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Optimal taxation and intergovernmental transfer in a dynamic model with multiple levels of government

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Abstract

In this paper, we study the optimal choices of the federal income tax, federal transfers, and local taxes in a dynamic model of capital accumulation and with explicit game structures among multiple private agents, multiple local governments, and the federal government. In general, the optimal local property tax is zero if the local property tax is constrained to be nonnegative, whereas the optimal local consumption tax is always positive. When the local consumption tax is chosen optimally, the federal income tax can be either positive or negative. For most reasonable parameter values, our numerical calculations have shown that with a positive local consumption tax there exists a reverse transfer from local governments to the federal government.

JEL classification: E0; H2; H4; H5; H7; O4; R5

Keywords: Income tax; Property tax; Consumption tax; Intergovernmental transfers; Capital accumulation; Fiscal federalism

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1. Introduction

In an earlier contribution to optimal taxation and revenue-sharing in the context of fiscal federalism, Gordon (1983) has utilized a static model to consider how local governments set the rules of local taxes including tax rates and types of taxes in a decentralized form of decision-making, while allowing the federal government the role of correcting externalities through grants, revenue-sharing, and regulations on local tax bases. Recently, Persson and Tabellini (1996a, b) have considered risk-sharing and redistribution across local governments in a federation in static models involving risk.

Following Gordon (1983), and Persson and Tabellini (1996a, b), we analyze the optimal choices of federal taxes, federal transfers, and local taxes in a dynamic model of capital accumulation with multiple levels of government.[1] This study considers multiple private agents and multiple local governments and it allows us to see how the optimal choices of federal taxation and federal transfers relate to heterogeneity across different private agents and different local governments.

Our approach goes one step further in bringing the existing optimal taxation study closer to reality. In the existing literature on optimal income and commodity taxation,[2] the government is often taken to be a single identity, without introducing the structure of tax assignments and expenditure assignments among multiple levels of government. But in reality, the income tax is collected by central governments in Europe and jointly by the federal government and state governments in the United States; the property tax is collected by local governments, and the commodity tax is collected by both central governments and local governments in Europe, and by local governments in the United States. In most developed countries, each level of government has the power to determine tax rates and tax bases. In addition, there exist intergovernmental transfers in various forms among different levels of government in every country of reasonable population size. It is natural to see how the structure of fiscal federalism affects the structure of optimal taxation and intergovernmental transfers.

Our dynamic approach is timely given that the design of tax assignments, expenditure assignments, and intergovernmental transfers among different levels of government has received considerable attention in the 1990s in the context of fiscal federalism, public sector reforms, and economic growth for both developing and developed countries. One of the most important goals of

[1] See Zou (1994, 1996), Brueckner (1996), Devarajan et al. (1996, 1998), Davoodi and Zou (1998), and Zhang and Zou (1998) for related dynamic approaches to multi-level government spending, intergovernmental transfers, federal taxes, and local taxes in a ‘federation’.

[2] Classical contributions include, for example, Ramsey (1927), Mirrlees (1971), Diamond and Mirrlees (1971), Atkinson and Stiglitz (1972, 1976), and Samuelson (1986). For comprehensive literature reviews see Atkinson and Stiglitz (1980), and Myles (1995).

establishing a sound intergovernmental fiscal relationship is to promote local as well as national economic growth (see Rivlin, 1992; Bird, 1993; Gramlich, 1993; Oates, 1993, 1999). Our dynamic model is a small step toward linking fiscal federalism and optimal economic growth within the context of some specific institutional arrangements in a federation.

The paper is organized as follows. In Section 2 we set up the general framework for the dynamic Stackelberg (leader-follower) games (i) between local governments (the leaders) and private agents (the followers), and (ii) between the federal government (the leader) on one side and local governments (the followers) and private agents (the followers) on the other. Our analysis will focus on open-loop equilibria because closed-loop specifications would be more difficult, and would also raise problems with time consistency. In this general, abstract form, we derive some results regarding the optimal choices of local taxes, the federal income tax, and federal transfers. In Section 3, we show how the optimal choices of the income tax, the consumption tax, and the property tax with one level of government are different from the corresponding choices with two levels of government. In Sections 4 and 5, through a concrete example, we see how the optimal choices of taxes and federal transfers with two levels of government can be computed. Since explicit solutions to complicated dynamic games are hard to come by, we provide some comparative analysis based on numerical calculations. We conclude this paper in Section 6.

2. The framework

Following Gordon (1983), and Persson and Tabellini (1996a, b), we examine a two-tier federal system with many local governments. For our dynamic analysis, it suffices to assume that there are two agents, two local governments, and the federal government in the economy. Agent 1 lives in locality 1, and agent 2 lives in locality 2. The federal government levies a uniform income tax on agents 1 and 2 at a flat rate of τ_{f}, whereas local government i ($i = 1,2$) levies a consumption tax, $\tau^i_{c_i}$, and a property or capital tax,[3] $\tau^i_{k_i}$. Federal public spending is f in the national jurisdiction, and local public spending in local jurisdiction i ($i=1,2$) is s_i. Local government i ($i=1,2$) also receives a matching federal transfer, gs_i, with g as the uniform matching rate.

2.1. *Agents* ($i = 1,2$)

As in Arrow and Kurz (1970), Barro (1990), Turnovsky (1995), and Turnovsky and Fisher (1995), government expenditures are introduced into

[3] In this model, capital includes real estate property, and the property tax can be viewed as a tax on capital (Mieszkowski, 1972). But, in general, these two are not equivalent because capital is more mobile than real estate property from a local government's perspective.

the representative agent's utility function. Unlike those studies, the model introduces public expenditures at both the federal and local levels. The agent derives a positive, but diminishing, marginal utility from the expenditures of both the federal and local governments and private consumption. Let f, s_i, and c_i be federal expenditure, local expenditure, and private consumption, respectively. If the utility function $u(c_i, f, s_i)$ is twice differentiable, the assumption is equivalent to

$$u_{c_i} > 0, \quad u_f > 0, \quad u_{s_i} > 0, \quad u_{c_i c_i} < 0, \quad u_{ff} < 0, \quad u_{s_i s_i} < 0. \tag{1}$$

In addition, $u(c_i, f, s_i)$ satisfies the Inada conditions

$$\lim_{s_i \to 0} u_{s_i} = \infty, \quad \lim_{f \to 0} u_f = \infty, \quad \lim_{c_i \to 0} u_{c_i} = \infty,$$

$$\lim_{s_i \to \infty} u_{s_i} = 0, \quad \lim_{f \to \infty} u_f = 0, \quad \lim_{c_i \to \infty} u_{c_i} = 0. \tag{2}$$

Agent i's discounted utility is given by

$$U_i = \int_0^\infty u_i(c_i, f, s_i) \mathrm{e}^{-\rho t} \, \mathrm{d}t, \tag{3}$$

where ρ is the positive, constant time preference.

Again following Arrow and Kurz (1970), Barro (1990), and Turnovsky (1995), agent i's output, y_i, is produced by a constant-return-to-scale production function with four inputs: capital stock, k_i, labor input l_i, federal government expenditure, f, and local government expenditure, s_i, namely

$$y_i = y_i(k_i, l_i, f, s_i).$$

For simplicity, we assume that the agent's labor input is fixed at one unit: $l_i = 1$. Therefore, we just write agent i's production function as

$$y_i = y_i(k_i, f, s_i). \tag{4}$$

The marginal productivity of private capital stock, federal government expenditure, and local government expenditure are positive and decreasing:

$$y_{k_i} > 0, \quad y_f > 0, \quad y_{s_i} > 0, \quad y_{c_i c_i} < 0, \quad y_{ff} < 0, \quad y_{s_i s_i} < 0. \tag{5}$$

Federal government expenditure, f, is financed by the income tax on the agent. Local government expenditure, s_i, is the sum of the consumption tax,[4] $\tau^i_{c_i} c_i$, the capital tax, $\tau^i_{k_i} k_i$, and federal government's transfer, $g s_i$. τ_f, $\tau^i_{c_i}$,

[4] Consumption tax has been analyzed recently in growth models with one level of government by King and Rebelo (1990), Rebelo (1991), and Jones et al. (1993), and Turnovsky (1995).

and $\tau^i_{k_i}$ are the federal income tax rate, local consumption tax rate, and local capital or property tax rate, respectively, and g is the matching rate of federal grant for local spending.[5] Hence, the budget constraints for the federal government and local government i $(i = 1, 2)$ can be written as follows:

$$f + gs_1 + gs_2 = \tau_f y^1 + \tau_f y^2, \tag{6}$$

$$s_i - gs_i = \tau^i_{c_i} c_i + \tau^i_{k_i} k_i, \tag{7}$$

respectively.

