The Fogel Approach to Health and Growth

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Abstract

According to Robert Fogel (1994a, 1994b), nutrition is the driving force for the increase in health human capital, which in turn has significantly promoted economic growth in the long run. In this paper, we take Fogel's finding to extend the standard Ramsey model by including the effect of consumption on nutrition and health human capital formation. It is demonstrated that there exist multiple equilibria in the modified Ramsey model with a subsistence level of consumption. That is to say, different countries may end up with different levels of long-run consumption, nutrition, health human capital, and physical capital.

Key words: Health Human Capital; Consumption; Economic Growth; Poverty Trap

JEL Classification: D990, E210, I120.

1. Introduction

The relationship between health and economic growth has long been attracting researchers as well as practitioners from many disciplines including economics, sociology, physiology, and others. Among the literature focusing on health and economic growth, there is a set of fairly quantitative historical case studies (Fogel, 1994a, 1994b, 2002, 2004; Strauss and Thomas, 1998) claiming that nutritional improvement is the driving force behind improvements in the health component of human capital (henceforth referred to as “health human capital”) and hence the primary force promoting economic growth in the long term. For instance, based on his series of studies, Fogel concluded that health and nutritional improvements may have accounted for more than 50% of British annual per capita income growth rate of about 1.15% in the 200-year period from 1780 to 1979 (Fogel, 1994a, 1994b, 2002). Using a methodology similar to Fogel’s, Sohn (2000) argued that improved nutrition has increased available labor inputs in the Republic of Korea by one percent a year or more between 1962 and 1995. Strauss and Thomas (1998) also indicated that nutrition and caloric intakes are positively related to health, measured in height and body mass index (BMI), in the case of post World War II America, Brazil and Vietnam. These studies indicate that health improvements derived from consumption increases, and hence nutritional improvements, are the main force that enhances long-run economic growth. If an increase in consumption can improve health human capital, can this category of health human capital be a driving force that motivates persistent long-run economic growth? If yes, then what is the mechanism by which health human capital motivates long-run growth in the theoretical framework? If not, then how does health human capital enhance long-run economic growth? This is the first question this paper addresses.

There is an alternative view to Fogel’s, however. For the interaction between health and income, health is always regarded as another kind of human capital, like education. Nevertheless, health differs from education in many respects[*] and the effect of health on economic growth is different from the effects of other kinds of human capital. First, since health can generate positive utility of its own, using health services is also a kind of consumption (Grossman, 1972).This implies that health cannot motivate long-run economic growth but is only a by-product of such growth (Baumol, 1967, Zon & Muysken, 2001, 2003), which seems to be contrary to Fogel’s result. One should note here that Zon & Muysken (2001, 2003) only analyzed the health human capital derived from health investment but not that from consumption improvement and hence nutritional improvement. If Fogel’s conclusion is true, does it imply that the health human capital

[*] Strauss and Thomas (1998) claimed that, at the microeconomic level, there are at least three aspects that distinguish health from most other human capital: first, health will vary over one’s life course compared with education, which is almost constant in one’s life cycle. Second, health is multidimensional, and it is important to differentiate among these dimensions. Third, measuring health is difficult, and in many cases, measurement error is likely to be correlated with outcomes of interest like income.

derived from nutritional improvement embodied in the increase in income and consumption can motivate the long-run economic growth? As a by-product of economic growth, how does the category of health become one main force improving economic growth in the long run? This paper aims to 1) explore how and how much the health improvement derived from increased income and consumption affects long-run economic growth and 2) explain the rationality of Fogel's conclusion that health improvements from increasing nutritional intake affects long-run economic growth in a theoretical framework. In addition, this paper sheds light on the mechanism of economic growth when consumption affects health and hence labor productivity, and to see whether this kind of human capital will bring endogenous economic growth, like education human capital, or whether it remains a by-product of economic growth, like what Baumol (1967) and Zon and Muysken (2001, 2003) have concluded.

Needless to say, in order to comprehend the effects of health on growth, it is a prerequisite that we explore clearly the complicated interaction mechanism between the two since there are so many channels through which health and growth affect each other. For instance, healthier populations tend to have higher labor productivity, which facilitates growth (Strauss & Thomas, 1998; Barro, 1996; Bloom, etc. 2004). Health improvement is also inclined to increase education human capital and hence improve growth (Howitt, 2000). In addition, health may influence economic growth through mortality, longevity, fertility, and population composition as well as other channels (Kalemli-Ozcan, Ryder & Weil, 2000; Kalemli-Ozcan, 2002, 2003; Morand, 2004). On the other hand, growth is able to influence health through increasing income. For example, economic growth increases income per capita, and people with higher income can increase their health investment (medical care and cure) and hence improve their health (Grossman, 1972). People with higher income are also able to improve their nutritional intake, which can improve their health (Strauss and Thomas, 1998, Fogel, 1994a, 1994b, 2002). It is beyond the scope of the present paper to explore all possible mechanisms through which health and growth interact with each other. Instead, we focus on one of the mechanisms suggested by Fogel that nutritional improvement can enhance the improvement of health human capital and we analyze the effects of health improvement on long-run economic growth when consumption affects labor productivity via health. By assuming consumption not only increases agents' utility but improves agents' health, we investigate the relationship between consumption, health, and capital accumulation, and discuss the effect of health on economic growth in an extended Ramsey model.

Under the assumption that consumption improves health, this paper shows that, first, the category of health human capital derived from consumption improvement and hence nutritional improvement is not the cause but rather the by-product of economic growth except for the case of a linear health generation function with sufficiently large marginal health productivity of consumption[*], which is suggested by Baumol (1967) and argued by Zon and Muysken (2001, 2003). Second, health human capital is able to magnify the economic growth driven by exogenous technology, which is consistent with Fogel's finding that nutritional improvement is the main force that enhances health human capital improvement and hence economic growth in the long term. We also formulate the proportion of economic growth from health improvement to total economic growth. By this formulation, under some reasonable parameter values, the contribution of health improvement derived from nutrition to the total economic growth is very close to Fogel's (1994a, 1994b, 2002) estimates. In addition, through some special health generation functions, the paper displays the possibility of the existence of multiple equilibria of capital stock, health, and consumption and the existence of the poverty trap, which is consistent with the reality that rich countries may end up with higher capital, better health, and higher consumption than poor countries.

This paper is organized as follows. Section 2 reviews the existing literature on health and growth. Section 3 presents a theoretical model with health generated by consumption. Section 4 analyzes the dynamics of physical capital and health human capital in an exogenous neoclassical growth model. Multiple equilibria and a poverty trap have been found in this framework with different health generation functions. Section 5 presents our conclusions.

2. Literature Review

[*]It is easy to prove that health derived from consumption improvement and hence nutritional improvement can motivate endogenous economic growth in the case of a linear health generation function with large enough marginal health productivity of consumption, however we do not include this case in this paper.

There are increasing theoretical and empirical investigations of the effect of health on economic growth. The empirical studies mainly form three categories (Jamison, et al., 2004). The first comprises the historical case studies that may be more or less quantitative (Fogel, 1994a, 1994b, 2002; Strauss and Thomas, 1998; Sohn, 2000). As stated above, these studies all concluded that nutritional improvement is the main force that enhances health human capital improvement and hence economic growth in the long term. The second category is characterized by many "micro" studies which involve either household surveys that include one or more measures of health status along with other extensive information, or the assessment of the impact of specific diseases. Strauss and Thomas (1998) provided a major review (extensively updated by Thomas, 2001), and Savedoff and Schultz (2000) surveyed methods used in the household studies and summarized findings of recent analyses from five Latin American countries. Recent studies include Liu et al (2003) on China and Laxminarayan (2004) on Vietnam. This literature confirms that health is positively associated with productivity on the micro level, which is consistent with our assumption that health human capital constitutes a type of production factor. The third category studied the relationship between health and economic growth from a macroeconomic perspective. These studies mainly rely on cross-national data to assess the impact of health at the national level, measured in life expectancy, adult survival rates, adult mortality rates or other indexes, on income growth rates and most confirmed that health is positively related to growth (World Bank, 1980; Hicks, 1979; Wheeler, 1980; Barro, 1996; Sachs & Warner, 1997; Bloom and Williamson, 1998; Casas, 2000; Mayer, et al, 2000; Arora, 2001; Bhargava, et al, 2001; Bloom et al., 2004; Lorentzen, McMillan and Wacziarg, 2005; McDonald and Roberts, 2006). On the microeconomic and macroeconomic contribution of health to economic growth and development, Shurcke et al. (2006) reviewed recent evidence.

