The Effects of Patent Protection: A Growth Model with Status Preference

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Abstract

We build a growth model with status preference to explore the effects of patent protection on innovation, inequality and social welfare. The main results are as follows. There is a non-monotonic relationship between patent protection and innovation. In addition, the effect of patent protection on social welfare is non-monotonic when the strength of status preference is small, whereas patent protection lowers social welfare when the strength of status preference is large. Finally, strengthening patent protection enlarges wealth inequality when agents have different time and status preferences.

JEL Classification: O31, O34, O40

Keywords: Patent Protection; Status Preference; Innovation; Inequality; Social Welfare

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1 Introduction

Conventional wisdom argues that the patent system encourages innovation. This argument has been questioned by various empirical studies such as Kortum and Lerner (1998), Hall and Ziedonis (2001) and Sakakibara and Branstetter (2001), which document that patent protection may retard innovation. Recently, Lerner (2009) and Qian (2007) report that there is an inverted-U shape relationship between patent protection and innovation. Thus, the important theoretical question of how patents will impact innovation and social welfare remains unsolved.

We develop a growth model with status preference, in which the Marginal Rate of Substitution (MRS) between assets and consumption is decreasing in the amount of assets, to investigate the effects of patent protection (patent breadth) on innovation, inequality and social welfare. On the one hand, as in the standard literature, patent protection promotes innovation by raising the value of innovation. On the other hand, patent protection reduces the MRS between assets and consumption, and thereby discouraging the accumulation of assets and innovation.\(^1\) We define this as the substitution effect of patent protection on innovation. When the degree of patent protection is low (high), the MRS is small (large), and therefore the positive (negative) effect of patent protection dominates. As a result, the relationship between patent protection and innovation is non-monotonic.

It is shown numerically that the effect of patent protection on social welfare relies on status preference. Strengthening patent protection reduces social welfare when the strength of status preference is large (i.e., the substitution effect of patent protection on innovation is great), whereas there is a non-monotonic relationship between patent protection and social welfare when the strength of status preference is small.

Various macroeconomic papers study the link between patent protection and innovation in the framework of endogenous growth theory.\(^2\) Goh and Oliver (2002),

\(^1\)As in the standard endogenous growth models, the total assets are equal to the value of patents.
\(^2\)Indeed, there are also a number of microeconomic perspectives in the literature (e.g., Green and Scotchmer, 1995; Scotchmer, 1996; O’Donoghue et al, 1998; and Segal and Whinston, 2007) analyzing how patent protection affects innovation.
Kwan and Lai (2003), O’Donoghue and Zweimuller (2004), Horii and Iwaisako (2007), Furukawa (2007, 2010), Futagami and Iwaisako (2007), Akiyama and Furukawa (2009), Chu (2009), Chu et al (2012), Chen and Iyigun (2011) and Chu and Pan (2012) can be used to explain the fact that stringent patent protection may stifle innovation and economic growth. Our paper provides a novel channel through the substitution effect that gives rise to a non-monotonic effect of patent protection on innovation and social welfare, complementary to the existing ones. This paper also relates to models with wealth preference (for example, Zou, 1994, 1995, 1998; Bakshi and Chen, 1996; Corneo and Jeanne, 1997; Futagami and Shibata, 1998; Smith, 1999, 2001; Luo et al, 2009). These models provide an interpretation for many economic phenomena such as savings, growth and assets pricing. To the best of our knowledge, however, the existing models with wealth preference do not address the issue of patent protection. Our paper contributes to this literature by exploring the impacts of patent protection on innovation, inequality and social welfare.

This paper also relates to the literature on patent protection and inequality, such as that by Adams (2008), Parello (2008) and Cozzi and Galli (2011). Adams emphasizes that intellectual property rights and openness are positively correlated with income inequality in developing countries. Parello argues that strengthening patent protection widens wage inequality in developed countries, whereas it may raise or lower wage inequality in developing countries. On the other side of the coin, Cozzi and Galli stress that strong patent protection increases wage inequality. Unlike this literature, our paper focus on the effect of reinforcing patent protection on wealth inequality.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 characterizes equilibrium and analyzes the effect of patent protection on innovation. A non-monotonic relationship between patent protection and innovation

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3 One implication of these models is that social welfare might be low when patent protection is strengthening.

4 Corneo and Jeanne, Futagami and Shibata focus on the relative wealth (the status), while Zou, Smith and Luo et al give attention to the absolute wealth. Furthermore, it is useful to note that there is a wealth of evidence supporting the existence of status preference; see Heffetz and Frank (2010).
is generated, due to the existence of the substitution effect of patent protection on innovation. Section 4 shows by simulation that the effect of patent protection on social welfare varies, depending on the strength of status preference. Section 5 presents an extended model with asymmetric agents to investigate the effect of patent protection on wealth inequality. Section 6 draws a conclusion.

