The Spirit of Capitalism and Consumption Inequality

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Abstract

In this paper we argue that Max Weber's (1904-05, German; 1958) theory on "spirit of capitalism" can be modeled via a direct preference for wealth. We incorporate the spirit of capitalism into a general equilibrium consumption-portfolio choice model to examine the effects on consumption inequality, equilibrium interest rate, and equity premium—an unexplored area in the literature. We provide closed-form solutions to help disentangle the effects of the spirit of capitalism in driving the key results. Quantitatively, we show that a small degree of the spirit of capitalism can improve the model's predictions in all three dimensions (consumption inequality, equilibrium interest rate, and equity premium) simultaneously. We show our results are robust to more general specifications on the income process and to incorporating macroeconomic rare disasters. Finally, we compare the spirit of capitalism with a closely related hypothesis, habit formation, and find that they have opposite effects on equilibrium asset returns and consumption inequality.

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1 Introduction

The importance of work ethic and religious beliefs on economic activity can be traced back to Max Weber (1904-1905, German; 1958), who famously argued that one important way the Protestant ethic can influence agents' economic behavior is by fostering the "spirit of capitalism"–the enjoyment of the accumulation of wealth regardless of its effect on consumption smoothing. McCleary and Barro (2006, 2019) provide solid empirical evidence based on cross-country data to show that certain religious beliefs do contribute to economic growth. Despite the strong evidence that the spirit of capitalism (or more broadly, religious beliefs) may influence individual decisions and macroeconomic outcomes, we still lack a general framework that can be used to study a range of macroeconomic topics. In particular, while the spirit of capitalism affects economic outcomes by influencing agents' work and saving behaviors, important and unexamined questions remain concerning this channel, including how the spirit of capitalism affects aggregate savings, the equilibrium interest rate, the distribution of consumption, and equity premium.

In this paper, we provide a general framework that incorporates the spirit of capitalism into modern heterogeneous-agent general-equilibrium models (defined in the next paragraph) to study the implications of the spirit of capitalism on consumption, savings, and asset returns. From a long historical perspective, we provide detailed evidence to argue that the spirit of capitalism can be modeled as a direct preference for wealth, which means that an agent with the spirit of capitalism accumulates wealth not only for consumption but also for the sake of accumulation itself.[1] Our approach allows us to embed the spirit of capitalism in modern equilibrium consumption-saving models and still derive analytical solutions to the models. In this way, our framework can help disentangle different driving forces on consumption and savings, and illustrate how the spirit of capitalism interacts with other conventional factors–such as patience, risk aversion, and intertemporal substitution–in determining the equilibrium asset returns and consumption inequality. When compared with a standard model that does not include the spirit of capitalism, our model is more consistent with the micro data on the distribution of consumption, as well as the aggregate data on the equilibrium interest rate and equity premium.

To capture as many key factors in consumption and saving decisions as possible, we construct a general equilibrium model featuring heterogenous agents, incomplete markets, multiple assets, and recursive preferences. The typical agent makes optimal consumption-saving and portfolio-choice decisions in an economy with uninsurable labor income risks. The incomplete markets hypothesis (Bewley 1986; Huggett 1993) leads agents to save for both consumption-smoothing

[1]This view is supported theoretically by Alaoui and Sandroni (2018). They argue that the Protestant ethic originated in particular in the Calvinist doctrine of predestination, and prove that the Protestant ethic represented by a direct preference for wealth is mathematically equivalent to modeling the Calvinist doctrine of predestination using a Kreps-Porteus type recursive utility.

and precautionary purposes. The multiple-asset setting (Merton 1971; Viceira 2001; Wang 2009) allows the agent to allocate wealth between risk-free and risky assets reflecting part of the agent's risk attitude and uncertainty. The recursive preference (Epstein-Zin 1989) separates risk aversion from intertemporal substitution which play different roles in consumption and portfolio-choice decisions. The general equilibrium with heterogenous agents (Wang 2003) link individual savings to the equilibrium interest rate and also allow us to study the equity premium in the presence of risky assets. Incorporating the spirit of capitalism into such a rich framework allows us to study how it interacts with each of these important factors in driving savings, asset returns, and consumption inequality in equilibrium.

We deliver three sets of results in this paper. First, we derive closed-form solutions for our consumption-portfolio choice model featuring the spirit of capitalism, recursive exponential utility, uninsurable labor income, and multiple assets.[2] These closed-form solutions help us understand how the spirit of capitalism interacts with the discount rate, the elasticity of intertemporal substitution (EIS), and constant absolute risk aversion (CARA) in determining the aggregate saving and equilibrium risk-free rate as well as the relative consumption inequality. At the individual level, we show that stronger spirit of capitalism affects the optimal saving level through four distinct channels: (i) by increasing the agent's effective patience and raising the patience-induced saving; (ii) by driving up the agent's precautionary saving; (iii) by decreasing individual savings through lowering the certainty-equivalent wealth due to the existence of risky assets; and (iv) by reducing the amount of saving for "a rainy day." We then prove the existence and uniqueness of a general equilibrium in the vein of Bewley (1986), Huggett (1993), and Wang (2003), and show that the spirit of capitalism lowers the equilibrium risk-free rate by raising the individual's aggregate saving through the first two channels while washing out the effects from the third and fourth channels.

Furthermore, we prove that the relative consumption inequality, defined as the relative dispersion of consumption to income, is an increasing function of the degree of the spirit of capitalism. This result can be understood using our derived consumption function, which shows an increased sensitivity of consumption to *unanticipated* income shocks in the presence of the spirit of capitalism. In addition to the spirit of capitalism, we show the relative consumption inequality also depends on the equilibrium interest rate, the spirit of capitalism, and the persistence of the income process.

Second, we show quantitatively that introducing the spirit of capitalism into the model helps explain the observed low risk-free rate, the high relative consumption inequality, and the large equity premium in the data. In particular, to explain the average real risk-free rate in the United

[2]We use the negative exponential function (i.e., CARA functional form) to characterize the household's preference for risk. The main reason for adopting this specification in our paper is that incomplete markets generally imply that aggregate dynamics depend on the wealth distribution, and this "curse of dimensionality" can be overcome by adopting a CARA-Gaussian specification because it ensures that risk-taking and therefore investment is independent of wealth.

States from 1980 to 2016, a rational expectations model without the spirit of capitalism would require the coefficient of risk aversion parameter to be as high as 35 to 106 when the EIS takes reasonable values from 0.5 to 1.5.[3] However, when agents care even slightly about the spirit of capitalism, the model can generate a low equilibrium interest rate with a reasonable level of risk aversion. Furthermore, even with a small amount of the spirit of capitalism, the model can generate the realistic relative consumption inequality. In addition, the model's performance in generating a reasonable equity premium is greatly improved when incorporating the spirit of capitalism. We also show that these results are robust to the more general income specification proposed in Wang (2004) and in the presence of rare disasters in the vein of Rietz (1988) and Barro (2009).

Third, we compare the implications of the spirit of capitalism with the implications of the traditional internal habit formation model on the equilibrium risk-free rate and relative consumption inequality. We conduct this comparison because these two hypotheses both influence the agent's consumption-portfolio but from two different angles: one from the wealth angle and the other from the consumption angle. We show that the two models have opposite effects on the equilibrium consumption inequality and on asset returns. Habit formation increases the equilibrium risk-free rate and raises the relative consumption inequality, while the spirit of capitalism does the opposite.

Our paper contributes to three branches of literature. First, it complements the empirical literature on the relationship between religion and economic outcomes by providing a theoretical framework. In particular, Keister (2003) uses the National Longitudinal Survey of Youth (1979 cohort) to show that religion has both indirect and direct effects on wealth. Guiso et al. (2003) use the World Values Survey to show that, on average, religion is good for developing attitudes that promote economic growth. Barro (2004), Barro and McCleary (2003), and McCleary and Barro (2006, 2019) combine various data sources and empirically show that economic growth responds positively to religious beliefs (notably those involving hell and heaven). Andersen et al. (2017) use county-level historical data in England to argue that counties that were more exposed to Cistercian monasteries experienced faster productivity growth from the 13th century onward. These empirical papers have argued different possible channels through which religion could influence economic outcomes. The theoretical framework we build focuses on a particular aspect: religion's influence on the spirit of capitalism or the direct preference for wealth accumulation.[4]

Second, our paper contributes to the theoretical literature on the spirit of capitalism in two

[3]We normalize the mean consumption level to be 1, so the coefficient of relative risk aversion is equal to the coefficient of absolute risk aversion. While estimates of the EIS fluctuate wildly, risk aversion coefficients of above 5 are generally not considered reasonable in the macroeconomic and finance literature.

[4]Though it is not our primary focus on which religion is the true origin of the spirit of capitalism, there are different views among Max Weber, Werner Sombart, and Milton Friedman. While Weber takes the Protestant ethic as the origin of the capitalist spirit, Sombart (1911, 1915) takes the Jewish religion and culture as the origin of the capitalist spirit, and Friedman (1987) provides support for Sombart's view.

significant ways. Zou (1994, 1995), Fershtman et al. (1996), Smith (1999), and Corneo and Jeanne (2001) examine the importance of the spirit of capitalism for capital accumulation and economic growth. Bakshi and Chen (1996), Smith (2001), and Gong and Zou (2002) study how incorporating the spirit of capitalism into otherwise standard models affects asset prices. Luo et al. (2009) find that incorporating the spirit of capitalism helps resolve the excess sensitivity puzzle and the excess smoothness puzzle in the literature on aggregate consumption. Karnizova (2010) studies the implications of the spirit of capitalism for business cycle fluctuations.[5] We expand this literature by offering a framework to study the implications on consumption inequality and the equilibrium asset returns. In addition, although these authors also use a direct preference for wealth to model the spirit of capitalism, we are the first to provide a comprehensive review of the origin of the spirit of capitalism based on a long line of sociological and economic literature to justify why this modeling strategy is consistent with Weber's original work on the spirit of capitalism.[6]

Third, our paper contributes to the broad literature on consumption inequality by examining the implication of the spirit of capitalism on consumption inequality and asset returns. Krueger and Perri (2006) use a calibrated incomplete-markets model with limited commitment to explain the observed difference between consumption and income inequality. Blundell et al. (2008) show that consumption and income inequality diverged during the 1980-2004 period, driven by the degrees of income persistence and consumption insurance. Luo et al. (2020) show that a reasonable degree of ambiguity aversion can help explain both the low real interest rate and consumption dispersion. For a recent review of consumption and income inequality issues, please see Attanasio and Pistaferri (2016). Our paper is the first to explore theoretically the implications of the spirit of capitalism on consumption inequality. Broadly speaking, our paper shows that culture and religion have meaningful implications on consumption inequality and asset returns through influencing agents' attitude on wealth accumulation.

The rest of paper is organized as follows. Section 2 argues, from a historical perspective, why Weber's idea of the spirit of capitalism can be modeled as a direct preference for wealth. Section 3 introduces the spirit of capitalism into a consumption-portfolio choice model with recursive exponential utility and incomplete markets, derives analytical optimal consumption-portfolio solutions, and shows key theoretical properties. Section 4 presents our quantitative results showing how our

[5] In a related literature on the quest for status, Saez and Stantcheva (2018), Michaillat and Saez (2015, 2019) introduce wealth in utility with the justification that wealth is a maker of social status and show it generates a few benefits, such as resolving several anomalies in New Keynesian models, generating results consistent with permanent liquidity traps, and helping address policy questions related to capital taxation.

[6] Our paper is also related to Doepke and Zilibotti (2009), who show that children's preferences (including patience and taste for leisure) can be shaped by their parents and that different preferences lead to different occupational choices and future earning profiles. This research line could further go back to Becker and Mulligan's (1997) endogenous determination of time preference.

model with the spirit of capitalism can account for both a lower equilibrium interest rate, high risk premia, and a higher relative consumption inequality. Section 5 provides further discussion on the presence of rare macroeconomic disasters and alternative income specifications, and finds that our main conclusions obtained in the benchmark model are robust to these alternative specifications. In addition, we also compare our model with the spirit of capitalism with the internal habit formation model. Section 6 concludes.

2 The Spirit of Capitalism: A Historical Perspective

To explain why the spirit of capitalism (henceforth SOC) can be modeled as a direct preference for wealth, in this section, we provide a comprehensive review of the origin of the SOC based on the descriptions of sociologists. In In the Appendix (Extended Literature Review), we provide additional evidence based on various related economic theories by economists to show that the spirit of capitalism can be modeled as a direct preference for wealth.

It is the main proposition in the sociological studies by Weber, Sombart, and Simmel that in a capitalist economy wealth accumulation is not only for consumption, but also for the sake of accumulation. According to Max Weber, this so-called capitalist spirit is the essential characteristic which distinguishes the capitalist economy from a traditional, pre-capitalist society:

"At all periods of history, wherever it was possible, there has been ruthless acquisition, bound to no ethical norms whatever." But only in a capitalist economy, "man is dominated by the making of money, by acquisition as the ultimate purpose of his life. *Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs.* This reversal of what we should call the natural relationship, so irrational from a naive point of view, is evidently *a leading principle of capitalism* as it is foreign to all people not under capitalist influence" (Weber, 1958, p.53).

Weber's idea is clear: capital accumulation in a capitalist economy is motivated not only by the maximization of long-run consumption, but also by the enjoyment (utility) from enhancing wealth itself. That is to say, the utility function of a typical capitalist can be defined as $U(c, w)$ where c represents consumption and w represents wealth; or stated differently, the utility function of a capitalist can be separated to two components: $U(c, w) = u(c) + b \cdot v(w)$, where $b > 0$ is used to measure the intensity of the capitalist spirit. Not only this, Weber also explicitly rejects the utility function defined only on consumption such as $U(c)$ as the objective function of the capitalist:

> "The summum bonum of this ethic, the earning of more and more money, combined with the strict avoidance of all spontaneous enjoyment of life, is above all completely devoid of any eudemonistic, not to say hedonistic, admixture. It is thought of so purely as an end in itself, that from the point of view of the happiness of, or utility to, the

single individual, it appears entirely transcendental and absolutely irrational" (Weber, p.53).

Sombart (1967) refines and broadens the concept of the capitalist spirit by dividing it into two parts: the love of money (or accumulation for the sake of accumulation) and the spirit of undertaking (or the spirit of enterprise). According to Sombart, "if not the whole of European history, then surely at least the history of the capitalist spirit, must begin with the struggle between Gods and men for the possession of that accursed thing, gold" (p. 25). Sombart uses two terms to describe this human desire: the love of money (p. 50) and the greed of gold (p. 25). But this desire is not enough for the capitalist economic system to emerge. An equally important part is the spirit of enterprise (p. 63) or "the spirit of undertaking" (p. 51). In the mathematical model in which $U(c, w) = u(c) + b \cdot v(w)$, $b \cdot v(w)$ can represent these two parts of the capitalist spirit: the love of money directly and the spirit of enterprise indirectly - indirectly because enterprise and undertaking can be measured and approximated by wealth and financial success. In this book, Sombart devotes a great portion to "tell the tale of extraordinary unions into which the greed of gold and the spirit of enterprise entered, and it will unfold the history of their offspring, the spirit of capitalist undertaking" (p. 63). "The love of money united with enterprise, and the capitalist spirit was the result" (p. 50).

Simmel (1900) has made a profound philosophical study on money. One central theme in his inquiry is to show how money as means grows into ends. In his view, money is first of all the purest example of the tool and an absolute means for various purposes (p. 210-211). But since the utilization of money has unlimited possibilities, it becomes an abstract tool, which in turn leads to "the surplus value of money": "The wealthy man enjoys advantages beyond the enjoyment of what he can buy with his money... he moves in an ideal atmosphere of unquestioned privilege." "Wealth, indeed is often regarded as a kind of moral merit" (p. 217). In this process, the desire for money is "developed into a psychological value absolute, into a complete engrossing final purpose governing our practical consciousness" (p. 232). Thus money as the extreme example of a means becomes an absolute means and an end.

"Money's value as a *means* increases with its *value* as a means right up to the point at which it is valid as an absolute value and the consciousness of purpose in it comes to an end. The inner polarity of the essence of money lies in its being the absolute means and thereby becoming psychologically the absolute purpose for most people, which makes it, in a strange way, a symbol in which the major regulators of practical life are frozen" (p. 232). Or stated differently, money-making turns out to be for both consumption (the means) and for money itself (the end); hence, the objective function should be characterized by: $u(c) + b \cdot v(w)$. This is a precise representation of "people's most remarkable psychological mania for accumulation... In such cases, value is located not in the subjective reflex of ownership that is normally the reason for acquisition and possession, but

in simple objective fact, that merely having these things in their possession is valuable for such people" (p. 239).