The theoretical studies on the relationship between health and growth started to appear in the last 20 years and are becoming more frequent. Early theoretical studies on this topic mainly focused on the provision of health services from a microeconomic demand perspective and did not analyze the effect of health in the form of human capital driving economic growth and development (Grossman, 1972; Muurinen, 1982; Forster, 1989; Ehrilch and Chuma, 1990; Johansson & Lofgren, 1995; Mertzer, 1997). Barro

(1996) first studied the macroeconomic effects of health as one of the most important components in human capital in a theoretical framework. In a three-sector neoclassical growth model incorporating a concept of health human capital as well as schooling capital, Barro analyzed the effects of health human capital on schooling capital and physical capital and the interaction between these three factors, and further discussed the effects of public policy in the case of health services as a publicly subsidized private good and as a public good. Muysken, et al. (1999) also investigated the growth implications of endogenous health on steady-state growth and transition dynamics in a standard neo-classical growth framework.

Extending the Lucas (1988) endogenous growth model to include health investment and taking into account that health services can provide utility, Zon and Muysken (2001, 2003) discussed the macroeconomic effects of health investment on economic growth. Compared to Barro (1996), besides the effect of health on labor productivity, Zon and Muysken (2001, 2003) considered three other channels through which health influences economic growth: 1) health increases the accumulation of education human capital; 2) health services increase an agent's utility; and 3) health improvement increases longevity and hence leads to an aging population. While the first two effects of health on labor productivity and on education human capital accumulation tend to facilitate economic growth, the last two effects suggest that health investment may exceed the optimal level at which the marginal contribution of health investment to growth equals the marginal cost. This may crowd out resources which could have been used for physical capital investment. Therefore, in such a situation, health investment may impede the progress of economic growth in the end. By introducing the effects of skill-driven technological change (henceforth SDTC) into the Zon and Muysken (2001, 2003) framework, Hosoya (2002, 2003) further investigated the relationships among economic growth, average health level, labor allocation, and longevity of the population in an endogenous growth model that integrates SDTC and human capital accumulation through formal schooling with health human capital accumulation. In addition, through integrating the accumulation of human capital, innovation in medical technology, health and longevity into a four-sector (education, consumption goods, R & D sector devoted to health research, and health goods) endogenous growth model with "keeping up with the Jones" preferences and an altruism utility function, Sanso and Asia (2006) also studied the bidirectional interaction between health and economic growth and concluded that health, by influencing longevity, may become an endogenous growth source.

In order to explain the real-world situation that rich countries may end up with higher capital, better health, and higher consumption than poor countries, the existence of multiple steady states and the poverty trap are also important issues in the literature on the relationship between health and economic development. Chakraborty (2004) and Bunzel and Qiao (2005) introduced endogenous mortality risk into a two-period overlapping generations model to study the effect of health (measured in mortality) on economic growth and confirmed the existence of multiple steady states. Hemmi, Tabata and Futagami (2006) studied the interaction between decisions on financing after-retirement health shocks and precautionary saving motives, and demonstrated that, at low levels of income, individuals choose not to save to finance the cost of after-retirement health shocks. However, once individuals become sufficiently rich, they do choose to save to finance the cost of these shocks. Therefore, this change in the individual saving behavior may also give rise to multiple steady state equilibria and result in the poverty trap.

The results of those theoretical papers are important for understanding the results of the two latter empirical studies discussed above, and also for both further academic research and policy. However, interpreting the results of the first empirical studies in the context of the theoretical framework and understanding how health improvements derived from consumption and nutritional improvement will influence the long term growth and hence economic policy is not clear. Through considering the effect of health improvements derived from consumption and nutritional improvement on economic growth, this paper fills the gap in the literature by proposing a carefully constructed theoretical framework to study the relationship between economic growth and health. We believe that the theoretical models can help us to analyze the relationships of health and long-run growth effectively, comprehensively, and systematically. Given the importance of the issues investigated here, our results will provide both the public and the private sectors with policy guidelines towards a better allocation of resources to improve the health of a population, reduce income inequality, and at the same time maintain economic growth.

3. The Benchmark Model

The goal of this paper is to consider the effect of health derived from consumption–which thus leads to nutritional improvement—on income and economic growth. Specifically, we mainly focus on the international mechanism that links health and economic growth where consumption affects health human capital. To this end, we consider an economy such as the following: there is an agent with infinite life in the economy. The agent has a quantity of physical capital and one unit of labor. The agent's income comes from his output which is produced with the use of two factors: physical capital and the agent's labor. Labor ability is determined by agent's health human capital. The agent decides how to divide the output between consumption and investment. When the agent consumes the product, he can attain utility. At the same time, as per Fogel's studies (1994a, 1994b and 2002), more consumption brings more nutrition to the agent (assuming the agent consumes food), and hence the agent also attains health human capital through consumption, which can improve his productivity in the next period. When the agent uses his product to invest, he can increase physical capital, which also makes him increase his production in the next period.

Suppose the instantaneous utility function is $u(.)$, and the subjective future discounting rate $\beta \in (0,1)$, then the agent lifetime utility function is given by

$$\int_{t=0}^{\infty} u(c(t)) e^{\beta t} dt , \tag{1}$$

Without loss of generality, we assume $u'(.) > 0$ and $u''(.) > 0$. The main channel through which health affects the economic growth lies in the production function, in which health can improve the efficiency of labor productivity. In the paper, the production function is assumed as follows

$$y = f(k, hl), \tag{2}$$

where y denotes the agent's product, k denotes physical capital, h denotes health human capital, and l labor supply. Compared with the commonly used neoclassical production function, the uniqueness of the above production function lies in the fact that we include health human capital in the production function. In fact, the existing literature points out several channels by which better health will raise productivity and output. Most directly, healthier workers have more energy and endurance and are able to work harder and longer. People with healthier bodies tend to be less susceptible to disease and have lower absenteeism. The fact that labor productivity is positively associated with health has been confirmed both in empirical micro- and macro-economic studies, especially in low-income settings (Strauss and Thomas, 1998, Bloom, et al, 2004). In addition, there are some indirect channels through which health influences productivity. For instance, improvements in health raise the incentive to acquire schooling, since investment in schooling can be amortized over a longer working life. Healthier students also have lower absenteeism and higher cognitive function, and thus receive a better education for a given level of schooling (Howitt, 2005; Kalemli-Ozcan, et al., 2000; Weil, 2006). All these factors lead to healthier people with higher productivity. Therefore, it is rational and natural for the health variable to enter the production function, just as Barro (1996), Issa (2003), Hosoya (2002, 2003), Muysken, et al. (1999), Zon and Muysken (2001, 2003), Weil (2006) and others have argued. Furthermore, just as Fogel observed (2002, p.24), the contribution of nutrition and health to economic growth may be thought of as labor-enhancing technological changes. Zon and Muysken (2001, p. xiii) also considered the contribution of health human capital to production ability as Harrod-neutral technical change.

