ESSAY FOUR

Optimal Taxation of Foreign Capital Income in A Growing Economy

with Different Governments
I. Introduction

Foreign direct investment and taxation have been comprehensively studied in static settings by many people, e.g., MacDougall [1960], Hamada [1966], Jones [1967], Feldstein and Hartman [1979]. The dynamic counterparts of these models are relatively few. In this short essay, by integrating the dynamics of both domestic capital accumulation and foreign direct investment into an intertemporal growth model, we will make some effort to study the optimal taxation of foreign capital income from the viewpoint of the host country. Throughout this paper, it is assumed that there are two countries: home country and foreign country; only foreign country undertakes direct investment in home country, and home country levies tax on foreign capital earnings.

In section II, with some ad hoc assumptions on home country's saving rate and tax rate, we are going to set up the basic dynamic model describing domestic capital accumulation and foreign direct investment in the home country. The stability of the equilibrium and some comparative statics will also be studied. Section III introduces two types of governments: open door government and close door government (to be defined later). In the framework of optimal choice on the part of governments, it is shown that the optimal aggregate capital (i.e. the sum of domestic and foreign capital) is still determined by the modified golden rule as in Cass [1965], and the optimal tax rate on foreign capital income is the same for both kinds of governments. While the optimal saving rate under close door government is higher than under open door government, the corresponding equilibrium per capita
consumption may be higher or lower than under open door government. The moral here is not against close door government. In fact, close door government may do better than the open door government in maintaining higher consumption for its country if the population growth rate is low, and the difference in the technological efficiency between foreign capital and domestic capital is small.

II. A Basic Model

As a convention, the home country and foreign country produce only one good which can either be consumed or invested. At any given time, the home country's total capital stock, $K$, is divided into domestically owned capital, $K_d$, and foreign owned capital, $δK_f$. Here the parameter $δ$ takes values which are larger than or equal to one; that is to say, foreign capital (or multinational corporations' investment) is more efficient than, or as equally efficient as, its home country's domestic capital. This can be justified if we take into account the technological advantage possessed by multinational corporations (see Caves [1982]). The parameter $δ$ is definitely larger than one if we consider developed countries' foreign investment in developing countries. As for another input of production, the home country's labor force at any time is $L$, which is growing at exogenously given rate $n$. Write the capital stock in per capita term,

$$k = k_d + δk_f \quad δ ≥ 1$$

(1)

where $k = K/L$, $k_d = K_d/L$, and $k_f = K_f/L$.  

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The production function, \( f(k) \), is the standard neoclassical one with continuously differentiable first and second order derivatives, and \( f'(k) > 0, f''(k) < 0, f(0) = 0, f'(0) = \infty \).

If the home country's tax rate on foreign capital income is \( \tau \), \((1-\tau)f'(k)k_f\) would be repatriated to foreign country by foreign investors (or retained by foreign investors). Thus in the home country the per capita gross national product, \( y \), is

\[
y = f(k) - (1 - \tau)f'(k)k_f = f(k) + (\tau - 1)f'(k)k_f
\]  

(2)

Suppose that home country's saving rate is \( s \), domestic capital accumulation by home country is described by following dynamics:

\[
\dot{k}_d = sy - nk_d = s[f(k) + (\tau - 1)f'(k)k_f] - nk_d
\]  

(3)

Foreign investors maximize expected profits and adjust their investment according to the return differential between the world interest rate and host country's (here home country's) capital returns:

\[
\dot{k}_f = \beta[(1-\tau)f'(k) - r^*]
\]  

(4)

where the constant \( \beta \) is larger than zero, \((1-\tau)f'(k)\) is net return (after tax return) on foreign capital, and \( r^* \) is the alternative net return on foreign capital in the world capital market.

Giving the parameters \( s \) and \( \tau \), equations (3) and (4) constitute a pair of dynamic systems in \( k_d \) and \( k_f \). In equilibrium, we have
\[ s \ddot{y} - n \ddot{k}_d = 0 \quad (5) \]
\[(1-\tau)f'(\ddot{k}) - r^* = 0 \quad (6)\]

where a bar above a variable denotes the equilibrium value of the variable.