Of course, to define the utility function on both consumption and wealth accumulation is also a way to model man not only as an economics animal, but also as a political animal. Ever since Aristotle (1958), we are taught that "man is by nature an animal of intended to live in a polis." Wealth provides man not only consumption means but also political power and social prestige. Possession of wealth is, to a considerable degree, a measure and standard of a person's success in a society. Thus wealth accumulation directly enters the utility function of a representative agent of a capitalist economy. In a book entitled *The Anatomy of Power*, Galbraith (1983), following the tradition of sociology and political science, classifies wealth as one of three resources of political power. "In the past, so great was the prestige of property that... it accorded power to its possessor. What the man of wealth said or believed attracted the belief of others as a matter of course." To this day, "wealth per se no longer gives automatic access to conditional power. The rich man who now seeks such influence hires a public relations firm to win others to his beliefs. Or he contributes to a politician or a political action committee that reflects his views. Or he goes into politics himself and uses his property not to purchase votes but to persuade voters" (p. 49-50).

This analysis agrees with what Lord Acton's (1988) contention that "power goes with property" (p. 572). Frank Knight (1942) also says that, in a laisser-faire capitalist economy, "freedom of accumulation not only carries with it the possibility of cumulative increase in the inequality of economic power... in addition, economic power confers power in other forms, including the political" (p.82). Seeking a high social position and power has been long recognized as the most important motivation in wealth accumulation in the analysis of capitalism by Max Weber, Werner Sombart, and Georg Simmel. Weber (1958) explicitly states that "the desire for the power and recognition which the mere fact of wealth brings plays its part" in wealth accumulation (p. 70). Simmel (1900) stresses "the honors acquired by wealth and the moral esteem that it enjoys" (p. 220). "The pure potentiality of money as a means is distilled in a general conception of power and significance which becomes effective as real power and significance for the owner of money" (p. 218). And the use of his money "makes him an object of respect and deference beyond his actual economic income" (p. 219)

It is worth noting that the analysis of the capitalist spirit by Weber, Simmel, and Sombart is based on the money-making mentality in the capitalist era and its comparison to ancient, primitive and feudalist societies. The accumulation spirit and the love for money came to dominate the economic and social life only at a relatively late stage of our human history: "The extent to which money becomes absolute for the consciousness of value depends on the major transformation of economic interest from primitive production to industrial enterprise. Modern man and the ancient Greek have such different attitudes towards money largely because formerly it served only

consumption whereas now it essentially serves production" (Simmel, 1990, 2nd enlarged edition, p.232).

The spirit of capitalism stands in striking contrast to the traditional mentality. In the long history before the full development of modern capitalism, acquisition for the sake of acquisition was condemned by most moral and religious teachings and the arts of money-making were sanctioned or constrained. As Weber makes it clear, in a traditional society, the normal situation for mankind is rather that the rationally acquisitive activities are oriented to a traditionally fixed standard of living. In contrast, in the capitalist era, the traditional practice is broken down and acquisition has been freed from any definite limit and becomes an endless process. "This attitude toward acquisition is 'rationalized' in the form in which Weber is interested in it, by holding it to be an ethical duty for its own sake. The SOC looks upon money-making activities not as a means or a necessary evil, but as an ethically enjoined end in itself. To earn money is an ethical obligation for its own sake" (Talcott Parsons, 1948, p. 514). The SOC not only views money acquisition as an end, it also disregards all traditional norms and ways to make money. "Only the ultimate end, the maximization of money, is sacred, the particular means are not, but are chosen anew according to the exigencies of each particular situation" (Parsons, p. 514). Closely related and as an extension of the spirit of capitalism, hard work is regarded as a duty and a moral obligation.

In addition, the key idea of the SOC –accumulating wealth for the sake of accumulation — has not only been taken by the eminent sociologists, but also been deeply connected with various theories by economists. In the Appendix, we document how the key idea of the SOC relates to Adam Smith's theory on frugality and savings, N.W. Senior's abstinence theory of capital, John Stuart Mill's theory on the spirit of accumulation, Karl Marx's description on the nature of capitalist accumulation, John Maynard Keynes' theory on the psychology of the capitalist society, as well as great thoughts by Alfred Marshall, Gustav Cassel, Thorstein Veblen, and Joseph A. Schumpeter.

3 A Consumption-Portfolio Model with The Spirit of Capitalism

In this section, we lay out a general framework on consumption-portfolio choice with the SOC. The model is a general equilibrium model in continuous time with recursive utility, multiple assets, and uninsurable labor income. To help explain the key structure of the model, we will introduce each of the key elements one by one, starting with specifications of the recursive utility and the SOC, followed by investment opportunity and labor income.

3.1 Recursive Exponential Utility with The Spirit of Capitalism

Although the expected power utility model has many attractive features, it implies that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion.

Conceptually, risk aversion (attitudes towards atemporal risks) and intertemporal substitution (attitudes towards shifts in consumption over time) capture two distinct aspects of decision-making and need not be so tightly connected.[7] By contrast, the class of recursive utility functions (Epstein and Zin 1989) enable one to disentangle risk aversion from intertemporal substitution, which is important for us to better understand how they interact with the SOC and then affect the equilibrium dynamics of consumption, saving, and asset returns. In this paper, we assume that agents in our model economy have the Kreps-Porteus type preference with *recursive exponential utility* (REU): for every stochastic consumption-financial wealth stream, $\{c_t, w_t\}_{t=0}^{\infty}$, the utility stream, $\{V(U_t)\}_{t=0}^{\infty}$, is recursively defined by:[8]

$$V(U_t) = \left(1 - e^{-\beta \Delta t}\right) V(c_t, w_t) + e^{-\beta \Delta t} V\left(\mathbb{CE}_t\left[U_{t+\Delta t}\right]\right), \tag{1}$$

where Δt is the time interval, $\beta > 0$ is the agent's subjective discount rate,

$$V(U_t) = -\psi \exp\left(-\frac{1}{\psi} U_t\right), \tag{2}$$

$$V(c_t, w_t) = -\psi \exp\left(-\frac{1}{\psi}\left(c_t + b w_t\right)\right), \tag{3}$$

$$\mathbb{CE}_t\left[U_{t+\Delta t}\right] = g^{-1}\left(\mathbb{E}_t\left[g\left(U_{t+\Delta t}\right)\right]\right), \tag{4}$$

is the certainty equivalent of $U_{t+\Delta t}$ conditional on the period t information, and

$$g(U_{t+\Delta t}) = -\frac{1}{\gamma} \exp\left(-\gamma U_{t+\Delta t}\right). \tag{5}$$

In (1)-(5), $\psi > 0$ governs the elasticity of intertemporal substitution (EIS), $\gamma > 0$ governs the coefficient of CARA, and $b \geq 0$ governs the strength of the spirit of capitalism.[9] It is worth noting that here we assume that the utility function depends on absolute wealth, not relative wealth. The main justification for this specification is that the average amount of asset holdings and thus the corresponding (expected) average level of financial wealth are constant in general equilibrium. A high value of ψ corresponds to a strong willingness to substitute consumption over time, and a high value of γ implies a low willingness to substitute consumption across states of nature. Note that

[7] Risk aversion describes the agent's reluctance to substitute consumption across different states of the world and is meaningful even in a static setting. By contrast, intertemporal substitution describes the agent's willingness to substitute consumption over time and is meaningful even in a deterministic setting.

[8] Skiadas (Chapter 6, 2009) axiomatizes and systematically characterizes this type of recursive exponential utility (or transition-invariant recursive utility.) Skiadas (2009) also compares this type of recursive utility with the scale-invariant Kreps-Porteus recursive utility (e.g., the Epstein-Zin parametric utility form). See Angeletos and Calvet (2006) for an application of REU in a business cycles model and Luo et al. (2020) for an application in a robust control problem.

[9] It is well-known that the CARA utility specification is tractable for deriving optimal policies and constructing general equilibrium in different settings. See Caballero (1990), Wang (2003), and Angeletos and Calvet (2006).

if $\psi = 1/\gamma$, the functions V and g are the same and the recursive utility reduces to the standard time-separable expected utility function used in Caballero (1990), Wang (2003), and Luo, et al. (2009).

The CARA-Gaussian specification has both advantages and disadvantages. It implies, for instance, that optimal investment in the risky asset is independent of individual wealth; our framework is thus not suitable to analyze the wealth distribution issues. The CARA-Gaussian specification, however, is probably not essential for the main insights of the paper and brings the benefit of tractability. As will be shown in this section, the cross-sectional distribution of consumption and wealth - an infinite dimensional object - will not be a relevant state variable for aggregate dynamics, and general equilibrium will be characterized in closed-form.

It is worth noting again that the utility function we proposed in (1) to model the SOC depends on absolute wealth, w_t. In the literature, however, some studies document and examine the importance of concerns about relative wealth on individuals' well-being. For example, see Duesnberry (1949), Robson (1992), Clark and Oswald (1996), Bakshi and Chen (1996), Corneo and Jeanne (2001), and Roussanov (2010). They assume that individuals care about their social status, which is in turn determined by their relative wealth. Within our recursive CARA setting, we may also model the SOC by assuming that the instantaneous utility function depends on relative wealth:[10]

$$V\left(c_t, w_t, \overline{w}_t\right) = -\psi \exp\left(-\frac{1}{\psi}\left[c_t + b\left(w_t - \vartheta \overline{w}_t\right)\right]\right), \tag{6}$$

where $\overline{w}_t$ is the per capita aggregate wealth, b governs the degree of the SOC, and ϑ measure the importance of average wealth in determining the SOC. It is not difficult to show that we can still analytically solve the model with relative wealth and construct a general equilibrium after imposing that the equilibrium net supply of risky assets is zero. The main results obtained in our benchmark model still hold in the model with relative wealth.[11] To keep our model tractable, we focus on our benchmark model in the main text.

3.2 Specifications of Multiple Assets and Labor Income

Following Merton (1971), Viceira (2001), and Wang (2009), we assume that consumers can access two financial assets: one risk-free asset and one risky asset. Specifically, the typical consumer can purchase both a risk-free asset with a constant interest rate r and a risky asset (the market portfolio) with a risky return r_t^e. The instantaneous return dr_t^e of the risky market portfolio over dt is given by:

$$dr_t^e = (r + \pi)\, dt + \sigma_e dB_{e,t}, \tag{7}$$

[10] An alternative specification for the SOC can be written as a multiplicative form: $w_t/\overline{w}_t$. However, given the CARA additive setting adopted in this paper, the model with the multiplicative specification of the SOC is intractable.

[11] The detailed proof and deviation are available from the authors by request.

where π is the market risk premium, σ_e is the standard deviation of the market return, and $B_{e,t}$ is a standard Brownian motion. Let ρ_{ye} be the contemporaneous correlation between the labor income process and the return of the risky asset. If $\rho_{ye} = 0$, then the labor income risk is purely idiosyncratic, so the risky asset does not provide a hedge against declines in labor income. The agent's financial wealth evolution is then given by:

$$dw_t = (rw_t + y_t - c_t)\,dt + \alpha_t\,(\pi dt + \sigma_e dB_{e,t})\,, \tag{8}$$

where α_t denotes the amount of wealth that the investor allocates to the market portfolio at time t.

Furthermore, we assume that the uninsurable labor income (y_t) follows an Ornstein-Uhlenbeck process:[12]

$$dy_t = (\mu - \rho y_t)\,dt + \sigma_y dB_t, \tag{9}$$

where $\mu = \rho\overline{y}$, $\overline{y}$ is the unconditional mean of y_t, σ_y is the unconditional volatility of the income change over an incremental unit of time, $\sigma_y^2/(2\rho)$ is the unconditional variance of y_t, the persistence coefficient ρ governs the speed of convergence or divergence from the steady state, and B_t is a standard Brownian motion.

3.3 The Optimization Problem

The optimization problem under rational expectations (RE) can thus be written as:

$$V(J_t) = \max_{(c_t,\alpha_t)} \left\{ \left(1 - e^{-\delta\Delta t}\right) V(c_t, w_t) + e^{-\delta\Delta t} V\left(\mathbb{CE}_t\left[J_{t+\Delta t}\right]\right) \right\}, \tag{10}$$

subject to (8)-(9). An educated guess is that $J_t = -\alpha_0 - \alpha_1 w_t - \alpha_2 y_t$. Following the standard procedure for solving the dynamic programming problem with recursive utility, we can obtain the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\beta V(J_t) = \max_{\{c_t,\alpha_t\}} \left\{ \beta V(c_t, w_t) + \mathcal{D}V(s_t) \right\}, \tag{11}$$

where

$$\mathcal{D}V(J_t) = V'(J_t)\left((\partial J)^T \cdot \mathbb{E}_t[ds_t] + \upsilon_t^T \cdot \Sigma \cdot \partial J - \frac{\gamma}{2}\left[(\partial J)^T \cdot \Sigma \cdot \partial J \right] \right),$$

$s_t = \begin{bmatrix} w_t & y_t \end{bmatrix}^T$, $ds_t = \begin{bmatrix} dw_t & dy_t \end{bmatrix}^T$, $\partial J = \begin{bmatrix} J_w & J_y \end{bmatrix}^T$, and $\Sigma = \begin{bmatrix} \alpha_t^2\sigma_e^2 & \rho_{ye}\sigma_y\alpha_t\sigma_e \\ \rho_{ye}\sigma_y\alpha_t\sigma_e & \sigma_y^2 \end{bmatrix}$. Finally, the transversality condition, $\lim_{t\to\infty}\{\mathbb{E}\,|\exp(-\delta t)\,V_t|\} = 0$, holds at the optimum. (See Online Appendix B for the detailed derivation.)

[12] In this paper, we abstract from income growth. It is worth noting that higher income growth generates higher risk-free rates. However, within our REU-OU framework, assuming constant income growth leads to time-varying risk-free rates, which greatly complicates our model.

3.4 Optimal Consumption-Portfolio Rules

We can now solve (11) and obtain the consumption and portfolio rules under the SOC. The following proposition summarizes the solution:

Proposition 1 *With the SOC, the decision rules for consumption and portfolio choices, the saving function, as well as the value function are given by:*

(i) The optimal consumption rule is:

$$c_t^* = rw_t + (r+b)\, h_t + \Psi - \Gamma + \Pi, \tag{12}$$

where

$$h_t = \frac{1}{r+b+\rho}\left[y_t + \frac{\mu}{r+b} - \frac{\pi\rho_{ye}\sigma_y}{(r+b)\,\sigma_e}\right] \tag{13}$$

is the risk-adjusted human wealth;

$$\Psi = \left(\frac{\beta}{r+b} - 1\right)\psi \tag{14}$$

measures the effects of impatience on consumption and saving;

$$\Gamma \equiv \left(1-\rho_{ey}^2\right)\frac{(r+b)\,\gamma}{2}\left(\frac{\sigma_y}{r+b+\rho}\right)^2 \tag{15}$$

is the investor's precautionary saving demand; and

$$\Pi = \frac{\pi^2}{2\,(r+b)\,\gamma\sigma_e^2} \tag{16}$$

is the additional increase in the investor's certainty equivalent wealth due to the presence of the risky asset.

(ii) The optimal portfolio rule is:

$$\alpha^* = \frac{\pi}{(r+b)\,\gamma\sigma_e^2} - \frac{\rho_{ye}\sigma_y}{(r+b+\rho)\,\sigma_e}, \tag{17}$$

where the first term is the standard speculation demand for the risky asset and the second term is the labor income-hedging demand.