In addition, we assume that

$$f_h > 0, f_k > 0, f_{hh} < 0, f_{kk} < 0\,, f_{hk} > 0\,, f_{kk} f_{hh} > f_{hk}{}^2 \tag{3}$$

which implies that the marginal productivity of physical capital and health human capital are positive but diminishing, and the production function exhibits convex technology in h and k.

The second main aspect of the interaction mechanism between health and economic growth in this paper lies in the effect of income on health through consumption and hence through nutritional improvement. As most economists observe, there are three main ways of improving individuals' health: First, sufficient nutrition is indispensable to maintaining a healthy body. Fogel (1994a, b, 2002) and Strauss and Thomas (1998) indicate that--measured in life expectation, in height, and in the ratio of height to weight--an increase in nutrition is the main factor which improve populations' health in the long run in many countries, including England, France, the United States, Vietnam and others. In the case of the underdeveloped periods of developed countries or the presently low- and middle-income countries, the main approach to improve health is still to increase nutrition and caloric intakes which are mainly embodied in food consumption. The second approach to improving health is health investment (Grossman, 1972; Strauss and Thomas, 1998; Zon and Muysken, 2001, 2003). According to Grossman (1972), health investment includes the consumer's time devoted to health care and market goods such as medical care, diet, exercise, recreation, and housing, which is obviously included in total consumption. Moreover, health investment may also include individuals' medical cure activities when he/she is stricken with disease or infection, since these activities can shorten the duration of ill health and avoid accidental death caused by illness (Zon and Muysken, 2003). The third way of improving health may be related to the individual's knowledge of health protection and life behavior (Howitt, 2005; Sanso and Asia, 2006).

Since the goal of this paper is to study the relationship between health and growth in the long term, we mainly focus on the health derived from improvements in nutrition and consumption. In the long term, as Fogel (1994a, 1994b, 2000) and Strauss and Thomas (1998) indicated, income and hence total consumption exert the greatest force on motivating improvements in health. To this end, we assume that health is mainly determined by agents' consumption, and people with more consumption will be much healthier, although other important factors also determine health quality. Furthermore, we assume that the health generation function is as follows[*]

$$h = h(c) \tag{4}$$

We assume that marginal productivity of consumption is nonnegative and nonincreasing, namely

[*] Note that in equation (3), health is considered as a flow variable rather than a stock variable and hence no depreciation is allowed. However, even if in the case that health is a stock variable and there exists health capital depreciation, the general conclusion of this paper is not affected.

$$h'(c) \geq 0, h''(c) \leq 0 \tag{5}$$

Here we assume that the function $h(c)$ is nondecreasing. Thus, with the increase of consumption, the health h will at least not decrease. Alternatively, we can assume that it is not a monotonic function. For example, there exists a consumption level, $\bar{c} > 0$, such that $h(c)$ increases when consumption is greater than $\bar{c}$; and $h(c)$ remains constant when consumption is less than $\bar{c}$. That is to say, we have $h'(c) \geq 0$, when $c > \bar{c}$; and $h'(c) = 0$, otherwise. We will discuss this type of health generation function involving a minimum consumption level in section 3.2.

If we further assume that the agent supplies unit labor inelastically at any time, then l equals 1. By the above assumption, the agent's physical capital accumulation equation is

$$\dot{k} = f(k, h(c)) - c - \delta k \tag{6}$$

where δ denotes the physical capital depreciation rate. A dot over a variable denotes the derivative of the variable with respect to time. The agent's optimization problem is that, given the initial physical capital, by choosing his consumption path, c, and his capital accumulation path, k, the agent maximizes his lifetime utility, i.e.

$$\max_{c,k} \int_{t=0}^{\infty} u(c(t)) e^{\beta t} dt$$

$$s.t.: \dot{k} = f(k, h(c)) - c - \delta k$$
$$given\ k_0$$

In order to solve the consumer's optimization problem, we define the Hamiltonian associated with the optimization problem

$$H = u(c) + \lambda[f(k, h(c)) - c - \delta k] \tag{8}$$

where λ is the costate variable representing the marginal utility of physical capital investment measured in utility. By the Pontryagin's Principle, we obtain the first-order conditions

$$\lambda = u'(c) + \lambda f_h(k, h(c)) h'(c) \tag{8}$$

$$\dot{\lambda} = \lambda\left[\beta + \delta - f_k(k, h(c))\right] \tag{9}$$

and the transversality condition $\lim_{t\to\infty} \lambda k e^{-\beta t} = 0$.

Proposition 1: under the above assumptions on the utility function, production function and health generation function, if and only if a pair of real numbers, ($c(t)$, $k(t)$), satisfies

$$1 > f_h(k, h(c))h'(c) \tag{10}$$

then the pair ($c(t)$, $k(t)$) satisfying equations (6), (8), (9) and the transversality condition maximizes the objective function.

Proof: (see appendix A)

Equation (8) asserts that the marginal value of physical capital investment equals the marginal value of consumption, which is the sum of the marginal utility of consumption and the contribution of consumption to production. From equation (8), we can express λ as a function of consumption and capital stock, $\lambda(c, k)$.

$$\lambda = \frac{u'(c)}{1 - f_h(k, h(c))h'(c)} \tag{11}$$

In equation (11), $f_h(k, h(c))h'(c)$ denotes the increase in production brought by increasing the unit consumption through increasing health human capital and hence improving productivity, and $1 - f_h(k, h(c))h'(c)$ denotes the cost of increasing the unit consumption measured in consumption goods, the right side of equation (11) represents the marginal value of increasing the unit consumption or/and the marginal cost of increasing the unit investment measured in utility. The left side of (10) represents the marginal value of investment. Therefore, Equation (11) implies that the agent divides his/her income between investment and consumption subject to the condition that the marginal value of investment equals the marginal cost. Compared with the standard Ramsey model, the uniqueness of this consumption optimal condition is that there is an additional $f_h(k, h(c))h'(c)$ in the denominator of the right side in the equation (11). If the consumption has no effect on health, i.e. $h'(c) = 0$, then equation (11) is the same as in standard Ramsey model.

By equation (11), we can understand why it must be that $1 > f_h(k, h(c))h'(c)$ for equation (10) when an agent's investment is optimal. Given any positive investment, as we can see from equation (11), if $1 \le f_h(k,h(c))h'(c)$, then the marginal value of investment measured in utility will be negative or zero. Since the marginal utility of consumption, $u'(c)$, is definitely positive, a decrease in investment or/and an increase in consumption always increases the utility. Therefore, if $1 \le f_h(k,h(c))h'(c)$, the agent who maximizes lifetime utility will keep increasing his/her consumption and decreasing his/her investment till the marginal value of investment becomes positive and equals the marginal cost of investment.

By the equation (11), we have the following short-run effects of consumption and capital stock on the marginal value of capital

$$\lambda_c = \frac{u_{cc}[1 - f_h(k,h(c))h'(c)] + u_c[f_{hh}(k,h(c))(h'(c))^2 + f_h(k,h(c))h''(c)]}{\left[1 - f_h(k,h(c))h'(c)\right]^2} < 0 \tag{12}$$

$$\lambda_k = \frac{u_c f_{hk}(k,h(c))h'(c)}{\left[1 - f_h(k,h(c))h'(c)\right]^2} > 0 \tag{13}$$

From equations (12) and (13), it is clear that when consumption increases, the marginal value of investment will decrease, which is the same as the standard Ramsey model. The difference between the two models is that the marginal value of investment decreases more in this model than in the standard Ramsey model, which results from the decreasing marginal health productivity of consumption ($u_c f_h(k,h(c))h''(c)$) and the decreasing marginal productivity of health ($u_c f_{hh}(k,h(c))(h'(c))^2$). However, when capital stock increases, the marginal value of investment will increase, compared with being constant in the standard Ramsey model. The intuition of this result is very obvious: in the standard Ramsey model, since the marginal cost of investment, $u'(c)$, has no relation to capital stock, the marginal value of the optimal investment, which equals $u'(c)$, has no relation to capital stock. But in our model, the marginal cost of investment, $u'(c)/[1 - f_h(k,h(c))h'(c)]$, is determined not only by consumption but also by capital stock. When capital stock increases, the marginal productivity of capital will increase, and hence the decrease in production brought by increasing the unit consumption will decrease. Consequently, with capital stock increasing, the marginal value of the optimal consumption and/or the marginal cost of the optimal investment will decrease, which results in the increasing marginal value of the optimal investment, λ.

