth suffers from serious limitations in its applicability to socialist reality. First of all, traditional optimal growth models are based on insufficient understanding or misunderstanding of the nature of the social planners. This point has been emphasized by Janos Kornai in his various studies [Kornai (1982), (1986), (1988)]. With both political power and economic resources in their control, social planners are not constrained or directed to choose the optimum feasible growth path with respect to the only criterion: maximizing social welfare. "Such an unworldly bureaucracy never existed in the past and will never exist in the future. Political bureaucracies have inner conflicts reflecting the divisions of society and the diverse pressures of various social groups. They pursue their own individual and group interests, including the interests of the particular specialized agency to which they belong. Power creates an irresistable temptation to make use of it. A bureaucrat must be interventionist because that is his role in the society; it is dictated by his situation" [Kornai (1986) pp. 1726-27]. In practice, it is manifested that social planners are often investment growth rate maximizers [Grosfeld (1987)], and their personal interests are more connected to persistent expansion of their organizations than to the increase in people's consumption. In their investment strategies, "the highest priority is placed on industry, and within industry on heavy industry, and within heavy industry on the part related to the military. ... Among the neglected, non-priority sectors, one typically finds agriculture, and even more so all the branches of the tertiary or service sector, such as transport and telecommunication, housing, other communal services, domestic trade, and health." This diversion of resources from consumption to investment takes place not provisionally for two or three years, but for decades, for twenty, thirty, or forty years. [Kornai (1988) p.244].

In this paper we intend to offer a simple alternative model to capture certain essential aspects of socialist economic growth. The most important feature of the model is defining the social planners' objective function in both per capita consumption and *per capita capital stock*. The justification and the set-up of the model are done in section II. In its abstract form, this modelling was presented by Mordecai Kurz in 1968. That paper has long been neglected in economics profession partly because, we guess, Kurz has not offered any justification for the so-called wealth effects model. We can find a realistic setting for Kurz model in socialist economic growth.

In section III, we demonstrate that this simple model provides deep microfoundation for the understanding of "investment hunger" and "expansion-drive" studied by Kornai (1980). Section IV derives a theory of political investment cycle from our basic model. It is shown that the investment rates (or accumulation rates in the terminology of socialist economics) are related to different political regimes in socialist countries. In section V, we will look at the empirical data on investment rates from 1952 to 1985 in China. The variations on investment rates throughout those years can be substantially explained by the change of political power at the top level of government: an evidence supporting the theory of political investment cycle.

II. The Model and Its Justification

We define the instantaneous utility function of social planners at a given time t as the summation of two parts: $u(c_t) + \pi v(k_t)$. c_t is consumption per capita and k_t is capital stock per capita at time t. Social planners derive positive utility from both consumption enjoyed by the people and capital stock owned by the state, so the first order derivatives of functions u(.) and v(.) are positive. The Greek letter π is a positive constant which measures the importance of capital accumulation from the point of view of the social planners. In later sections, we will allow π to take different values, and its effects on capital accumulation, investment rate and consumption will be studied. Furthermore, for technical reason, we assume that the second order derivatives of u(.) and v(.) are negative, and

$$\lim_{c_t \to 0} u'(c_t) = \infty$$

which guarantee the sufficiency of the first order conditions for optimization, and exclude the corner solution of zero consumption.

In modelling the social planners' preference, we hold that social planners do care about people's consumption, and the improvement in the living standard of the people seems to justify their manipulation of political power and economic resources in a socialist economy. But it is more important to note that social planners' own interest lies more directly in the expansion of the firm and public organization of which they are in charge. Social planners are not just a group of persons in the central planning bureau, they consist of all persons involved in the plan-making, from the managers at the bottom to the ministers at the top. According to Janos Kornai (1981, 1986, 1988), the first and the most important motivation for accelerated capital accumulation is social planners' identification with their own jobs. An expansion of the firm or organization under their direct control is always a source of satisfaction. The second motivation is prestige. "A larger organization brings more prestige, and also more power" [Kornai (1981)]. "The simple urge to exert power over people, and to exercise some discretion over the allocation of physical resources can also make managers strive for higher investment levels for their firm" [Kornai (1988) p.264]. So "it is important to note that investment hunger and expansion drive characterize not only the behavior of the top manager and his subordinates in a particular firm, but also *the attitude of economic agents at all levels of the bureaucratic hierarchy in a socialist system.... the general ideology of the system favors expansion, and no claimant's application for funds is ever regarded as unreasonable or unethical by anyone in the hierarchy. On the contrary, everyone considers such a request as the natural and normal behavior within the system.*" [Kornai (1988) pp.264-265, italic added.]

This approach to socialist planners is essentially the same as the one used in the analysis of bureaucrats in western democracies. For example, Orzechowski (1977) defines the bureaucrat's utility function directly on the output produced by his bureau and the capital stock or labor in his control. And the striving for more budget revenue in western public sectors resembles the investment hunger and expansion drive in socialist economy.

With these discussions, we might call π appeared in social planners' utility function as the measure of the degree of expansion drive. Large value of π means the social planners are highly expansion-oriented; and zero value of π brings us back to Ramsay-Cass-Koopman's mathematical utopia of socialism. (Phelps (1961) presents the golden rule of capital accumulation in "a fable for growthmen". In reality, where we can find the King of the Kingdom of Solovia?)

To proceed with our model, we assume that the social planners maximize following intertemporal utility with discounting: (for notation convenience, we omit the time subscript t of all variables from now on.)

$$\int_0^{\infty} [u(c) + \pi v(k)] e^{-\rho t} dt \qquad \rho > 0 . \qquad (1)$$

where ρ is the social planners' subjective rate of discount.

There is a standard neoclassical production function f(k) in the economy with $f'(k) > 0$, and $f''(k) < 0$. Capital is subject to depreciation rate δ. The population growth rate is exogenously given as n. So capital accumulation in per capita term follows the dynamic equation:

$$\dot{k} = f(k) - c - (n + \delta)k \qquad (2)$$

Social planners maximize (1) subject to the dynamic constraint (2). The current value Hamiltonian H is defined by

$$H = u(c) + \pi v(k) + \lambda[f(k) - c - (n + \delta)k] \qquad (3)$$

The optimal pathes for consumption and investment are

$$\dot{c} = \frac{1}{-u''(c)} [\pi v'(k) + u'(c)(f'(k) - n - \delta - \rho)] \tag{4}$$

$$\dot{k} = f(k) - c - (n + \delta)k \tag{5}$$

$$\lim_{t \to \infty} e^{-\rho t}\lambda k = 0 \tag{6}$$

We are going to make detailed analysis of above dynamics in next section.