Agent i maximizes a discounted utility over an infinite time horizon

$$\text{Max} \int_0^\infty u_i(c_i, f, s_i) \mathrm{e}^{-\rho t} \, \mathrm{d}t$$

subject to his budget constraint

$$\frac{\mathrm{d}k_i}{\mathrm{d}t} = (1 - \tau_f) y_i(k_i, f, s_i) - (1 + \tau^i_{c_i}) c_i - (\delta + \tau^i_{k_i}) k_i. \tag{8}$$

His initial capital stock is given by $k_i(0) = k_{i0}$.

Solving the optimization problem, we obtain the first-order conditions

$$\frac{\mathrm{d}k_i}{\mathrm{d}t} = (1 - \tau_f) y_i(k_i, f, s_i) - (1 + \tau^i_{c_i}) c_i - (\delta + \tau^i_{k_i}) k_i,$$

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}t} = -\lambda_i \left[(1 - \tau_f) \frac{\partial y_i}{\partial k_i} - \rho - \delta - \tau^i_{k_i} \right], \tag{9}$$

$$u_{c_i} = (1 + \tau^i_{c_i}) \lambda_i. \tag{10}$$

From Eq. (10), we have

$$c_i = c_i(\lambda_i, k_i, \tau^i_{c_i}, f, s_i). \tag{11}$$

At the steady state, we have

$$(1 - \tau_f) y_i(k_i, f, s_i) - (1 + \tau^i_{c_i}) c_i(\lambda_i, k_i, \tau^i_{c_i}, f, s_i) - (\delta + \tau^i_{k_i}) k_i = 0, \tag{12}$$

$$(1 - \tau_f) \frac{\partial y_i}{\partial k_i} - \rho - \delta - \tau^i_{k_i} = 0. \tag{13}$$

[5] The matching grant here is justified on the ground that the federal government intends to provide incentives for local public services. It can also be viewed as the way to transfer resources among different levels of government. A straightforward extension is to allow externalities (benefit spillover) in local public services in both utility functions and production functions.

2.2. Local governments ($i = 1, 2$)

In each locality, the local government and the private agent play the Stackelberg game with the local government as the leader and private agent the follower.[6] At the same time, in this section we assume that the local government is the follower in the Stackelberg game with the federal government.[7] That is to say, given the federal income tax rate, the federal matching grant, and federal spending, the local government maximizes the agent's welfare by fully incorporating the agent's first-order conditions in Section 2.1 into its own maximization. Specifically, the local government will choose its optimal taxes, $\tau^i_{c_i}$ and $\tau^i_{k_i}$, and its public expenditure, s_i, private capital stock, k_i, and the marginal utility of private wealth, λ_i, to maximize the agent's steady-state welfare:

$$\underset{\tau^i_{c_i}, \tau^i_{k_i}, s_i,}{\text{Max}}\ u_i(c_i, f, s_i)$$

subject to the steady-state conditions for individual i, (10), (12), and (13), and the budget constraint of locality i, (7).

Now, the first-order conditions of locality i are as follows:

$$\frac{\partial L_i}{\partial c_i} = \frac{\partial u_i(c_i, f, s_i)}{\partial c_i} - \theta^i_1(1 + \tau^i_{c_i}) + \theta^i_4 \tau^i_{c_i} + \theta^i_3 u_{c_i c_i} = 0, \tag{14}$$

$$\begin{aligned}\frac{\partial L_i}{\partial s_i} &= \frac{\partial u_i(c_i, f, s_i)}{\partial s_i} + \theta^i_1(1 - \tau_f)\frac{\partial y_i(k_i, f, s_i)}{\partial s_i} \\ &\quad + \theta^i_2(1 - \tau_f)\frac{\partial^2 y_i}{\partial k_i \partial s_i} + \theta^i_4(g_i - 1) = 0,\end{aligned} \tag{15}$$

$$\frac{\partial L_i}{\partial \lambda_i} = -\theta^i_3 \lambda_i (1 + \tau^i_{c_i}) = 0. \tag{16}$$

$$\frac{\partial L_i}{\partial k_i} = \theta^i_1\left[(1 - \tau_f)\frac{\partial y_i}{\partial k_i} - \delta - \tau^i_{k_i}\right] + \theta^i_2(1 - \tau_f)\frac{\partial^2 y_i}{\partial k_i^2} + \theta^i_4 \tau^i_{k_i} = 0, \tag{17}$$

[6] See a similar technique used by Turnovsky and Brock (1980), Judd (1985), Chamley (1985, 1986), and Lucas (1990) in the context of the representative agent model with one level of government.

[7] The choices of federal government as the leader and local governments as followers are natural in the light of recent policy discussions on how to harden the budget constraint on local governments and how to avoid bailouts of local governments by the federal government in both developing and developed countries. See Wildasin (1998).

$$\frac{\partial L_i}{\partial \tau_{c_i}^i} = -\theta_1^i c_i - \theta_3^i \lambda_i + \theta_4^i c_i + \theta_6^i = 0, \tag{18}$$

$$\theta_6^i \tau_{c_i}^i = 0, \quad \theta_6^i \geq 0, \tag{19}$$

$$\frac{\partial L_i}{\partial \tau_{k_i}^i} = -\theta_1^i k_i - \theta_2^i + \theta_4^i k_i + \theta_5^i = 0, \tag{20}$$

$$\theta_5^i \tau_{k_i}^i = 0, \quad \tau_{k_i}^i \geq 0, \ \theta_5^i \geq 0, \tag{21}$$

where θ_1^i is the multiplier associated with Eq. (12); θ_2^i is the multiplier associated with Eq. (13); θ_3^i is the multiplier associated with Eq. (10); θ_4^i is the multiplier associated with locality's budget constraint equation (7); θ_1^i is the multiplier associated with the nonnegative constraint on the property tax, $\tau_{k_i}^i \geq 0; \theta_6^i$ is the multiplier associated with the nonnegative constraint on the consumption tax, $\tau_{c_i}^i \geq 0$.

Proposition 1. The optimal steady-state property taxes in the two localities are zero. [8]

Proof. Suppose $\tau_{k_i}^i \neq 0$. From Eqs. (18) and (20), we have

$$\text{sign}(\theta_2^i) = \text{sign}(-\theta_1^i + \theta_4^i) = -\text{sign}(\theta_6^i) \leq 0.$$

But from Eq. (17), we have

$$\theta_1^i \rho + \theta_2^i (1 - \tau_f) \frac{\partial^2 y_i}{\partial k_i^2} + \theta_4^i \tau_{k_i}^i = 0$$

and $\theta_1^i \rho \geq 0$, $\theta_2^i (1 - \tau_f) \partial^2 y_i / \partial k_i^2 \geq 0$, and $\theta_4^i \tau_{k_i}^i \geq 0$. Therefore, we must have

$$\theta_1^i \rho = 0, \quad \theta_2^i (1 - \tau_f) \frac{\partial^2 y_i}{\partial k_i^2} = 0, \quad \theta_4^i \tau_{k_i}^i = 0.$$

We also know

$$\theta_1^i = \theta_2^i = \theta_3^i = \theta_4^i = \theta_5^i = 0.$$

[8] The optimal property tax would be negative without the nonnegative constraint on the property tax, $\tau_{k_i}^i \geq 0$.

Then, from Eq. (15)

$$\frac{\partial u_i(c_i, f, s_i)}{\partial s_i} = 0,$$

which is impossible by our assumption. Hence, we must have $\tau^i_{k_i} = 0$. $\square$

The intuition is as follows. For local governments, the consumption tax is less distortionary than the capital (property) tax in its adverse effect on private production and capital accumulation. Without the nonnegative constraint on the property tax, $\tau^i_{k_i} \geq 0$, the optimal local property tax is negative. The proof is left as an exercise for the reader.

2.3. The federal government

Given the optimal choices of local governments and agents, the federal government as the leader in its Stackelberg game with both the agents and local governments chooses $k_i, s_i, g_i, \tau^i_{c_i}$ $(i = 1,2)$, τ_f, and f to maximize the weighted steady-state welfare of the two agents in the two localities with χ_i $(i = 1,2)$ as the weights.:

$$\max_{k_i,s_i,g_i,\tau^i_{c_i},\tau^i_{k_i},\tau_f} \chi_1 u_1(c_1, f, s_1) + \chi_2 u_2(c_2, f, s_2)$$

subject to the first-order conditions of the two localities: Eqs. (10), (7), (12)–(21), and the budget constraint (6).