2 The Model

2.1 Preferences

In this model economy there exist $L$ workers and each of them inelastically provides one unit of labor. Agent $i$ maximizes discounted utility:

$$U_i(t) = \int_0^\infty u_i \left[ c_i(t), \frac{a_i(t)}{\bar{a}(t)} \right] e^{-\rho t} dt = \int_0^\infty \frac{\{c_i(t)^\mu [V(a_i(t)/\bar{a}(t))]^{\nu}\}^{1-\gamma} - 1}{1-\gamma} e^{-\rho t} dt,$$

where $\gamma$ represents the inverse of the rate of intertemporal substitution, and $\rho$ represents time preference. $c_i(t)$ and $a_i(t)$ represents respectively consumption and assets of agent $i$, and $\bar{a}(t)$ represents the average level of wealth in the economy. Following Futagami and Shibata (1998) we assume that $V$ is a monotonically increasing function of $a_i(t)/\bar{a}(t)$, and that $1 - \mu (1 - \gamma) > 0$ holds. The assumption that instantaneous utility depends on the status (the person’s relative wealth position in the society) captures the idea of Hume, Marx, Veblen and others.\footnote{Hume (1978) states: “One of the most considerable of these passions is that of love or esteem in others, which therefore proceeds from a sympathy with the pleasure of the possessor. But the possessor has also a secondary satisfaction in riches arising from love and esteem he acquires by them, and this satisfaction is nothing but a second reflection of that original pleasure, which proceeded from himself. This secondary satisfaction or vanity becomes one of the principal recommendations of riches, and is the chief reason, why we either desire them for ourselves, or esteem them in others.” We took this from Futagami and Shibata (1998).}

The individual’s budget constraint is described as:

$$\dot{a}_i(t) = r(t) a_i(t) + w(t) - c_i(t),$$

\footnote{We took this from Futagami and Shibata (1998).}
where \(r(t)\) and \(w(t)\) denote the interest rate and the wage rate, respectively. A dot over a variable denotes time derivative. Here we normalize the price of consumption (the final good) to be unitary. We drop the time index as long as it does not cause confusion.

The maximization of (1) subject to (2) gives rise to the Euler equation on balanced growth path.\(^6\)

\[
\frac{\dot{c}_i}{c_i} = \frac{1}{1 - \mu (1 - \gamma)} \left[ \frac{\partial u_i}{\partial a_i} + (r - \rho) \right]
= \frac{1}{1 - \mu (1 - \gamma)} \left[ \frac{\nu V'(1) c_i}{\mu V(1) \bar{a}} + (r - \rho) \right] = \frac{\theta c_i / \bar{a} + (r - \rho)}{1 - \mu (1 - \gamma)}. \tag{3}
\]

It is useful to note that \(\frac{\partial u_i}{\partial a_i} / \frac{\partial a_i}{\partial c_i}\) is the MRS between assets and consumption. Moreover, as in Futagami and Shibata (1998), \(\theta \geq 0\) measures the strength of status preference. Clearly when \(\theta\) equates zero, (3) becomes the standard Euler equation.

In the meantime, the transversality condition of this dynamic optimization is given by:

\[
\lim_{t \to \infty} \lambda_i (t) a_i (t) = 0, \tag{4}
\]

where \(\lambda_i (t)\) is the co-state variable of \(a_i (t)\). Equation (4) implies \(\rho - g \mu (1 - \gamma) > 0\) in equilibrium.

### 2.2 Production

The final good sector is perfectly competitive. In this sector firms employ intermediate goods and labor to produce the final good using the following technology:

\[
Y = \int_0^N k_j ^{1-\alpha} dj \cdot \Lambda^\alpha, \tag{5}
\]

\(^6\)The Euler equation \(\frac{\dot{c}_i}{c_i} = \frac{1}{1 - \mu (1 - \gamma)} \left[ \frac{\partial u_i}{\partial a_i} + (r - \rho) + \nu (1 - \gamma) \frac{V'(a_i / \bar{a}) a_i}{V(a_i / \bar{a}) \bar{a}} \left( \frac{a_i}{\bar{a}} - \frac{n}{\bar{n}} \right) \right]\) collapses to (3), since \(a_i = \bar{a}\) in symmetric equilibrium.
where $N$ is the number of intermediate goods, $k_j$ is the quantity used of intermediate good $i$.

The maximization of a firm’s profit yields the demand for intermediate goods:

$$ k_j = \left[ \frac{(1 - \alpha)}{\chi_j} \right]^{1/\alpha} L, \quad (6) $$

where $\chi_j$ is the price of intermediate good $j$.