By equations (6), (8), (9) and (11), we derive the dynamic equation of capital stock as follows

$$\dot{c} = -\frac{\lambda}{\lambda_c}\left[f_k(k,h(c)) - \delta - \beta\right] - \frac{\lambda_k}{\lambda_c}\left[f(k,h(c)) - c - \delta k\right] \tag{14}$$

Equations (6) and (14) determine the accumulation paths for capital stock and consumption. In the following sections, we analyze the dynamic behavior for the physical capital accumulation, consumption, and hence health human capital accumulation.

4. Dynamics of Physical Capital, Consumption and Health Human Capital

By equations (6) and (14), the consumption and the capital stock approach the steady-state value when $\dot{c} = \dot{k} = 0$. It can be characterized as

$$f(k,h(c)) - c - \delta k = 0 \tag{15}$$

$$f_k(k,h(c)) - \delta - \beta = 0 \tag{16}$$

Under the assumption of the neoclassical production function, the existence of a steady state is obvious, but we cannot guarantee its uniqueness. We will give examples for the existence of a unique steady state and multiple steady states in the following section. About the stability of the steady state, we have the following theorem:

Theorem 1: If and only if the steady state (c^*, k^*) satisfying equations (15) and (16) satisfies

$$\beta h'(c^*) f_{kh}(k^*,h(c^*)) + [1 - f_h(k^*,h(c^*))h'(c^*)] f_{kk}(k^*,h(c^*)) < 0, \tag{17}$$

then the steady state (c^*, k^*) in the economy is saddle-point stable. Otherwise, the steady state is divergent.

Proof: see Appendix B.

In general, however, we still cannot determine the stability and the uniqueness of the steady state. In order to further analyze the dynamic characters of the economy, we need to take further assumption on the form of the production and health generation function. Without loss of generality, we assume the production function is a linearly homogeneous function in k and h, i.e.

$$y = f(k,h) = Ag(k,h)\,, \tag{2'}$$

where A represents a technology parameter. Function $g(k,\ h)$ satisfies the following characteristics:

$$\omega g(k,h) = g(\omega k, \omega h),\ for\ \forall\,\omega > 0\,, \tag{18}$$

By assumption (3), the production function (2') satisfies

$$g_k > 0, g_h > 0, g_{kk} < 0, g_{hh} < 0, g_{hk} > 0, g_{kk}g_{hh} > g_{hk}{}^2 \tag{3'}$$

$$g(0,h) = g(k,0) = 0, \lim_{x_i \to 0} g(x_1,x_2) = +\infty, \lim_{x_i \to +\infty} g(x_1,x_2) = 0, i = 1, 2 \tag{3''}$$

Furthermore, we can rewrite the production function as follows

$$\hat{y} = y/h = Ag(k/h,1) \equiv A\hat{g}(\hat{k})\,, \tag{19}$$

A hat over a variable denotes the ratio of the variable to health. Obviously, $\hat{g}(\hat{k})$ satisfies

$$\hat{g}(0) = 0, \lim_{x \to \infty} \hat{g}(x) = +\infty, \hat{g}'(0) = +\infty, \lim_{x \to \infty} \hat{g}'(x) = 0 \tag{3'''}$$

Therefore, we can rewrite the equation (15) and (16) as follows

$$A\hat{g}(\hat{k}) - \hat{c} - \delta\hat{k} = 0 \tag{20}$$

$$\hat{g}_k(\hat{k}) = (\delta + \beta)/A \tag{21}$$

Under the assumptions of (2'), (3') and (3''), we have the following theorem 2 and proposition 2:

Theorem 2: Under the assumptions of (2'), (3') and (3''), there exist unique $\hat{k}^*$ and $\hat{c}^*$ that satisfy equations (20) and (21).

Proof: (Omit)

Proposition 2: Under the assumptions of (2'), (3') and (3''), the steady state (k^*, c^*, h^*) of the economy satisfying equations (15) and (16) is saddle-point stable if and only if

$$h'(c^*) < h(c^*)/c^* \Leftrightarrow \varepsilon_{hc} \equiv c^* h'(c^*)/h(c^*) < 1 \tag{22}$$

Otherwise, the steady state is divergent.

Proof: (See appendix C)

By Theorem 2 and Proposition 2, under the assumption of a linearly homogeneous production function, the stability and the uniqueness of the steady state are totally determined by the health generation function. In the following section, we will discuss these questions in the context of the various forms of the health generation function.

4.1 Unique Steady State: the Effect of Technology Progress on the Health and Labor Productivity.

In this subsection, we assume that the health generation function is a neoclassical function that satisfies

$$h(0) = 0, h'(c) \geq 0, h''(c) \leq 0, \lim_{c \to 0^+} h'(c) = +\infty, \lim_{c \to +\infty} h'(c) = 0 \tag{5'}$$

By this assumption, we have, following Theorem 2 and Proposition 2:

Theorem 3: Under the assumptions of (2'), (3'), (3'') and (5), there exists one and only one equilibrium of (k^*, c^*, h^*) which satisfies equation (15) and (16) and this steady state is saddle-point stable in the economy.

Proof: (See appendix D)

The economic intuition of Theorem 3 can be understood easily by looking at Figure 1. In Figure 1, Beeline *OE* denotes that the ratio of consumption to health human capital is constant and equals $\hat{c}^*$ and Curve *OEB* denotes the health generation function. By Theorem 2, in the steady state, the ratio of consumption to health human capital equals $\hat{c}^*$ and hence the equilibrium of (c^*, h^*) must be on the Beeline *OE*. At the same time, equilibrium (c^*, h^*) also must be on the health generation function *OEB*. According to

Figure 1, it is obvious that there exists one and only one equilibrium of (k^*, c^*, h^*) which satisfies equations (15) and (16). Furthermore, Figure 1 also indicates that this steady state is stable.

Insert Figure1 here

Theorem 3 indicates that *the health human capital derived from nutrition and consumption can not drive the long-run persistent economic growth if there is no exogenous technology progress*, although consumption can improve the health human capital and hence improve productivity in this model. *The category of health human capital, however, can magnify the economic growth derived from exogenous technology progress*. The economic intuition of the result can be explained easily through Figure 2.