III. The Dynamics of the Model and the Properties of the Equilibrium

As noted by Kurz (1968), the dynamic systems (4) and (5) may easily result in multiple equilibria, and some equilibrium points are saddle point stable, some are sources. To see this, denote the equilibrium values of consumption and capital as c^* and k^*, and linearize the systems around these values:

$$\begin{bmatrix} \dot{c} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} (n+\delta+\rho) - f'(k^*) & \dfrac{\pi v''(k^*) + u'(c^*)f''(k^*)}{-u''(c^*)} \\ -1 & f'(k^*) - n - \delta \end{bmatrix} \begin{bmatrix} c - c^* \\ k - k^* \end{bmatrix} \tag{7}$$

Denote the 2x2 matrix as M. The trace of the matrix

$$\text{Trace of } M = \rho > 0\ . \tag{8}$$

As the trace is the sum of the two characteristic roots of the systems, at least one of the roots is positive. Therefore we can not have stable equilibrium point.

Next the determinant of the matrix is

$$\Delta = [n+\delta+\rho-f'(k^*)][f'(k^*)-n-\delta] - \frac{\pi v''(k^*)+u'(c^*)f''(k^*)}{u''(c^*)} \qquad (9)$$

The second term on the right hand side of (9) is negative; the first term is positive or negative depending on the capital stock is smaller or larger than the golden rule capital as pointed out by Kurz (1968). If the steady state capital stock is equal to or larger than the golden rule capital, $f'(k)$ is equal to or less than $n + \delta$; the first term on the right hand side of (9) is also negative because $[n+\delta+\rho-f'(k^*)]$ is positive as shown below in proposition one. In this case, Δ is negative. For Δ is the product of two characteristic roots, negative Δ implies that one root is positive and one negative. If Δ is positive, then both roots will be positive as existence of two negative roots contradicts (8). For this section, we will focus on the case that Δ is negative, that is to say, there exists a unique optimal path in the neighborhood of the equilibrium. Furthermore, we assume that there exists only one equilibrium for the systems. A numerical example is presented in next section before discussing the political investment cycle.

Of course, if the time discount rate is very small, the first term on the right hand side of (9) is negative; so is Δ. To our knowledge, there does not exist any convincing evidence for the social planners to discount future. We have a few comments about this at the end of this section.

The properties of the unique saddle point equilibrium follow in order:

Property One: The equilibrium capital stock is larger than the modified golden rule one, i.e. $f'(k^*) < n + \delta + \rho$.

To show this, note that, in steady state, we have

$$\frac{1}{-u''(c^*)} \{\pi v'(k^*) + u'(c^*)[f'(k^*)-n-\delta-\rho]\} = 0 \tag{10}$$

$$f(k^*) - c^* - (n + \delta)k^* = 0 \tag{11}$$

From (10),

$$f'(k^*) = n + \delta + \rho - \frac{\pi v'(k^*)}{u'(c^*)} < n + \delta + \rho = f'(k^{mg}) \tag{12}$$

where k^{mg} denotes the modified golden rule amount of capital. From (12), it is clear $k^* > k^{mg}$ as $f''(.)$ is negative. The explanation is simple. Since social planners benefit directly from the expansion of the economic organizations, and consumers' welfare over the infinite horizon is not the only criterion for the plan-makings, the short-run consumption will be partly sacrified for the expansion drive. It is quite possible that, as shown in next numerical example, consumption is permanently sacrificed in this kind of models: equilibrium consumption is lower than the golden rule one and capital is over-accumulated --- dynamic inefficiency.

Property Two: The higher the value π, the higher the steady state capital. Totally differentiating equations (10) and (11), we have

$$\begin{bmatrix} dc \\ dk \end{bmatrix} = M \begin{bmatrix} \dfrac{v'(k^*)}{u''(c^*)} d\pi \\ 0 \end{bmatrix} \qquad (13)$$

It is simple to show that

$$\frac{dk}{d\pi} = \frac{1}{\Delta} \frac{v'(k^*)}{u''(c^*)} \qquad (14)$$

which is positive as the economy is on the unique optimal convergent path. As for the steady state consumption, the sign is ambiguous depending whether the equilibrium capital is higher or less than the golden rule capital.

The effects of π on investment and consumption on the unique optimal path can also be analyzed. From (7), the solutions of the linearized systems for the behavior of the capital stock and consumption are:

$$k_t = k^* - (k^* - k_0)e^{\theta t} \qquad (15)$$

$$\dot{k}_t = -\theta(k^* - k_t) \qquad (16)$$

$$c_t = c^* + (f'(k^*) - n - \delta - \theta)(k_t - k^*) \qquad (17)$$

where θ is the negative root of the dynamic system,

$$\theta = \frac{1}{2} [\rho - \sqrt{\rho^2 - 4\Delta}] \qquad (18)$$

From (16) and (17), it is clear that, through its positive effect on steady state capital, k^*, high value of π leads to high investment and low consumption on the optimal path for all k_t less than k^*. But we should note that π may also affect θ and c^*. If increase in π tends to lower θ, i.e. θ becomes more negative, then the investment will be unambiguously high as a result of high π.

Property Three: The higher the value π, the higher the steady state investment rate (or saving rate).

In steady state, investment is just $(n + \delta)k^*$. Let investment rate (or saving rate) be s, then

$$s = \frac{(n+\delta)k^*}{f(k^*)} \qquad (19)$$

So

$$\frac{ds}{d\pi} = \frac{(n+\delta)}{(f(k^*))^2} [\, f(k^*) - f'(k^*)k^* \,] \frac{dk}{d\pi} \qquad (20)$$

which is positive since $dk/d\pi$ is positive and $[f(k^*)-f'(k^*)k^*]$ is also positive.