(iii) The saving function is:

$$d_t^* = x_t - \Psi + \Gamma + \Pi, \tag{18}$$

respectively, where $x_t \equiv \rho\,(y_t - \overline{y})\,/\,(r+b+\rho)$ *is the demand for savings "for a rainy day".*

(iv) The associated value function is

$$V(w_t, y_t) = -\psi \exp\left(\frac{1}{\psi}\left[\alpha_0 + (r+b)\, w_t + \frac{r+b}{r+b+\rho}y_t\right]\right), \tag{19}$$

where

$$\alpha_0 = -\Psi + \Gamma - \Pi - \psi \ln\left(\frac{r+b}{\beta}\right) - \frac{\mu}{r+b+\rho} + \frac{\rho_{ye}\sigma_y\pi}{(r+b+\rho)\,\sigma_e}.$$

Proof. See Online Appendix B for the derivation. ■

Expression (12) clearly shows that current consumption is determined by the annuity value of total wealth, the sum of financial wealth (w_t) and human wealth (h) as well as three additional components, (14)-(16), governing the effects of relative impatience, the precautionary motive, and the presence of the risky asset, respectively. We can see from (12) that this solution is identical to that obtained in the model without the SOC but with an interest rate of $r + b$ rather than r. In other words, from a *partial equilibrium* perspective (i.e., for given r), the SOC increases the effective rate of interest.[13] As argued in Luo et al. (2009), in the presence of the SOC, investing additional dollars yields psychi returns because the additional increase in wealth raises utility; consequently, the effective interest rate, $r+b$, can thus be viewed as the "psychological" rate at which households used to discount future labor income when computing human wealth.

Using the effective rate, we can follow the literature (e.g., Wang 2004, 2009) and construct human wealth as the discount present value of the current and future labor incomes. Specifically, in our incomplete markets economy, there exists a unique stochastic discount factor ζ_t under the *minimal* martingale measure $\mathbb{Q}$ satisfying:

$$d\zeta_t = -\zeta_t \left[(r+b)\, dt + \frac{\pi \rho_{ye}}{\sigma_e} dB_{e,t} \right],$$

where $\zeta_0 = 1$. The human wealth under incomplete markets is then defined as follows:

$$h(y_t) = \mathbb{E}_t^{\mathbb{P}} \left[\int_t^{\infty} \frac{\zeta_s}{\zeta_t} y_s ds \middle| \mathcal{F}_t^y \right] = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\infty} e^{-(r+b)(s-t)} y_s ds \middle| \mathcal{F}_t^y \right], \tag{20}$$

where $\mathbb{Q}$ is the risk-neutral probability measure with respect to $\mathbb{P}$ for the original income process, (9). Using (9), the above expression of $h(y)$ can thus be simplified as (13).

From (12) and (13), it is also clear that the stronger the SOC, the more consumption responds initially to changes in current income (y_t) because the marginal propensity of consumption out of current income is increasing with the SOC, i.e., $\partial \left(\frac{r+b}{r+b+\rho} \right) / \partial b > 0$. That is, if we take the SOC into account, optimal consumption is more sensitive to unanticipated income shocks. More specifically, the responses of consumption to either an increase or a decrease in current income are stronger, which makes consumption more volatile and more dispersed. This response is referred to as "making hay while the sun shines" in the literature. See, e.g., Flavin (1981) and Jappelli and Pistaferri (2010).

Expression (14) shows that the presence of the SOC strengthens the relative importance of the interest rate to the discount rate, which is equivalent to increasing the degree of patience and thus makes saving and future consumption more attractive. It is worth mentioning that this equivalence

[13] In the next section, we will explore how the SOC affects the risk-free rate in a general equilibrium.

result is consistent with a conclusion about the link between the degree of patience and the spirit of capitalism obtained in Doepke and Zilibotti (2008). They argued that families with a strong spirit of capitalism are much easier to develop patience and work ethic, which become the key determinants of the success of industrialization. From (14), it is also clear that if the household is impatient relative to the effective interest rate ($\beta > r + b$), the higher the EIS, the stronger the demand for consumption. If $\beta > r + b$, households want consumption to fall over time, and a higher EIS implies that consumption will be allowed to fall faster for a given value of $\beta/(r + b)$; as a result, consumption must initially be high.

Expression (15) shows that the demand for precautionary saving is defined as the amount of saving induced by the combination of uninsurable labor income risk and risk aversion. It is clear from (15) that the precautionary saving demand is larger for a more volatile income innovation (higher σ_y) and a larger persistence coefficient (lower ρ).[14] In addition, since the risky asset can be used to hedge labor income risk (provided the correlation is not zero), it will reduce the precautionary saving demand arising from income uncertainty by a factor $1 - \rho_{ye}^2 \in (0, 1)$. It is straightforward to show that for given r, the SOC can increase the precautionary saving demand when $\rho > r + b$.[15]

From (16), we can see that stronger SOC reduces the increase in the household's certainty equivalent wealth due to the presence of risky assets because the effective risk-free rate makes investment in the risky asset less attractive. Expression (12) also shows that the presence of the risky asset in the agent's investment opportunity has two effects on current consumption. First, it reduces the risk-adjusted certainty equivalent human wealth by $\pi\rho_{ye}\sigma_y/(r(r + b + \rho)\sigma_e)$ because the agent faces more risk when holding the risky asset. Second, it increases current consumption because it offers a higher expected return. In general equilibrium, the second effect dominates the first effect. (See the next section for the general equilibrium analysis.)

Expression (17) clearly shows that the SOC reduces the standard speculation demand of the risky asset, while it increases the hedging demand for the risky asset. It is straightforward to show that for given asset returns, the SOC increases the overall demand for the risky asset if $\alpha^* > 0$.

Furthermore, from (18), we can see that the presence of the $\pi\alpha^*$ term has the potential to increase saving because it offers a higher expected return. Combining these two effects, it is clear from (18) that the net effect of the risky asset on current saving is governed by $\Pi > 0$ defined in (16). It is thus clear from (18) that there are four saving motives in our model. The first three saving motives–x_t, Ψ, and Γ–are the demand for savings "for a rainy day", the saving demand due to relative patience, and the precautionary saving demand respectively. The fourth term, Π,

[14] As argued in Caballero (1990) and Wang (2003), a more persistent income shock takes a longer time to wear off and thus induces a stronger precautionary saving demand by a prudent forward-looking consumer.

[15] In the next section, we will show that this condition is always satisfied for our estimates of the income process.

captures the additional saving demand due to the higher expected return of the risky asset.

In summary, we see the SOC affects each key component in the consumption-saving rule – it increases the sensitivity of consumption to income shocks; it increases the effective degree of patience and then interacts with the EIS in determining the saving demand due to relative patience; it raises precautionary demand; and it reduces the household's certainty equivalent wealth (which lowers consumption). In addition, the SOC lowers the standard speculation demand, while increasing the labor-hedging demand for risky assets, with a negative net effect on the risky asset demand if $\alpha^* > 0$.

3.5 General Equilibrium Implications

As in Bewley (1986), Huggett (1993), and Wang (2003), we assume that the economy is populated by a continuum of *ex ante* identical, but *ex post* heterogeneous consumers, with each agent having the saving function, (18). In addition, we also assume that the risk-free asset in our model economy is a pure-consumption loan and is in zero net supply, and that the net supply of the risky asset is $\alpha_s \geq 0$. We first consider the equilibrium in the market for the risky asset. Assuming the equilibrium condition in the market for the risky asset is:

$$\alpha_s = \frac{\pi}{(r+b)\,\gamma\sigma_e^2} - \frac{\rho_{ye}\sigma_y}{(r+b+\rho)\,\sigma_e}, \tag{21}$$

for a given risk free rate, r.

In the model economy, the initial cross-sectional distribution of income is assumed to be its stationary distribution $\Phi(\cdot)$. By the law of large numbers (LLN), provided that the spaces of agents and the probability space are constructed appropriately, aggregate income and the cross-sectional distribution of permanent income $\Phi(\cdot)$ will be constant over time. Using the individual saving function (18), we have the following aggregation result on savings:

Proposition 2 *The total savings demand "for a rainy day" in our model equals zero for any positive interest rate. That is,* $F_t(r) = \int_{y_t} x_t(r)\, d\Phi(y_t) = 0$, *for* $r > 0$.

Proof. Given that labor income is a stationary process, the LLN can be directly applied. The proof is the same as that in Wang (2003). ■

Using this result, from (18), after aggregating across all consumers, the expression for total savings can be written as:

$$D^{total}(r) \equiv \Gamma(r) - \Psi(r) - \Pi(r), \tag{22}$$

where $\Psi(r)$, $\Gamma(r)$, and $\Pi(r)$ are given in (14), (15), and (16), respectively. Note that compared with the original Huggett (1993), Calvet (1997), and Wang (2003) models, here $\Pi(r)$ is an additional

term due to the positive net supply of the risky asset in this model. Since the net supply of the risk-free asset is zero in equilibrium, an equilibrium interest rate r^* satisfies:

$$D(r^*) \equiv \Gamma(r^*) - \Psi(r^*) = 0, \tag{23}$$

where $D(r^*)$ denotes the amount of saving in the risk-free asset.

The following proposition proves that an equilibrium exists and that the PIH is satisfied.

Proposition 3 *In equilibrium, each consumer's optimal consumption-portfolio rules are described by:*

$$c_t^* = r^* w_t + (r^* + b) h_t, \tag{24}$$

and

$$\alpha^* = \alpha_s, \tag{25}$$

respectively, where h_t is defined in (13). Furthermore, in equilibrium, the evolution equations of c_t^* and w_t are:

$$dc_t^* = r^* dw_t + \frac{r^* + b}{r^* + b + \rho} dy_t, \tag{26}$$

$$dw_t^* = (x_t + \Pi)\, dt + \alpha_s \sigma_e dB_{e,t}, \tag{27}$$

respectively, and the risk premium is

$$\pi^* = (r^* + b)\, \gamma \sigma_e \left(\frac{\rho_{ye} \sigma_y}{r^* + b + \rho} + \alpha_s \sigma_e \right). \tag{28}$$

If $\rho > \beta$, the equilibrium, (r^*, π^*), is unique.

Proof. Online Appendix B for the proof. ■

The intuition behind this proposition is similar to that in Wang (2003) and Luo et al. (2020). With an individual's constant total precautionary savings demand $\Gamma(r)$, for any $r > 0$, the equilibrium interest rate r^* must be at a level with the property that individual's dissavings demand due to impatience is exactly balanced by their precautionary-savings in the risk-free asset, $\Gamma(r^*) = \Psi(r^*)$. It is clear from (23) that a high value of ψ would amplify the relative importance of the dissaving effect $\Psi(r)$ for the equilibrium interest rate. The intuition behind this result is simple. When ψ is higher, consumption growth responds less to changes in the interest rate. In order to clear the market, the consumer must be offered a higher equilibrium risk free rate in order to be induced to save more and make his consumption tomorrow even more in excess of what it is today (less smoothing). Figure 1 shows how the equilibrium risk-free rate is unique and is decreasing with the value of b when $\gamma = 5$, $\psi = 0.8$, $\delta = 0.048$, $\sigma_y = 0.162$, and $\rho = 0.102$.[16]

[16] In Section 4.1, we will discuss the choice of these preference parameters and provide more details about how to estimate the income process using the U.S. panel data. The main result here is robust to the choices of these parameter values.

From the equilibrium condition, (23):

$$\frac{1}{2}\left(r^*+b\right)\gamma\left(\frac{\sigma_y}{r^*+b+\rho}\right)^2-\left(\frac{\beta}{r^*+b}-1\right)\psi=0, \tag{29}$$

it is straightforward to show that

$$\frac{dr^*}{db}=-1\ <0; \tag{30}$$

so that r^* is decreasing in the degree of the SOC, b. In other words, an increase in b has no effect on the values of Γ and Ψ in general equilibrium. In addition, it is straightforward to see that:

$$\frac{dr^*}{d\gamma}<0 \text{ and } \frac{dr^*}{d\psi}>0.$$

That is, the equilibrium interest rate decreases with the degree of risk aversion and increases with the degree of intertemporal substitution. From (26) and (27), we can conclude that although both the CARA model and the linear-quadratic (LQ) model lead to the PIH in general equilibrium, both risk aversion and intertemporal substitution play roles in affecting the dynamics of consumption and wealth in the CARA model via the equilibrium interest rate channel.

Note that mathematically, the cross-sectional dispersion of consumption and wealth (relative to income), our measures of consumption and wealth inequality, can be measured by the relative volatility of consumption and wealth to income, as our model satisfies a mixing condition in the steady state. In addition, as in the typical Bewley-Huggett type economy, the cross-sectional distribution of consumption and wealth in our model economy are the same as the long-run stationary distribution of individual household consumption and wealth. The following result is then immediate.

Proposition 4 *The relative inequality of consumption to income is:*

$$\mu\equiv\frac{\mathrm{sd}\left(dc_t^*\right)}{\mathrm{sd}\left(dy_t\right)}=\frac{r^*+b}{r^*+b+\rho}\sqrt{1+\left(r^*+b+\rho\right)^2\left(\frac{\sigma_e}{\sigma_y}\alpha_s\right)^2+2\left(r^*+b+\rho\right)\frac{\rho_{ye}\sigma_e}{\sigma_y}\alpha_s}, \tag{31}$$

where $\mathrm{sd}\left(\cdot\right)$ *denotes standard deviation. Furthermore, the relative inequality of wealth to income is:*

$$\mu_{wy}\equiv\frac{\mathrm{sd}\left(dw_t^*\right)}{\mathrm{sd}\left(dy_t\right)}=\alpha_s\frac{\sigma_e}{\sigma_y}. \tag{32}$$

Proof. Using (26), the proof is straightforward. ■

Given the complexity of the expressions of the endogenous interest rate and (31), we will quantitatively evaluate the effects of the SOC on the equilibrium asset returns and the relative inequality of consumption to income in the next section. It is worth noting that it is clear from (31) that when $\alpha_s=0$,

$$\mu=\frac{r^*+b}{r^*+b+\rho}, \tag{33}$$

which is clearly increasing with the degree of the SOC, b, for a given equilibrium risk-free rate. The main intuition of this result is that the presence of the SOC increases the "psychological" or effective rate of interest at which households discount future labor income when computing human wealth and compute the annuity value of human wealth. Furthermore, (26) clearly shows that the larger the value of b, the more consumption responds initially to changes in current income because $\partial\mu/\partial b > 0$. That is, the optimal consumption is more sensitive to unanticipated income shocks (either positive or negative) in the presence of the SOC; consequently, equilibrium consumption becomes more volatile and more dispersed. Figure 2 illustrates how the marginal propensity of consumption (the MPC) out of current income varies with the degree of the SOC. It is clear from the figure that this MPC is more volatile when the degree of the SOC becomes stronger.

As documented in many empirical studies in the PIH literature, consumption growth are robustly correlated with predictable changes in labor income at both the micro- and macro-levels. For example, Flavin (1981) estimates the joint consumption-income equations, and finds evidence of excess sensitivity of consumption to predicted income growth. Parker (1999) uses a psuedo-natural experiment provided by the pattern of Social Security tax withholding to test whether household consumption responds to expected changes in take-home pay. Souleles (1999) studies the relationship between consumption and the receipt of income tax funds. These studies find evidence of excess sensitivity and are thus interpreted as strong evidence against the PIH.[17] To examine how the SOC affects the excess sensitivity of equilibrium consumption to the *anticipated* change in income in our model, we rewrite (26) as follows:

$$dc_t^* = r^*\Pi dt + \frac{b}{r^*+b+\rho}\mathbb{E}_t\left[dy_t\right] + \left(\frac{r^*+b}{r^*+b+\rho}\sigma_y dB_t + r^*\alpha_s\sigma_e dB_{e,t}\right), \tag{34}$$

where $\mathbb{E}_t\left[dy_t\right]$ is the anticipated change in income and we use the fact that $dy_t = \mathbb{E}_t\left[dy_t\right] + \sigma_y dB_t$. It is clear from (34) that with the SOC, the anticipated growth of labor income can be used to predict changes in equilibrium consumption because of $b > 0$. That is, excess sensitivity can arise in equilibrium for consumers with the SOC even in the absence of borrowing constraints. Our SOC model can therefore be an alternative explanation for apparent excess sensitivity of consumption to anticipated changes in income that is consistent with the PIH in general equilibrium.

Finally, (32) clearly shows that once the net supply of the risky asset is exogenously set, the relative wealth inequality is determined by the interaction of the three parameters, α_s, σ_e, and σ_y, pinned down from the data. However, in the next section on quantitative analysis, we find that the implied relative wealth inequality is also consistent with the empirical evidence. Given these features, our model is not the ideal one to examine the effects of the SOC on the wealth inequality.

[17] In addition, in the literature following Hall (1978), excess sensitivity was generally held to result from the presence of borrowing constraints. See Jappelli and Pistaferri (2010) for a recent survey on the consumption response to income changes.