To simplify, we assume patent length to be infinite.\(^7\) Suppose that any firm can produce one unit of intermediate goods by using one unit of the final good. Following Goh and Oliver (2002), we introduce patent breadth $B \geq 1$ as the policy variable such that\(^8\)

$$ \chi_j = B. \quad (7) $$

That is, the wider the patent breadth, the greater the firm’s ability to raise the price. Combining (6) and (7), we obtain

$$ \pi = \pi_j = (B - 1) \left( \frac{1 - \alpha}{B} \right)^{1/\alpha} L, \quad (8) $$

where $\pi_j$ is the profit of the firm producing intermediate good $j$.

\section*{2.3 R&D}

Innovators can discover a new design of intermediate goods by inputting $\eta$ units of the final good. More formally, the equation of knowledge accumulation is\(^9\)

$$ \dot{N} = \frac{Z}{\eta}, \quad (9) $$

where $Z$ is the resources devoted to innovation.

\(^7\)Finite patent length would not change the main results, however.

\(^8\)Equation (6) suggests that the monopoly price is equal to $\frac{1}{1-\alpha}$. It follows that $B \in (1, \frac{1}{1-\alpha}]$. We restrict our attention to the case $B < \frac{1}{1-\alpha}$ in the following analysis.

\(^9\)This refers to the lab-equipment innovation specification in Rivera-Batiz and Romer (1991).
3 Patent Protection and Innovation

Denote the value of a new patent at time $t$ as $P(t)$. Then in equilibrium

$$P(t) = \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi(\tau)d\tau = (B - 1)\left(\frac{1 - \alpha}{B}\right)^{1/\alpha} \frac{L}{r}. \quad (10)$$

Free entry into R&D business suggests that, in equilibrium

$$P = P(t) = \eta. \quad (11)$$

Combining (10) and (11), we derive

$$r = (B - 1)\left(\frac{1 - \alpha}{B}\right)^{1/\alpha} \frac{L}{\eta}. \quad (12)$$

Differentiating (12) with respect to the patent policy instruments, $B$, results in

$$\frac{dr}{dB} = \frac{1 - (1 - \alpha)B}{\alpha B} \left(\frac{1 - \alpha}{B}\right)^{1/\alpha} \frac{L}{\eta}. \quad (13)$$

Taking advantage of (13), we state the following lemma:

**Lemma 1** The interest rate rises with patent breadth. Moreover, $\frac{dr}{dB}|_{B=1} = (1 - \alpha)^{1/\alpha} L/\eta > 0$, $\frac{dr}{dB}|_{B=1/(1-\alpha)} = 0$.

Stringent patent protection (broad patent breadth) raises the value of innovation, therefore driving up the interest rate (the return to assets).

It is useful to note that the equilibrium growth rate becomes $\frac{r - \rho}{1 - \mu(1 - \gamma)}$, if there is no status preference (e.g., $\nu = 0$). In this case, Lemma 1 implies

**Lemma 2** Patent protection promotes innovation, if there is no status preference.

Like Futagami and Shibata (1998), we only focus on symmetric equilibrium, in which $c_i = c$ and $a_i = \bar{a} = a$. Thus in equilibrium the resource constraint is
where \( g = \frac{\dot{N}}{N} \). In addition, the total assets owned by households equal the value of all patents. That is,

\[
aL = \int_0^N Pdj = \eta N. \tag{15}
\]

Note that in equilibrium \( \theta = \frac{\nu V'(a_i/\pi)}{\nu V'(1)} = \frac{\nu V'(1)}{\nu V(1)} \) is a constant. Thus the MRS between assets and consumption is also a constant:

\[
\theta \frac{c}{a} = \theta \frac{(1 - \alpha)(1 - \alpha)/\alpha L(B + \alpha - 1)/B^{1/\alpha} - g\eta}{\eta}. \tag{16}
\]

Differentiating (16) with respect to \( B \), we reveal

\[
\frac{\partial(\theta c/a)}{\partial B} = -\theta \frac{(1 - \alpha/\alpha)}{B} \frac{(B - 1)L}{\alpha \eta B}. \tag{17}
\]

As a consequence, we have

\textbf{Lemma 3} The MRS between assets and consumption decreases with the degree of patent protection, i.e., \( \frac{\partial(\theta c/a)}{\partial B} \leq 0 \). Moreover, \( \frac{\partial(\theta c/a)}{\partial B} \big|_{B=1} = 0 \), \( \frac{\partial(\theta c/a)}{\partial B} \big|_{B=1/(1-\alpha)} = -\theta(1 - \alpha)^{2/\alpha}L/\eta < 0 \).