The three properties stated above reveal how social planners' preferences exert effects on the growth pattern in a socialist economy. In Cass model, we know, the form of social welfare functions does not enter to the determination of equilibrium capital stock. Even if we interpret the social welfare function as the social planners' own preference, the equilibrium capital and consumption are still independent of social planners' preference as long as their preference are defined only on consumption. Recall from Cass (1965), in equilibrium,

$$f'(k^{mg}) = n + \delta + \rho \qquad (21)$$

$$f(k^{mg}) - c - (n + \delta)k^{mg} = 0 \qquad (22)$$

so social welfare function itself plays no role in the determination of k^{mg} in Cass model. (Please compare (21) and (22) with equilibrium conditions (10) and (11).) The invention by Kurz (1968) provides us a rich picture for the link between preference and economic growth. Of course, as a positive approach, Kurz model with proper justification is much more realistic than Cass model when applied to socialist economy.

Another point is worth commenting here. In almost all growth models, it is assumed that social planners or representative agent in a capitalist economy systematically discount the future. We must admit that this imposition on the social planners has no empirical relevance. It seems that, for social planners, future is equally important as today if not more important. Socialism and communism are the doctrine promising bright future and advocating hardship today. Every schoolboy in socialist country knows that there exists a higher stage of socialism where prevailing principle is "from each according to his ability, to each according to his needs". Investment hunger and expansion drive are often justified as a sacrifice of current generation for the betterment of the future generations even though they are carried out more in the interest of social planners. In this point, social planners and the people share the same moral. This is why investment hunger and expansion-drive are not regarded immoral or unethical even by the common people. Ramsay (1928), and Stigler and Becker (1978) have questioned the appropriability about imposing positive discount rate. What is more ridiculous is Uzawa (1968). In Uzawa's time preference model, the discount rate is positively related to the current utility enjoyed by the representative agent in a capitalist economy. The logical consequence is that rich people perceive future darker than the poor people, and the homeless deem future bright.

IV. A Numerical Example and an Illustration of Political Investment Cycle

Even though the modified Kurz model gives us interesting results, the existence of multiple equilibria brings about erratic dynamics even with simple preferences and technology. Here we show that, if preference is the popular logarithm functions of consumption and capital, and technology is standard Cobb-Douglas, there exists a unique equilibrium and a unique optimal path.

Now the social planners maximize

$$\int_0^\infty [\log c + \pi \log k] e^{-\rho t} dt \qquad (23)$$

s.t.

$$\dot{k} = k^\alpha - c - nk \qquad (24)$$

where $0 < \alpha < 1$, and we have set δ equal to zero for simplicity. The corresponding optimal conditions are

$$\dot{c} = \frac{c}{k} [\pi c - \alpha k^\alpha - (n+\rho)k] \qquad (25)$$

$$\dot{k} = k^{\alpha} - nk - c \tag{26}$$

Set the time derivatives of c and k equal to zero in (25) and (26), the unique optimal equilibrium point is

$$k^{*} = \left[\frac{\pi + \alpha}{n\pi + n + \rho}\right]^{\frac{1}{1-\alpha}} \tag{27}$$

$$c^{*} = k^{*\alpha} - nk^{*} \tag{28}$$

The determinant of the corresponding matrix M is

$$\Delta = \frac{c^{*}}{k^{*}} \{\pi[\alpha k^{*\alpha-1} - n] + [\alpha^{2} k^{*\alpha-1} - (n+\rho)]\} \tag{29}$$

Upon substitution,

$$\Delta = \frac{(n+\rho-n\alpha)(\pi n+n+\rho)[\pi(\alpha-1)-\alpha(1-\alpha)]}{(\pi+\alpha)^{2}} < 0 \tag{30}$$

as $0 < \alpha < 1$. So there is one negative characteristic root and one positive root, the equilibrium is saddle point stable.

It is straight forward to check that

$$\frac{dk}{d\pi} = (1-\alpha)k^{*\alpha} \frac{\rho+(1-\alpha)n}{(\pi n+n+\rho)^{2}} > 0 \tag{31}$$

$$\frac{ds}{d\pi} = n(1-\alpha)k^{*(-\alpha)} \frac{dk}{d\pi} > 0 \tag{32}$$

Next we are going to see when high degree of expansion drive leads to dynamic inefficiency. From Phelps (1961), the golden rule capital stock at which consumption is maximized is given as (for Cobb-Douglass technology)

$$k^g = [\frac{\alpha}{n}]^{\frac{1}{1-\alpha}} \qquad (33)$$

For k^* is larger than k^g, it is required that

$$\frac{\pi + \alpha}{n\pi + n + \rho} > \frac{\alpha}{n} \qquad (34)$$

which is the same as require that

$$\pi > \frac{\alpha\rho}{1 - \alpha n} \qquad (35)$$

For $\alpha = 0.25$, $\rho = 0.05$, and $n = 0.01$, π should be larger than 0.0125. which is not a strict requirment. So, in this case, people's consumption is not only sacrificed on the dynamic path converging to the steady state, but also sacrificed in the steady state. Shortage of consumption goods can persist for ever.

Suppose there are two groups of social planners in the economy. Following the convention, we may call one group as "softliners" or the "right", and other group as "hardliners" or the "left". They alternatively control the plan-makings. It is known in socialist countries such as Hungary that shifts of resources towards consumption rather than investment always come about as a result of "softline" rule; the "hardliners" or the "left" are always more expansion-oriented. [see Kornai (1988), pp. 283-284] In our model, if we denote π_l as the expansion desire of the "left", and π_r as the expansion desire of the "right", and let $\pi_l > \pi_r > \alpha\rho/(1-\alpha n)$, then

the "left" maximize

$$\int_0^\infty [\log c_l + \pi_l \log k_l] e^{-\rho t} dt \qquad (36)$$

s.t.

$$\dot{k}_l = k_l^\alpha - c_l - nk_l \qquad (24)$$

the "right" maximize

$$\int_0^\infty [\log c_r + \pi_r \log k_r] e^{-\rho t} dt \qquad (37)$$

s.t.

$$\dot{k}_r = k_r^\alpha - c_r - nk_r \qquad (24)$$

The initial capital stock is the same for both groups: $k_0 = \bar{k}$. To avoid the problem of time inconsistency, we assume that the "left" and the "right" all commit to the optimal programs they calculate at time zero, and make no change later on.