Define the Lagrangian function

$$\begin{aligned}
L = {} & \chi_1 u_1(c_1, f, s_1) + \chi_2 u_2(c_2, f, s_2) \\
& + \sum_{i=1}^{2} \xi^i_1 [(1-\tau_f) y_i(k_i, f, s_i) - (1+\tau^i_{c_i}) c_i - \delta k_i] \\
& + \sum_{i=1}^{2} \xi^i_2 \left[(1-\tau_f) \frac{\partial y_i}{\partial k_i} - \rho - \delta \right] \\
& + \sum_{i=1}^{2} \xi^i_3 \left[\frac{\partial u_i(c_i, f, s_i)}{\partial c_i} - \theta^i_1 (1+\tau^i_{c_i}) + \theta^i_4 \tau^i_{c_i} \right] \\
& + \sum_{i=1}^{2} \xi^i_4 \left[\frac{\partial u_i(c_i, f, s_i)}{\partial s_i} + \theta^i_1 (1-\tau_f) \frac{\partial y_i(k_i, f, s_i)}{\partial s_i} \right. \\
& \left. + \theta^i_2 (1-\tau_f) \frac{\partial^2 y_i}{\partial k_i \partial s_i} + \theta^i_4 (g-1) \right]
\end{aligned}$$

$$+\sum_{i=1}^{2}\xi_5^i\left[\theta_1^i\rho+\theta_2^i(1-\tau_f)\frac{\partial^2 y_i}{\partial k_i^2}\right]+\sum_{i=1}^{2}\xi_{10}^i[\tau_{c_i}^i c_i-s_i+gs_i]$$
$$+\sum_{i=1}^{2}\xi_6^i[-\theta_1^i c_i+\theta_4^i c_i+\theta_6^i]+\sum_{i=1}^{2}\xi_7^i[\theta_6^i\tau_{c_i}^i]+\sum_{i=1}^{2}\xi_8^i\theta_6^i+\sum_{i=1}^{2}\xi_9^i\tau_{c_i}^i$$
$$+\eta[\tau_f y^1+\tau_f y^2-f-gs_1-gs_2],$$

where $\xi_1^i,\xi_2^i,\xi_3^i,\xi_4^i,\xi_5^i,\xi_6^i,\xi_7^i,\xi_8^i,\xi_9^i$, and ξ_{10}^i are the multipliers associated with Eqs. (12)–(19), and (7), respectively, and η is the multiplier associated with the federal budget constraint (6). Now, we have the first-order conditions for federal optimization

$$\frac{\partial L}{\partial\tau_f}=-\sum_{i=1}^{2}\xi_1^i y_i(k_i,f,s_i)-\sum_{i=1}^{2}\xi_4^i\left[\theta_1^i\frac{\partial y_i(k_i,f,s_i)}{\partial s_i}+\theta_2^i\frac{\partial^2 y_i}{\partial k_i\partial s_i}\right]$$
$$-\sum_{i=1}^{2}\xi_5^i\theta_2^i\frac{\partial^2 y_i}{\partial k_i^2}+\eta(y^1+y^2)-\sum_{i=1}^{2}\xi_2^i\frac{\partial y_i}{\partial k_i}=0, \tag{22}$$

$$v\tau_f=0,\quad v\geq 0,$$

$$\frac{\partial L}{\partial g}=\sum_{i=1}^{2}\xi_4^i\theta_4^i+\sum_{i=1}^{2}\xi_{10}^i s_i-\eta(s_1+s_2)+\kappa=0,\quad \kappa g=0,\ \kappa\geq 0, \tag{23}$$

$$\frac{\partial L}{\partial\tau_{c_i}^i}=-\xi_1^i c_i+\xi_3^i(-\theta_1^i+\theta_4^i)+\xi_{10}^i c_i+\xi_7^i\theta_6^i+\xi_9^i=0,$$
$$\xi_9^i\tau_{c_i}^i=0,\ \xi_9^i\geq 0, \tag{24}$$

$$\frac{\partial L}{\partial\theta_6^i}=\xi_6^i+\xi_7^i\tau_{c_i}^i+\xi_8^i=0,\quad \xi_8^i\theta_6^i=0,\ \xi_9^i\geq 0, \tag{25}$$

$$\frac{\partial L}{\partial\theta_2^i}=\xi_4^i(1-\tau_f)\frac{\partial^2 y_i}{\partial k_i\partial s_i}+\xi_5^i(1-\tau_f)\frac{\partial^2 y_i}{\partial k_i^2}=0, \tag{26}$$

$$\frac{\partial L}{\partial\theta_4^i}=\xi_3^i\tau_{c_i}^i+\xi_4^i(g-1)+\xi_6^i c_i=0, \tag{27}$$

$$\frac{\partial L}{\partial\theta_1^i}=-\xi_3^i(1+\tau_{c_i}^i)+\xi_4^i(1-\tau_f)\frac{\partial y_i(k_i,f,s_i)}{\partial s_i}+\xi_5^i\rho-\xi_6^i c_i=0, \tag{28}$$

$$
\begin{aligned}
\frac{\partial L}{\partial k_i} &= \xi_1^i \left[(1-\tau_f)\frac{\partial y_i}{\partial k_i} - \delta\right] + \xi_4^i \left[\theta_1^i(1-\tau_f)\frac{\partial y_i^2(k_i,f,s_i)}{\partial s_i \partial k_i}\right. \\
&\quad \left. + \theta_2^i(1-\tau_f)\ \frac{\partial^3 y_i}{\partial k_i^2 \partial s_i}\right] + \xi_5^i\theta_2^i(1-\tau_f)\frac{\partial^3 y_i}{\partial k_i^3} + \eta\tau_f\frac{\partial y_i}{\partial k_i} \\
&\quad + \xi_2^i(1-\tau_f)\frac{\partial^2 y_i}{\partial k_i^2} \\
&= 0, \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial f} &= \chi_1\frac{\partial u_1(c_1,f,s_1)}{\partial f} + \chi_2\frac{\partial u_2(c_2,f,s_2)}{\partial f} + \sum_{i=1}^{2}\xi_1^i(1-\tau_f)\frac{\partial y_i(k_i,f,s_i)}{\partial f} \\
&\quad + \sum_{i=1}^{2}\xi_2^i(1-\tau_f)\frac{\partial^2 y_i}{\partial k_i \partial f} + \sum_{i=1}^{2}\xi_3^i\frac{\partial u_i^2(c_i,f,s_i)}{\partial c_i\partial f} \\
&\quad + \sum_{i=1}^{2}\xi_4^i\left[\frac{\partial u_i^2(c_i,f,s_i)}{\partial s_i\partial f} + \theta_1^i(1-\tau_f)\frac{\partial y_i^2(k_i,f,s_i)}{\partial s_i\partial f}\right. \\
&\quad \left. + \theta_2^i(1-\tau_f)\ \frac{\partial^3 y_i}{\partial k_i\partial s_i\partial f}\right] \\
&\quad + \sum_{i=1}^{2}\xi_5^i\theta_2^i(1-\tau_f)\frac{\partial^3 y_i}{\partial k_i^2\partial f} + \eta\left[\tau_f^1\frac{\partial y_1(k_i,f,s_i)}{\partial f}\right. \\
&\quad \left. + \tau_f\ \frac{\partial y_2(k_i,f,s_i)}{\partial f} - 1\right], \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial c_i} &= \chi_i\frac{\partial u_i(c_i,f,s_i)}{\partial c_i} - \xi_1^i(1+\tau_{c_i}^i) + \xi_3^i\frac{\partial u_i^2(c_i,f,s_i)}{\partial c_i^2} \\
&\quad + \xi_4^i\frac{\partial u_i^2(c_i,f,s_i)}{\partial s_i\partial c_i} + \xi_{10}^i\tau_{c_i}^i + \xi_6^i(\theta_4^i - \theta_1^i) = 0, \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial s_i} &= \chi_i\frac{\partial u_i(c_i,f,s_i)}{\partial s_i} + \xi_1^i(1-\tau_f)\frac{\partial y_i(k_i,f,s_i)}{\partial s_i} + \xi_2^i(1-\tau_f)\frac{\partial^2 y_i}{\partial k_i\partial s_i} \\
&\quad + \xi_4^i\left[\frac{\partial u_i^2(c_i,f,s_i)}{\partial c_i\partial s_i} + \theta_1^i(1-\tau_f)\frac{\partial y_i^2(k_i,f,s_i)}{\partial s_i^2} + \theta_2^i(1-\tau_f)\frac{\partial^3 y_i}{\partial k_i\partial s_i^2}\right] \\
&\quad + \xi_5^i\theta_2^i(1-\tau_f)\frac{\partial^3 y_i}{\partial k_i^2\partial s_i} + \xi_{10}^i(g-1) - \eta g + \xi_3^i\frac{\partial u_i^2(c_i,f,s_i)}{\partial c_i\partial s_i} \\
&\quad + \eta\left[\tau_f^1\frac{\partial y_1(k_i,f,s_i)}{\partial s_1} + \tau_f\frac{\partial y_2(k_i,f,s_i)}{\partial s_1}\right] \\
&= 0 \qquad (32)
\end{aligned}
$$

plus Eqs. (12)–(19), and (7).