We refer to \( \frac{\partial(\theta c/a)}{\partial B} \leq 0 \) as the substitution effect of patent protection on innovation. In other words, patent protection lowers the growth rate through lowering the MRS between assets and consumption.

We are now ready to explore the relationship between patent protection and innovation. Clearly, the equilibrium growth rate is:
\[
g = \frac{\dot{N}}{N} = \frac{\dot{c}}{c} = \frac{\theta c/a + r - \rho}{1 - \mu (1 - \gamma)}.
\]  
\text{(18)}

Equations (16) and (18) imply that

\[
\frac{dg}{dB} = \frac{\partial (\theta c/a)}{\partial B} + \frac{dr}{dB} = \frac{\partial (\theta c/a)}{\partial B} + \frac{dr}{dB}.
\]

\text{where } \frac{\partial (\theta c/a)}{\partial g} = -\theta < 0. \text{ Consequently, the effect of patent breadth is straightforward, due to Lemmas 1 and 3. The positive effect of rising interest rate dominates when } B > 1, \text{ while the negative effect of declining MRS is dominant when } B \to 1/(1 - \alpha). \text{ Thus there is a non-monotonic relationship between patent breadth and innovation.}

\textbf{Proposition 1} The relationship between patent protection and innovation is non-monotonic.

\textbf{Proof.} See the Appendix. \hfill \blacksquare

A marginal change in patent breadth does not affect the growth rate when \( B = 1/(1 - \alpha) \), because the monopoly price maximizes profits. At the same time, a large value of innovation lowers the growth rate, owing to the substitution effect. Therefore, finite patent breadth results in the maximization of the growth rate.

Proposition 1 says that intermediate \( B^* \) maximizes the growth rate \( g \). In this case, we examine how \( B^* \) changes when the strength of status preference \( \theta \) changes.

\textbf{Proposition 2} The degree of patent breadth maximizing the growth rate decreases with the strength of status preference. That is, \( \frac{\partial B^*}{\partial \theta} < 0. \)

\textbf{Proof.} Since \( B^* = \frac{1+\theta}{1+\theta-\alpha} \), \( \frac{\partial B^*}{\partial \theta} = -\frac{\alpha}{(1+\theta-\alpha)^2} < 0. \) \hfill \blacksquare

Apparently, the larger the \( \theta \), the greater the marginal change in the MRS between assets and consumption. It means that bigger \( \theta \) leads to higher substitution effect. Therefore, the result in Proposition 2 is established.
In many developing countries, individuals strive for the accumulation of assets.\textsuperscript{10} To some extent, Proposition 2 implies that patent protection in developing countries should be weaker than in developed countries.\textsuperscript{11}

4 Social Welfare

In this section, we quantitatively analyze the effect of patent protection on social welfare. Using (1) and (18), we find:

\[
S = L \cdot U = \frac{N(0)^{\mu(1-\gamma)} L^{1-\mu(1-\gamma)} [V(1)]^{\nu(1-\gamma)}}{1-\gamma} - \frac{L}{\rho (1-\gamma)}, \tag{20}
\]

where \(S\) is social welfare, and \(W = \frac{[(1-\alpha)^{1/\alpha} L(B+\alpha-1)]^{1/\alpha}}{(\rho - g\mu(1-\gamma))} \cdot [\gamma(1-\gamma)]^{\mu(1-\gamma)-1}\). As a result, we state

**Proposition 3** Strengthening patent protection reduces social welfare when \(B = \frac{1}{1-\alpha}\).

**Proof.** See the Appendix. \(\blacksquare\)

As usual, stringent patent protection decreases social welfare through monopoly pricing. It moreover lowers social welfare via stifling growth when \(B = \frac{1}{1-\alpha}\). Accordingly, social welfare goes down when patent protection is strengthening, if \(B = \frac{1}{1-\alpha}\).

The qualitative analysis is complicated, thus we use a quantitative method to explore the effect of patent protection on social welfare for \(B \in (1, \frac{1}{1-\alpha})\).\textsuperscript{12} To do this, we first calibrate the structural parameters to quantify the model. Following

\[
\frac{dS}{dB}\bigg|_{B=1} = \frac{\mu N(0)^{\mu(1-\gamma)} L^{1-\mu(1-\gamma)} [V(1)]^{\nu(1-\gamma)} \left[ \frac{L(B+\alpha-1)(1-\gamma)^{1/\alpha}}{1-\alpha} - g\eta \right]^{\mu(1-\gamma)-1}}{[\rho - g\mu(1-\gamma)]^2} \cdot \frac{\alpha (1-\alpha)^{(1-\alpha)/\alpha} L [1 - \mu (1-\gamma) (1-\theta)] - \theta g\mu \frac{dg}{dB}}{1 - \mu (1-\gamma) + \theta} \bigg|_{B=1}.
\]

\textsuperscript{10}Zou (1994) discusses how the capitalist spirit contributes to development of Japan, South Korea, Singapore, China Hongkong and China Taiwang.