From calculations above, it is easy to obtain that in steady state

$$k_l^* > k_r^* \ , \ c_l^* < c_r^* \qquad (38)$$

From (32), the steady state investment rate for the "left" is always larger than the one for the "right". If the "left" is in the power, the economy experiences higher investment and lower consumption; if the "right" is in the power, consumption is relatively high and investment relatively lower. The cyclical change on consumption, investment and investment rate can be diagramatically shown below:

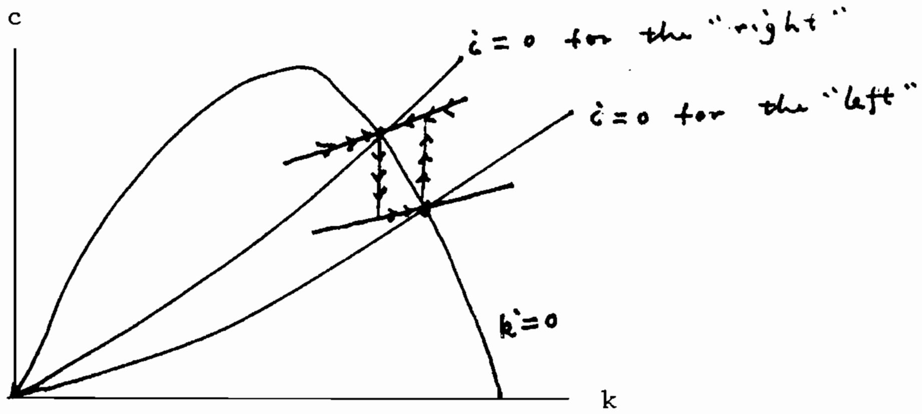

Figure I

where E_l and E_r are equilibrium points for the "left" and the "right" respectively. If the economy is currently in E_l, the power shift from the "left" to the "right" results in immmediate upward jump of consumption and reduction of investment; the new long-run equilibrium is E_r where capital stock is lower than, but consumption is higher than, the equilibrium levels at E_l. The investment rates fluctuate following the political power shifts. This is an demonstration of political investment cycles at steady states. Investment cycles can also happen on the pathes converging to the steady states. In Figure II, P_r and P_l are the optimal convergent pathes for the "right" and the "left" respectively.

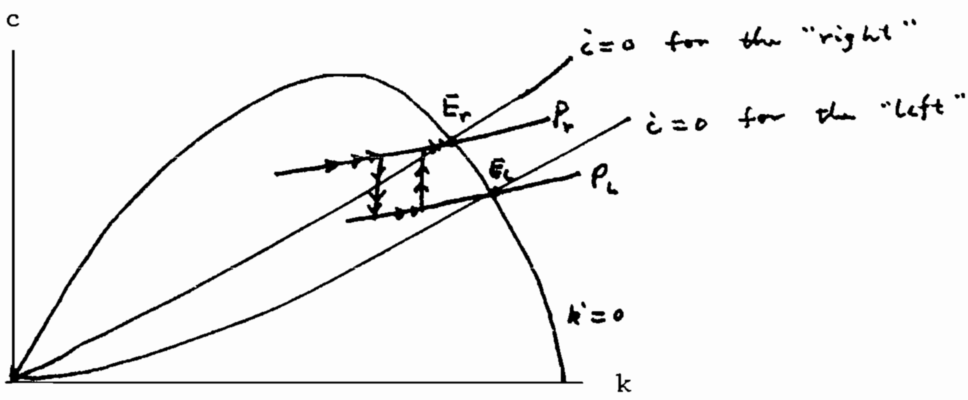

Figure II

If the economy is initially on the path for the "right" P_r, a change in political regimes from the "right" to the "left" leads to immediate downward jump from the path P_r to the path P_l. Througout time, the economy follows a zigzag path, and investment rates fluctuate accordingly on the path.

V. Historical Evidence

In this section we present a preliminary empirical study of the effects of political change on the investment rate in China. The labels for different lines in the communist party, the "right" and the "left", are popularly known in China. The "left" are strong and dogmatic adherents of socialism; they advocate centralization of economic activities, rapid abolishtion of private ownership in industrial sector, and rapid transition of agricultural sector from private ownership to collective ownership, and then to state-ownership. Chairman Mao is the symbol of the "left". The "right" are more often associated with the economic policies of capitalist flavor such as reliance on market mechanism and material incentive in planned sectors, allowance of private plots and contract systems in agricutural production. The prominant members of this line are Liu Shaoqi, president of China from 195 to 1966, Deng Xiaoping, and they were called as the capitalist representatives in the communist party during Cultural Revolution. The power struggles between the "left" and the "right" have shaped the history of China in the past four decades, and their effects can be seen from every aspects of Chinese society. Our present focus is their effects on the investment rates.

Following table contains relevent data for our analysis.

year	investment rate as % of GNP	consumption rate as % of GNP	productive investment as % of total investment	power regimes as a dummy variable
1952	21.4	78.6	50.8	0
1953	23.1	76.9	49.4	0
1954	25.5	74.5	50.3	0
1955	22.9	77.1	51.4	0
1956	24.4	75.6	71.0	0
1957	24.9	75.1	58.8	0
1958	33.9	66.1	82.3	1
1959	43.8	56.2	86.9	1
1960	39.6	60.4	97.4	1
1961	19.2	80.8	78.5	0
1962	10.4	89.6	63.6	0
1963	17.5	82.5	63.9	0
1964	22.2	77.8	60.8	0
1965	27.1	72.9	70.7	0
1966	30.6	69.4	68.9	1
1967	21.3	78.7	82.2	1
1968	21.1	78.9	78.5	1
1969	23.2	76.8	76.2	1
1970	32.9	67.1	71.8	1
1971	34.1	65.9	76.2	1
1972	31.6	68.4	78.7	1
1973	32.9	67.1	73.7	1
1974	32.3	67.7	75.4	1
1975	33.9	66.1	73.4	1
1976	30.9	69.1	79.3	1
1977	32.3	67.7	70.9	1
1978	36.5	63.5	71.8	1
1979	34.6	65.4	64.1	0
1980	31.5	68.5	54.5	0
1981	28.3	71.7	46.8	0
1982	28.8	71.2	46.4	0
1983	29.7	70.3	52.5	0
1984	31.2	68.8	58.6	0
1985	33.7	66.3	57.7	0

Source.---Statistical Year Book of China, 1986

In the table, the power to make economic planning is represented by a dummy variable; and a value of zero means that the "right" control the planning board, a value of one means that the "left" control the planning. The term "productive investment" is special to Marxist and socialist economics, and needs some explaination. It refers to investment directly serving material production or meeting the needs of material production. Its counterpart is non-productive investment which includes investment on public utilities, housing, public health, social welfare, education, etc.. Since non-productive investment is more or less related to people's consumption, especially durable and public consumption, the percentage of productive investment in total investment outlay is a more accurate measure of accumulation. The fluctuations of investment rate and productive investment rate are depicted in figure III below.