Proposition 2. If the local consumption tax is strictly positive, then the federal transfer to localities can be negative or positive.

The proof is a mechanical, tedious demonstration that a negative value of g and a positive value of g are both consistent with the first-order conditions for federal maximization. Perhaps it first appears surprising that the federal transfer to localities can be negative. The intuition is quite convincing. With a less distortionary consumption tax available at the local level, and with the Stackelberg game between the federal government (the leader) and local governments (the followers), the federal government can levy a smaller income tax, which is more distortionary than the local consumption tax. At the same time, the federal government can 'force' a reverse transfer from local governments to the federal government. In this way, part of the local consumption tax finances federal spending.

Furthermore, we cannot exclude the possibility that the federal income tax is negative when the local consumption tax is available.

Proposition 3. If the local consumption tax is strictly positive, the federal income tax can be positive or negative.

Again, the proof of this proposition follows from a mechanical procedure that both a negative value of τ_f and a positive value of τ_f are consistent with the first-order conditions for federal optimization. In this case, the local consumption tax is utilized to finance both federal spending and federal subsidy for private production. For a concrete example, see Table 2 in Section 4.

Proposition 3 stands in sharp contrast to the result from the optimal taxation model where there is only one government. In the one government model, we can show that, if a consumption tax is available, it is always optimal to set a positive consumption tax, while levying no income tax. To illustrate this point, we turn to the analysis of optimal consumption tax, income tax, and property tax with one government. This analysis also provides a benchmark for our normative discussions in the context of multiple levels of government.

3. Taxes with one government

Suppose there are one individual and one government.[9] The individual has a preference defined on private consumption good, c, and public good, G:

$$U(c, G). \tag{33}$$

[9] The model in this section is an extension of Chamley (1986) and Lucas (1990) with a consumption tax.

For simplicity, we take the utility function to be separable, that is

$$U(c,G)=u(c)+v(G).$$

Output, y, is produced by the production function

$$y(k,G) \tag{34}$$

which is defined on private capital, k, and public good, G.

It is further assumed that both the utility function and the production function satisfy the usual Inada conditions.

The government collects a consumption tax, $\tau_c c$, an income tax, $\tau_y y$, and a property tax, $\tau_k k$. Hence, we have the balanced budget constraint for the government

$$\tau_c c+\tau_y y+\tau_k k=G. \tag{35}$$

3.1. Individual maximization

The budget constraint for individual can be written as

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t}=(1-\tau_y)y(k,G)-(1+\tau_c)c-(\delta+\tau_k)k. \tag{36}$$

The representative agent chooses his consumption path, $c(t)$, and capital accumulation path, $k(t)$, to maximize his discounted utility, namely

$$\mathrm{Max}\int_0^\infty [u(c)+v(G)]\mathrm{e}^{-\rho t}\,\mathrm{d}t$$

subject to the budget constraint (36). The initial capital stock, $k(0)=k_0$, is given.

Define the Hamiltonian as

$$H=u(c)+v(G)+\lambda((1-\tau_y)y(k,G)-(1+\tau_c)c-(\delta+\tau_k)k).$$

The first-order conditions are

$$\frac{\partial u(c)}{\partial c}=(1+\tau_c)\lambda, \tag{37}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t}=-\lambda\left\{(1-\tau_y)\frac{\partial y_i(k,G)}{\partial k}-\rho-\delta-\tau_k\right\} \tag{38}$$

and the transversality condition is

$$\lim_{t\to\infty} \lambda k \mathrm{e}^{-\rho t} = 0.$$

From condition (37), we have

$$c = c(\tau_c, \lambda).$$

At the steady state, we have

$$(1-\tau_y)\frac{\partial y_i(k,G)}{\partial k} - \rho - \delta - \tau_k = 0, \tag{39}$$

$$(1-\tau_y)y(k,G) - (1+\tau_c)c(\tau_c,\lambda) - (\delta+\tau_k)k = 0. \tag{40}$$

3.2. Government maximization

Given the optimal choices for the individual, the government chooses its public service, G, and its taxes τ_y, τ_k, and τ_c to maximize the steady-state agent's welfare subject to its budget constraint, that is

$$\max U(c(\tau_c,\lambda),G) = u(c(\tau_c,\lambda)) + v(G)$$

subject to the individual's first-order conditions (37), (39), (40), the budget constraints of the government, (35), and the nonnegative constraints for the tax rates:

$$\tau_y \geq 0, \quad \tau_k \geq 0, \ \tau_c \geq 0.$$

Now, define the Lagrangian function as

$$\begin{aligned} L = {} & U(c(\tau_c,\lambda),G) + \mu_1\left[(1-\tau_y)\frac{\partial y(k,G)}{\partial k} - \rho - \delta - \tau_k\right] \\ & + \mu_2[(1-\tau_y)y(k,G) - (1+\tau_c)c(\tau_c,\lambda) - (\delta+\tau_k)k] \\ & + \mu_3[\tau_c c(\tau_c,\lambda) + \tau_y y + \tau_k k - G] + \mu_4\tau_k + \mu_5\tau_c + \mu_6\tau_y, \end{aligned}$$

where μ_1 is the multiplier associated with Eq. (39), μ_2 is the multiplier associated with Eq. (40), μ_3 is the multiplier associated with Eq. (35), μ_4, μ_5 and μ_6 are the multipliers associated with the nonnegative constraints of tax rates, respectively.

The first-order conditions are

$$\frac{\partial L}{\partial \tau_y} = -\mu_1\frac{\partial y(k,G)}{\partial k} - \mu_2 y(k,G) + \mu_3 y + \mu_6 = 0, \tag{41}$$

$$\mu_6\tau_y = 0, \quad \mu_6 \geq 0, \tag{42}$$

$$\frac{\partial L}{\partial \lambda} = \frac{\partial U(c,G)}{\partial c}\frac{\partial c}{\partial \lambda} - \mu_2(1+\tau_c)\frac{\partial c}{\partial \lambda} + \mu_3\tau_c\frac{\partial c}{\partial \lambda} = 0, \tag{43}$$

$$\frac{\partial L}{\partial k} = \mu_1(1-\tau_y)\frac{\partial^2 y(k,G)}{\partial k^2} + \mu_2\left\{(1-\tau_y)\frac{\partial y(k,G)}{\partial k} - (\delta+\tau_k)\right\} + \mu_3\tau_k = 0, \tag{44}$$

$$\frac{\partial L}{\partial G} = \frac{\partial u(c,G)}{\partial G} + \mu_1(1-\tau_y)\frac{\partial^2 y(k,G)}{\partial k \partial G} + \mu_2(1-\tau_y)\frac{\partial y(k,G)}{\partial G} - \mu_3 = 0, \tag{45}$$

$$\frac{\partial L}{\partial \tau_c} = -\mu_2 c + \mu_3 c + \frac{\partial U(c,G)}{\partial c}\frac{\partial c}{\partial \tau_c} - \mu_2(1+\tau_c)\frac{\partial c}{\partial \tau_c} + \mu_3\tau_c\frac{\partial c}{\partial \tau_c} + \mu_5 = 0, \tag{46}$$

$$\mu_5\tau_c = 0, \quad \mu_5 \geq 0, \tag{47}$$

$$\frac{\partial L}{\partial \tau_k} = -\mu_1 - \mu_2 k + \mu_3 k + \mu_4 = 0, \tag{48}$$

$$\mu_4\tau_k = 0, \quad \mu_4 \geq 0. \tag{49}$$

Proposition 4. The steady-state property tax and income tax are zero, and the steady-state consumption tax is positive.

Proof. Step 1: it is obvious that all three taxes cannot be positive at the same time.