4 Model's Quantitative Implications

In this section, we evaluate the quantitative implications of the SOC in explaining the equilibrium interest rate, consumption inequality, and the risk premium. Our parameterization is based on the benchmark model. We provide a robustness check by allowing alternative specifications in the next section.

4.1 Parameterization and Data Moments

Labor Income Parameters. To estimate the labor-income process specified in Section 2, we use the Panel Study of Income Dynamics (PSID) to first estimate a discrete AR(1) process and then calculate the key coefficients based on the estimated AR(1) process. Specifically, we follow Blundell et al.(2008) and Luo et al. (2020) to apply a two-step panel regression to estimate the following AR(1) process:[18]

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sigma\varepsilon_t,\ t \geq 1,\ \ |\phi_1| < 1, \tag{36}$$

where $\varepsilon_t \sim N(0,1)$, $\phi_0 = (1-\phi_1)\,\overline{y}$, $\overline{y}$ is the mean of y_t, and the initial level of labor income y_0 are given. Once we have estimates of ϕ_1 and σ, we can recover the drift and diffusion coefficients in the Ornstein-Uhlenbeck process specified in (9) by rewriting (36) in the time interval $[t, t+\Delta t]$ as:

$$y_{t+\Delta t} = \phi_0 + \phi_1 y_t + \sigma\sqrt{\Delta t}\varepsilon_{t+\Delta t}, \tag{37}$$

where $\phi_1 = \exp(-\rho\Delta t)$, $\sigma = \sigma_y\sqrt{(1-\exp(-2\rho\Delta t))/(2\rho\Delta t)}$, and $\varepsilon_{t+\Delta t}$ is the time-$(t+\Delta t)$ standard normal distributed innovation to income.[19] As the time interval, Δt, converges to 0, (37) reduces to the Ornstein-Uhlenbeck process, (9). The estimation results and the recovered persistence and volatility coefficients in (9) are reported in the top panel of Table 1. The detailed description on the data is in Online Appendix A.

Other Parameters. The rest of the key parameters in the benchmark model are most taken from the literature and reported in Table 2. Specifically, following the literature on consumption and asset pricing (e.g., Campbell 2003, Bansal and Yaron 2004), we set the coefficient of risk aversion (γ) and the discount factor (β) to be 6 and 0.048, respectively. The magnitude of the EIS (ψ) is a key issue in macroeconomics and asset pricing. For example, Vissing-Jorgensen and Attanasio (2003) estimate the EIS to be well in excess of one. Bansal and Yaron (2004) show that a small, persistent

[18] Adopting a more general ARMA(1,1) specification,

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sigma(\varepsilon_t + \varrho\varepsilon_{t-1}), \tag{35}$$

does not change the main quantitative conclusions. The results obtained using this ARMA(1,1) are available from the coressponding author by request.

[19] Note that here we use the fact that $\Delta B_t = \varepsilon_t\sqrt{\Delta t}$, where ΔB_t represents the increment of a Wiener process.

component of consumption growth can have quantitatively important implications for asset prices if the representative agent has Epstein–Zin preferences with the EIS equaling 1.5. Campbell (2003), on the other hand, estimates its value to be well below one.[20] Crump et al. (2015) find that the EIS is precisely and robustly estimated to be around 0.8 in the general population using the newly released FRBNY Survey of Consumer Expectations. Here we choose $\psi = 0.8$ as the benchmark value.

In addition, estimating the correlation between individual labor income and the equity return is complicated by the lack of panel data on household portfolio choice, and we find several estimates in the literature: Viceira (2001) adopts $\rho_{ye} = 0.35$ when simulating a life-cycle consumption-portfolio choice model. Davis and Willen (2000) estimate that the correlation is between 0.1 and 0.3 for college-educated men, and is 0.25 or more for college-educated women. Here, we follow Viceira (2001) and set $\rho_{ye} = 0.35$ as a benchmark value. Following Campbell (2003), we measure the asset return volatility σ_e using stock market data between 1980 and 2016 and the resulting value is 0.17. The values of the risk premium (π) will be determined endogenously in equilibrium.

Key Data Moments. We examine the model's implications on the risk-free rate, consumption inequality, wealth inequality, and equity premium. These key moments are measured using various data. We follow Campbell (2003) to calculate the real risk-free rate based on the real 3-month Treasury yields. The equity premium is calculated as the difference between the average equity return and the average risk-free rate. The empirical counterpart of the relative consumption and wealth inequality is measured using the PSID data. Please see Online Appendix A for details in constructing a panel data with household-level consumption, income, and wealth. The values of these key moments are reported in the first row of Table 3.

4.2 Effects of the SOC in Matching the Key Moments

Figure 3 shows that the equilibrium interest rate decreases with the degree of the SOC for different plausible values of γ, ψ, β, and ρ_{ye} in the benchmark model. It is clear from the figure that a small degree of the SOC can have significant effects on the equilibrium interest rate. For example, if b is increased from 0 to 0.02, r^* falls from 4.1% to 2.1%.[21] In addition, the figure also shows that the interest rate decreases with γ, and increases with ψ, β, and ρ_{ye} for different values of b. We can

[20] Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1).

[21] Note that this value of the SOC parameter (b) is relatively small given that the coefficient attached to the consumption term is normalized to 1. In addition, this value is also well below that obtained in Bakshi and Chen (1996). In a model with the SOC and CRRA preferences, they find that when the relative weight of the degree of the SOC compared to the degree of risk aversion to consumption is about $1/3 - 1/2$, the model can explain the joint behavior of aggregate consumption and the equity return observed in the U.S. economy. He at al. (2020) show a growth model calibrated to the U.S. economy suggests a value of $b = 0.09$.

also see from the figure that the correlation between labor income risk and the equity return risk have only minor effects on the equilibrium rate of interest.

To put this into a statistical perspective, we compare the model's predictions on the key targets under different specifications. As reported in the left panel in Table 3, the average real risk-free rate in $1980-2016$ is 2.1%. Without the SOC (i.e., $b=0$), it requires the degree of risk aversion to be as high as 57 to generate a risk-free rate as observed in the data when the EIS is 0.8. When the EIS is relatively lower, say, $\varphi=0.5$, it still requires the degree of risk aversion to be 35 to generate the observed real risk-free rate.[22] In contrast, when there is a small degree of the SOC, the model can generate an equilibrium interest rate of 2.1% with much lower values of the coefficient of risk aversion. As reported in the third column of Table 3, with $\gamma=6$, $\psi=0.8$, and $b=0.02$, the model generates the same interest rate as in the standard model with $\gamma=57$. Note that $\gamma=6$ is in the middle of the range of reasonable values for the coefficient of relative risk aversion used in most macro-asset pricing models. For example, Caballero (1990) and Wang (2004, 2009) set γ to be 2 or 3, while Bansal and Yaron (2004) set γ to be 10. More generally, Figure 4 shows the relationship between risk aversion parameter γ and the SOC parameter ϑ to generate an interest rate of 2.1% for different values of EIS parameter ψ.

Turning to the relative consumption inequality, as measured by $(\mu=\mathrm{sd}\left(dc_t^*\right)/\mathrm{sd}\left(dy_t\right))$, the average value in $1980-2016$ is 0.4 as reported in the middle panel of Table 3. Without the SOC, even the degree of risk aversion is set at 57 (the level which generates a real risk-free rate as in the data without the SOC), the model implied relative consumption inequality is only 0.22, which is about half of that in the data. In contrast, when there is a small degree of the SOC ($b=0.02$), the model can generate a relative consumption inequality as in the data with a reasonable degree of risk aversion $\gamma=6$. In other words, incorporating a small amount of the SOC could greatly improve the model's predictions on both the equilibrium real risk-free rate and the relative consumption inequality.

Furthermore, incorporating the SOC also helps produce more realistic equity premium. As reported in the right panel of Table 3, under the same degree of the SOC ($b=0.02$), the model generates an equity premium of 4.3%, which is close to the actual equity premium in the data. In contrast, without the SOC, the model can only produce less than half of the observed equity premium for the same degree of risk aversion, 6.

Finally, our model implied relative wealth inequality is also consistent with the data. As also reported in the Table 3, at $b=0.02$, our model's implied relative wealth inequality exactly matches the data. In our model, the relative wealth inequality is determined by the interaction of the three parameters, α_s, σ_e, and σ_y. It is worth noting that in our model, α_s, σ_e, and σ_y are exogenously

[22] Note that since we set the mean income level to be 1, the coefficient of CRRA evaluated at this level is equal to the coefficient of CARA.

calibrated or estimated using the data. (See Tables 1 and 2 for the details.) As a result, we cannot examine how the SOC affects the relative wealth inequality in our model.

5 Further Discussions

In this section we provide three extensions to the benchmark model as robustness checks. First, we introduce rare macroeconomic disasters into the model and check the robustness of the results. Second, we assume a more general labor-income process and examine whether our key results still hold. Third, we compare the SOC with habit formation, which represents an alternative attitude toward consumption and saving, and show they lead to completely different results on consumption inequality and the equilibrium interest rate. For all three discussions, we provide analytical solutions.

5.1 Incorporating Rare Macroeconomic Disasters

In our benchmark model, we assumed that the income process takes a diffusive form. In this subsection, we consider an important extension in which the households might experience rare macroeconomic disasters, and examine how the SOC affects the equilibrium risk-free rate and relative consumption inequality. Rietz (1988) first introduced rare disasters into a consumption-based asset pricing model and showed that it can help resolve the equity premium puzzle. Barro (2009) and Barro and Ursua (2011) showed that the presence of rare macroeconomic disasters has the potential to explain an array of macro-asset pricing puzzles. To model rare disasters (e.g., financial crisis), we assume that the labor income process facing the agents in our model economy not only include a diffusion component but also includes a Poisson jump process.[23] As argued in Barro (2009) and Barro and Ursua (2011), the probability and size distribution of rare disasters are difficult to quantify empirically because the relevant events are rare and possibly absent in short samples. In this paper, we therefore consider a simple model in which the rare event of a sharp decline in income is a Poisson process with a constant occurring probability per unit of time, λ (i.e., Poisson arrivals). When the event happens, labor income is reduced by a constant amount φ.[24] Specifically, we assume that the dynamics of labor income is now governed by:

$$dy_t = (\mu - \rho y_t)\, dt + \sigma_y dB_t - \varphi dq, \tag{38}$$

where B_t reflects "normal" economic fluctuations and q is a pure jump process. That is, $dq = 1$ when the jump happens and $dq = 0$ otherwise. It is worth noting that we can use this Poisson process to

[23] Barro (2009) and Barro and Ursua (2011) considered the similiar output jump process within a discrete-time Lucas-type asset pricing setting.

[24] Note that here we can also interpret φ as a fraction of the mean of income because we have assumed that the mean level of income ($\overline{y}$) is 1.

model financial crises. When a financial crisis happens, the mean income of the economy is dropped by φ. The reason that we consider jumps with constant size is to keep our heterogeneous-agent incomplete markets model tractable. Note that allowing for a stochastic φ introduces an additional demand for precautionary savings, and does not change our main results. The HJB for the model with rare disasters can be written as:[25]

$$\delta f\left(J_t\right)=\max_{c_t}\left\{\delta f\left(c_t,w_t\right)+f'\left(J_t\right)\left(\partial J\cdot\mathbb{E}\left[ds_t\right]-\frac{1}{2}\gamma\left[\partial J\cdot\sigma_y^2\cdot\left(\partial J\right)^T\right]\right)+\lambda\left[f\left(J\left(w_t,y_t-\varphi\right)\right)-f\left(J\left(w_t,y_t\right)\right)\right]\right\}$$

where $f\left(J_t\right)=\left(-\psi\right)\exp\left(-J_t/\psi\right)$, $f'\left(J_t\right)=\exp\left(-J_t/\psi\right)$, $s_t=\left[\begin{array}{cc} w_t & y_t \end{array}\right]^T$, $ds_t=\left[\begin{array}{cc} dw_t & dy_t \end{array}\right]^T$, and $\partial J=\left[\begin{array}{cc} J_w & J_y \end{array}\right]$. For simplicity, here we assume that the supply of the risky asset is zero. The budget constraint for the typical consumer becomes: $dw_t=\left(rw_t+y_t-c_t\right)dt$. Following the same procedure, we can solve the above optimization problem with rare events and obtain the consumption and saving functions. The following proposition summarizes the solution:

Proposition 5 *Under rare events and the SOC, the consumption function is:*

$$c_t=rw_t+\frac{r+b}{r+b+\rho}y_t+\frac{1}{r+b+\rho}\left(\mu-\varphi\lambda\right)+\Psi-\Gamma-\Lambda, \tag{39}$$

and the saving function is:

$$d_t=x_t-\Psi+\Gamma+\Lambda, \tag{40}$$

where $x_t=\rho\left(y_t-\overline{y}\right)/\left(r+b+\rho\right)$ is the demand for savings for a rainy day", $\overline{y}=\left(\mu-\varphi\lambda\right)/\rho$ is the mean income, $\Psi=\psi\left(\beta/(r+b)-1\right)$, $\Gamma=0.5\gamma(r+b)\sigma_y^2/\left(r+b+\rho\right)^2$, and

$$\Lambda=\lambda\frac{\psi}{r+b}\left[\exp\left(\frac{r+b}{\psi}\frac{\varphi}{r+b+\rho}\right)-1\right] \tag{41}$$

is the additional saving demand due to the presence of rare disasters.

Proof. See Online Appendix C. ■

In the presence of rare events, Λ is the capitalized value of the expected future declines to labor income, capitalized at a higher rate than the risk-free rate reflecting the agents' degree of the SOC and aversion to intertemporal substitution.[26] It is clear from (41) that the presence of rare events increases individual saving via the interaction between frequency (λ) and size (φ) of the rare events.

Following the same procedure and the definition of general equilibrium adopted in the benchmark model, we can also obtain the general equilibrium implications of the SOC for the risk-free rate

[25] For simplicity, here we assume that the net supply of the risky asset is zero. It is worth noting that the assumption of a positive net supply of the risky asset does not change our main conclusions if the equity return is not correlated with the Poisson jump in the labor income process.

[26] Note that in the expected utility case this term is affected by the degree of risk aversion because we do not distinguish intertemporal substitution from risk aversion in this case.

and consumption inequality in the presence of rare disasters. The following proposition summarizes the general equilibrium results:

Proposition 6 *There exists one equilibrium with an interest rate* $r^* \in (0, \beta)$ *such that*

$$D(r^*) = -\Psi + \Gamma + \Lambda = 0. \tag{42}$$

In any such equilibrium, each agent's optimal consumption-portfolio rules are described by:

$$c_t^* = r^* w_t + \frac{r^* + b}{r^* + b + \rho}\left(y_t + \frac{\mu}{r^*}\right). \tag{43}$$

Furthermore, in this equilibrium, the relative volatility of consumption growth to income growth is:

$$\mu \equiv \frac{\mathrm{sd}(dc_t^*)}{\mathrm{sd}(dy_t)} = \frac{r^* + b}{r^* + b + \rho}, \tag{44}$$

where $\mathrm{sd}(dy_t) = \sqrt{\left(\sigma_y^2 + \lambda\varphi^2\right) dt}$.

Proof. The proof is the same as that in the benchmark model in Section 3. ■

We make two comments on these theoretical results. First, from expressions (41) and (42), we can see that the presence of rare disasters has the potential to further drive down the risk-free rate because it generates an additional saving term Λ. In addition, (44) clearly shows that introducing rare disasters can indirectly lower the relative consumption inequality through generating a lower equilibrium interest rate. However, for a given equilibrium risk-free rate, introducing rare disasters has no effect on the relative consumption inequality.

To fully explore the general equilibrium effect of the SOC in the presence of rare disasters, we need to do a quantitative analysis. We adopt the disaster parameters estimated by Barro and Ursua (2011). When applying to disasters of size 10% or larger, they estimate that the disaster probability for GDP (i.e., the empirical frequency of entry into disaster states) is 3.7%. In addition, they estimate that the average disaster size, subject to the threshold of 10%, is 0.21 for GDP. We therefore set the probability (λ) and size (φ) to be 3.7% and 0.21, respectively, as the baseline values in our quantitative analysis. Figure 5 shows that how the equilibrium risk-free rate decreases with the degree of the SOC for different plausible values of λ, φ, and ψ when other parameters are set to their baseline values. It is clear from the figure that a small degree of the SOC can have significant effects on the equilibrium risk-free rate for plausible values of the probability (λ) and size (φ) as well as the EIS (ψ). That is, the main result in our benchmark model still holds in the presence of rare disasters. Furthermore, it is also clear from the figure that the probability and size do not have significant effects on the equilibrium risk-free rate. For example, if b is increased from 0 to 0.017, r^* falls from 3.9% to 2.1%, given $\gamma = 6$, $\psi = 0.8$, $\beta = 0.048$, $\lambda = 3.7\%$, and $\varphi = 0.21$. As is clear from (44), the quantitative implication of the SOC on the relative consumption inequality is the same as that obtained in the benchmark model once the equilibrium risk-free rate is reached to its empirical counterpart.