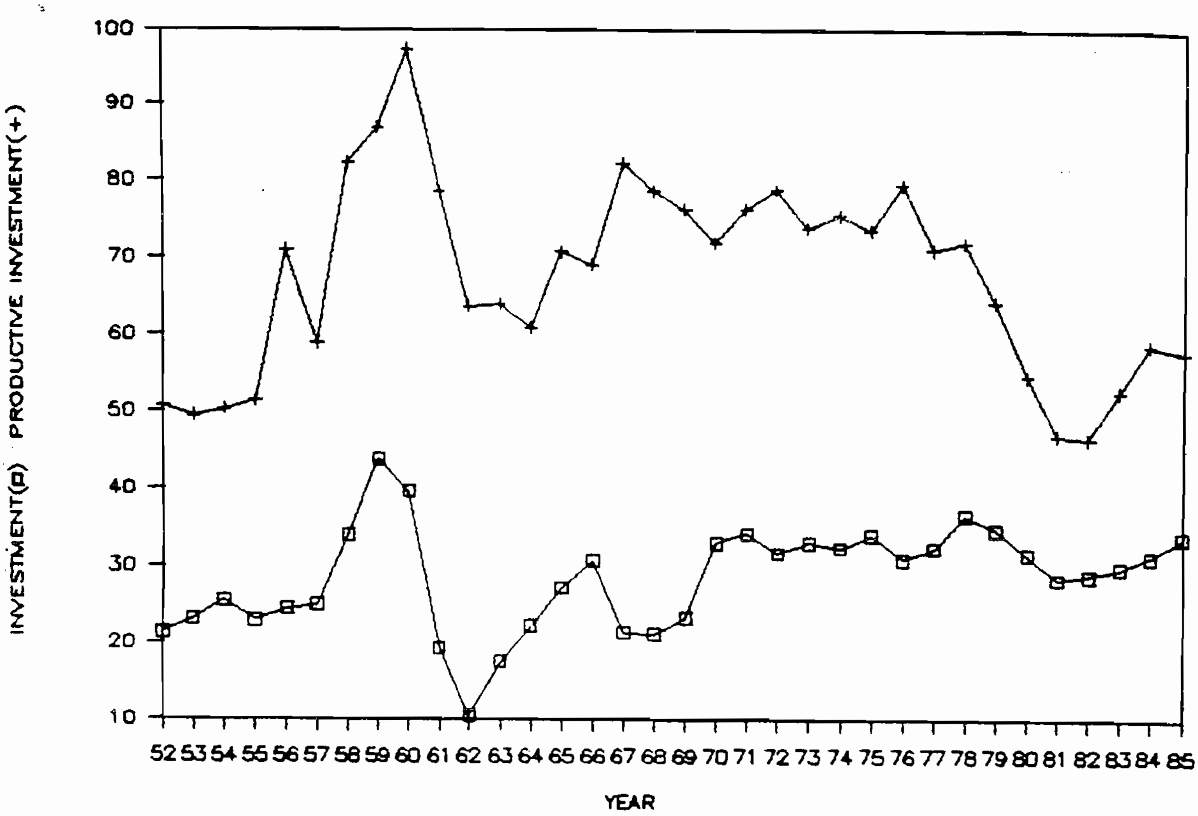

Figure III

For the period of 1952-57, economic decision-makings were more under the control of the "right" as Mao had no total domination in the planning processes, and political life was more democratic in the communist party. The investment rates were in the range of 21.4% and 25.5%. The average share of productive investment in total investment was 55.3%. Both production and consumption went up rapidly in those six years, and those investment rates were later ragarded as the optimal or normal ones by Chen Yuan, the chief economist in the communist party, who was a member of the "right" and was in direct charge of economic planning for that period.

The year 1957 was a turning-point in the political climate of China. The anti-"rightest" movement launched by Mao produced fundamental effects on political and economic life in China. With the beginning of "Great Leap Forward" in 1958 and the movement of people's communes some time later, economic planning was dominated by the ideology of the "left". The investment rate jumped up to 33.9%, 43.8% and 39.6% in the year 1958, 1959 and 1960, respectively. The average share of productive investment for the three years was up to 88.3%. High investment rates and natural calamities in this period caused poverty, hunger and death all over China. Facing economic disaster, Mao retreated from plan-making and even admitted mistake in 1962. The power of plan-making shifted back to the hands of the "right", and Chinese economy entered a period of adjustments.

From 1963 to 1965, the average investment rate was set at 22.7% and productive investment only accounted for 64% of total investment. President Liu Shaoqi even introduced many programs in agricultural production which later became important ingredients of economic reforms under Deng Xiaoping.

The life of the "right" was short-lived. The next ten years, 1966-76, were "Great Cultural Revolution", and Mao and the "left" were in absolute control of plan-making. Except for the years 1967-69 when the economy was almost paralyzed with the destructive political turmoils, the investment rate on the average was above 31% , and 75% of the investment was for productive purpose. After Mao's death, Hua Guofang, the successor chosen by Mao, continued the expansion-drive of the "left", and even started a "Foreign Leap Forward" from 1977 to 1979 --- large scale imports of foreign technology. The average investment rate was above 34%.

In 1979, political power began to shift back to the "right", and Deng Xiaoping and the "reformers" came to the forefront. But the ideology of the "left" was still deeply affected the planning; the effect of "Foreign Leap Forward" still kept the investment rate at a high level of 34.6%. At that year, the proportion of productive investment in total investment began to decrease. From 1981 to 1985, the average investment rate was lowered to 30.8%, and the average share of productive investment was at historical low level - 52.4%. That is to say, large proportion of investment was diverted to the improvement of residential conditions, service sectors, public health and education.

Simple sketches above illustrate that investment rates and political changes are closely related in China. It is convenient to test how much fluctuations in investment rates and productive investment rates can be explained by the political changes in China's socialist history. Here we report a few results of regression equations.

$$I_t = \underset{(2.84)}{11.23} + \underset{(2.05)}{4.08D_t} + \underset{(3.36)}{0.55I_{t-1}} \qquad (39)$$

$$R^2 = 0.50, \qquad DW = 1.13$$

$$PI_t = \underset{(4.47)}{31.25} + \underset{(5.17)}{12.36D_t} + \underset{(4.35)}{0.45PI_{t-1}} \qquad (40)$$

$$R^2 = 0.73, \qquad DW = 1.93$$

where I_t = investment rate at time t, D_t = dummy variable of political change PI_t= share of productive investment in total investment at time t.