Insert Figure2 here

Figure 2(A) indicates the determination of equilibrium (c^*, h^*), which is the same as in Figure 1 except that Beeline OE_1 denotes the equilibrium of $\hat{c}^*$ under the technology level A_1 and Beeline OE_2 denotes the equilibrium of $\hat{c}^*$ after technology progresses to A_2. Figure 2(B) indicates the determination of equilibrium of (k^*, h^*) and y^*. Isoquant curve is used to represent combination of health and capital that provide the same ratio of output to technology. In figure 2(B), Curves I, II and III denote three isoquant curves, whose level of output per unit of technology is y_3/A_2, y_2/A_2 and y_1/A_1, respectively. Beeline *OA* and *OC* in figure 2(B) denote that the ratio of physical capital to health human capital equals to $\hat{k}_1^*$ and $\hat{k}_2^*$, which are the equilibrium of $\hat{k}$ in the technology level A_1 and A_2, respectively. When the technology level is A_1, the steady state in Figure 2(A) is Point E_1 and the equilibrium of (c, h) is (c_1^*, h_1^*), and the steady state in Figure 2(B) is Point *A* and the equilibrium of (k, h) is (k_1^*, h_1^*), correspondingly. Therefore, the level of output per unit of technology under the technology level A_1 is y_1/A_1. When the technology level improves from A_1 to A_2, by equations (20) and (21), the ratio of consumption to health human capital will improve from $\hat{c}_1^*$ to $\hat{c}_2^*$ and the ratio of physical capital to health human capital from $\hat{k}_1^*$ to $\hat{k}_2^*$. As a result, the steady state in figure 2(A) will change from Point E_1 to Point E_2 and health human capital will improve from h_1^* to h_2^*. In Figure 2(B), correspondingly, the production state will change from Point A to Point C and physical capital will improve from k_1^* to k_2^*. Therefore, the equilibrium level of output per unit of technology will improve from Curve III to Curve I. From Figure 2(B) we can see, however, that if the health human capital remains unchanged, the production state will change from Point A to Point B and hence the level of output per unit of technology will only improve from Curve III to Curve II. Since the health human capital improves, the production state continues to change from Point B to Point C and the level of output per unit of technology continues to improve from Curve II to Curve I. Thereby, the difference between Curve I and Curve II is the contribution of health human capital to the output growth.

According to the above results, we can understand Fogel's results on health and long-run economic growth. Fogel has claimed (in 1994a, 1994b, 2002) that "the combined effort of the increase in dietary energy available for work, and of the increased human efficiency in transforming dietary energy into work output, appears to account for 50 percent of British economic growth since 1790" (Fogel, 1994a, p388), and he further considered that "the impact of nutrition on long-term economic growth accounts for most of the previously unmeasured increase in British total factor productivity." (Fogel, 2002, p27) By the above analysis we know that, although health derived from consumption and hence nutritional improvement cannot motivate long-run endogenous economic growth, the category of health human capital can magnify the economic growth from exogenous technology progress and hence contribute to long-run economic growth. We can even estimate the contribution of health on economic growth by the following analysis.

By equation (2'), we obtain

$$x_y = \dot{y}/y = x_A + \varepsilon_{yh} x_h + \varepsilon_{yk} x_k \tag{23}$$

where x_i denotes the growth rate of variable i, ε_{yk} denotes the partial elasticity of output with respect to physical capital and ε_{yh} denotes the partial elasticity of output with respect to Labor. In equation (23), the total factor productivity in the Solow model is x_A and the Solow residual is $x_A + \varepsilon_{yk} x_k$. By equations (4), (20) and (21), we obtain

$$x_h = \frac{\left[1-\delta/(\delta+\beta)\right]}{-\varepsilon_{\hat{g}'}\left[1/\varepsilon_{yk}-\delta/(\delta+\beta)\right](1/\varepsilon_h-1)}x_A \tag{24}$$

$$x_k = \frac{\left[1/\varepsilon_{yk}-\delta/(\delta+\beta)\right](1/\varepsilon_h-1)+\left[1-\delta/(\delta+\beta)\right]}{-\varepsilon_{\hat{g}'}\left[1/\varepsilon_{yk}-\delta/(\delta+\beta)\right](1/\varepsilon_h-1)}x_A \tag{25}$$

where $\varepsilon_h = h'(c)c/h(c)$ denotes the elasticity of health output with respect to consumption and $\varepsilon_{\hat{g}'} = \hat{g}''(\hat{k})\hat{k}\big/\hat{g}'(\hat{k})$ denotes the elasticity of marginal physical capital productivity with respect to physical capital. Since $\varepsilon_{\hat{g}'} < 0$, by equation (24), if technology level improves, health human capital and hence the productivity of labor will also increase. The ratio of economic growth derived from health human capital improvement to the total growth rate of per capita income, R_{hy}, and the ratio to Solow residual, R_{hS}, respectively are

$$R_{hy} \equiv \frac{\varepsilon_{yh}x_h}{x_A+\varepsilon_{yh}x_h+\varepsilon_{yk}x_k} = \frac{\varepsilon_{yh}}{\left(\varepsilon_{yk}-\varepsilon_{\hat{g}'}\right)(1/\varepsilon_h-1)\left[1/\varepsilon_{yk}-\delta/(\delta+\beta)\right]\Big/\left[1-\delta/(\delta+\beta)\right]+1} \tag{26}$$

$$R_{hS} \equiv \frac{\varepsilon_{yh}x_h}{x_A+\varepsilon_{yh}x_h} = \frac{\varepsilon_{yh}\left[1-\delta/(\delta+\beta)\right]}{-\varepsilon_{\hat{g}'}\left[1/\varepsilon_{yk}-\delta/(\delta+\beta)\right](1/\varepsilon_h-1)+\varepsilon_{yh}\left[1-\delta/(\delta+\beta)\right]} \tag{27}$$

By equations (26) and (27), we can compare the contribution in our model of health human capital improvement to economic growth to Fogel's results. In order to illustrate the effect of health human capital derived from consumption and hence nutritional improvement on the long-run economic growth when there is exogenous technology progress, by equations (26) and (27), some estimated values of R_{hy} and R_{hS} for various constellations of the parameter values are given in Table 1 and 2, respectively. In these two tables, according to the real economy, we take the capital depreciation rate (δ) and the discount rate (β) to both equal 0.1. The partial elasticity of output with respective to physical capital (ε_{yk}) varies from 0.1 to 0.75, the partial elasticity of output with respect to labor (ε_{yh}) from 0.1 to 0.9, and the health output elasticity of consumption (ε_h) from 0.1 to 1.[*]

Insert Table 1 here

[*] By the assumption of the Cobb-Douglas production function, we use $\varepsilon_{\hat{g}'} = \varepsilon_{yk} - 1$ in the table 1.

Table 1 indicates that the contribution of health to economic growth (R_{hy}) increases when the elasticity of health generation with respect to consumption (i.e. ε_h) increases and first increases and then decreases when the elasticity of output with respect to capital (i.e. ε_{yk}) increases. If the elasticity of output with respect to capital is taken as 0.35, which is very close to the real level in developed countries, and ε_h changes from 0.1 to 0.9, R_{hy} will change from 0.015 to 0.427. Especially, if ε_h is taken 0.9 and ε_{yk} is taken from 0.2 to 0.6, the estimate of R_{hy} in our model is less than but much closer to Fogel's estimate of R_{hy}. Since the assumption in the above estimate that only technology levels increase and other parameters remain unchanged is not consistent with the progress of real economic growth, the estimate of R_{hy} in our model may be even closer to Fogel's if we include those factors in our estimate.

Table 2 indicates the contribution of health in explaining the Solow residuals (R_{hS}) which increase both with increases in the elasticity of health generation with respect to consumption (i.e. ε_h) and with increases in the elasticity of output with respect to capital (i.e. ε_{yk}). If the elasticity of output with respect to capital is taken as 0.35 which is very close to the real level in developed countries, and ε_h changes from 0.1 to 0.9, R_{hS} will change from 0.023 to 0.656. Specifically if ε_h and ε_{yk} are very large, the contribution of health to economic growth can explain most of the Solow residuals, which is identical to one of Fogel's conclusions.

Insert Table2 here

4.2 The Existence of Multiple Steady States: Health and Poverty Traps

In this subsection, we consider the case of an economy with multiple steady states, a case that lends itself to explaining the presence of poverty traps in the real world, that is, how rich countries may end up with higher capital, better health, and higher consumption than poor countries. This circumstance will appear in the presence of the minimum consumption requirement in the health generation function.