To say the stability of the equilibrium, we linearize equations (3) and (4) around the equilibrium values of \(k_d\) and \(k_F\):

\[
\begin{bmatrix}
\dot{k}_d \\
\dot{k}_F
\end{bmatrix} =
\begin{bmatrix}
s\delta y/\delta k_d - n & s\delta y/\delta k_F \\
\beta(1-\tau)f''(\ddot{k}) & \beta(1-\tau)f''(\ddot{k})\delta
\end{bmatrix}
\begin{bmatrix}
k_d - \ddot{k}_d \\
k_F - \ddot{k}_F
\end{bmatrix} \quad (7)
\]

It is easy to see that both \(\delta y/\delta k_d\) and \(\delta y/\delta k_F\) in (7) are positive, by differentiating (2).

\[
\begin{align*}
\delta y/\delta k_d &= f'(k) + (\tau - 1)f''(k)k_F > 0 \\
\delta y/\delta k_F &= f'(k)(\delta - 1 + \tau) + (\tau - 1)f''(k)\delta k_F^2 > 0.
\end{align*}
\]

which is simply to say, increase in capital stock always brings about more national income. Knowing the signs of \(\delta y/\delta k_d\) and \(\delta y/\delta k_F\), we are ready to analyze the properties of the equilibrium. Firstly, we will show that there exists a stable equilibrium if the growth rate of labor force is larger than the product of saving rate \(s\) and marginal contribution of domestic capital to national product \(\delta y/\delta k_d\), i.e.

\[ n > s\delta y/\delta k_d \quad (8) \]

Suppose that inequality (8) holds, the trace of the matrix in (7) is negative as \([s\delta y/\delta k_d - n] + [\beta(1-\tau)f''(k)\delta] < 0\); and the determinant of the matrix is positive as \([s\delta y/\delta k_d - n][\beta(1-\tau)f''(k)\delta] - s\delta y/\delta k_F\beta(1-\tau)f''(k) > 0\). So we obtain two negative eigenvalues for the linearized dynamic systems, and the systems are stable. The phase diagram is in figure 1 below.
The curve for equation $\frac{dk_f}{dt} = 0$ is downward sloping, its slope is given by $(-1/\delta)$. With the assumption that $n > s\delta y/\delta k_d$, the curve for $\frac{dk_d}{dt} = 0$ is upward sloping, and its slope is $(n - s\delta y/\delta k_d)/s\delta y/\delta k_f$. Below $\frac{dk_f}{dt} = 0$, for any given amount of $k_f$, $k_d$ is relatively small, and $(1-\tau)f'(k)$ is higher than $r^*$, so $k_f$ will increase. The converse argument goes for capital stock above the line of $\frac{dk_f}{dt} = 0$. Similarly, below $\frac{dk_d}{dt} = 0$; for any given $k_f$, $k_d$ is relatively small. To maintain equilibrium, $k_d$ has to increase. And vice versa. Therefore the dynamic path spirals inward clockwise in $(k_d, k_f)$ space of figure 1.

In this model, an increase in tax rate on foreign capital income will always reduce foreign direct investment in the home country, but its effect on home country's capital stock is ambiguous. This can be easily seen from figure 2. As a result of higher tax $\tau$, $\frac{dk_f}{dt} = 0$ curve shifts inward because, for any initial equilibrium values of $k_d$ and $k_f$, $[(1-\tau)f'(k) - r^*]$ is less than zero; foreign investors will reduce their investment in host country. For $\frac{dk_d}{dt} = 0$ curve, higher tax rate raises more tax revenue and leads to higher national income initially, the curve shifts downward accordingly. In new equilibrium, $k_f$ is reduced, and $k_d$ may go up or down. In the diagram, we depict a
situation where both $k_f$ and $k_d$ decrease as a result of higher tax on foreign capital.

![Figure 2: Tax Increase](image)

The economic explanation for above results is simple: High rate of capital tax reduces foreign capital stock. The total tax revenue may increase or decrease depending on the elasticity of foreign investment with respect to tax (a case of familiar Laffer curve of taxation). If high tax raises more total tax revenue, $k_d$ is expected to go up; if high tax lowers tax revenue, $k_d$ will go down. This naturally leads to the optimal taxation by the home country’s government, which we will deal with in next section.