Equations (39) and (40) both show that political changes have substantial effects on investment rate and productive investment rate. The positive coefficients say that a "left" regime causes high rates, and a "right" regime leads to low rates. As investment projects often last for a few years, the lagged variables also help to explain the rates.

If we exclude the politically abnormal years 1967-69, then political changes alone can explain about half of the variations in the investment rates:

$$I_t = \underset{(18.17)}{25.36} + \underset{(5.19)}{8.9D_t} \qquad (41)$$

$$R^2 = 0.43, \qquad DW = 0.74$$

$$PI_t = \underset{(28.14)}{58.32} + \underset{(6.51)}{19.12D_t} \qquad (42)$$

$$R^2 = 0.57 \qquad DW = 0.93$$

Two points should be added to our analysis of political investment cycles in China. First, the "right" and the "left" are both expansionists because they share the nature of the social planners, the difference lies only in the degree of the expansion drive. Throughout time, there is a tendency for social planners to increase the investment rates; this can be seen from regressing the investment rates against a time variable:

$$I_t = \underset{(1.02)}{5.07} + \underset{(2.21)}{4.22D_t} + \underset{(2.56)}{0.48I_{t-1}} + \underset{(1.135)}{0.114TIME} \qquad (43)$$

$$R^2 = 0.52 \qquad DW = 1.13$$

Secondly, political factors as an exogeneous variable can not fully explaine all fluctuations in investment rates; a theory expoused by T. Bauer (1978) and Janos Kornai (1980, 1988), which we may rightly call as model of endogeneous investment cycles, has developed to explain the investment cycles under the same political regime. These two theories on investment cycles should be taken as complementary, and "they can be usefully and effectively placed side by side and, taken together, they do a good job not only of explaining the regular pattern of the cycle, but also of explaining its irregularities" [Kornai (1988), P.284].

VI. Conclusion

In a dynamic optimization framework, we have tried to demonstrate expansion drive, investment hunger and investment cycles from a rational choice model of social planners. In our own opinion, this approach is superior to many existing ad hoc models in both theoretical and empirical studies on socialist growth and investment as the latter lack firm microfoundation. This approach is also superior to many optimization models for social planners as the latter start from very naive assumptions about the social planners.

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Essay Six

Endogenous Investment Cycles in Centrally Planned Economy

Introduction

Investment cycles in centrally planned economies have received considerable studies in the East and the West. Contrary to the prediction made by the founders of socialism that socialist planning will eliminate the anarchic state of the capitalist economy and socialist economy will be free from the cyclical movements in output, employment, and growth, many studies have shown that certain socialist macroeconomic indicators, in particular the investment rate, are subject to cyclical fluctuations in almost all socialist economies. The explanations of investment cycles can be divided to two approaches. One is political approach which explains investment fluctuations by political changes. A typical study is A. Ungvarszki's work on Hungary investment cycles, she goes through the whole economic history of Hungary in the last thirty-five years, and shows that each economic upswing or downswing has been associated with political changes [see Kornai (1988) for a summary of Ungvarszki's study]. Another explanation of investment cycle is economic approach, the notable works are Bauer (1978) and Kornai (1980, 1988). Many empirical studies have followed this line, e.g., Grosfeld (1987) and Roland (1987). The main idea of this approach is following: social planners adjust investment rates according to the shortage intensity in the economy; when shortage is below or around normal state, social planners approve and initiate large numbers of projects. High investment rate will soon overheat the economy, and shortage intensity will exceed the normal level and become intolerable. Social planners will react to halt all new investment, and cut or stop quite a few existing investment projects. After this slowdown process lasts for a while, shortage intensity is reduced, and more slack is revealed.

There comes a new wave of optimism, and a new cycle starts again.

In my opinion, most of these theoretical and empirical cycle studies suffer from a common shortcoming: ad hoc assumption and absence of explicit optimization behavior on the part of social planners. This point is especially applicable to the case of economic approach to investment cycles. If social planners can perceive the effects of their action, which is possible from repeated practices, why do not they follow a smoother pattern of investment rates? Is the cycle itself a optimal choice of the social planners? if we make a comparison between cycle studies on socialist economies and the studies on capitalist economies, we can immediately notice that the latter often start with the rational choice of different parties and governments, e.g. Nordhaus (1975), McRae (1977), Alesina and Sachs (1988). In essay five, I have tried to model political investment cycle in centrally planned economy based on explicit optimization of different lines in the communist party. Here I hope to extend the model to the case of economic theory of investment cycle. We call this as endogeneous cycle because exogenous political shocks are assumed away from the beginning. Of course, these two cycles are often interwoven in practice as noted by Kornai (1988).

In section 1, I will focus on the dynamic relation between shortage intensity and investment rate. With some over-simplification, but without sacrifice of the essence, I reformulate Bauer-Kornai's cycle theory in the familiar Volterra-Lotka equations of cycle.

I begin section 2 with defining social planners' preference. it will be justified that social planners derive positive utility from high investment rate and negative utility or disutility from high shortage intensity in the economy. If their preference is convex and separable in investment rate and shortage intensity, then investment cycle is not possible under very reasonable assumption about the dynamic specification of shortage intensity. In fact, there exists a unique optimal path converging to equilibrium for the economy. So investment cycle can only be caused by exogeneous shocks such as political changes.

To get rational, endogeneous investment cycle, some deviation from usual assumption about social planners' preference is crucial. In section 3, I assume that social planners' utility function is convex, instead of concave, in investment. Then an exact Volterra-Lotka model can be derived from a well-defined objective function of the social planners.

1. A Formulation of Bauer-Kornai's Cycle Theory in Volterra-Lotka Model

I first present a stylized description of Bauer-Kornai's theory of investment cycle. The references are Bauer (1978) and Kornai (1980, 1988).