\textsuperscript{11}Deardoff (1992) and Grossman and Lai (2004) also stress that developing countries should implement weaker patent protection than developed countries.

\textsuperscript{12}Simple algebra results in
Chu (2009), we set the discount rate $\rho$ to 0.04, the rate of intertemporal substitution $1/\gamma$ to 0.42, the labor share $\alpha$ to 0.7, the average annual TFP growth rate $g$ to 1.33%, the real interest rate $r$ to 0.084 and the markup is about 3%. Without loss of generality, we unitize total labor force, i.e., $L = 1$. Moreover, we assume $V(1)$ and $N(0)$ to be 1 and 100 respectively for convenience. Using (12), we then pin down the innovation cost parameter $\eta$ to 0.061. Table 1 presents the calibrated values of parameters $\{\alpha, \rho, \mu, \gamma, \theta\}$ for $\theta \in (0, 3]$.\textsuperscript{13}

<table>
<thead>
<tr>
<th>Table 1: Calibrated Parameters</th>
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<tbody>
<tr>
<td>$\alpha$</td>
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<td>$\theta$</td>
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The simulation result of relationship between patent protection and social welfare is reported in Figures 1-3.\textsuperscript{14} Thus we have

**Claim 1** Strengthening patent protection lowers social welfare when the strength of status preference is large, whereas there is a non-monotonic effect of patent protection on social welfare when the strength of status preference is small.

When the strength of status preference is big (the substitution effect of patent protection on innovation is great), the positive effect of patent protection on social welfare via stimulating growth tends to be weak. Thus social welfare may go down when patent protection becomes strong. In contrast, the positive effect of patent protection may or may not improve social welfare, even if patent protection is initially low.

\textsuperscript{13}There is no estimate on the value of $\mu$. For simplicity, we only report the result when $\mu = 0.5$. The results are robust to different $\mu$, however. Furthermore, $\nu$ is determined once $\theta$ is given.

\textsuperscript{14}Obviously, $B \in (1, 10/3]$ if $\alpha = 0.7$. The result in Figures 1-3 is robust to the scale on the horizontal axis, however.
protection is large when the strength of status preference is big (the substitution effect of patent protection on innovation is less). Consequently, the relationship between patent protection and social welfare is non-monotonic.

Basu (1996) and Basu and Fernald (1997) documents that the aggregate profit share is about 3%, while Laitner and Stolyarov (2004) reports that the markup is about 1.1 (i.e. a 10% markup) in the US. Thus $B$ is between 1.03 and 1.1. By our simulation result, we conclude that a marginal increase in patent protection may raise or reduce social welfare even if the initial patent protection is low, depending on the strength of status preference.\(^\text{15}\)

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\(^{15}\)The literature does not provide a precise estimate for $\theta$. 

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Figure 1: The Effect of Patent Protection on Social Welfare ($\theta = 0.8$)

Figure 2: The Effect of Patent Protection on Social Welfare ($\theta = 1.9$)
5 Patent Protection and Wealth Inequality

In this section we extend the basic model to examine the effect of patent protection on wealth inequality. To this end we follow Futagami and Shibata (1998) to assume that there exist two types of agents who have different time and status preferences (see further assumptions below), and that the size of each type of agent is $L/2$. The same reasoning as before implies that, on the balanced growth path, Euler equation for type $i$ agent is

\[
\frac{\dot{c}_i}{c_i} = \frac{1}{1 - \mu (1 - \gamma)} \left[ \frac{\nu V_i' (a_i / \bar{a})}{\mu V_i (a_i / \bar{a})} \cdot \frac{c_i}{\bar{a}} + (r - \rho_i) \right]
\]

where $\epsilon_i = \frac{a_i}{\bar{a}}$ is the share of wealth of type $i$ agent, measuring wealth inequality.

Wealth inequality widens as $|\epsilon_1 - \epsilon_2|$ becomes large. Combining (2) and (21), we obtain the equilibrium growth rate:

\[
g_i = \frac{[1 + \sigma_i (\epsilon_i)] r + \sigma_i (\epsilon_i) w}{1 - \mu (1 - \gamma) + \sigma_i (\epsilon_i)},
\]
where \( \sigma_i (\epsilon_i) = \frac{\nu_i' (\epsilon_i)}{\mu_i (\epsilon_i)} \cdot \epsilon_i, \ r = (B - 1) \left( \frac{1-\alpha}{B} \right)^{1/\alpha} \frac{L}{\eta}, \ w = \alpha \left( \frac{1-\alpha}{B} \right)^{(1-\alpha)/\alpha} N \) and \( \bar{n} = \frac{\eta N}{L} \).