Step 2: any two taxes among the three cannot be positive at the same time. This can be seen as follows:

1. If $\tau_k > 0$, $\tau_c > 0$, from Eqs. (43), (46), and (48), we have

$$-\mu_2 + \mu_3 = 0, \quad \mu_1 = 0.$$

Then, from Eq. (44), we have

$$\mu_2\left[(1-\tau_y)\frac{\partial y(k,G)}{\partial k} - \delta\right] = 0.$$

Hence,

$$\mu_2 = \mu_3 = 0, \quad \mu_1 = 0.$$

Then, from (43),

$$\frac{\partial U(c,G)}{\partial c}\frac{\partial c}{\partial \lambda} = 0$$

which is impossible.

2. If $\tau_y > 0$, $\tau_c > 0$, from Eqs. (41), (43), and (46), we have

$$-\mu_2 + \mu_3 = 0, \quad \mu_1 = 0.$$

Then, again from Eq. (44), we have

$$\mu_2 = \mu_3 = 0, \quad \mu_1 = 0$$

which again requires the impossible:

$$\frac{\partial U(c,G)}{\partial c}\frac{\partial c}{\partial \lambda} = 0.$$

3. If $\tau_y > 0$, $\tau_k > 0$. from Eqs. (41), (43), and (48), we have

$$\mu_1 \frac{\partial y(k,G)}{\partial k} + \mu_2 y(k,G) - \mu_3 y = 0,$$
$$-\mu_1 - \mu_2 k + \mu_3 k = 0.$$

Then,

$$\frac{\partial y(k,G)}{\partial k} k = y$$

which is impossible.

Step 3: we prove that it is impossible to have $\tau_c = 0$. In fact, from Eqs. (43) and (46), we have

$$\text{sign}(-\mu_2 + \mu_3) = -\text{sign}(\mu_5) \le 0.$$

If $\tau_y > 0$, $\mu_6 = 0$, from Eq. (41), we have

$$\text{sign}(\mu_1) = \text{sign}(-\mu_2 + \mu_3) \le 0.$$

Furthermore, from Eq. (44), we have

$$\mu_1(1 - \tau_y)\frac{\partial^2 y(k,G)}{\partial k^2} + \mu_2 \rho = 0,$$

which leads to

$$\mu_1 = 0 = \mu_2.$$

But from (41), we must have

$$\mu_2 = \mu_3 = 0, \quad \mu_1 = 0,$$

which again contradicts (43). Therefore, $\tau_y = 0$.

If $\tau_k > 0$, from Eq. (48), we have

$$\text{sign}(\mu_1) = \text{sign}(-\mu_2 + \mu_3) \leq 0.$$

Combining with Eq. (44), we have

$$\mu_1(1 - \tau_y)\frac{\partial^2 y(k, G)}{\partial k^2} + \mu_2\rho + \mu_3\tau_k = 0,$$

which in turn implies

$$\mu_2 = \mu_3 = 0, \quad \mu_1 = 0.$$

This is again impossible. Hence $\tau_k = 0$.

Now since government spending, G, enters both the utility function and the production function, and since the Inada conditions hold, government spending must be financed by the consumption tax. Therefore, $\tau_c > 0$. □

The intuition is rather simple: the consumption tax is less distortionary than the income tax and the property tax. In a one-level government, it is optimal to only tax consumption to finance public spending. But when there are two levels of government, this result does not hold anymore, as shown in Section 2. Now we turn back to our analysis of optimal taxes and federal transfer when there are two levels of government.

4. An example

One of the difficulties with the optimal taxation literature is that we cannot make too much intuitive sense out of the numerous first-order conditions. Very often explicit solutions are very hard to obtain even in static models. In this section, we show that with some specific production and utility functions, we can obtain explicit solutions to the long-run optimal local taxes, optimal federal income tax, and optimal federal transfers in a dynamic framework. However, hard and tedious calculations with a sense of guessing and good luck are a prerequisite.

Let the production function be

$$y_i = A_i k_i^{\alpha} l_i^{\varepsilon} f^{\beta} s_i^{\gamma} = A_i k_i^{\alpha} f^{\beta} s_i^{\gamma},$$

where $\alpha + \varepsilon + \beta + \gamma = 1$, and $\alpha + \beta + \gamma < 1$ because $0 < \varepsilon < 1$ and $l_i = 1$ by our assumption.

Let the utility function be

$$u(c_i, f, s_i) = \ln c_i + \omega_1^i \ln f + \omega_2^i \ln s_i.$$

First, from private optimization, we have

$$\begin{aligned}
k_i &= \left(\frac{\rho + \delta + \tau_{k_i}^i}{\alpha(1 - \tau_f)}\right)^{1/(\alpha-1)} f^{\beta/(1-\alpha)} s_i^{\gamma/(1-\alpha)}, \quad i = 1, 2,\\
y_i &= \left(\frac{\rho + \delta + \tau_{k_i}^i}{\alpha(1 - \tau_f)}\right)^{\alpha/(\alpha-1)} f^{\beta/(1-\alpha)} s_i^{\gamma/(1-\alpha)}, \quad i = 1, 2,\\
c_i &= \frac{\rho + (1-\alpha)(\delta + \tau_{k_i}^i)}{\alpha(1 + \tau_{c_i}^i)} \left(\frac{\rho + \delta + \tau_{k_i}}{\alpha(1 - \tau_f)}\right)^{1/(\alpha-1)} f^{\beta/(1-\alpha)} s_i^{\gamma/(1-\alpha)}, \quad i = 1, 2.
\end{aligned} \tag{50}$$

Second, local government i maximizes

$$\max_{\tau_{c_i}^i, \tau_{k_i}^i, s_i,} \ln c_i + \omega_1^i \ln f + \omega_2^i \ln s_i$$

subject to (50) and its budget constraint

$$s_i - g s_i = \tau_{c_i}^i c_i + \tau_{k_i}^i k_i. \tag{51}$$

Substituting Eq. (50) into (51), we have

$$\begin{aligned}
s_i &= \frac{\tau_{c_i}^i}{1 - g} \frac{\rho + (1-\alpha)(\delta + \tau_{k_i}^i)}{\alpha(1 + \tau_{c_i}^i)} \left(\frac{\rho + \delta + \tau_{k_i}^i}{\alpha(1 - \tau_f)}\right)^{1/(\alpha-1)} f^{\beta/(1-\alpha)} s_i^{\gamma/(1-\alpha)}\\
&\quad + \frac{\tau_{k_i}^i}{1 - g} \left(\frac{\rho + \delta + \tau_{k_i}^i}{\alpha(1 - \tau_f)}\right)^{1/(\alpha-1)} f^{\beta/(1-\alpha)} s_i^{\gamma/(1-\alpha)}\\
&= \left(\frac{\tau_{c_i}^i(\rho + (1-\alpha)\delta) + (\tau_{c_i}^i + \alpha)\tau_{k_i}^i}{\alpha(1 + \tau_{c_i}^i)}\right)\\
&\quad \left(\frac{\rho + \delta + \tau_{k_i}^i}{\alpha(1 - \tau_f)}\right)^{1/(\alpha-1)} f^{\beta/(1-\alpha)} s_i^{\gamma/(1-\alpha)} \frac{1}{1 - g}.
\end{aligned}$$

Therefore, we get

$$s_i = \left(\frac{\tau^i_{c_i}(\rho + (1-\alpha)\delta) + (\tau^i_{c_i} + \alpha)\tau^i_{k_i}}{\alpha(1+\tau^i_{c_i})}\right)^{(1-\alpha)/(1-\alpha-\gamma)} \left(\frac{1}{1-g}\right)^{(1-\alpha)/(1-\alpha-\gamma)} \times \left(\frac{\rho + \delta + \tau^i_{k_i}}{\alpha(1-\tau_f)}\right)^{-1/(1-\alpha-\gamma)} f^{\beta/(1-\alpha-\gamma)}. \tag{52}$$

Now, the objective function for the local government can be rewritten as

$$\begin{aligned}
\ln c_i + \omega^i_2 \ln s_i &= \ln(\rho + (1-\alpha)(\delta + \tau^i_{k_i})) - \ln(1+\tau^i_{c_i}) \\
&\quad + \frac{1}{\alpha - 1}\ln(\rho + \delta + \tau^i_{k_i}) \\
&\quad + \left(\frac{\gamma}{1-\alpha} + \omega^i_2\right)\ln s_i + \text{constant} \\
&= \ln(\rho + (1-\alpha)(\delta + \tau^i_{k_i})) - \ln(1+\tau^i_{c_i}) \\
&\quad + \frac{1}{\alpha - 1}\ln(\rho + \delta + \tau^i_{k_i}) + \frac{\omega^i_2(1-\alpha)+\gamma}{1-\alpha-\gamma} \\
&\quad \times [\ln(\tau^i_{c_i}(\rho + (1-\alpha)\delta) + (\tau^i_{c_i} + \alpha)\tau^i_{k_i}) - \ln(1+\tau^i_{c_i})] \\
&\quad - \frac{\omega^i_2(1-\alpha)+\gamma}{(1-\alpha)(1-\alpha-\gamma)}\ln(\rho + \delta + \tau^i_{k_i}) + \text{constant}.
\end{aligned} \tag{53}$$