In the upswing of investment cycle, the economy is in a state of *run-up* and *rush*. An increasing number of investment projects is approved, and many new investments are initiated, and the existing projects are accelarated and expanded. This process continues until the economy hits the "tolerance limits": (1) high rate of investment leads to serious shortage of consumption goods that various forms of protest by consumers take place; (2) large number of investment projects compete with each other in investment inputs, and shortage of certain materials, late deliveries, bottleneck and many other shortage phenomena become intolerable; (3) continued high rate of investment brings about large foreign trade deficit and debt accumulation. All these send alarm signals to social planners, and they react to suddenly halt all new investment projects and cut or stop many existing projects. The economy enters into a state of *slowdown* and the investment rate is often substantially reduced.

After some time, the shortage intensity is reduced considerably; producers find a relatively small volume of unfilled orders, consumers are more often in a short queue. So "there is too much slack" in the economy, "more could be squeezed out of investment sphere. This increasingly optimistic spirit suddenly becomes a strong determonation and a new impetus is given to investment activity. The cycle starts again" [Kornai (1980) pp.213-14].

It seems to me that the essential parts of this cycle theory can be summarized in two dynamic equations. The first equation governs the dynamics of shortage intensity: if investment rate exceeds some normal rate, shortage tends to intensify; if investment rate is below the normal level, shortage intensity mitigates. The second equation describes the behavior of social planners: facing high and untolerable shortage phenomena, social planners lower investment rate; and when shortage intensity is well below certain critical or normal level, they raise the investment rate. Let me denote investment rate as k and shortage intensity as z. Also denote the normal investment rate as k^* and normal shortage intensity as z^*. k^* and z^* are assumed to be constant here. The concept of normality has been well studied by Kornai (1971, 1980, 1982) and plays a central role in the understanding of centrally planned economies. For elaboration, I refer readers to Kornai's original works.

Mathematically we write the change in the shortage intensity as a positive and linear function of the difference between actual investment rate and normal investment rate:

$$\frac{\dot{z}}{z} = A(k - k^*) \qquad A > 0 \qquad (1)$$

The equation regarding social planners' behavior takes following form:

$$\frac{\dot{k}}{k} = B(z^* - z) \qquad B > 0 \qquad (2)$$

Without affecting the result qualitatively, I set the constants A and B to unity. In their forms, equations (1) and (2) are just Volterra-Lotka model in terms of shortage intensity and investment rate, and their application can be found in predator-prey model of biology and Goodwin cycle of economics [Goodwin (1967)]. Following the discussion in Hirsch and Smale (1974), I depict the phase diagram in figure 1.

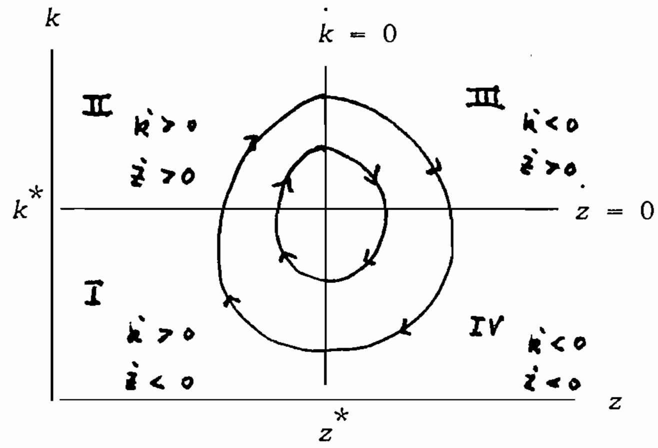

Figure 1

The equilibrium points are given by (z^*, k^*) and (0, 0). The interesting equilibrium is the normal levels of shortage and investment. It is proved that (z^*, k^*) is a stable equilibrium, and there exists no limit cycle in the model, and every trajectory of these two equations is a closed orbit except the two equiliria, see Hirsh and Smale (1974). Therefore,, for any given initial shortage intensity and investment rate other than (z^*, k^*), shortage and investment will oscillate cyclically.

In region I, as z is less than z^*, and k less than k^*, investment rate goes up and shortage is expected to go down. This region is more or less like the period of *run-up* called by Tamas Bauer. Region II is Bauer's *rush* period, investment rate exceeds the normal rate, and shortage level increases; sooner or later, the tolerance limit will be hit and the social planners begin to "pull the brake". So in region III, investment rate is reduced and shortage continues to rise for a while. This *slowdown* process is typically manifested in region IV where both investment rate and shortage intensity begin to decrease.

Now the questions. Is equation (2) really reflects the optimal choice of the social planners? Why should social planners stick to the cyclical path instead of some smooth one? We take up these questions in next two sections.

2. The Case of No Investment Cycle

To derive social planners' optimal decision rule from explicit rational choice, we first define social planners' objective function. Social planners are a large group of people who are in charge of all economic organizations and management in a socialist economy. With both political power and economic resources in their control, "they pursue their own individual and group interests, including the interests of the particular specialized agency to which they belong. Power creates an irresistable temptation to make use of it" [Kornai (1986)]. It is a common phenomenon that social planners in all socialist economies have placed the highest priority on capital accumulation, and "investment hunger" and "expansion-drive" are the most important characteristics of socialist economic growth. According to Kornai (1981, 1986, 1988), it is in their own interests for social planners to accelerate investment and capital accumulation, because the expansion of the organization or firms is a way for social planners to identify themselves with their own jobs; "a larger organization brings more prestige, and also more power" [Kornai (1981)].

Of course, social planners do care about shortage intensity caused by high rate of investment. Chronic and excessive shortage often brings about consumers's dissatisfaction with social planners' manipulation of the power, and the overextended investment policy may lead to complaints, wild strikes or demonstration, or open revolt.

Taking these two aspects into account, social planners enjoy positive utility from investment rate and derive disutility from shortage intensity if the shortage intensity is very high. Using the notations in last section, I write their instantaneous utility function as $u(k) - v(z)$. I further assume that functions $u(.)$ and $v(.)$ are twice continuously differentiable, with the property $u'(.) \geq 0$, $u''(.) < 0$, and $v''(.) > 0$. As for the first order derivative of function $v(.)$, it is reasonable to assume that $v'(.) > 0$ for large value of shortage intensity; if z is below certain level, $v'(.)$ may well be negative. Since $v'(z)$ is continuous by assumption, there exists at least one value z^* such that

$$v'(z^*) = 0 \qquad (3)$$

For $v''(z)$ is strictly larger than zero for all z, z^* satisfying (3) is unique. Equation (3) will be useful when I discuss the equilibrium later.