5.2 More General Income Specification

In our benchmark model, we assumed that labor income follows an AR(1) process. In this section, we consider an alternative realistic income specification. In the first case, we follow Wang (2004) and assume that labor income has two distinct components:

$$y_t = y_{1,t} + y_{2,t},$$

where

$$dy_{1,t} = (\mu_1 - \rho_1 y_{1,t})\, dt + \sigma_1 dB_{1,t}, \tag{45}$$

$$dy_{2,t} = (\mu_2 - \rho_2 y_{2,t})\, dt + \sigma_2 dB_{2,t}, \tag{46}$$

and the two Brownian innovations, $dB_{1,t}$ and $dB_{2,t}$, are independent, and are also independent of the innovation to the equity return. All the other notations are similar to those we used in our benchmark model. Without loss of generality, we assume that $\rho_1 < \rho_2$. For simplicity, here we assume that agents can distinguish the two individual components. Our main results still hold if we assume that the two individual components are indistinguishable.

Following the same procedure and the definition of general equilibrium adopted in the benchmark model, we can also obtain the general equilibrium implications of the SOC under this alternative income specification. The following proposition summarizes the general equilibrium results:

Proposition 7 *There exists one equilibrium with an interest rate $r^* \in (0, \beta)$ such that the amount of aggregate saving in the risk free asset is zero*

$$\pi^* = (r^* + b)\, \gamma \alpha_s \sigma_e^2. \tag{47}$$

In any such equilibrium, each agent's optimal consumption-portfolio rules are described by:

$$c_t^* = r^* w_t + \frac{r^* + b}{r^* + b + \rho_1} \left(y_{1,t} + \frac{\mu_1}{r^*} \right) + \frac{r^* + b}{r^* + b + \rho_2} \left(y_{2,t} + \frac{\mu_2}{r^*} \right). \tag{48}$$

Furthermore, in this equilibrium, the evolution equations of wealth and consumption are

$$dw_t^* = (f_t + \Pi)\, dt + \alpha_s \sigma_e dB_{e,t}, \tag{49}$$

$$dc_t^* = \left(\frac{r^* + b}{r^* + b + \rho_1} \sigma_1 + \frac{r^* + b}{r^* + b + \rho_2} \sigma_2 \right) dt + r^* \alpha_s \sigma_e dB_{e,t}, \tag{50}$$

respectively, where $f_t = \rho_1 (y_{1,t} - \overline{y}_1) / (r^* + \rho_1) + \rho_2 (y_{2,t} - \overline{y}_2) / (r^* + \rho_2)$ and $\Pi = \pi^2 / \left[2 (r + b) \gamma \sigma_e^2 \right]$. Finally, the relative volatility of consumption growth to income growth is

$$\mu \equiv \frac{\mathrm{sd}\,(dc_t^*)}{\mathrm{sd}\,(dy_t)} = \frac{1}{\sqrt{\sigma_1^2 + \sigma_2^2}} \sqrt{\left(\frac{r^* + b}{r^* + b + \rho_1} \sigma_1 \right)^2 + \left(\frac{r^* + b}{r^* + b + \rho_2} \sigma_2 \right)^2 + (r^* \sigma_e \alpha_s)^2}. \tag{51}$$

Proof. The proof is the same as that in the benchmark model in Section 3. See Online Appendix D for the detailed derivation. ■

Based on these analytical solutions, we can evaluate the implications of the SOC on the risk-free rate and relative consumption inequality. We follow the approach in Blundell et al. (2008) to estimate the parameters which are reported in the bottom panel of Table 1. Using the above estimated parameter values, we can quantitatively evaluate how SOC affects the equilibrium risk-free rate. Figure 6 shows that the equilibrium interest rate decreases with the degree of SC for different plausible values of ψ.[27] It is clear from the figure that similar to our benchmark model, a small degree of SOC can have significant effects on the equilibrium interest rate. For example, when $\gamma = 6$, $\psi = 0.8$, $\beta = 0.048$, we only need $b = 0.016$ for the model to generate an interest rate of 2.1%, which is the empirical counterpart. If we do not consider SOC, the risk aversion parameter γ need to be as high as 30 to generate the observed risk-free rate.

Using the formula (51), we can quantitatively evaluate the implications of the SOC for the relative consumption inequality. For example, without SOC, after setting $\gamma = 30$ to obtain that $r^* = 2.1\%$, we can then compute that $\mu = 0.35$, which is below the average value $\mu = 0.4$. In contrast, when we consider the SOC and set $b = 0.016$ and $\gamma = 6$, the same values we used above to generate a risk-free rate of 2.1%, the model implies a relative consumption inequality of $\mu = 0.42$, which is very close to the empirical counterpart.

5.3 Comparing with Internal Habit Formation

It has long been recognized by economists that preferences may not be intertemporally separable. In particular, high past consumption generates a desire for high current consumption and preferences may thus display habit formation (or intertemporal complementarity) features.[28] The notion that past consumption may affect current utility is very old. Deusenberry (1949) is probably the first to examine the implications of habit formation. Several papers have shown the importance of allowing for habit formation (HF) in utilities when examining consumption dynamics and equilibrium asset returns. For example, Sundaresan (1989), Constantinides (1990), and Ingersoll (1992) find that models with habit formation can obtain a high equity premium with low degrees of risk aversion and smooth consumption dynamics. As argued in Ingersoll (2011), the model with the SOC is close in spirit to the habit formation models mentioned above, but it is the interaction future consumption rather than past consumption that is of concern here because wealth is part of total resources that matters for financing future consumption.

With HF preferences, households try to smooth consumption growth (roughly speaking), rather

[27] The other parameter values are set as follows: $\psi = 0.8$ and $\beta = 0.048$.

[28] For simplicity, here we only discuss internal habit formation and do not consider the catch-up-with-Joneses effect (i.e., external habit formation).

than the level of consumption; the result is that consumption tends to respond slowly to changes in income shock. In this section, we compare the different implications of HF and the SOC with a general equilibrium framework. Following Sundaresan (1989), Ingersoll (1992), Alessie and Lusardi (1997), and Smith (1999), we assume that habit formation takes a subtractive form and the momentary utility function of households depends on the difference between current consumption and the habit stock. Specifically, we assume that the momentary utility function takes the following form:

$$V\left(c_t-\theta z_t\right)=(-\psi)\exp\left(-\frac{1}{\psi}\left(c_t-\theta z_t\right)\right), \tag{52}$$

where $\theta\in(0,1)$ governs the degree of habit formation, and z_t is the habit stock and is governed by the following equation:

$$z_t=z_0+\int_0^t\exp\left(-\kappa\left(t-s\right)\right)c_s ds, \text{ or } dh_t=\kappa\left(c_t-z_t\right)dt. \tag{53}$$

In other words, $V\left(c_t-\theta z_t\right)$ is dependent on not only current consumption at t, but also on the habit stock (i.e., the weighted average of past consumption). Here $\kappa>0$ is a smoothing constant. The higher κ is, the less weight is put on past consumption in determining the habit stock. We can then introduce HF into our REU model specified in Section 2 by assuming that the recursive utility takes the following form:

$$V\left(J_t\right)=\max_{\left(c_t,\alpha_t\right)}\left\{\left(1-e^{-\beta\Delta t}\right)V\left(c_t-\theta z_t\right)+e^{-\beta\Delta t}V\left(\mathbb{CE}_t\left[J_{t+\Delta t}\right]\right)\right\}, \tag{54}$$

where the definitions of $V\left(J_t\right)$ and $\mathbb{CE}_t\left[J_{t+\Delta t}\right]$ are the same as that in the benchmark model.

Following the same solution method and the definition of general equilibrium in our benchmark model, we can construct an general equilibrium under HF. The following proposition summarizes the solution and key properties:

Proposition 8 *There exists a unique equilibrium with an interest rate $r^*\in(0,\beta)$ such that $D\left(r^*\right)=0$, and*

$$\pi^*=\eta r^*\gamma\sigma_e\left(\frac{\rho_{ye}\sigma_y}{r^*+\rho}+\alpha_s\sigma_e\right). \tag{55}$$

(i) In any such equilibrium, each agent's optimal consumption-portfolio rules are described by:

$$c_t^*=\eta r^*\left(w_t+h_t\right), \tag{56}$$

and

$$\alpha^*=\alpha_s\geq 0, \tag{57}$$

respectively, where

$$\eta=1-\frac{\theta\kappa}{r+\kappa}\in(0,1), \tag{58}$$

governs the impact of HF on consumption and portfolio rules,

$$h_t = \frac{1}{r^* + \rho}\left(y_t + \frac{\mu}{r^*} - \frac{\pi \rho_{ye} \sigma_y}{r^* \sigma_e}\right)$$

is the risk-adjusted human wealth; and the equilibrium risk-free rate, r^*, is endogenously determined by:

$$\Gamma(r^*) = \Psi(r^*), \tag{59}$$

where

$$\Gamma(r^*) = \frac{1}{2}\eta^2 r^* \gamma \frac{\left(1 - \rho_{ey}^2\right)\sigma_y^2}{\left(r^* + \rho\right)^2} \tag{60}$$

is the precautionary saving demand and $\Psi(r^*) = \psi(\beta/r^* - 1)$ is the dissaving effect due to relative impatience;

(ii) the equilibrium risk-free rate, r^*, increases with the strength of habit formation (measured by θ and κ). That is,

$$\frac{dr^*}{d\theta} > 0 \text{ and } \frac{dr^*}{d\kappa} > 0.$$

Proof. See Online Appendix E for the derivation. ■

We make a couple of comments here. First, from (59), it is clear that the only difference in the equilibrium condition between the HF model and the otherwise standard model is the presence of η^2 defined in (58). When $\theta = 0$ or $\kappa = 0$, the two models are identical. Second, we can see from (59) and (60) that HF affects the precautionary savings demand because $\eta^2 < 1$, while it has no effect on the dissaving effect due to relative impatience; consequently, HF drives up the equilibrium risk-free rate. That is, the stronger the habit persistence (higher θ), the higher the equilibrium interest rate.[29] Thus, we can conclude that the SOC and HF have opposite effects on the demand for precautionary savings, which then leads to opposite effects on the equilibrium interest rate. The SOC reduces the equilibrium interest rate, while HF increases it.

Figure 7 illustrates how the general equilibrium interest rates vary with θ for different values of κ.[30] We can see from the figure that r^* increases as the degree of habit persistence increases, i.e., either θ or κ increase. For example, for given $\kappa = 0.2$, if θ is raised from 0 to 0.5, r^* increases from 3.44 percent to 3.89 percent. In other words, the presence of habit formation actually exaggerates the risk-free rate puzzle we observed in the U.S. economy.

The following proposition summarizes the implications of habit formation for the relative volatility of consumption and wealth to income:

[29] In a partial equilibrium model, Alessie and Lusardi (1997) and Smith (2002) show that the stronger the habit, the smaller the effect of income uncertainty on the precautionary saving term.

[30] Here the other parameter values are set to be the same as in the benchmark model.

Proposition 9 *In equilibrium, the evolution equations of c_t^* and w_t^* are:*

$$dc_t^* = \eta r^* \left(dw_t + \frac{1}{r^* + \rho} dy_t \right), \tag{61}$$

$$dw_t^* = (x_t + \Pi)\, dt + \alpha_s \sigma_e dB_{e,t}, \tag{62}$$

respectively, where $x_t = \rho (y_t - \overline{y}) / (r^* + \rho)$ is the demand for savings "for a rainy day", and $\Pi = \pi^2 / \left(2r^* \gamma \sigma_e^2\right)$ is the additional increase in the agent's certainty equivalent wealth due to the presence of the risky asset. Using (61), the relative inequality of consumption to income is:

$$\mu \equiv \frac{\mathrm{sd}\,(dc_t^*)}{\mathrm{sd}\,(dy_t)} = \frac{\eta r^*}{r^* + \rho} \sqrt{1 + (r^* + \rho)^2 \left(\frac{\sigma_e}{\sigma_y} \alpha_s \right)^2 + 2 (r^* + \rho) \frac{\rho_{ye} \sigma_e}{\sigma_y} \alpha_s}, \tag{63}$$

where $\mathrm{sd}\,(\cdot)$ denotes standard deviation. Furthermore, the relative inequality of wealth to income is the same as that obtained in the benchmark model.

Proof. Using (56), the derivation is straightforward. ■

As shown above, it is clear from (59) and (60) that habit formation has the potential to drive up the equilibrium risk-free rate, and thus cannot help resolve the risk-free rate puzzle. If we increase the degree of risk aversion to generate the observed r^* (2.1%), it is clear from (63) that habit formation will further drive down the relative consumption inequality given that $\eta < 1$, which makes the model fit the data worse in this dimension. For example, given $\gamma = 50$, $\theta = 0.2$, and $\kappa = 0.2$, using the equilibrium conditions for the risk-free rate and equity premium, we can pin down $r^* = 2.1\%$ and $\pi^* = 12.7\%$, respectively. However, these two values are obtained at the cost of setting γ to be as high as 50, which is much higher than the plausible values adopted in most macro-finance literature. Furthermore, if $r^* = 2.1\%$ is reached, the presence of HF (i.e., $\eta < 1$) makes μ be reduced to 0.16, which is even lower than that obtained in the model without HF and is also well below the empirical counterpart.

6 Conclusion

We have argued in this paper that the spirit of capitalism emphasized by sociologists and economists can be modeled as a direct preference for wealth. We construct a general framework to study the implications of the spirit of capitalism on aggregate savings, the equilibrium interest rate, consumption inequality, and equity premium—an unexplored area in the literature. The model includes many key factors in determining consumption, saving, and portfolio choices, but can still be solved analytically. We use the closed-form solutions to help understand how the spirit of capitalism interacts with other key factors in driving the equilibrium risk-free rate, consumption

inequality, and equity premium. We show a small and reasonable amount of the spirit of capitalism can significantly improve the model's predictions on all three key dimensions.

We also consider two alternative specifications: (1) a rare disaster specification and (2) a two-component income specification, and show that our main conclusions still hold under these two alternative specifications. In addition, we compare the spirit of capitalism with a closely-related hypothesis, habit formation in consumption, by examining their equilibrium implications for asset returns and consumption inequality, and find that the spirit of capitalism is more consistent than the habit formation in explaining the low risk-free rate and high consumption inequality in the data.

7 Appendix: Extended Literature Review

In this appendix, we provide additional evidence based on a wide range of economic theories by various economists to show that the spirit of capitalism can be modeled as a direct preference for wealth.

7.1 Adam Smith: Frugality and Savings

Adam Smith's (1776) book, *An Inquiry into the Nature and Causes of the Wealth of Nations*, always reminds us of the division of labor and the working of the invisible hand as the main causes of wealth generation. But Adam Smith also pays particular attention to the role of frugality and parsimony— a critical aspect of the economic-man's mentality— in wealth accumulation. In this sense, the capitalist-spirit approach to capital accumulation has been also taken by Adam Smith:

> "*The principle which prompts to save, is the desire of bettering our condition, a desire which though generally calm and dispassionate, comes with us from the womb, and never leaves us till we go into the grave...An augmentation of fortune is the means by which the greater part of men propose and wish to better their condition. It is the means of the most vulgar and the most obvious; and the most likely way of augmenting their fortune, is to save and accumulate some part of what they acquire, either regularly and annually, or upon some extraordinary occasions. Though the principle of expense, therefore prevails in almost all men upon some occasions, and in some men upon almost all occasions, yet in the greater part of men, taking the whole course of their life at an average, the principle of frugality seems not only to predominate, but to predominate very greatly.*"(pp.324–25)

It is interesting to note that Adam Smith seems to take saving and frugality as the means to improve one's conditions. But he immediately points out that the habit of saving or capital

accumulation is innate: it "comes with us from the womb, and never leaves us till we go into the grave." Thus the habit of saving develops as an end in itself and the economic objective of one's life often turns out to be saving for the sake of savings. Adam Smith is so occupied by the moral of savings that he declares "*every prodigal appears to be a public enemy, and every frugal man a public benefactor*" (p.324). Naturally, both consumption and capital accumulation should be included to the utility function of Adam Smith's economic man or the representative agent in a capitalist economy.