In the subsection 3.1, we assume that the health generation function is a new classical function, which implies that an agent with any level of consumption possesses health human capital that can provide labor productivity. In fact, just as Fogel (1994b) claimed, an individual always needs to obtain a minimum amount of energy required to maintain his/her basal metabolism, such as to maintain body temperature and to sustain the functioning of the heart, liver, brain and other organs. For example, for adult males aged 20-39 living today in moderate climates, the basal metabolic rate (BMR) normally ranges between 1350 and 2000 kcal per day. Furthermore, since BMR does not allow for the energy required to eat and digest food, nor does it for essential hygiene, an individual cannot survive on the calories needed for basal metabolism. Fogel regarded that a survival diet is 1.27 BMR, which is not sufficient to maintain long-run health but represents the short-term maintenance level of totally inactive dependent people (Fogel, 1994b, p.6). By these conclusions, in a word, only by taking in nutrition above the minimum amount of consumption and nutritional requirements, can an individual maintain a level of health human capital that can provide labor productivity. Therefore, in a health generation function, there is a possibly minimum consumption requirement below which the agent's health will be zero, namely we assume that the health generation function is

$$h=\begin{cases} h(c) & if\ c\le \underline{c} \\ 0 & if\ c<\underline{c} \end{cases} \quad \text{and} \quad h'(c)\ge 0, h''(c)\le 0, \lim_{c\to \underline{c}^+} h'(c)=+\infty, \lim_{c\to +\infty} h'(c)=0 \tag{5''}$$

where $\underline{c}$ is the minimum consumption requirement that provides the amount of energy required to maintain the individual's basal activity. The other assumption is the same as in subsection 3.1.

Under the assumptions of (2'), (3') and (3''), similar to subsection 3.1, we can prove that there exist unit $\hat{k}^*$ and $\hat{c}^*$ satisfying equations (20) and (21). By this result, we have Theorem 4.

Theorem 4: under the assumptions of (2'), (3') and (3'') on the production function and of (5'') on the health generation function, if and only if

$$h(h'^{-1}(1/\hat{c}^*)) > h'^{-1}(1/\hat{c}^*)/\hat{c}^*\ , \tag{28}$$

there exist two equilibria in the economy. Where $h'^{-1}(.)$ denotes the inverted function of $h'(.)$ and $\hat{c}^*$ is determined by the equations of (20) and (21). Furthermore, the lower steady state is unstable and the higher steady state is stable.

Proof: We prove Theorem 4 based on Figure 3.

In Figure 3, Beeline E_1E_2 denotes that the ratio of consumption to health human capital is constant and equals $\hat{c}^*$ and Curve E_1BE_2 denotes the health generation function. At point B corresponding to which per capita consumption is c_1, the tangent of the health generation function has the same slope as the Beeline E_1E_2, i.e. $h'(c_1) = 1/\hat{c}^*$. Therefore $c_1 = h'^{-1}(1/\hat{c}^*)$.

Insert Figure3 here

First, by Theorem 2, in the steady state, the ratio of consumption to health human capital equals $\hat{c}^*$ and hence equilibrium of (c^*, h^*) must be on the Beeline E_1E_2. Furthermore, equilibrium of (c^*, h^*) must also be on the health generation function E_1BE_2. It is obvious that there exists an equilibrium in the economy if and only if Curve E_1BE_2 and Beeline E_1E_2 have a point of intersection. By geometry, Curve E_1BE_2 and Beeline E_1E_2 have two points of intersection if and only if $h(c_1) > c_1/\hat{c}^*$, i.e. $h(h'^{-1}(1/\hat{c}^*)) > h'^{-1}(1/\hat{c}^*)/\hat{c}^*$.

Second, since in the lower steady state E_1, the slope of the health generation function E_1OE_2 is greater than Beeline E_1E_2 (i.e. $h'(c^*_{low}) < h(c^*_{low})/c^*_{low}$), and in the high steady state E_1, the slope of health generation function E_1OE_2 is less than Beeline E_1E_2 (i.e. $h'(c^*_{low}) < h(c^*_{low})/c^*_{low}$), by Proposition 2, it must be that the lower steady state is unstable and the higher steady state is stable.

Theorem 4 implies that when an economy is above the lower steady state, the economy will eventually converge to the higher steady state and enter the developed phase. However, if an economy is below the lower steady state, then the economy will eventually converge to the zero point and hence fall into the poverty trap in which an agent has lower and lower consumption and holds poorer and poorer health human capital. Therefore, there is a poverty trap in the economy when there is a minimum consumption requirement in the health generation function. This conclusion implies that even being faced with the same technology level, these countries with health human capital less than a certain lever will always be in this poverty trap unless some exogenous factors improve the people's health to above the critical health level. By this result we conclude that the way to help these countries to get away from the poverty trap is to help these countries to improve the people's health.

Insert Figure4 here

We will show the impact of technology progress on economic growth and the poverty trap through Figure 4. In Figure 4, Curve $E_{low}{}^{1}E_{low}{}^{2}E_{high}{}^{1}E_{high}{}^{2}$ denotes the health generation function, Beeline $OE_{low}{}^{1}E_{high}{}^{1}$ denotes the equilibrium ratio of health to per capita consumption in the case of the technology level A^{1}, and $OE_{low}{}^{2}E_{high}{}^{2}$ in the case of the technology level A_2. Figure 4 indicates that, when technology is A_1, the stable steady state is $E_{high}{}^{1}$ and the unstable steady state is $E_{low}{}^{1}$, which implies that an economy above $E_{low}{}^{1}$ will converge to the developed state $E_{high}{}^{1}$ and an economy below $E_{low}{}^{1}$ will fall into the poverty trap. When the technology level improves from A_1 to A_2, first, the stable steady state will improve from $E_{high}{}^{1}$ to $E_{high}{}^{2}$, the economic implication of which is the same as in subsection 3.1 (we won't explain it further here). Second, the unstable steady state, i.e. the boundary of the poverty trap will decrease from $E_{low}{}^{1}$ to $E_{low}{}^{2}$. This implies that an economy above $E_{low}{}^{2}$ but below $E_{low}{}^{1}$, which falls into the poverty trap before technology improvement, will escape from the poverty trap and enter the development phase after technology level improves from A_1 to A_2. These results provide the second method in which the country in poverty can get away from poverty: to improve its own technology level.

Insert Table3 here

The above results of this subsection have an economic implication for explaining health and poverty traps in developing countries and polarization in the real economy, which is displayed in Table 3, and for economic policy advice in developing countries. By the above analysis, if the initial capital stock of an economy is below the lower steady state, this economy definitely falls into the poverty trap, in which the economy has a lower per capita consumption and a lower health state. The co-existence of low health and poverty results from the interaction of some economic forces: low capital stock which leads to low per capita output makes people unable to obtain adequate consumption and nutrition, which results in low health human capital and low labor productivity and hence low capital accumulation and economic development. This vicious circle between health poverty and low development will continue forever unless there are some other forces to break the chain between low health, the productivity and low economic development. By the above analysis, we know that there are two ways to break the vicious circle: to improve the people's health state or to improve the technology level.

5. Conclusions

In this paper, based on Fogel's research and assuming that consumption not only raises agents' utility but also increases agents' health, we study the relationship between capital accumulation and consumption and discuss the effect on long-run economic growth of health human capital derived from consumption in an extended Ramsey model. First, this study indicates that health human capital derived from consumption can neither motivate endogenous economic growth nor be the driving source promoting long-run economic growth. This result is the same as conclusion reached by Baumol (1967) and Zon & Muysken (2001 and 2003). However, the study also finds that this category of health human capital can enhance economic growth through exogenous technology progress. Furthermore, comparing the results of this paper to Fogel's work, the degree to which health enhances economic growth is consistent with Fogel's estimates on the effect of health from nutritional improvement on long-run economic growth in England.