Another interesting case is the effects of change of saving rate on foreign and domestic capital. Suppose that saving rate $s$ is up. The curve $\frac{dk_f}{dt} = 0$ is unchanged, but $\frac{dk_d}{dt} = 0$ shifts downward. In the new equilibrium, $k_d$ is higher, and $k_f$ is lower. See figure 3.
This approach to study foreign direct investment and domestic capital accumulation in the home country is much in the spirit of Findlay [1978]. In reality, many countries, especially many developing countries, do make deliberate choices about the tax on foreign capital income and saving for domestic investment. Thus a more elaborate model is needed to describe the behaviors of governments in many countries. That will be our next task.

III. Optimal Choices of Taxation and Saving by the Governments

Following Bardhan (1967), we define governments' preferences on per capita consumption and foreign capital stock in home country. The preferences are separable in consumption and capital:

\[ U(c, k_f) = u(c) + v(k_f) \]  \hspace{1cm} (9)

An open door government is defined to be one deriving nonnegative utility from foreign capital's presence in home country, that is to say, \[ v'(k_f) \geq 0 \]
for open door government. On the contrary, a close door government dislikes foreign capital and derives disutility from foreign capital, i.e., $v'(k_f) < 0$. Of course, for both governments, $u'(c) > 0$. In practice, we should have no difficulty in identifying these two kinds of governments.

Let both kinds of governments maximize their long-run welfare by choosing tax rate on foreign capital income and domestic saving rate subject to the two dynamic constraints: equations (3) and (4). Write out the optimal problem fully:

$$\max \int_0^\infty \left[ u(c) + v(k_f) \right] e^{\rho t} dt$$

subject to

$$c = (1 - s)y = (1 - s)[f(k) + (r - 1)f'(k)k_f]$$

$$k_d - s[f(k) + (r - 1)f'(k)k_f] + nk_d$$

$$k_f - \beta[(1 - r)f'(k) - r^*]$$

where $\rho$ is the time discount rate, and $0 < \rho < 1$.

To avoid boundary solution $s = 1$, and to have sufficiency of the first order conditions, the following assumptions are made: $u'(0) = \infty$, $u''(c) < 0$ and $v''(k_f) < 0$. By substituting expression (11) into the objective function, we have the Hamiltonian function $H$,

$$H = e^{\rho t} \left\{ u[(1 - s)(f(k) + (r - 1)f'(k)k_f)] + v(k_f) + \lambda_1[s(f(k) + (r - 1)f'(k)k_f)] 
- nk_d + \lambda_2[\beta(l - r)f'(k) - r^*] \right\}$$

(12)
The first order conditions are

\[
\frac{\partial H}{\partial s} = e^{-\rho t}[u'(c)y + \lambda_1 y] = 0 \tag{13}
\]

\[
\frac{\partial H}{\partial r} = e^{-\rho t}[u'(c)(1-s)f'(k)k_f + \lambda_1 sf'(k)k_f - \lambda_2 \beta f'(k)] = 0 \tag{14}
\]

\[
\frac{\partial H}{\partial \lambda_1} = -\frac{d(e^{-\rho t}\lambda_1)}{dt} \tag{15}
\]

\[
\frac{\partial H}{\partial \lambda_2} = -\frac{d(e^{-\rho t}\lambda_2)}{dt} \tag{16}
\]

and dynamic constraints (3) and (4), plus transversality conditions.

By equation (13) and (14), we get the shadow prices of domestic capital and foreign capital:

\[
\lambda_1 = u'(c) \tag{17}
\]

\[
\lambda_2 = u'(c)k_f/\beta \tag{18}
\]

With (17) and (18), optimal conditions (15) and (17) can be written as

\[
\frac{u''(c)}{u'(c)} \frac{dc}{dt} = n + \rho - f'(k) \tag{19}
\]

\[
d\lambda_2/dt = -[u'(c)(\delta - 1 + \tau)f'(k) + \nu'(k_f)] + \rho u'(c)k_f/\beta \tag{20}
\]

In steady state, we obtain

\[
\beta'(1 - \tau)f'(k) - \lambda_2 = 0 \tag{21}
\]

\[
s[f(k) + (\tau - 1)f'(k)k_f] - nk_d = 0 \tag{22}
\]

\[
n + \rho - f'(k) = 0 \tag{23}
\]

\[
-[(\delta - 1 + \tau)f'(k)u'(c) + \nu'(k_f)] + u'(c)k_f \rho/\beta = 0 \tag{24}
\]

Given specific forms of production function and utility function, we can
solve these equations and find out steady state levels of $\tau$, $s$, $k_d$, and $k_f$.