The role of frugality and parsimony in wealth accumulation is also stressed in Adam Smith's *The Theory of Moral Sentiments* (1759). "The methods of improving our fortune, which it principally recommends to us, are those which expose to no loss or hazard; real knowledge and skill in our trade or profession, assiduity and industry in the exercise of it, frugality, and even some degree of parsimony, in all our expenses" (Smith, 1759, p. 213). This practice of frugality is highly approved by "the spectator". "Hence arises that eminent esteem with which all men naturally regard a steady perserverance in the practice of frugality, industry, and application, though directly to no other purpose than the acquisition of fortune" (pp. 189-190).

7.2 N.W. Senior: Abstinence Theory of Capital

Nassau W. Senior (1836) directly identifies capital as abstinence and frugality. In his theory of production, there are three instruments of production: labor, natural agents such as land, and abstinence (p. 58). Why is abstinence so important? Senior answers as follows: "The most laborious population, inhabiting the most fertile territory, if they devoted all their labor to the production of immediate result, and consumed its produce as it arouse, would soon find their utmost exertions insufficient to produce even the mere necessaries of existence" (p. 58). Therefore, "without which the two others are inefficient, we shall give the name of Abstinence" (p. 58).

The role of abstinence in economic development is greatly emphasized by Senior. Even though abstinence and exertion have been existing in different degree in almost all societies, it is the most effective element of production, and it "is the slowest in its increase, and the least generally diffused" (p. 60). "Among nations, those that are the least civilized, and among the different classes of the same nation those which are worst educated, are always the most improvident, and consequently the least abstinent" (p. 60). After some cross-country examination of the profit rates, Senior found that the profits were lower in Holland and in England than in any other part of the globe. The reason for this, according to Senior, is that "abstinence with them (the English and the Dutch, added) is a cheap instrument of production, and they use it to the utmost" (p. 196).

But abstinence and frugality often lead to the habit of accumulation for the sake of accumulation: "capitals are generally formed from small beginnings by acts of accumulation, which become in time habitual. The capitalist soon regards the increase of his capital as the great business of his life; and

considers the greater part of his profit more as a means to that end than as a subject of enjoyment" (p. 192). Here it is clear that Senior defines the objective (utility) function of the capitalist on capital accumulation in addition to the enjoyment of consumption. In equation (1.1), we can take the parameter b as the degree of abstinence and $b \cdot v(w)$ as the enjoyment of accumulation for the sake of accumulation in the sense of Senior.

7.3 John Stuart Mill: the Spirit of Accumulation

J.S. Mill (1848, 1909) calls the accumulation desire as "the spirit of accumulation" in his analysis of capital accumulation and economic growth over time and across countries. In Mill's time, while this spirit was low in many underdeveloped parts of the world, the spirit of accumulation was so strong in the more prosperous countries of Europe, that the signs of rapidly increasing wealth met every eye (p. 173). There were various causes for this strong spirit of accumulation in England, and, among them, puritanism was one important factor: rapid capital accumulation in England was "greatly aided by that extreme incapacity of the people for personal enjoyment, which is a characteristic of countries over which puritanism has passed" (p. 174).

The comparison between Holland and England made by J.S. Mills also illustrates the importance of the frugal habit and the accumulation spirit in wealth creation: "the effective desire of accumulation has never reached so high a pitch in England as it did in Holland, where, there being no rich idle class to set example of a reckless expenditure, and the mercantile classes, who possessed the substantial power on which social influence always waits, being left to establish their own scale of living and standard of propriety, their habits remained frugal and unostentatious" (p.175).

Mill has also stressed the role of wealth and property in providing man not only consumption means but also political power and social prestige. The effect of the desire for social and political power on wealth accumulation was particularly significant in the modern history of Great Britain: "The earlier decline of feudalism having removed or much weakened invidious distinctions between the originally trading classes and those who have been accustomed to despise them; and a polity having growing up which made wealth the real source of political influence; its acquisition was invested with a factitious value, independent of its intrinsic utility. It becomes synonymous with power; and since power with the common herd of mankind *gives* power, wealth became the chief source of personal consideration, and the measure and stamp of success in life. To get out of one rank in society into the next above it, is the great aim of English middle-class life, and the acquisition of wealth the means" (p. 174). The capitalist institutions in Great Britain not only "give a most direct and potent stimulus to the desire of acquiring wealth," but "by the scope they have allowed to individual freedom of action, have encouraged personal activity and self-reliance, while by the liberty they confer of association and combination, they facilitate industrial enterprise on a large scale" (p. 174).

7.4 Karl Marx: The Nature of Capitalist Accumulation

Karl Marx regards the instinctive nature of accumulation by capitalists as an essential part of capitalism. In *Capital* (1977), Marx divides the surplus value or profits possessed by a capitalist into two parts: one part of the surplus value is consumed by the capitalist as revenue, and the other part employed as capital, i.e. it is accumulated (p. 738). But capitalist is "capital personified" (p. 739), his function is to accumulate capital for the sake of accumulation: "*Except as capital personified, the capitalist has no historical value, and no right to that existence." "But, in so far as he is capital personified, his objective is not the acquisition and enjoyment of use values*" (p. 739, italics added) or the utility of personal consumption, $u(c)$, "but the acquisition and augmentation of exchange-values" (p. 739) or the accumulation of capital $u(c)+b\cdot v(w)$. "As such, he shares with the miser an absolute drive towards self-enrichment" (p. 739). According to Marx, accumulation for the sake of accumulation has its origin in commodity production and monetary exchange. Its early form is money hoarding. The hoarding drive is boundless in its nature. Qualitatively and formally considered, money is independent of all limits, that is it is the universal representative of material wealth because it is directly convertible into any other commodity. But at the same time every actual sum of money is limited in amount, and therefore has only a limited efficacy as a means of purchase. This contradiction between the quantitative limitation and the qualitative lack of limitation of money keeps driving the hoarder back to his Sisyphean task: accumulation. He is in the same situation as a world conqueror, who discovers a new boundary with each country he annexes (Marx, pp. 230-231).

In order that gold be held as money, and made to form a hoard, it must be prevented from circulating, or from dissolving into the means of purchasing enjoyment. The hoarder therefore sacrifices the lusts to the fetish of gold. He takes the gospel of abstinence very seriously. On the other hand, he cannot withdraw any more from circulation, in the shape of money, than he has thrown into it, in the shape of commodities. The more he produce, the more he can sell. Work, thrift and greed are therefore his three cardinal virtue, and to sell much and buy little is sum of his political economy (Marx, p.213).

This accumulation drive is determined by two factors. The first factor is the pressure of external competition which forces capitalists to accumulate:

> "what appears in the miser as the mania of an individual is in the capitalist the effect of a social mechanism in which he is merely a cog...competition subordinates every individual capitalist to the immanent laws of capitalist production, as external and coercive laws. It compels him to keep extending his capital, so as to preserve it, and he can only extend it by means of progressive accumulation. "(p. 739)

The second and the much more important factor is the capitalist's internal impulse and desire

towards accumulation:

> "Accumulate, accumulate! That is Moses and the prophets! 'Industry furnishes the material which saving accumulates.' Therefore save, save, i.e. reconvert the greatest possible portion of surplus-value or surplus product into capital! Accumulation for the sake of accumulation, production for the sake of production: this was the formula in which classical economics expressed the historical mission of the bourgeoisie in the period of its domination. "(p. 733)

Or as John Ramsay MacCulloch (1830) in his the *Principles of Political Economy* states: "The inextinguishable passion for gain, the *auri sacra fames* (accursed hunger for gold), will always lead capitalists" (p.179).

Therefore, to Marx, there perhaps exists no better way to describe the capitalist preference than defining the utility function as $u(c) + b \cdot v(w)$, where b interpreted as the degree of internal accumulation drive.

Marx thinks that the spirit of capitalism, i.e. accumulation for the sake of accumulation and production for the sake of production, is the fundamental force leading to the creation of enormous wealth and the progress of our humankind. In the *Manifesto to the Communist Party* (1848), Marx and Engles eulogize this accumulation drive and enormous material progress achieved by the "capitalist mode of production":

> "The bourgeoisie, during its rule of scarce one hundred years, has created more massive and more colossal productive forces than have all preceding generations together. Subjection of Nature's forces to man, machinery, application of chemistry to industry and agriculture, steam-navigation, canalization of rivers, whole population conjured out of the ground — what earlier century had even a presentiment that such productive forces slumbered in the lap of social labor? "(Howe, ed., 1976, p. 35).

Similar expression can be found in *Capital*. As the capitalist accumulates for the sake of accumulation, "in this way he spurs on the development of society's production forces, and the creation of those material conditions of production which alone can form the real basis of a higher form of society, a society in which the full and free development of every individual forms the ruling principle. Only as a personification of capital is the capitalist respectable" (*Capital*, p. 739).

7.5 John Maynard Keynes: the Psychology of the Capitalist Society

John Maynard Keynes (1919) develops the same idea in his statement of the "psychology" of the capitalist society. He says that "Europe was so organized socially and economically as to secure

the maximum accumulation of capital. While there was some continuous improvement in the daily conditions of life of the mass of the population, society was so framed as to throw a great part of the increased income into the control of the *class least likely to consume it.* The new rich of the nineteenth century were not brought up to large expenditures, and *preferred the power which investment gave them to the pleasures of immediate consumption*...Herein lay, in fact, the main justification of the capitalist system. If the rich had spent their new wealth on their own enjoyments, the world would long ago have found such a regime intolerable. But like bees they saved and accumulated, not less to the advantage of the whole community because they themselves held narrow ends in prospect." (p. 16, italic added.)

Keynes continues to describe the savings behavior of the capitalist class: They "were allowed to call the best part of the cake theirs and were theoretically free to consume it, on the tacit underlying condition that they consumed very little of it in practice. *The duty of 'saving' became nine-tenths of virtue and the growth of the cake the object of true religion...* And so the cake increased; but to what end was not clearly contemplated. Individuals would be exhorted not so much to abstain as to defer, and to cultivate the pleasures of security and anticipation. Saving was for old age or for your children; but this was only in theory- the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you" (p. 12, italic added). Here Keynes directly identifies both consumption and capital accumulation (the increase of the cake) as the objective of capitalists. Very often in the past as well as at present time, saving is said to be for the future consumption. It seems that the ultimate purpose of saving is for the consumption. "*But this was only in theory*," this is not true in reality! The truth, in Keynes' words, is that *the duty of 'saving' became nine-tenths of virtue and the growth of the cake the object of true religion.*

For Keynes, this habit of saving for the sake of savings played a fundamental role in the enormous progress of living condition for the common people and the equilibrium of social life in the capitalist society. "The accumulation habit of Europe...were the necessary condition of the greatest of the external factors which maintained the European equipose" (p. 19), and "the immense accumulation of fixed capital which, to the great benefit of mankind,... could never have come about in a Society where wealth was divided equitably" (pp. 16-17). "In fact, it was precisely the *inequality* of the distribution of wealth which made possible those vast accumulations of fixed wealth and of capital improvements which distinguished that age from all others" (p. 16). Just because of the capitalists' habit of accumulation for the sake of accumulation, and not because of their accumulation for the sake of consumption, the general public benefited from capital accumulation and accepted the inequality.

7.6 Alfred Marshall and Gustav Cassel

Marshall (8th edition, 1920) explain savings by various factors. "The accumulation of wealth is governed by a great variety of causes: by custom, by habits of self-control and realizing the future, and above all by the power of family affection" (p. 196). "That men labour and save chiefly for their families, and not for themselves, is shown by the fact that they seldom spend, after they have retired from work, more than the income that comes in from their savings, preferring to leave their stored-up wealth intact for their families" (pp. 189-190). But at the same time, Marshall points out the importance of social distinction in the explanation why some people save for the sake of savings. In Marshall's view, to achieve social distinction is the main purpose of life, savings are just the mean to the end. He cites approvally Nassau Senior: "the desire for distinction...comes with us from the cradle and never leaves us till we go into grave" (p. 73). With this, it is not difficult to understand even the "irrational" savings behavior:

> "There are indeed some who find an intense pleasure in seeing their hoards of wealth grow up under their hands, with scarcely any thought for the happiness that may be got from its use by themselves or by others. They are prompted partly by the instinct of the chase, by the desire to outstrip their rivals; by the ambition to have shown ability in getting the wealth, and to acquire power and social position by its possession. And sometimes the force of habit, started when they were really in need of money, has given them, by sort of reflex action, an artificial and unreasoning pleasure in amassing wealth for its own sake. "(p. 189)

Gustav Cassel (1924) rejects the life-cycle and intergenerational-transfer motives as the whole explanation of savings. He lists various factors, in particular, the desire for higher social recognition in motivating savings:

> "There is a formation of capital for which it is hardly possible to assign any concern about the future as motive. It cannot be said of the leading capitalists who satisfy all their wants of any consequence, and have a capital the returns on which guarantees this satisfaction of wants for all time to them and their families, yet constantly set aside large sum to increase their wealth, that they save out of the concern about the future. In these cases there must be some other motive. It is the economic interest of the capitalist to increase his wealth, and this in time becomes an end in itself. The motives that are at work are numerous. The senseless cupidity that in times finds its sole pleasure in contemplating the growth of its wealth, and may very well be described as an abnormal sluggishness of spirit and a pathological impoverishment of the emotional life, is certainly not the sole explanation. The desire of splendor and of the higher

position in the society which the possession of great wealth assures, the stimulation of jealousy of other men, he healthy joy of the strong man in successful work as such, in ruling large masses, in influence especially — these are all factors that have to be taken into account. "(pp. 228-29)

7.7 Thorstein Veblen

Veblen (1899) is strongly against the economic theory of consumption-motivated accumulation (p. 34) and he takes the objective of accumulation mainly as a symbol of honor, social distinction and "respect from one's neighbor":

> "The end of acquisition and accumulation is conventionally held to be the consumption of the goods accumulated...This is at least felt to be the economically legitimate end of acquisition, which alone it is incumbent on the theory to take account of... But it is only when taken in a sense far removed from its naive meaning that consumption of goods can be said to afford the incentive from which accumulation invariably proceeds. The motive lies at the root of ownership. The possession of wealth confers honor, it is an invidious distinction. Nothing equally cogent can be said for the consumption of goods, nor for any other conceivable incentive to acquisition, and especially not for an incentive to the accumulation of wealth. "(p. 35)

In this sense, it is not difficult to understand why "no approach to a definite attainment is possible" or why there is no limit for the accumulation process, "since the struggle is substantially a race for reputability on the basis of an invidious comparison" (p. 39) and since "property set out with being booty held as trophies of the successful raid" (p. 36)

In addition to honor and respect, "the power conferred by wealth also afford a motive to accumulation" (p. 39). "Under the regime of individual ownership the most available means of visibly achieving a purpose is that afforded by the acquisition and accumulation" (p. 40). Therefore, "the propensity for achievement — the instinct of workmanship— tends more and more to shape itself into a straining to excel others in pecuniary achievement. Relative success, tested by an invidious pecuniary comparison with other men, becomes the conventional end of action" (p. 40).

Veblen (1915, 1939) applies his theory to explain economic development in Imperial Germany while making a comparison between the industrial revolution in England and the one in Germany. This is a very interesting point because Veblen presents a contrast between conspicuous consumption in the English society and an accumulation drive in Imperial Germany. Veblen's thinking can be easily represented by the utility function defined on both consumption and accumulation: $(1-b)\cdot u(c)+b\cdot v(w)$; here $b(0<b<1)$ is the weight assigned to consumption in a society. With economic progress in the English society, b became greater or $1-b$ became smaller, more and more

weight was thrown to accumulation instead of consumption. This is Veblen's idea of the "penalty of taking the lead"! Veblen (1899, 1915) predicts the same fate for the American society. But in Imperial Germany (and later Veblen extended his study to Japan), the imperial power based on accumulation restrained this shift towards conspicuous consumption. Since we cannot offer a better summary than Joseph Dorfman (1939), let us make a paraphrase from Dorfman:

"Germany avoids this 'penalty of taking the lead' in modern industry under private property by virtue of its more recent entrance among the great powers. Relatively free from the dead hand of funded capital, her technology could develop at a faster pace than in the hands of its birth. However, the surplus is devoted not to the conspicuous consumption or waste of resources human and inanimate of the leisure class in English society, but to the intangible glory of the reigning house. The objective of a dynastic Sate is insatiable dominion, and wealth accumulation is organized to enhance fighting capacity." (Dorfman, p. xviii) Thus in Imperial Germany, the will to power and dominance kept capital accumulation as an end it itself. This is the reason why Imperial Germany and Imperial Japan "made rapid stride in the machine technology as to outstrip the democratic English-speaking peoples from whom emanated the modern industrial process.