Second, in the presence of a minimum consumption requirement in the health generation function, the study argues that multiple steady states appear in the economy, which lends itself to explaining the presence of the poverty trap in the real world in which rich countries may end up with higher capital, better health, and higher consumption than poor countries. These results imply that a poor state of health may be one of those key factors that make certain poor countries relapse into—or remain in—a poverty trap. The study suggests that there are two methods to break off a poverty trap: improving the state of the people's health or/and improving the technology level. Therefore, in the case of a constant technology level, improving people's health in poor countries is the only way to help poor countries break out of the poverty trap.

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Appendix:

A. The proof of Proposition 1:

First, we prove equation (10) is the necessary condition.

By the Hamiltonian, we have

$$\partial H/\partial c = u'(c) + \lambda[f_h(k, h(c))h'(c) - 1] \tag{A.1}$$

$$\partial H/\partial k = \lambda\left[f_k(k,h(c)) - \delta\right] \tag{A.2}$$

and

$$\partial^2 H/\partial c^2 = u''(c) + \lambda[f_h(k,h(c))h''(c) + f_{hh}(k,h(c))(h'(c))^2] \tag{A.3}$$

$$\partial^2 H/\partial c\partial k = \lambda f_{ck}(k,h(c))h'(c) \tag{A.4}$$

$$\partial^2 H/\partial k^2 = \lambda f_{kk}(k,h(c)) \tag{A.5}$$

If the objective function attains its maximum when (*c, k*) satisfies the first-order condition, then the Hamiltonian must be nonpositive. Therefore, $\partial^2 H/\partial c^2$ and $\partial^2 H/\partial k^2$ must be nonpositive and the determinant of Haisier second-order matrix must be nonnegative. By the assumption (3), $f_{kk} < 0$, in order that $\partial^2 H/\partial k^2 \le 0$, there must be $\lambda \ge 0$, which result in $f_h(k,h(c))h'(c) < 1$.

Second, we prove it is the sufficient condition: first, when $f_h(k,h(c))h'(c) < 1$, then $\lambda > 0$. It is obvious that $\partial^2 H/\partial c^2$ and $\partial^2 H/\partial k^2$ are positive. Second, the determinant of the Haisier matrix is

$$\begin{vmatrix} H_{cc} & H_{ck} \\ H_{kc} & H_{kk} \end{vmatrix} = \lambda f_{kk}\left\{u'' + \lambda[h''f_h + h'^2 f_{hh}]\right\} - \left(\lambda h' f_{hk}\right)^2 = \lambda f_{kk}u'' + \lambda^2\left[f_{kk}f_h h'' + h'^2\left(f_{kk}f_{hh} - f_{hk}{}^2\right)\right]$$

By the assumption of the utility function, production function and health generation function, the determinant of Haisier matrix must be positive. Therefore, (*c*, *k*) satisfying equations (6), (8), (9) and the transversality condition maximizes the objective function.

B. The Proof of Theorem 1.

Linearize the system (6) and (14) around the steady state

$$\begin{pmatrix} \dot{k} \\ \dot{c} \end{pmatrix} = J \begin{pmatrix} k - k^* \\ c - c^* \end{pmatrix} \tag{B.1}$$

Where

$$J = \begin{pmatrix} \beta & f_h(k,h(c))h'(c) - 1 \\ -\dfrac{\lambda}{\lambda_c} f_{kk} - \dfrac{\lambda_k}{\lambda_c}\beta & 0 \end{pmatrix} \tag{B.2}$$

is the coefficient matrix associated with the above linear system. The eigenvalues μ_1 and μ_2 of the matrix *J* satisfy

$$\mu_1 + \mu_2 = \beta \tag{B.3}$$

$$\mu_1\mu_2 = -\frac{\lambda}{\lambda_c}\{\beta h'(c) f_{hk}(k,h(c)) - [h'(c) f_h(k,h(c)) - 1] f_{kk}(k,h(c))\} \tag{B.4}$$

Thus, the saddle-point stability requires that

$$\beta h'(c) f_{hk}(k,h(c)) - [h'(c) f_h(k,h(c)) - 1] f_{kk}(k,h(c)) < 0 \tag{B.5}$$

Otherwise, if

$$\beta h'(c) f_{hk}(k,h(c)) - [h'(c) f_h(k,h(c)) - 1] f_{kk}(k,h(c)) \geq 0 \tag{B.5}$$

then $\mu_1\mu_2 \geq 0$. Since $\mu_1 + \mu_2 = \beta > 0$, therefore, both μ_1 and μ_2 are nonnegative, then the steady state is divergent.

C. The proof of Proposition 2

By the assumptions of (3'), (2'), (3'') and (18), we have

$$\hat{y} = y/h = Ag(k/h,1) \equiv A\hat{g}(\hat{k}) \Leftrightarrow y = hA\hat{g}(\hat{k})$$

and

$$f_k = A\hat{g}'(\hat{k}),\ f_h = A\hat{g}(\hat{k}) - A\hat{k}\hat{g}'(\hat{k}),\ f_{kk} = A\hat{g}''(\hat{k})/h,\ \ f_{kh} = -A\hat{k}\hat{g}''(\hat{k})/h$$

By Theorem 1, we have

$$\beta h'(c) f_{kh}(k,h(c)) + [1 - f_h(k,h(c))h'(c)] f_{kk}(k,h(c)) < 0 \Leftrightarrow$$
$$h'(c)\left[\beta f_{kh}(k,h(c)) - f_{kk}(k,h(c)) f_h(k,h(c))\right] < -f_{kk}(k,h(c))$$

and hence

$$h'(c) < -\frac{f_{kk}(k,h(c))}{\beta f_{kh}(k,h(c)) - f_{kk}(k,h(c)) f_h(k,h(c))}$$
$$= \frac{-A\hat{g}''(\hat{k})/h}{-\beta A\hat{k}\hat{g}''(\hat{k})/h - \left[A\hat{g}(\hat{k}) - A\hat{k}\hat{g}'(\hat{k})\right] A\hat{g}''(\hat{k})/h} = \frac{1}{\hat{c}} = \frac{h(c)}{c}$$

D. The Proof of Theorem 3.

By equations (20) and (3'''), we have

$$k^*/h^* = \hat{k}^* = \hat{g}_k^{-1}((\delta + \beta)/A) \tag{C.1}$$

By equations (C.1) and (21), we get

$$c^*/h^* = \hat{c}^* = A\hat{g}(\hat{k}^*) - \delta\hat{k}^* = A\hat{g}\left[\hat{g}_k{}^{-1}((\delta+\beta)/A)\right] - \delta\hat{g}_k{}^{-1}((\delta+\beta)/A) \qquad \text{(C.2)}$$

Since $h = h(c)$, we have

$$c^* = \hat{c}^* h(c^*) \qquad \text{(C.3)}$$

Let $\varphi(c) = \hat{c}^* h(c) - c$, by the assumption (5'), $\lim_{x\to 0^+}\varphi(x) > 0, \lim_{x\to\infty}\varphi(x) < 0$. Since $\varphi(c)$ is continuous, therefore, as seen from Figure A1, there exists an exclusive c^* satisfying equation (C.3). By equations (C.1) and (C.2), there is an exclusive k^* and h^* that satisfy equations (20) and (21). Then Proposition 2 is proved.