Without going to special technology and preference, following interesting results can be easily derived.

**Result 1:** Optimal aggregate capital stock is the same under both kinds of governments, and its level is determined by the modified golden rule.

This is just a statement of equation (23), which is exactly the modified golden rule as in Cass (1965), and, in particular, independent of government preferences. This result may be a little bit surprising for our model is so different from the original Cass model.

**Result 2:** The optimal tax rate on foreign capital income is the same for both kinds of governments:

$$
\tau = \frac{n + \rho - r^*}{n + \rho}
$$

(25).

To see this, we substitute the modified golden rule into equation (21). The result follows immediately. From intuition, the close door government should tax more on foreign capital income than the open door government. That is possible in the present model only if the close door government has higher discount rate than the open door government. Another way to obtain different tax rates for different governments is to follow Kurz (1968) and introduce both domestic capital and foreign capital into utility functions ... so called wealth effects. We will discuss this possible extension at the end of this paper.
From (25), we note that the higher the time discount rate, the higher the tax rate:

\[ \frac{\partial r}{\partial \rho} = \frac{r^*}{(n + \rho)^2} > 0 \]  \hspace{1cm} (26)

Similarly, the faster the population growth, the more the governments need to provide people with the same consumption and capital goods. So governments tax more to meet the growing demand:

\[ \frac{\partial r}{\partial n} = \frac{r^*}{(n + \rho)^2} > 0 \]  \hspace{1cm} (27)

But obviously enough, the higher the alternative return on foreign capital, the less the tax rate:

\[ \frac{\partial r}{\partial r^*} = - \frac{1}{(n + \rho)} < 0 \]  \hspace{1cm} (28)

Result 3: At optimum, the marginal benefit from foreign capital is equal to the marginal cost.

This comes from equation (24),

\[ u'(c)[(\delta - 1)f'(k)] + u'(c)r f'(k) + v'(k_f) = u'(c)k_{f^0} \frac{\rho}{\beta} \]

On the left hand side of above equation, the first term is the gain in technological efficiency contributed by foreign capital (recall \( \delta \geq 1 \)), the second term is the tax revenue from foreign capital, and the third term is the direct utility gain (loss) if \( v'(k_f) > 0 \) (\( v'(k_f) < 0 \)). So the left hand side is the net marginal benefit resulting from foreign capital. The right hand side, \( u'(c)k_{f^0} \frac{\rho}{\beta} \), is just \( \lambda_2 \), the imputed price of foreign capital.

Result 4: Foreign capital per capita under open door government is higher than under close door government.

This is what we expect. To show this result, we denote foreign capital per capita under open door and close door governments as \( k_{f^0} \) and \( k_{f^c} \).
respectively, consumption per capita as \( c^o \) and \( c^c \), and saving rates as \( s^o \) and \( s^c \). We write equation (24) for open door government and close door government separately (note \( f'(k) = n + \rho \) in steady state):

\[
u'(c^o)[(\delta - 1 + \tau)(n + \rho) - k^o_f \rho / \beta] = -\nu'(k^o_f)
\]

and

\[
u'(c^c)[(\delta - 1 + \tau)(n + \rho) - k^c_f \rho / \beta] = -\nu'(k^c_f)
\]

For open door government, \( \nu'(k^c_f) > 0 \). That requires

\[((\delta - 1 + \tau)(n + \rho) - k^o_f \rho / \beta) < 0. \]

For close door government, \( \nu'(k^c_f) < 0 \). That requires

\[((\delta - 1 + \tau)(n + \rho) - k^c_f \rho / \beta) > 0. \]

Therefore,

\[
k^o_f > (\delta - 1 + \tau)(n + \rho) \beta / \rho > k^c_f
\]

which is what we want to demonstrate.