Note that \( \sigma_i \), denoting the elasticity of utility derived from status for agent \( i \) also captures the strength of status preference.\(^{16}\)

In equilibrium \( g_1 (\epsilon_1) = g_2 (\epsilon_2) \). It is then followed by\(^{17}\)

\[
\begin{align*}
F (\epsilon_1, r, \frac{w}{a}) &= \mu (1 - \gamma) [\sigma_2 (\epsilon_2) - \sigma_1 (\epsilon_1)] r + \frac{w}{a} \left\{ \frac{\sigma_1 (\epsilon_1) [1 - \mu (1 - \gamma) + \sigma_2 (\epsilon_2)]}{\epsilon_1} 
- \frac{\sigma_2 (\epsilon_2) [1 - \mu (1 - \gamma) + \sigma_1 (\epsilon_1)]}{\epsilon_2} \right\} - \rho_1 [1 - \mu (1 - \gamma) + \sigma_2 (\epsilon_2)] \\
+ \rho_2 [1 - \mu (1 - \gamma) + \sigma_1 (\epsilon_1)] &= 0. \tag{23}
\end{align*}
\]

Differentiating (23) with respect to \( B \) leads to

\[
\frac{d\epsilon_1}{dB} = - \frac{1}{\partial F / \partial \epsilon_1} \cdot \frac{L}{\eta} \left( \frac{1-\alpha}{B} \right)^{1/\alpha} \left\{ \frac{\mu (1 - \gamma) [\sigma_2 - \sigma_1]}{\epsilon_1} \frac{1 - (1 - \alpha) B}{\alpha B} 
- \frac{\sigma_1 [1 - \mu (1 - \gamma) + \sigma_2]}{\epsilon_1} + \frac{\sigma_2 [1 - \mu (1 - \gamma) + \sigma_1]}{\epsilon_2} \right\}. \tag{24}
\]

Patent protection affects wealth inequality by affecting the interest revenue of two types of agents, which is featured by \(- \frac{1}{\partial F / \partial \epsilon_1} \cdot \frac{L}{\eta} \left( \frac{1-\alpha}{B} \right)^{1/\alpha} \mu (1 - \gamma) (\sigma_2 - \sigma_1) \frac{1 - (1 - \alpha) B}{\alpha B} \).

Moreover, \(- \frac{1}{\partial F / \partial \epsilon_1} \cdot \frac{L}{\eta} \left( \frac{1-\alpha}{B} \right)^{1/\alpha} \left\{ - \frac{\sigma_1 [1 - \mu (1 - \gamma) + \sigma_2]}{\epsilon_1} + \frac{\sigma_2 [1 - \mu (1 - \gamma) + \sigma_1]}{\epsilon_2} \right\} \) illustrates that increasing patent protection decreases the ratio of wage to assets, therefore influencing wealth inequality.

Usually, the poor are more impatient than the rich.\(^{18}\) In the meantime, poor people strive more for social status than do rich people (e.g., Zou, 1994). Thus it is natural to assume that \( \sigma_1 (\epsilon_1) > \sigma_2 (\epsilon_2) \) and \( \rho_1 > \rho_2 \) hold, when \( \epsilon_1 < 1 < \epsilon_2 \).

Suppose \( \epsilon_1 < 1 \) to be satisfied in equilibrium. In turn, (24) results in

**Proposition 4** *Strengthening patent protection enlarges wealth inequality.*

**Proof.** See the Appendix. \( \blacksquare \)

\(^{16}\)We suppose \( \frac{d\alpha_i}{\delta \epsilon_i} < 0 \) to ensure that the equilibrium is always stable in this paper. See Futagami and Shibata (1998) for details.

\(^{17}\)Clearly, \( \epsilon_1 + \epsilon_2 = 2 \), because the quantity of each type of agent is the same.

\(^{18}\)See Lawrance (1991) for the evidence.
On the one hand, stringent patent protection reduces \( \frac{w}{a} \), thereby widening wealth inequality. This outcome would discourage the poor to accumulate wealth. On the other hand, great patent protection drives the interest rate up, thus probably narrowing wealth inequality.\(^{19}\) The first effect is dominant, however.