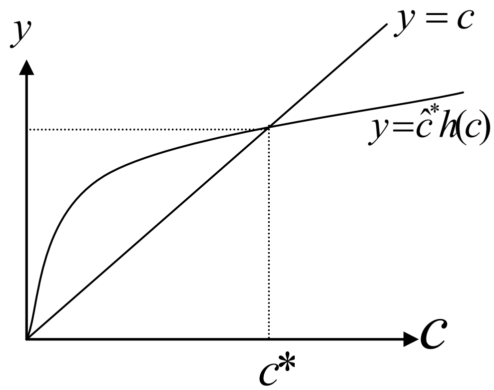


Figure A1: The Resolve of $\hat{c}^* h(c) = c$

Table 1: the contribution of health to economic growth: Estimate of R_{hy}

ε_{yk} \ ε_h	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.005	0.012	0.020	0.031	0.045	0.066	0.098	0.157	0.289
0.2	0.010	0.022	0.036	0.055	0.080	0.114	0.165	0.246	0.400
0.25	0.011	0.026	0.043	0.065	0.093	0.132	0.188	0.273	0.422
0.3	0.013	0.030	0.050	0.074	0.105	0.147	0.204	0.290	***0.430***
0.35	**0.015**	**0.033**	**0.054**	**0.080**	**0.114**	**0.157**	**0.215**	**0.298**	**0.427**
0.4	0.017	0.035	0.058	0.086	0.120	0.163	***0.221***	***0.300***	0.415
0.5	0.018	0.038	***0.063***	***0.091***	***0.125***	***0.167***	0.219	0.286	0.375
0.6	***0.018***	***0.039***	0.062	0.089	0.120	0.157	0.200	0.253	0.318
0.65	0.018	0.038	0.060	0.085	0.114	0.147	0.185	0.230	0.284
0.7	0.017	0.036	0.056	0.080	0.105	0.134	0.167	0.205	0.249
0.8	0.014	0.029	0.044	0.062	0.080	0.100	0.122	0.145	0.171
0.9	0.008	0.017	0.026	0.035	0.045	0.055	0.066	0.077	0.088

Table 2: the contribution of health to explain the Solow residuals: Estimate of R_{hS}

	ε_h	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ε_{yk}										

0.1		0.006	0.013	0.022	0.034	0.050	0.073	0.109	0.174	0.321
0.2		0.012	0.027	0.045	0.069	0.100	0.143	0.206	0.308	0.500
0.25		0.016	0.034	0.058	0.087	0.125	0.176	0.250	0.364	0.563
0.3		0.019	0.042	0.070	0.105	0.150	0.209	0.292	0.414	0.614
0.35		**0.023**	**0.050**	**0.083**	**0.124**	**0.175**	**0.241**	**0.331**	**0.459**	**0.656**
0.4		0.027	0.059	0.097	0.143	0.200	0.273	0.368	0.500	0.692
0.5		0.036	0.077	0.125	0.182	0.250	0.333	0.438	0.571	0.750
0.6		0.045	0.097	0.155	0.222	0.300	0.391	0.500	0.632	0.794
0.65		0.051	0.107	0.171	0.243	0.325	0.419	0.529	0.658	0.813
0.7		0.056	0.119	0.188	0.264	0.350	0.447	0.557	0.683	0.829
0.8		0.069	0.143	0.222	0.308	0.400	0.500	0.609	0.727	0.857
0.9		0.083	0.170	0.260	0.353	0.450	0.551	0.656	0.766	0.880

Table 3: Life expectancy and mortality, by country development category, (1995-2000)

Development category	Annual Average Income (US dollars)	Life Expectancy at birth (years)	Infant Mortality (death before age 1 per 1000 live birth)	Under five Mortality (deaths before age 5 per 1000 live births)
Least-Developed countries	296	51	100	159
Other Low-Income countries	538	59	80	120
Lower-Middle-Income Countries	1200	70	35	39
Upper-Middle-Income Countries	4900	71	26	35
High-Income Countries	25730	78	6	6
Memo: sub-Saharan Africa	500	51	92	151

Original Source: Human Development Report 2001. Table 8, and CMH calculations using World Development Indication of World Bank, 2001. Here cited from Jeffrey D. Sachs (2001, p2)

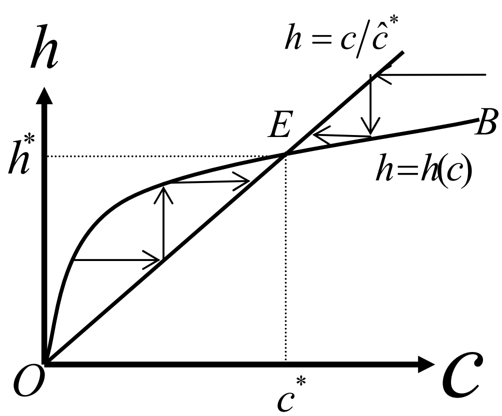
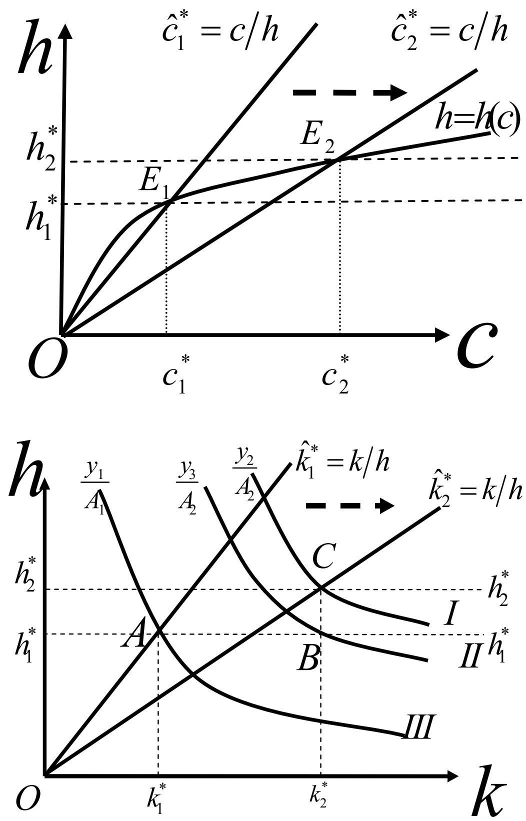


Figure 1: the Case of a Unique Equilibrium



(A)

(B)

Figure 2: Health Human Capital Increases the Growth Rate

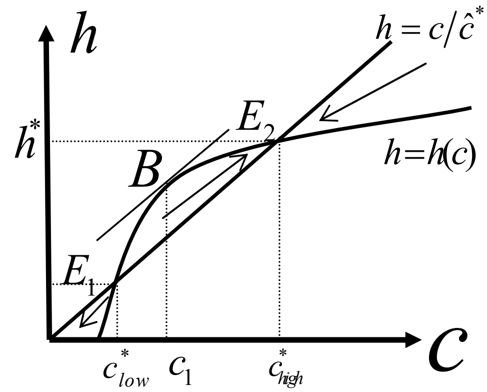


Figure 3: the Case of Two Equilibria: Presence of Subsistence level of Consumption in the Health Generation Function

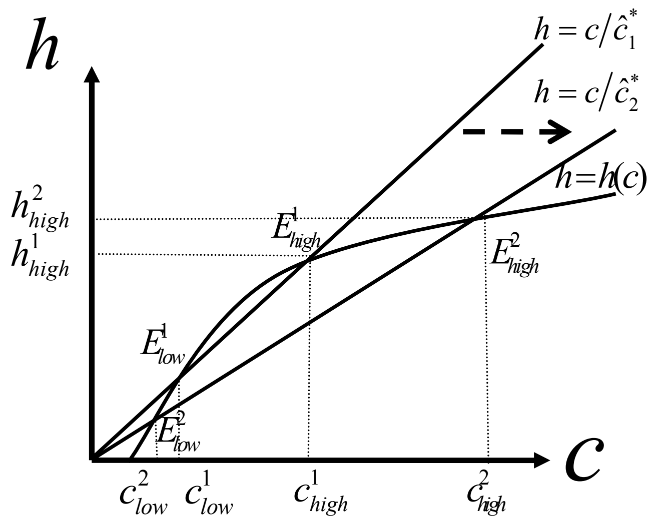


Figure 4: Impact of Technology Progress on Economic Growth and Poverty Trap: Presence of Subsistence Level of Consumption in Health Generation Function