The local government's optimization problem is equivalent to maximizing the expression in Eq. (53) with respect to $\tau^i_{c_i}$ and $\tau^i_{k_i}$. Thus, we have

$$\begin{aligned}
&\frac{1-\alpha}{\rho + (1-\alpha)(\delta + \tau^i_{k_i})} + \frac{\omega^i_2(1-\alpha)+\gamma}{1-\alpha-\gamma}\,\frac{\tau^i_{c_i} + \alpha}{\tau^i_{c_i}(\rho + (1-\alpha)\delta) + (\tau^i_{c_i} + \alpha)\tau^i_{k_i}} \\
&\quad - \frac{\omega^i_2(1-\alpha)+\gamma}{(1-\alpha)(1-\alpha-\gamma)}\,\frac{1}{\rho + \delta + \tau^i_{k_i}} + \frac{1}{\alpha - 1}\,\frac{1}{\rho + \delta + \tau^i_{k_i}} = 0, \\
&\quad - \frac{1}{1+\tau^i_{c_i}} + \frac{\omega^i_2(1-\alpha)+\gamma}{1-\alpha-\gamma} \\
&\quad \times \left[\frac{\rho + (1-\alpha)\delta + \tau^i_{k_i}}{\tau^i_{c_i}(\rho + (1-\alpha)\delta) + (\tau^i_{c_i} + \alpha)\tau^i_{k_i}} - \frac{1}{1+\tau^i_{c_i}}\right] = 0.
\end{aligned}$$

From the above two equations, we can determine $\tau^i_{c_i}$ and $\tau^i_{k_i}$ as

$$\begin{aligned}
\tau^i_{c_i} &= \frac{\omega^i_2(1-\alpha)+\gamma}{1-\alpha-\gamma}, \\
\tau^i_{k_i} &= 0.
\end{aligned} \tag{54}$$

Third, for the federal government, it maximizes the weighted steady-state welfare of the two agents in the two localities

$$\max \sum_{i=1,2} \chi_i(\ln c_i + \omega_1^i \ln f + \omega_2^i \ln s_i)$$

subject to the optimal behaviour of private agents and local governments and its budget constraint:

$$f + gs_1 + gs_2 = \tau_f y^1 + \tau_f y^2 \tag{55}$$

Substituting Eqs. (51) and (52) into Eq. (55) yields

$$\begin{aligned}
f = \sum_{i=1}^{2} \Bigg\{ & \tau_f \left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{\alpha/(\alpha-1)} \left(\frac{\tau_{c_i}^i \rho}{\alpha(1+\tau_{c_i}^i)}\right)^{\gamma/(1-\alpha-\gamma)} \\
& \times \left(\frac{1}{1-g}\right)^{\gamma/(1-\alpha-\gamma)} \left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{-1/(1-\alpha-\gamma)\gamma/(1-\alpha)} f^{\beta/(1-\alpha-\gamma)} \\
& - g \left(\frac{\tau_{c_i}^i \rho}{\alpha(1+\tau_{c_i}^i)}\right)^{(1-\alpha)/(1-\alpha-\gamma)} \left(\frac{1}{1-g}\right)^{(1-\alpha)/(1-\alpha-\gamma)} \\
& \times \left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{-1/(1-\alpha-\gamma)} f^{\beta/(1-\alpha-\gamma)} \Bigg\} \\
= \sum_{i=1}^{2} \Bigg\{ & f^{\beta/(1-\alpha-\gamma)} \Bigg\{ \tau_f \left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{\alpha/(\alpha-1)} \left(\frac{\tau_{c_i}^i \rho}{\alpha(1+\tau_{c_i}^i)}\right)^{\gamma/(1-\alpha-\gamma)} \\
& \times \left(\frac{1}{1-g}\right)^{\gamma/(1-\alpha-\gamma)} \left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{-1/(1-\alpha-\gamma)\gamma/(1-\alpha)} \\
& - g \left(\frac{\tau_{c_i}^i \rho}{\alpha(1+\tau_{c_i}^i)}\right)^{(1-\alpha)/(1-\alpha-\gamma)} \left(\frac{1}{1-g}\right)^{(1-\alpha)/(1-\alpha-\gamma)} \\
& \times \left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{-1/(1-\alpha-\gamma)} \Bigg\}\Bigg\},
\end{aligned}$$

$$\begin{aligned}
f^{(1-\alpha-\beta-\gamma)/(1-\alpha-\gamma)} = \sum_{i=1}^{2} \Bigg\{ & \tau_f \left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{\alpha/(\alpha-1)} \left(\frac{\tau_{c_i}^i \rho}{\alpha(1+\tau_{c_i}^i)}\right)^{\gamma/(1-\alpha-\gamma)} \\
& \times \left(\frac{1}{1-g}\right)^{\gamma/(1-\alpha-\gamma)} \left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{-1/(1-\alpha-\gamma)\gamma/(1-\alpha)}
\end{aligned}$$

$$-g\left(\frac{\tau^i_{c_i}\rho}{\alpha(1+\tau^i_{c_i})}\right)^{(1-\alpha)/(1-\alpha-\gamma)}\left(\frac{1}{1-g}\right)^{(1-\alpha)/(1-\alpha-\gamma)}$$

$$\times\left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{-1/(1-\alpha-\gamma)}\Bigg\}$$

$$=\sum_{i=1}^{2}\left\{\tau_f\frac{\rho}{\alpha(1-\tau_f)}A_i^{\gamma}-\frac{g}{1-g}A_i^{1-\alpha}\right\}$$

$$\left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{-1/(1-\alpha-\gamma)}\left(\frac{1}{1-g}\right)^{\gamma/(1-\alpha-\gamma)},$$

where $A_i = (\frac{\tau^i_{c_i}\rho}{\alpha(1+\tau^i_{c_i})})^{1/(1-\alpha-\gamma)}$.

Now, we have

$$f=\left\{\left[\tau_f\frac{\rho}{\alpha(1-\tau_f)}A_1^{\gamma}-\frac{g}{1-g}A_1^{1-\alpha}\right]\right.$$

$$\left.+\left[\tau_f\frac{\rho}{\alpha(1-\tau_f)}A_2^{\gamma}-\frac{g}{1-g}A_2^{1-\alpha}\right]\right\}^{(1-\alpha-\gamma)/(1-\alpha-\beta-\gamma)}$$

$$\times\left(\frac{\rho}{\alpha(1-\tau_f)}\right)^{1/(1-\alpha-\beta-\gamma)}\left(\frac{1}{1-g}\right)^{\gamma/(1-\alpha-\beta-\gamma)}. \tag{56}$$

Hence, the objective function of the federal government can be written as

$$\chi_1(\ln c_1+\theta_1^1\ln f+\theta_2^1\ln s_1)+\chi_2(\ln c_2+\theta_1^2\ln f+\theta_2^2\ln s_2)$$

$$=\chi_1\left\{\frac{1}{1-\alpha}\ln(1-\tau_f)+\left(\frac{\gamma}{1-\alpha}+\theta_2^1\right)\right.$$

$$\left.\left[-\frac{1-\alpha}{1-\alpha-\gamma}\ln(1-g)+\frac{1}{1-\alpha-\gamma}\ln(1-\tau_f)\right]\right\}$$

$$+\chi_2\left\{\frac{1}{1-\alpha}\ln(1-\tau_f)+\left(\frac{\gamma}{1-\alpha}+\theta_2^2\right)\right.$$

$$\left.\left[-\frac{1-\alpha}{1-\alpha-\gamma}\ln(1-g)+\frac{1}{1-\alpha-\gamma}\ln(1-\tau_f)\right]\right\}$$

$$+[\chi_1C_1+\chi_2C_2]\frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma}$$

$$\ln\left\{\left[\tau_f\frac{\rho}{\alpha(1-\tau_f)}(A_1^{\gamma}+A_2^{\gamma})-\frac{g}{1-g}(A_1^{1-\alpha}+A_2^{1-\alpha})\right]\right.$$
$$+[\chi_1C_1+\chi_2C_2]\left\{\frac{1}{1-\alpha-\beta-\gamma}\ln(1-\tau_f)\right.$$
$$\left.+\frac{-\gamma}{1-\alpha-\beta-\gamma}\ln(1-g)\right\}+\text{constant}, \tag{57}$$

where

$$C_i=\left(\frac{\gamma}{1-\alpha}+\theta_2^i\right)\frac{\beta}{1-\alpha-\gamma}+\frac{\gamma}{1-\alpha}+\theta_2^i, \quad i=1,2.$$