7.8 Joseph A. Schumpeter

In the Schmupeterian system of economic development and growth, entrepreneurs play the fundamental role. According to Schumpeter (1934, 1950), capitalism is by nature a form or method of economic change, which not only never is but never can be stationary (Schumpeter, 1950, p.82). The fundamental impulse that sets and keeps the capitalist engine in motion comes from the "process of destructive creation" or entrepreneurs' innovations. In *The Theory of Economic Development*, Schumpeter classifies those entrepreneurs' innovations as five cases: (1) The introduction of a new goods or of a new quality of a good. (2) The introduction of a new method of production. (3) The opening of a new market. (4) The conquest of a new source of supply of materials or half-manufactured goods. (5) The carrying out of the new organization of any industry (Schumpeter, 1934, p. 66).

Since entrepreneurs are central to the capitalist economic system, the representative-agent model of growth should have their role. But what is the objective function of entrepreneurs in the Schumpeterian system? Schumpeter rejects the traditional, hedonistic definition which has a utility function on consumption. Let us look at his analysis of the "psychology of the entrepreneurs" closely.

Schumpeter first states that an entrepreneur does have certain motive or objective, but "in *no* sense is his characteristic motivation of the hedonist kind" (Schumpeter, 1934, p.92), namely, the "wants as are capable of being satisfied by the consumption of goods." If we restrict the entrepreneur's wants to this consumption desire, "then it is no longer true that our type is acting

on a wish to satisfy his wants" (p.92). Schumpeter has the following observation and reasoning to reject the hedonistic definition of entrepreneurs' preference:

> "For unless we assume that individuals of our type are driven along by an insatiable craving for hedonist satisfaction, the operation of Gossen's law would in the case of business leaders soon put a stop to further effort. Experience teaches, however, that typical entrepreneurs retire from the arena only when and because their strength is spent and they feel no longer equal to their task. This does not seem to verify the picture of the economic man, balancing probable results against disutility of effort and reaching in due course a point of equilibrium beyond which he is not willing to go. Effort, in our case, does not seem to weigh at all in the sense of being felt as a reason to stop. "(p. 92)

Even though the consumption-hedonistic motive is not the main objective of an entrepreneur, "yet it points to another psychology of non-hedonist character." The entrepreneur is strongly motivated first of all by the "dream and the will to found a private kingdom, usually, though not necessarily, also a dynasty" (p. 93). "Then there is the will to conquer, the impulse to fight, to prove oneself superior to others, to succeed for the sake, not of the fruits of success, but of success itself, from this aspect, economic action becomes akin to sport... The financial result is a secondary consideration, or, at all events, mainly valued as an index of success and as a symptom of victory, the displaying of which very often is more important as a motive of large expenditure than the wish for the consumers' goods themselves" (p. 93). This drive for "industrial or commercial success is still the nearest approach to medieval lordship possible to modern man. Its fascination is specially strong for people who have no other chance of achieving social distinction. The sensation of power and independence loses nothing by the fact that both are largely illusion" (p. 93).

Therefore, according to Schumpeter, an entrepreneur's utility function should be defined on both consumption and the drive for success. But the latter can be represented by the financial success and wealth accumulation. So the utility function $u(c) + b \cdot v(w)$ can be used to approximate an entrepreneur's utility function and "the psychology of entrepreneurial activity" (p. 94). "Pecuniary gain is indeed a very accurate expression of success" (p. 94).

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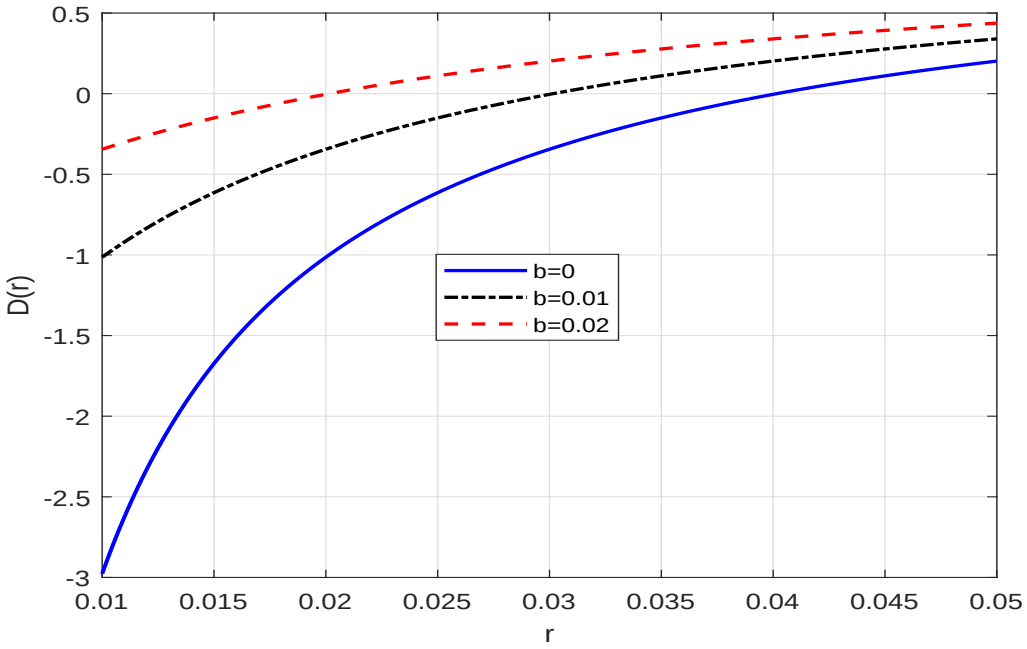


Figure 1: Effects of the SOC on Savings in the Risk-free Asset

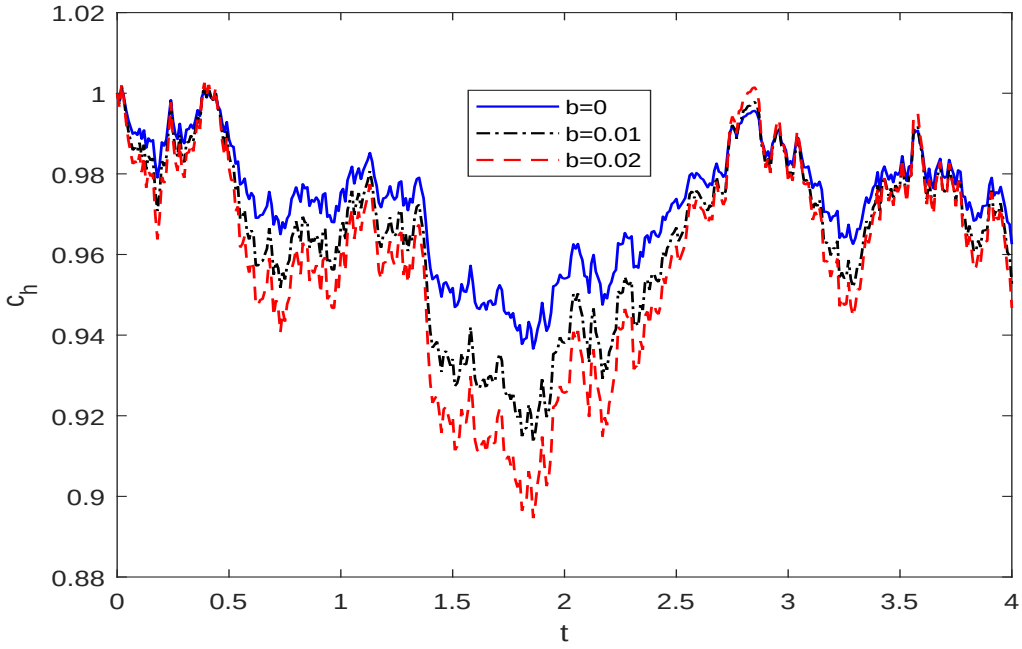


Figure 2: Effects of the SOC on the MPC out of Current Income

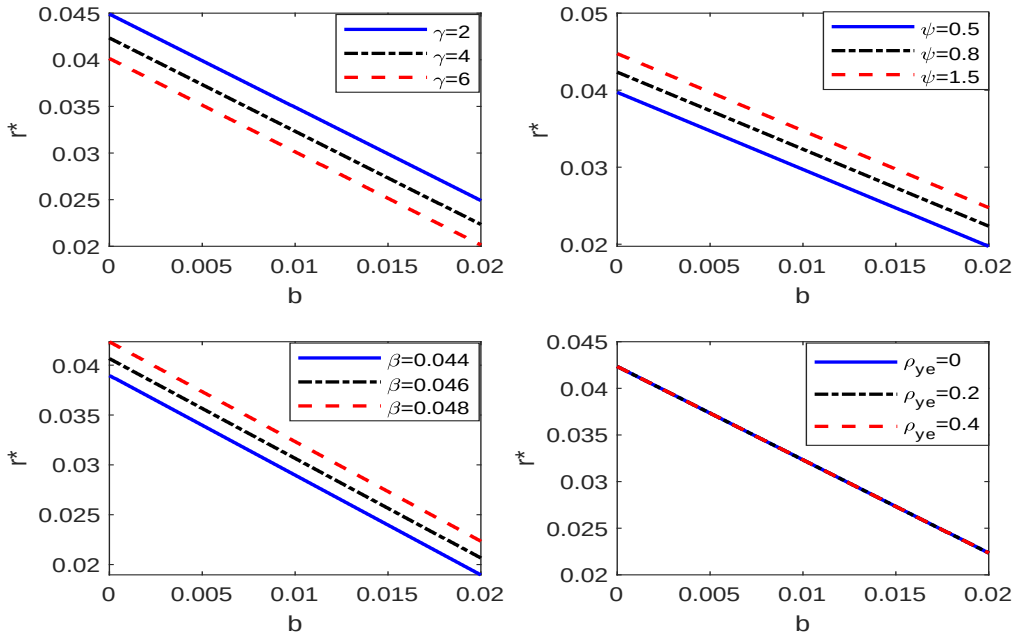


Figure 3: Effects of the SOC on the Equilibrium Risk-free Rate

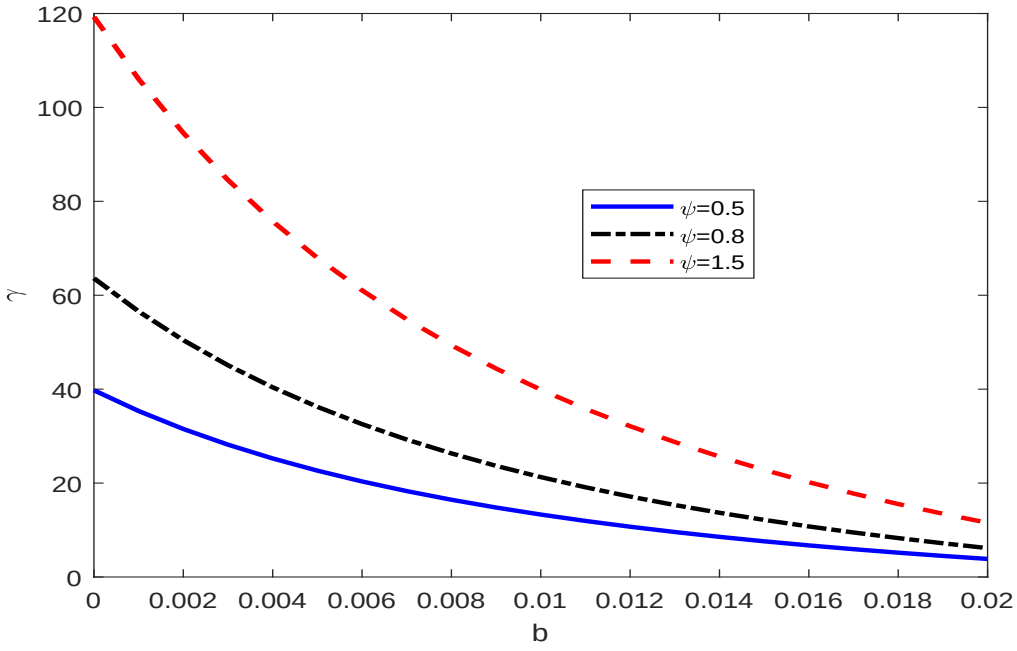


Figure 4: The Trade-off between b and γ

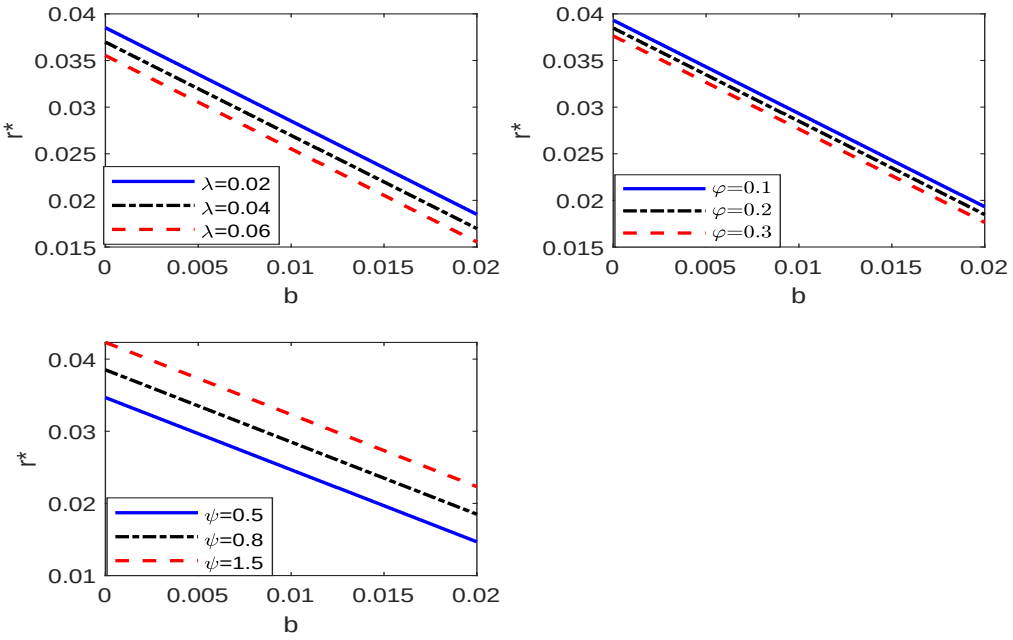


Figure 5: Effects of the SOC on the Equilibrium Risk-free Rate (with Rare Events)

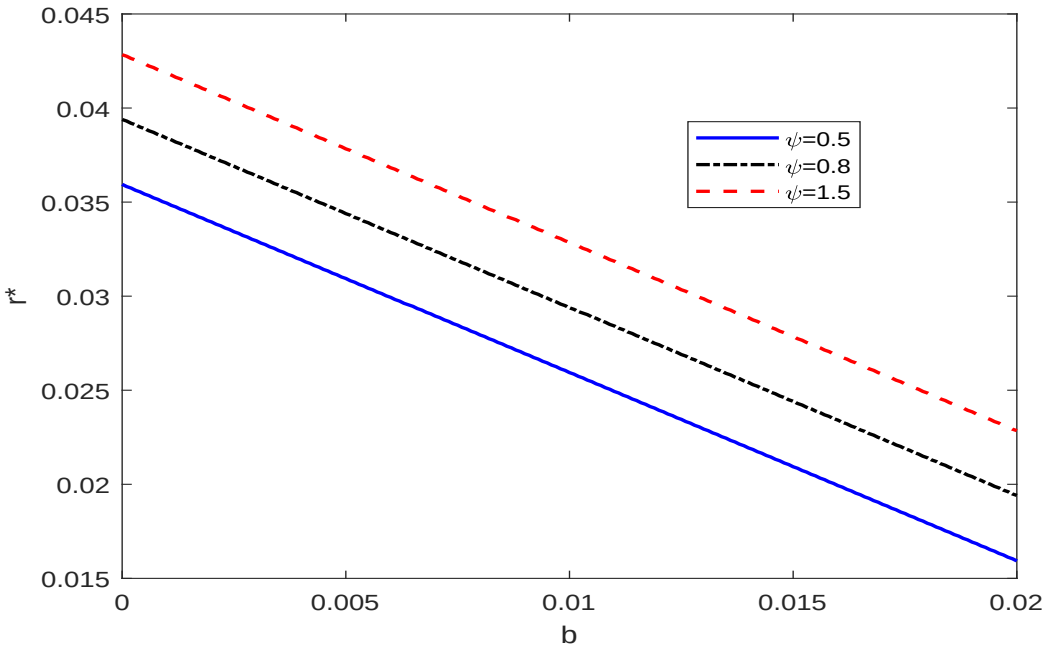


Figure 6: Effects of the SOC on the Equilibrium Risk-free Rate (2-Component Case)