We now explore the effect of patent protection on innovation in the case where agents are asymmetric. Using (22) we find

\[
\frac{dg}{dB} = \frac{dg_2}{dB} = \frac{\partial g_2}{\partial B} + \frac{\partial g_2}{\partial \epsilon_2} \frac{d\epsilon_2}{dB} \\
= \frac{[1 + \sigma_2] \frac{dr}{dB} + \frac{\sigma_2}{\epsilon_2} \frac{\partial (w/a)}{dB}}{1 - \mu (1 - \gamma) + \sigma_2} + \frac{\partial g_2}{\partial \epsilon_2} \frac{d\epsilon_2}{dB} \tag{25}
\]

It leads to

**Proposition 5** Strengthening patent protection stifles innovation when initial patent protection is already great.

**Proof.** See the Appendix. \( \blacksquare \)

The intuition behind Proposition 5 is the same as that behind Proposition 2 except that patent protection retards innovation by enlarging wealth inequality (i.e., \( \frac{\partial g_2}{\partial \epsilon_2} \frac{d\epsilon_2}{dB} < 0 \)) in the case of asymmetric agents.\(^{20}\) Propositions 4 and 5 suggests that reinforcing patent protection is harmful to economic growth and wealth distribution as patent protection is initially strong.

## 6 Conclusion

An endogenous growth with status preference has been constructed to examine the impact of patent protection on innovation, inequality and social welfare. As in the standard literature, patent protection stimulates innovation by enlarging the value

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\(^{19}\)The poor tends to accumulate more wealth than the rich owing to high interest rate.

\(^{20}\)Apparently, reinforcing patent protection may foster innovation when initial patent protection is weak. However, the proof is complicated since patent protection would indirectly affect innovation by affecting wealth inequality.
of innovation. In this model, the MRS between assets and consumption goes down when the amount of assets goes up. Great innovation value reduces individuals’ incentive to accumulate assets (innovation) owing to low MRS between assets and consumption, and thus stifling innovation. This is called the substitution effect of patent protection on innovation. Strengthening patent protection promotes innovation, owing to a small substitution effect, when initial patent protection is weak, whereas it hinders innovation, because of a large substitution effect, when initial patent protection is stringent. In addition, it has been shown that the degree of patent protection maximizing the innovation rate decreases with the strength of status preference. The reason is that the larger the strength of status preference, the bigger the substitution effect.

Furthermore, we have shown numerically that there is a non-monotonic relationship between patent protection and social welfare when the strength of status preference is small, whereas reinforcing patent protection is harmful to social welfare when the strength of status preference is large. The intuition is that the strength of status preference determines the substitution effect of patent protection on innovation, therefore determining the positive effect of patent protection on social welfare through promoting innovation.

We moreover investigate the effect of patent protection on wealth inequality by taking account of asymmetric agents who have different time and status preferences. It has been shown that strengthening patent protection enlarges wealth inequality. Thus not only does increasing patent stifle innovation, it also expands wealth inequality, when patent protection is initially great.

It is a complex assignment to investigate the effects of patent length on innovation and inequality. However, we expect that the qualitative results remain unchanged. This is left for future research.
References


Appendix: Proof of Propositions

Proof of Proposition 1.

**Proof.** Combining (12) and (18), we obtain

\[
g = \frac{1}{1 + \theta - \mu (1 - \gamma)} \left\{ \frac{L}{\eta} \left( \frac{1 - \alpha}{B} \right)^{1/\alpha} \left[ \left( \frac{\theta}{1 - \alpha} + 1 \right) B - (1 + \theta) \right] - \rho \right\}. \tag{A1}
\]

Differentiating \(g\) with respect to \(B\) leads to

\[
\frac{dg}{dB} = \frac{L}{[1 + \theta - \mu (1 - \gamma)] \eta} \left( \frac{1 - \alpha}{B} \right)^{1/\alpha} \frac{(1 + \theta) - (1 + \theta - \alpha) B}{\alpha B}. \tag{A2}
\]

Thus \(\frac{dg}{dB} > 0\) when \(B < B^*\), and \(\frac{dg}{dB} < 0\) when \(B > B^*\), where \(B^* = \frac{1 + \theta}{1 + \theta - \alpha} < \frac{1}{1 - \alpha}\). ■

Proof of Proposition 3.