**Result 5:** If \( (r^*/\delta) > n \), consumption per capita under open door government is lower than under close door government; if \( (r^*/\delta) < n \), consumption per capita under open door government is higher than under close door government. They are equal if \( (r^*/\delta) = n \).

**Proof:** In steady state,

\[
c = (1 - s)y = y - nk_d = f(k) + (\tau - 1) f'(k)k_f - nk_d \tag{30}
\]

Substituting \( f'(k) = n + \rho, \tau = (n + \rho \cdot r^* ) / (n + \rho), \) and \( k_f = (k \cdot k_d) / \delta \) into equation (30), we get

\[
c = f(k) - (r^*/\delta)k - (n - r^*/\delta)k_d \tag{31}
\]

Since \( k \) and \( f(k) \) are the same for both governments in steady state,

\[
c^o = f(k) - (r^*/\delta)k - (n - r^*/\delta)k_d^o
\]

and
\[ c^c = f(k) - (r^*/\delta)k - (n - r^*)k^c_d \]

So

\[ c^c - c^o = (n - r^*/\delta)(k^o_d - k^c_d). \]

We know \( (k^o_d - k^c_d) \) is negative as \( k^o_f > k^c_f \) (from equation (29)) and \( k \) is the same for both governments in steady state. Hence

\[ c^c > c^o \text{ if } n < r^*/\delta; \]
\[ c^c < c^o \text{ if } n > r^*/\delta; \]
\[ c^c = c^o \text{ if } n = r^*/\delta. \]

That is to say, for given world capital return \( r^* \) and home country's population growth rate \( n \), if foreign capital is much more technologically efficient than domestic capital in home country, the open door government provides more consumption than the close door government. This is because larger foreign capital under open door government brings about higher tax revenue and greater efficiency gain. It is of some interest to note a special case when \( \delta = 1 \), i.e. foreign capital and domestic capital have the same efficiency level. In this case, the comparison between domestic population growth rate and world capital return provides a simple criterion to assess the welfare enjoyed by the people. If population growth rate is larger than world interest rate, open door government will perform better by the criterion of consumption per capita. If population growth rate is smaller than the world interest rate, close door government will be favored by the people. This line of arguments by comparing population growth with capital return definitely reminds us of assessing dynamic efficiency in a close economy.

Result 6: Steady state saving rate under open door government is lower than
under close door government.

By substituting the same tax rate $\tau$ and modified golden rule into (22),

$$ s [f(k) - r^* k_f] - nk_d = 0 $$

Namely,

$$ s = \frac{nk_d}{f(k) - r^* k_f} $$

For the two governments, the saving rates are

$$ s^c = \frac{nk^c_d}{f(k) - r^* k^c_f} \quad \quad s^o = \frac{nk^o_d}{f(k) - r^* k^o_f} $$

Therefore

$$ s^c - s^o = \frac{n(k^o_f - k^c_f)[\delta f(k) - (1-\tau)f'(k)k]}{(f(k) - r^* k^c_f)(f(k) - r^* k^o_f)} \tag{32} $$

which is positive since $(k^o_f - k^c_f) > 0$, and $[\delta f(k) - (1-\tau)f'(k)k] > f(k) - f'(k)k = \text{marginal returns on labor input} > 0$. (Recall $\delta \geq 1$)

IV. A Possible Extension of the Optimal Choice Model

A straightforward extension of the model in last section is to define governments' preferences on both foreign capital and domestic capital --- a hybrid preference of Bardhan (1967) and Kurz (1968). The justification may be borrowed from essay three. For example, the governments' utility function may be separable in consumption, domestic capital and foreign capital:
$$U(c, k_d, k_f) = u(c) + v(k_f) + w(k_d).$$

Of course, for both governments, \( w'(k_d) > 0 \).

While this approach is interesting, we can hardly get any simple result from it owing to the existence of multiple equilibria. It will be true that optimal choices of capital stock and tax rate are different for different governments. It may be possible to find out a specific example as in the first three essays of this thesis to illustrate the differences in optimal choices between open door government and close door government. But we find that, even relying on a conventional approach in international economics, the results in section three are interesting and surprising.
References