The federal government's optimization problem is equivalent to maximizing the expression in Eq. (57) with respect to τ_f and g. Thus, we have

$$-\chi_1\frac{(1-\alpha)\theta_2^1-\gamma}{1-\alpha-\gamma}-\chi_2\frac{(1-\alpha)\theta_2^1-\gamma}{1-\alpha-\gamma}-[\chi_1C_1+\chi_2C_2]\frac{\gamma}{1-\alpha-\beta-\gamma}$$
$$[\chi_1C_1+\chi_2C_2]\frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma}\frac{\frac{1}{1-g}(A_1^{1-\alpha}+A_2^{1-\alpha})}{\tau_f\frac{\rho}{\alpha(1-\tau_f)}(A_1^{\gamma}+A_2^{\gamma})-\frac{g}{1-g}(A_1^{1-\alpha}+A_2^{1-\alpha})}$$
$$=0,$$

$$-\chi_1\frac{1+\theta_2^1}{1-\alpha-\gamma}-\chi_2\frac{1+\theta_2^2}{1-\alpha-\gamma}-[\chi_1C_1+\chi_2C_2]\frac{1}{1-\alpha-\beta-\gamma}$$
$$[\chi_1C_1+\chi_2C_2]\frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma}\frac{\frac{\rho}{\alpha(1-\tau_f)}(A_1^{\gamma}+A_2^{\gamma})}{\tau_f\frac{\rho}{\alpha(1-\tau_f)}(A_1^{\gamma}+A_2^{\gamma})-\frac{g}{1-g}(A_1^{1-\alpha}+A_2^{1-\alpha})}$$
$$=0.$$

From these derivations above, we have

Proposition 5. The optimal local taxes, optimal federal income tax, and optimal federal transfers are given by

$$\tau_{c_i}^i=\frac{\omega_2^i(1-\alpha)+\gamma}{1-\alpha-\gamma},$$

$$\tau_{k_i}^i=0,$$

$$g=1-\frac{(K-K_2+K_1)(A_1^{1-\alpha}+A_2^{1-\alpha})}{K_1[(A_1^{1-\alpha}+A_2^{1-\alpha})-\frac{\rho}{\alpha}(A_1^{\gamma}+A_2^{\gamma})]},$$

Table 1
Optimal tax rates and transfers versus the productivity of local spending

γ	0.0	0.05	0.1	0.15	0.20	0.25	0.30
τ_c^1	0.12	0.206154	0.30667	0.425455	0.568	0.742222	0.96
τ_c^2	0.1	0.184615	0.283333	0.4	0.54	0.711111	0.925
τ_f	0.244043	0.23142	0.216711	0.199333	0.178483	0.153002	0.121152
g	−0.753848	−0.42049	−0.329976	−0.300318	−0.298075	−0.313655	−0.344829
τ_k^1	0	0	0	0	0	0	0
τ_k^2	0	0	0	0	0	0	0

$$\tau_f = 1 - \frac{(K - K_2 + K_1)\frac{\rho}{\alpha}(A_1^{\gamma} + A_2^{\gamma})}{K_2[(A_1^{1-\alpha} + A_2^{1-\alpha}) - \frac{\rho}{\alpha}(A_1^{\gamma} + A_2^{\gamma})]},$$

where

$$K_1 = \chi_1 \frac{(1-\alpha)\theta_2^1 - \gamma}{1-\alpha-\gamma} + \chi_2 \frac{(1-\alpha)\theta_2^1 - \gamma}{1-\alpha-\gamma} + [\chi_1 C_1 + \chi_2 C_2] \frac{\gamma}{1-\alpha-\beta-\gamma},$$

$$K_2 = \chi_1 \frac{1+\theta_2^1}{1-\alpha-\gamma} + \chi_2 \frac{1+\theta_2^2}{1-\alpha-\gamma} + [\chi_1 C_1 + \chi_2 C_2] \frac{1}{1-\alpha-\beta-\gamma},$$

$$K = [\chi_1 C_1 + \chi_2 C_2] \frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma}.$$

In this proposition, as a result of specific assumptions on the utility function and technology, local consumption tax at locality i $(i=1,2)$ only depends on its own preference and technology parameters. However the optimal federal income tax and transfers clearly depend on all preference and technology parameters of the two localities plus the social-welfare weights assigned to agent 1 and agent 2.

In order to get some intuition out of these explicit solutions of optimal taxes and transfer schemes, let us make some numerical simulations.

In Table 1, we focus on how the change in the productivity of local public spending measured by the parameter γ affects the choices of taxes and transfers. All other exogenous parameters take the following values: $\alpha = 0.3$, $\beta=0.2$, $\omega_1^1=0.2$, $\omega_1^2=0.12$, $\omega_2^1=0.2$, $\omega_2^2=0.1$, $\chi_1=0.6$, $\chi_2=0.4$, and $\rho=0.05$. When $\gamma=0$, local public spending does not contribute to private production.[10] But at the same time, the productivity of federal public spending is set at a relatively high value: $\beta = 0.2$. Therefore, it is optimal to let localities 1 and 2 levy consumption taxes and transfer a large share of local revenues to

[10] Local spending does enter the utility function of the agents as $\omega_2^1 = 0.2$ and $\omega_2^2 = 0.1$.

Table 2
Optimal tax rates and transfers versus the productivity of private Capital

α	τ_c^1	τ_c^2	τ_f	g	τ_k^1	τ_k^2
0.10	0.26	0.2375	0.153934	−0.0232071	0	0
0.15	0.269333	0.246667	0.136664	−0.105997	0	0
0.20	0.28	0.257143	0.118314	−0.200658	0	0
0.25	0.292328	0.269231	0.09863336	−0.309935	0	0
0.30	0.306667	0.283333	0.0772916	−0.437486	0	0
0.35	0.323636	0.3	0.0538344	−0.588302	0	0
0.40	0.344	0.32	0.0276274	−0.769371	0	0
0.45	0.368889	0.344444	−0.00224604	−0.990787	0	0
0.50	0.4	0.375	−0.0371607	−1.26769	0	0
0.55	0.44	0.414286	−0.0792772	−1.62389	0	0

the federal government to finance federal spending. As local public spending gradually becomes more productive (i.e., γ rises from zero to 0.30), local consumption taxes rises sharply from around 10% to more than 90%. Please note the difference in the rates of the consumption tax between locality 1 and locality 2, which results from the difference in the effects of local spending on private utility in the two localities. That is to say, since $\omega_2^1 = 0.2 > \omega_2^2 = 0.1$, it is always optimal for locality 1 to levy a higher consumption tax than locality 2: $\tau_c^1 > \tau_c^2$. As federal income tax is more distortionary than pure consumption tax, the federal income tax rate gradually decreases from 24.4% to 12.1%. At the same time, federal transfers are always negative, and local consumption taxes are utilized to finance federal spending. These 'reverse' federal transfers are always optimal when a local consumption tax is available and when the federal government is the leader in its Stackelberg game with local governments and private agents.

It is also possible for the federal government to levy a negative income tax (provide a production subsidy) and 'force' the two local governments to tax consumption and remit their revenues to the federal government. Therefore, the less distortionary consumption tax can be used to finance federal spending and to subsidize private production. This case is illustrated in Table 2.

This table shows the responses of the optimal tax rates and federal transfers to changes in the productivity of private capital measured by the parameter α. All other preference and production parameters are fixed at the following values: $\beta = 0.1$, $\gamma = 0.1$, $\omega_1^1 = 0.1$, $\omega_1^2 = 0.12$, $\omega_2^1 = 0.1$, $\omega_2^2 = 0.1$, $\chi_1 = 0.6$, $\chi_2 = 0.4$, and $\rho = 0.05$. As α rises from 0.10 to 0.55, federal income tax decreases from 15.4% to −7.9%. The rate of federal transfers changes from −2.3% to −162%.