**Proof.** Equation (20) reveals

\[
\begin{align*}
\frac{dS}{dB} \bigg|_{B = \frac{1}{1 - \alpha}} &= \mu N (0)^{\mu(1-\gamma)} L^{1-\mu(1-\gamma)} [V (1)]^{\mu(1-\gamma)} \left[ \left( \frac{L(B+\alpha-1)(1-\alpha)}{1-\alpha} \right)^{1/\alpha} - \rho \eta \right]^{\mu(1-\gamma)-1} \\
& \cdot \left\{ - (1 - \alpha)^{2/\alpha} \left[ \rho - g\mu (1 - \gamma) \right] L \left[ 1 - \frac{\theta}{1 + \theta - \mu (1 - \gamma)} \right] \\
& + \left[ \frac{L (B + \alpha - 1)(1-\alpha)}{1 - \alpha} - \rho \eta \right] \frac{dg}{dB} \bigg|_{B = \frac{1}{1 - \alpha}} \right\} < 0, \tag{A3}
\end{align*}
\]

because \(\frac{dg}{dB} \bigg|_{B = \frac{1}{1 - \alpha}} < 0\), \(\frac{L(B+\alpha-1)(1-\alpha)}{1-\alpha} - \rho \eta > 0\), \(\rho - g\mu (1 - \gamma) > 0\) and \(1 - \mu (1 - \gamma) > 0\). ■

Proof of Proposition 4.
Proof. We first prove $\frac{\partial F}{\partial \epsilon_1} < 0$. Equation (23) reveals

$$
\frac{\partial F}{\partial \epsilon_1} = \left\{ -\mu (1 - \gamma) r + \frac{w}{\tilde{a}} \left[ \frac{[1 - \mu (1 - \gamma) + \sigma_2]}{\epsilon_1} - \frac{\sigma_2}{\epsilon_2} \right] + \rho_2 \right\} \frac{d\sigma_1}{d\epsilon_1}
+ \left\{ -\mu (1 - \gamma) r + \frac{w}{\tilde{a}} \left[ \frac{[1 - \mu (1 - \gamma) + \sigma_1]}{\epsilon_2} - \frac{\sigma_1}{\epsilon_1} \right] + \rho_1 \right\} \frac{d\sigma_2}{d\epsilon_2}
- \left\{ \frac{\sigma_1 [1 - \mu (1 - \gamma) + \sigma_2]}{\epsilon_1^2} + \frac{\sigma_2 [1 - \mu (1 - \gamma) + \sigma_1]}{\epsilon_2^2} \right\} \frac{w}{\tilde{a}}
$$

(A4)

Taking advantage of (22), we obtain $-\mu (1 - \gamma) r + \frac{w}{\tilde{a}} \left[ \frac{1 - \mu (1 - \gamma) + \sigma_j}{\epsilon_i} - \frac{\sigma_j}{\epsilon_j} \right] + \rho_j = [1 - \mu (1 - \gamma) + \sigma_j] \left( r + \frac{w}{\tilde{a}} g \right) > 0 \ (i \neq j \text{ and } i,j = 1,2)$. Thus $\frac{\partial F}{\partial \epsilon_1} < 0$.

It is clear that $\frac{de_1}{dB} < 0$ as $\gamma \leq 1$ by (24). Furthermore, using (24) we get

$$
\frac{de_1}{dB} \leq -\frac{1}{\partial F/\partial \epsilon_1} \cdot \frac{L}{\eta} \left( \frac{1 - \alpha}{B} \right)^{1/\alpha} \left[ -\frac{\sigma_1}{\epsilon_1} + \frac{\sigma_2}{\epsilon_2} - \frac{\sigma_1 \sigma_2 (\epsilon_2 - \epsilon_1)}{\epsilon_1 \epsilon_2} \right]
+ \mu (1 - \gamma) \left( \frac{1 - \epsilon_1}{\epsilon_1} + \frac{\epsilon_2 - 1}{\epsilon_2} \right) < 0,
$$

(A5)

when $\gamma > 1$. 

Proof of Proposition 5.

**Proof.** Equation (22) implies

$$
\frac{\partial g_i}{\partial \epsilon_i} = \frac{1}{[1 - \mu (1 - \gamma) + \sigma_i]^2} \left\{ -\mu (1 - \gamma) r + \frac{w}{\tilde{a}} \cdot \frac{1 - \mu (1 - \gamma)}{\epsilon_i} + \rho_i \right\} \frac{d\sigma_i}{d\epsilon_i}
- \frac{w}{\tilde{a}} \cdot \frac{1 - \mu (1 - \gamma) + \sigma_i}{\epsilon_i^2} \frac{d\sigma_i}{d\epsilon_i} < 0.
$$

(A6)

It follows that $\frac{\partial g_2}{\partial \epsilon_2} < 0$ due to $\frac{de_2}{dB} = -\frac{de_1}{dB} > 0$. At the same time, $\frac{\partial g_2}{\partial B}_{B=1/(1-\alpha)} = -\frac{\sigma_2/\epsilon_2}{1-\mu (1-\gamma)+\sigma_2} (1-\alpha)^{2/\alpha} L/\eta < 0$. As a consequence, $\frac{dg_2}{dB}_{B=1/(1-\alpha)} < 0$. In other words, strengthening patent protection stifles innovation when patent protection is initially strong. 

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