Social planners maximize their utility over a finite horizon [0, T]

$$\int_0^T [u(k) - v(z)]dt \qquad (4)$$

subject to the dynamic equation (2) in last section:

$$\frac{\dot{z}}{z} = (k - k^*) \qquad (5)$$

where k is control variable and z is state variable, the Hamiltonian function is

$$H(k, z, \lambda) = u(k) - v(z) + \lambda z(k - k^*) \qquad (6)$$

The first order conditions for maximization are

$$-\frac{u'(k)}{z} = \lambda \qquad (7)$$

$$-v'(z) + \lambda(k - k^*) = -\dot{\lambda} \qquad (8)$$

$$z(k - k^*) = \dot{z} \qquad (9)$$

where λ gives the marginal disutility of shortage intensity and is nonpositive from (7). At time T, the boundary condition requires that $\lambda(T) = 0$, which is the same as require that $u'(k(T)) = 0$; social planners should choose such a $k(T)$ at the end of time horizon, T.

Now I will focus on the dynamics of equations (7), (8) and (9). Substituting λ into (8), and manipulating some algebra, I obtain

$$\dot{k} = -\frac{v'(z)z}{u''(k)} \qquad (10)$$

$$\dot{z} = z(k - k^*) \qquad (9)$$

From (3), there exists unique strictly positive equilibrium point (z^*, k^*) for equations (9) and (10). It is easy to show that this equilibrium is a saddle point. Linearizing equations (9) and (10) around (z^*, k^*),

$$\begin{bmatrix} \dot{k} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{v''(z^*)z^*}{u''(k^*)} \\ z^* & 0 \end{bmatrix} \begin{bmatrix} k - k^* \\ z - z^* \end{bmatrix} \qquad (11)$$

The product of two characteristic roots is given by the determinant of the 2x2 matrix, Δ,

$$\Delta = \frac{v''(z^*)z^{*2}}{u''(k^*)} < 0 \qquad (12)$$

so there is one positive root and one negative root. For relatively large time horizon T, the path given by the positive root is a divergent path, which will eventually violates the boundedness of investment rate k. So the economy will follow a unique perfect foresight path as diagramatically shown in figuer 2.

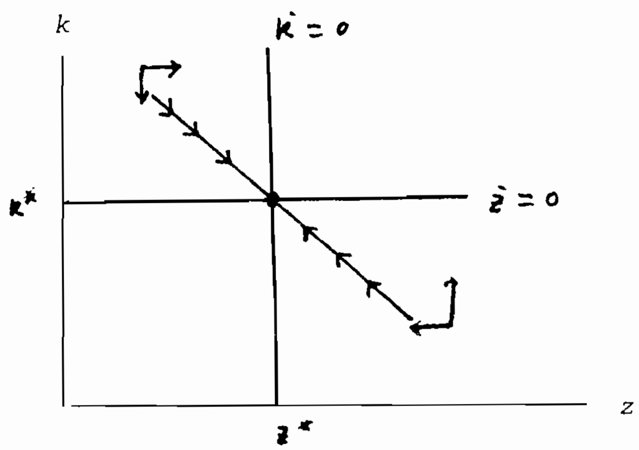

Figure 2

Shortage intensity and investment rate go up or down in opposite direction on the optimal path. In the long run, the economy will converge to the equilibrium state or the normal state. The reason is quite simple: as social planners's utility functions are concave in shortage and investment rate, a smoother pattern of shortage and investment provide higher utility. In this conventional, rational choice framework, investment cycle is not possible for social planners with convex preferences.

3. The Case of Investment Cycle

To get an exact Volterra-Lotka cycle of investment, I assume that social planners' utility functions need not satisfy the concave requirements. More specifically, social planners maximize a utility function taking following form:

$$u(k) - v(z) = k\log k - z + z^{*}\log z \qquad (12)$$

This function is convex in in k, but concave in z. If actual shortage is higher than the normal one, marginal utility of shortage is negative; if actual shortage is lower than the normal, marginal utility of shortage is positive. Suppose social planners maximize

$$W = \int_{0}^{T}[k\log k - z + z^{*}\log z]\, dt \qquad (13)$$

subject to

$$\dot{z} = z(k - k^{*}) \qquad (9)$$

I will first show that social planners never take a smooth pattern of shortage and investment as their optimal choice in this case. If social planners set the investment rate at $k_1 > k^*$ after some time t_1, then

$$z(t) = e^{(k_1 - k^*)t} \qquad (14)$$

The integral (13) is equal to

$$W = k_1 \log k_1 \ (T - t_1) - \frac{e^{(k_1 - k^*)T} - e^{(k_1 - k^*)t_1}}{(k_1 - k^*)} +$$

$$+ z^*(k_1 - k^*)(T^2 - t_1^2) + \text{some constant.} \qquad (15)$$

W will be a large negative number for large T because $(k_1 - k^*)$ is positive and the second term on the right hand side of (15) dominates all other terms.

Similarly, if social planners choose a investment rate k_1 less then k^*, the third term on the right hand side will become very negative and the second term approaches zero for large T. Also note that the third term will dominate the first term as square of T is far greater than T for large T. Therefore, W goes to negative for large T.

To find the optimal adjustment function for social planners, we follow the procedures outlined in section 2. The Hamiltonian for this special case is

$$H = k\log k - z + z^*\log z + \lambda z(k - k^*) \qquad (16)$$

The optimal conditions for this case upon substitution are

$$\frac{\dot{k}}{k} = (z^* - z) \qquad (16)$$

$$\frac{\dot{z}}{z} = (k - k^*) \qquad (17)$$

which are exactly the Volterra-Lotka equations discussed in section 1. If initial conditions of shortage intensity and investment rate are different from the equilibrium state (z^*, k^*), the economy will oscillate throughout time. In particular, the cycle is proved to be the rational choice of social planners.

Conclusion

Through some simple excercises, I have tried to demonstrate how cyclical phenomenon in centrally planned economy can be understood from the point of view of rationality on the part of social planners. With our simple set-up, it is shown that social planners' preferrences have to be nonconvex in investment in order to obtain rational or endogenous investment cycle. Endogenous cycle theory based on competitive capitalist economy has well developed with sound microfoundation, the counterpart for the socialist is still casted in very *ad hoc* models. This paper is a very preliminary step towards the theory of rational investment cycle in socialist economy.

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