Table 3
Optimal tax rates and transfers versus the productivity of federal spending

β	0.01	0.05	0.1	0.15	0.20	0.25	0.30
τ_c^1	0.30667	0.30667	0.30667	0.30667	0.30667	0.30667	0.30667
τ_c^2	0.283333	0.283333	0.283333	0.283333	0.283333	0.283333	0.283333
τ_f	0.33980	0.354832	0.373571	0.392311	0.41105	0.429789	0.448529
g	0	0	0	0	0	0	0
τ_k^1	0	0	0	0	0	0	0
τ_k^2	0	0	0	0	0	0	0

While there does exist a rationale for optimal reverse transfers from local to federal governments, the reality in developed countries shows the opposite. We will not argue whether reality is a violation of the theoretical optimality here. But it is interesting to note that, if we impose the condition $g \geq 0$, we have

Proposition 6. When $g \geq 0$, the optimal local taxes, federal income tax, and federal transfers are

$$\tau_{c_i}^i = \frac{\omega_2^i(1-\alpha)+\gamma}{1-\alpha-\gamma},$$

$$\tau_{k_i}^i = 0,$$

$$\tau_f = 1 - \frac{K_1}{K_2}\frac{\rho}{\alpha}\frac{A_1^\gamma + A_2^\gamma}{A_1^{1-\alpha}+A_2^{1-\alpha}},$$

$$g = 0.$$

Therefore, federal transfers are always zero. This ‘forced’ choice of federal transfers effectively precludes the federal government from taking the advantage of the less distortionary local consumption to finance federal spending and subsidize private production. A simple example is given in Table 3.

In Table 3, the measure of the productivity of federal spending rises from 0.01 to a very high value of 0.30. Other parameter values are set to be: $\alpha = 0.3$, $\gamma = 0.1$, $\omega_1^1 = 0.2$, $\omega_1^2 = 0.12$, $\omega_2^1 = 0.2$, $\omega_2^2 = 0.1$, $\chi_1 = 0.6$, $\chi_2 = 0.4$, and $\rho = 0.05$. It is clear that, since $g = 0$, federal spending can only be financed by the federal income tax. As its productivity increases, the federal income tax rate rises from 34% to 44%. As a result of our chosen utility and production functions, local consumption taxes remain constant since they are independent of the productivity of federal spending.

5. The case of a positive property tax

In most developed countries, the local property tax is positive. In order to generate a positive property tax in our model, it is easiest to set the local consumption tax at zero. In this case, it can be shown that

Proposition 7. If the local consumption tax is zero, then

$$\tau_k^1 = \frac{\sqrt{[4\alpha(\gamma+(1-\alpha)\omega_2^1](1-\alpha)(1+\omega_2^1)\rho^2+[\alpha^2(1+\omega_2^1)+\gamma+\omega_2^1-(3\omega_2^1+2)\alpha]^2\rho^2}}{2\alpha(1-\alpha)(1+\omega_2^1)} + \frac{[\alpha^2(1+\omega_2^1)+\gamma+\omega_2^1-(3\omega_2^1+2)\alpha]\rho}{2\alpha(1-\alpha)(1+\omega_2^1)}$$

$$\tau_k^2 = \frac{\sqrt{[4\alpha(\gamma+(1-\alpha)\omega_2^2](1-\alpha)(1+\omega_2^2)\rho^2+[\alpha^2(1+\omega_2^2)+\gamma+\omega_2^2-(3\omega_2^2+2)\alpha]^2\rho^2}}{2\alpha(1-\alpha)(1+\omega_2^2)} + \frac{[\alpha^2(1+\omega_2^2)+\gamma+\omega_2^2-(3\omega_2^2+2)\alpha]\rho}{2\alpha(1-\alpha)(1+\omega_2^2)}$$

$$g = 1 - \frac{(K-K_2+K_1)(B_1^{1-\alpha}+B_2^{1-\alpha})}{K_1[(B_1^{1-\alpha}+B_2^{1-\alpha})-\frac{\rho}{\alpha}(B_1^{\gamma}+B_2^{\gamma})]},$$

$$\tau_f = 1 - \frac{(K-K_2+K_1)\frac{\rho}{\alpha}(B_1^{\gamma}+B_2^{\gamma})}{K_2[(B_1^{1-\alpha}+B_2^{1-\alpha})-\frac{\rho}{\alpha}(B_1^{\gamma}+B_2^{\gamma})]},$$

where

$$B_i = (\tau_k^i)^{1/(1-\alpha-\gamma)},$$

$$K_1 = \chi_1\frac{(1-\alpha)\omega_2^1-\gamma}{1-\alpha-\gamma}+\chi_2\frac{(1-\alpha)\omega_2^1-\gamma}{1-\alpha-\gamma}+[\chi_1 C_1+\chi_2 C_2]\frac{\gamma}{1-\alpha-\beta-\gamma},$$

$$K_2 = \chi_1\frac{1+\omega_2^1}{1-\alpha-\gamma}+\chi_2\frac{1+\omega_2^2}{1-\alpha-\gamma}+[\chi_1 C_1+\chi_2 C_2]\frac{1}{1-\alpha-\beta-\gamma},$$

$$K = [\chi_1 C_1+\chi_2 C_2]\frac{1-\alpha-\gamma}{1-\alpha-\beta-\gamma}.$$

Table 4 illustrates how local property taxes change with respect to the productivity of private capital.

Table 4
Optimal tax rates and transfers versus private productivity

α	0.2	0.25	0.3	0.35	0.40
τ_k^1	0.0548904	0.0357433	0.0264897	0.0210318	0.0174299
τ_k^2	0.0446251	0.0305477	0.0232356	0.0187604	0.0157411
τ_f	0.278583	0.292879	0.292324	0.286295	0.277641
g	0.0161731	0.171493	0.228833	0.248805	0.249675
τ_c^1	0	0	0	0	0
τ_c^2	0	0	0	0	0

In Table 4, we have set all other parameter values at $\gamma=0.1$, $\beta=0.2$, $\omega_1^1=0.2$, $\omega_1^2=0.12$, $\omega_2^1=0.2$, $\omega_2^2=0.1$, $\chi_1=0.6$, $\chi_2=0.4$, and $\rho=0.05$. As the productivity of private capital, α, rises, optimal property taxes in both localities decrease steadily. In the meantime, federal transfers to local governments rise. The rising federal transfers are financed from the federal income tax, which first increases and then declines—a typical Laffer curve. The reason for this result is as follows. Local property taxes are always more distortionary than the federal income tax. As the productivity of private capital rises, it is optimal to reduce local property taxes and stimulate capital accumulation. To maintain local public spending, the federal government will raise its income tax and its transfers to localities. With a cut in the property tax in both localities, coupled with rising productivity, private capital accumulation accelerates. In the end, the tax base for the federal income tax expands, and the federal government can raise even more revenues through a reduced income tax rate. Therefore, the federal income tax is a Laffer curve of the productivity of private capital.

6. Conclusion

This paper has presented a fiscal federalism approach to the optimal federal income tax, local consumption tax, local property tax, and federal transfers in a dynamic model of capital accumulation with some complicated structure of the Stackelberg games among private agents, local governments, and the federal government. In general, the optimal long-run local property tax is zero if the local property tax is constrained to be nonnegative, whereas the optimal local consumption tax is always positive. When the local consumption tax is chosen optimally, the federal income tax can be either positive or negative. For most reasonable parameter values, our numerical calculations have shown that with a positive local consumption tax, there is a reverse transfer from local governments to the federal government. In this case, the local consumption tax is used to finance federal spending and even finance federal subsidy to private production if private capital is very productive relative to federal and local public expenditures. These results stand in sharp contrast to the real world where local governments usually receive transfers from the federal government. Of course, in light of a large body of literature on optimal taxation since the 1970s, it is not surprising to find that many optimal tax (subsidy) formulas have not been carried out in practice. Whereas the actual implementation of taxes and transfers in different countries are a result of political, economic, and historical circumstances, our theoretical inquiry does illustrate the potential welfare gain from using the local consumption tax to finance federal spending and even to subsidize private production. However, our finding of a negative federal transfer is, at least partially, a result of assuming away interjurisdictional benefit spillovers from local spending. In the presence of such external benefits, decentralized decisions about spending would typically lead to the underprovision of such goods, hence makes it efficient for the federal government to use a system of matching grants that reduce the marginal cost of local spending and, thus, encourage a higher level of spending on those goods that generate positive externalities.[11]

It is also worthwhile to note that, without the multi-tier government structure, namely, if there exists only a one-level government, it is always optimal to set both the property tax and the income tax to zero when there is a consumption tax. Once we have a two-tier system of federal government with the Stackelberg game between the federal government (the leader) and local governments (the followers) and private agents (the followers), the federal income tax can be positive.

In general, if the local consumption tax is set to zero, then the local property tax is positive. Furthermore, in this case, the federal income tax and federal transfers to local governments are always positive. This is because a federal income tax is less distortionary than a local property tax in its adverse effect on private capital accumulation.

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