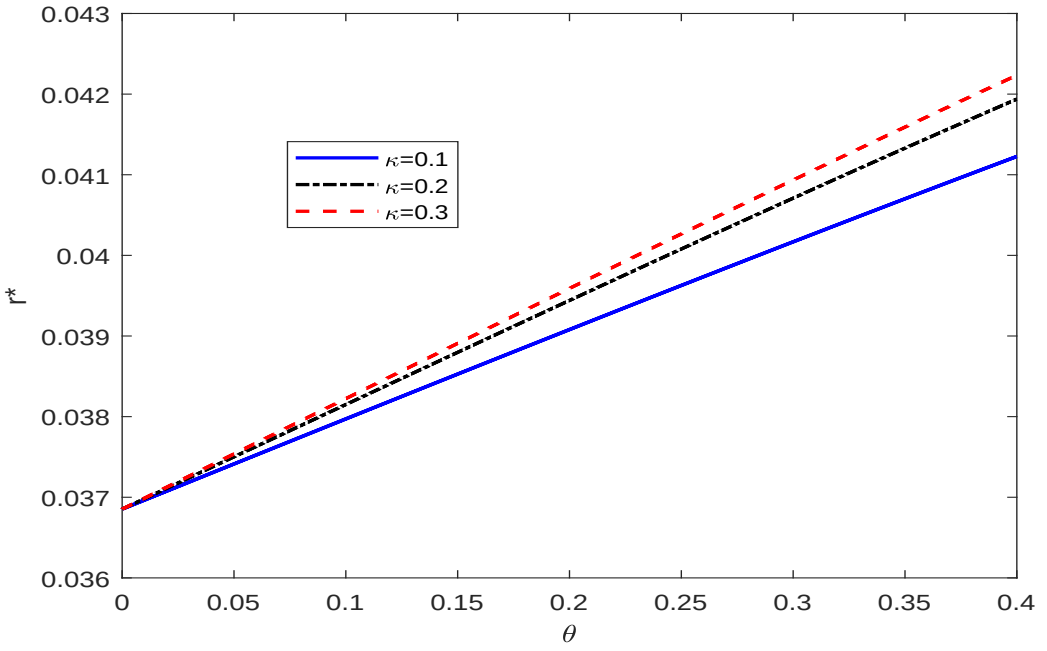


Figure 7: Effects of Habit Formation on the Equilibrium Risk-free Rate

Table 1: Estimation of Labor Income Parameters

	Parameter	Values
AR(1) Specification		
Discrete-time parameters		
Persistence	ϕ_1	0.903
Std. of shock	σ	0.154
Continuous-time parameters		
Persistence	ρ	0.102
Std. of income changes	σ_y	0.162
Two-Component Specification		
Discrete-time parameters		
Persistence	$\phi_{1,y}$	0.928
Persistence	$\phi_{2,y}$	0.361
Std. of shock	ω_1	0.133
Std. of shock	ω_2	0.194
Continuous-time parameters		
Persistence	ρ_1	0.075
Persistence	ρ_1	1.019
Std. of income changes	σ_1	0.138
Std. of income changes	σ_2	0.297

Table 2: Other Parameters

	Parameter	Values	Source
Benchmark Parameters			
Risk aversion	γ	6	Campbell (2003); Bansal and Yaron (2004)
Patience parameter	β	0.048	Campbell (2003); Bansal and Yaron (2004)
EIS parameter	ψ	0.8	Crump et al. (2015)
Correlation parameter	ρ_{ye}	0.35	Viceira (2001)
Std. of risky-asset return	σ_e	0.166	Data
Net supply of risky assets	α_s	5.03	Data
Rare-disaster Parameters			
Event probability	λ	0.037	Barro and Ursua (2011)
Event size	φ	0.21	Barro and Ursua (2011)
Habit Formation Model			
Degree of HF	ϑ	0.2	Constantinides (1990)
Smoothing parameter	κ	0.2	Constantinides (1990)

Table 3: Model's Quantitative Implications

	Real Risk-free Rate			Consumption Inequality		Wealth Inequality	Equity Premium	
Data	2.1%	2.1%	2.1%	0.40	0.40	5	5.8%	5.8%
Model	2.1%	2.1%	2.1%	0.22	0.40	5	4.3%	2.2%
b	0	0	0.02	0	0.02	0.02	0.02	0
γ	57	35	6	57	6	6	6	6
φ	0.8	0.5	0.8	0.8	0.8	0.8	0.8	0.8

Online Appendix for "The Spirit of Capitalism and Consumption Inequality" (Not for Publication)

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January 13, 2021

1 Online Appendix A: Data Description

This appendix describes the data we use in the paper. To measure the relative consumption inequality, we construct a panel of household expenditures and income based on the data in the Panel Study of Income Dynamics (PSID). As the PSID does not provide detailed expenditure information in the earlier sample period (i.e., prior to 2004), we follow the approach in Blundell et al. (2008) to estimate a demand function based on the data in the Consumer Expenditure Survey (CEX) and then impute a consumption measure in the PSID. For details, please see the detailed description in Luo et al. (2020).

The current analysis has also extended the previous analysis in two dimensions. First, we extend the previous data by including more recent years. Second, we re-estimate the consumption demand function–a function that links nondurable expenditures and food expenditures which is used to impute nondurable expenditures in the PSID–using a much more complete version of the CEX data. Due to the change in this demand function, there is an upward shift in the level of the imputed consumption in the PSID, but the scholastic properties are largely unchanged.

The relative inequality of wealth is constructed in a similar way though it is based on a shorter sample period due to the limited wealth information in the PSID. The net supply of risky assets

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is calculated as the ratio of risky assets to income. Risky assets are defined as total wealth minus safe assets which include checks, bonds, and vehicles. The standard deviation of risk-asset return is calculated based on the S&P500 and the NASDAQ indexes in 1980-2016.[1] Both measures are taken from the Haver database.

2 Online Appendix B: Solving the Consumption-Portfolio Choice Model with the SOC

2.1 Deriving the Optimal Consumption-Saving-Portfolio Rules

Given the optimization problem:

$$V\left(J_t\right)=\max_{\left(c_t,\alpha_t\right)}\left\{\left(1-e^{-\beta\Delta t}\right)V\left(c_t,w_t\right)+e^{-\beta\Delta t}V\left(\mathbb{C}\mathbb{E}_t\left[J_{t+\Delta t}\right]\right)\right\}, \tag{1}$$

subject to

$$dw_t=\left(rw_t+y_t-c_t\right)dt+\alpha_t\left(\pi dt+\sigma_e dB_{e,t}\right), \tag{2}$$

$$dy_t=\left(\mu-\rho y_t\right)dt+\sigma_y dB_t, \tag{3}$$

we first guess that $J_t=-\alpha_0-\alpha_1 w_t-\alpha_2 y_t$, and write $J_{t+\Delta t}$ as:[2]

$$\begin{aligned}J_{t+\Delta t}&\equiv J\left(w_{t+\Delta t},y_{t+\Delta t}\right)=-\alpha_0-\alpha_1 w_{t+\Delta t}-\alpha_2 y_{t+\Delta t}\\&\approx-\alpha_0-\left[\alpha_1 w_t+\alpha_1\left(rw_t+y_t-c_t+\alpha_t\pi\right)\Delta t+\alpha_1\sigma_e\alpha_t\Delta B_{e,t}\right]\\&\quad-\left[\alpha_2 y_t+\alpha_2\left(\mu-\rho y_t\right)\Delta t+\alpha_2\rho_{ye}\sigma_y\Delta B_{e,t}+\alpha_2\sqrt{1-\rho_{ye}^2}\sigma_y\Delta B_{i,t}\right].\end{aligned}$$

Using the above expression for $J_{t+\Delta t}$ and assume that the time interval Δt goes to infinitesimal dt, we can evaluate the certainty equivalent of $J_{t+\Delta t}$ as follows:

$$\begin{aligned}&\exp\left(-\gamma\mathbb{C}\mathbb{E}_t\right)=\mathbb{E}_t\left[\exp\left(-\gamma J_{t+\Delta t}\right)\right]\\&=\exp\left(-\gamma\mathbb{E}_t\left[-\alpha_1 w_{t+dt}-\alpha_2 y_{t+dt}\right]+\frac{1}{2}\gamma^2\,\mathrm{var}_t\left[-\alpha_1 w_{t+dt}-\alpha_2 y_{t+dt}\right]+\gamma\alpha_0\right)\\&=\exp\left(\gamma\alpha_0-\gamma\left(\partial J\right)^T\cdot\left(s_t+\mathbb{E}_t\left[ds_t\right]\right)+\frac{\gamma^2}{2}\left[\left(\partial J\right)^T\cdot\Sigma\cdot\partial J\right]dt\right)\\&=\exp\left(-\gamma J_t\right)\exp\left(-\gamma\left(\partial J\right)^T\cdot\mathbb{E}_t\left[ds_t\right]+\frac{\gamma^2}{2}\left[\left(\partial J\right)^T\cdot\Sigma\cdot\partial J\right]dt\right),\end{aligned}$$

[1] It is the average of the standard deviations using the S&P500 and the NASDAQ data respectively.

[2] Here $\Delta B_t=\sqrt{\Delta t}\epsilon$ and ϵ is a standard normal distributed variable.

where $s_t = \begin{bmatrix} w_t & y_t \end{bmatrix}^T$, $ds_t = \begin{bmatrix} dw_t & dy_t \end{bmatrix}^T$, $\partial J = \begin{bmatrix} J_w & J_y \end{bmatrix}^T$, and $\Sigma = \begin{bmatrix} \alpha_t^2\sigma_e^2 & \rho_{ye}\sigma_y\alpha_t\sigma_e \\ \rho_{ye}\sigma_y\alpha_t\sigma_e & \sigma_y^2 \end{bmatrix}$.
Solving this equation yields:

$$\mathbb{CE}_t\left[J_{t+dt}\right] = J_t + \left((\partial J)^T \cdot \mathbb{E}_t\left[ds_t\right] - \frac{\gamma}{2}\left[(\partial J)^T \cdot \Sigma \cdot \partial J\right] dt\right).$$

Substituting the expression of $\mathbb{CE}_t$ into (1), we can obtain the following HJB equation:

$$\beta V\left(J_t\right) = \max_{\{c_t,\alpha_t\}} \left\{\beta V\left(c_t, w_t\right) + \mathcal{D}V\left(s_t\right)\right\}, \tag{4}$$

where

$$\begin{aligned}\mathcal{D}V\left(J_t\right) &= V'\left(J_t\right)\left((\partial J)^T \cdot \mathbb{E}_t\left[ds_t\right] + \upsilon_t^T \cdot \Sigma \cdot \partial J - \frac{\gamma}{2}\left[(\partial J)^T \cdot \Sigma \cdot \partial J\right]\right) \\ &= V'\left(J_t\right)\begin{bmatrix} -\alpha_1\left(rw_t + y_t - c_t + \alpha_t\pi\right) - \alpha_2\left(\mu - \rho y_t\right) \\ -\frac{\gamma}{2}\left(\alpha_1^2\sigma_e^2\alpha_t^2 + 2\rho_{ey}\alpha_1\alpha_2\sigma_e\alpha_t\sigma_y + \alpha_2^2\sigma_y^2\right) \end{bmatrix}\end{aligned}$$

The first-order condition (FOC) for consumption (c_t) is then:

$$\beta V_c\left(c_t, w_t\right) + \alpha_1 V'\left(J_t\right) = 0.$$

Using the facts that $V\left(c_t, w_t\right) = (-\psi)\exp\left(-\left(c_t + bw_t\right)/\psi\right)$ and $J_t = -\alpha_0 - \alpha_1 w_t - \alpha_2 y_t$, we have:

$$c_t = -\psi\ln\left(-\frac{\alpha_1}{\beta}\right) - \alpha_0 - \left(\alpha_1 + b\right)w_t - \alpha_2 y_t. \tag{5}$$

The FOC for portfolio choice (α_t) is:

$$\alpha_t = -\frac{\pi}{\alpha_1\gamma\sigma_e^2} - \frac{\alpha_2\rho_{ye}\sigma_y}{\alpha_1\sigma_e}. \tag{6}$$

Matching the coefficients on the w_t, y_t, and constant terms yields:

$$\begin{aligned}&\alpha_1 = -\left(r + b\right), \alpha_2 = -\frac{r+b}{\rho + r + b}, \\ &\alpha_0 = \left(1 - \frac{\beta}{r+b}\right)\psi - \psi\ln\left(\frac{r+b}{\beta}\right) - \frac{\mu}{r+b+\rho} \\ &\quad + \frac{\rho_{ye}\sigma_y\pi}{\left(r+b+\rho\right)\sigma_e} - \frac{\pi^2}{2\left(r+b\right)\gamma\sigma_e^2} + \left(1 - \rho_{ey}^2\right)\frac{\left(r+b\right)\gamma}{2}\left(\frac{\sigma_y}{r+b+\rho}\right)^2\end{aligned}$$

Substituting the expressions of α_1 and α_2 into (27), we can obtain the following optimal portfolio rule:

$$\alpha^* = \frac{\pi}{\left(r+b\right)\gamma\sigma_e^2} - \frac{\rho_{ye}\sigma_y}{\left(r+b+\rho\right)\sigma_e}. \tag{7}$$

Similarly, substituting the expressions of α_0, α_1, and α_2 into (5), we can obtain the following consumption function:

$$c_t^* = rw_t + \frac{r+b}{r+b+\rho} y_t + \frac{\mu}{r+b+\rho} - \frac{\rho_{ye}\sigma_y \pi}{(r+b+\rho)\sigma_e} - \left(1 - \frac{\beta}{r+b}\right)\psi - \left(1-\rho_{ey}^2\right)\frac{(r+b)\gamma}{2}\left(\frac{\sigma_y}{r+b+\rho}\right)^2 + \frac{\pi^2}{2(r+b)\gamma\sigma_e^2},$$

which is just the consumption function in the main text after we using the definitions of $\Psi(r) \equiv \left(1 - \frac{\beta}{r+b}\right)\psi$, $\Gamma(r) \equiv \left(1-\rho_{ey}^2\right)\frac{(r+b)\gamma}{2}\left(\frac{\sigma_y}{r+b+\rho}\right)^2$, and $\Pi(r) \equiv \frac{\pi^2}{2(r+b)\gamma\sigma_e^2}$.

Using the above optimal consumption-portfolio rules, the individual saving function can be written as:

$$\begin{aligned} d_t^* &= rw_t + y_t - c_t^* + \pi\alpha^* \\ &= \frac{\rho}{r+b+\rho}\left(y_t - \frac{\mu}{\rho}\right) + \left(1 - \frac{\beta}{r+b}\right)\psi + \frac{1}{2}\left(1-\rho_{ey}^2\right)(r+b)\gamma\left(\frac{\sigma_y}{r+b+\rho}\right)^2 + \frac{\pi^2}{2(r+b)\gamma\sigma_e^2} \\ &= x_t - \Psi + \Gamma + \Pi, \end{aligned}$$

where $x_t = \rho(y_t - \overline{y})/(r+b+\rho)$ and $\overline{y} = \mu/\rho$.

2.2 Proof of the Existence and Uniqueness of the Equilibrium

Proof. There exists at least one equilibrium interest rate $r^* \in (0, \beta)$ in the our model; if $\beta < \rho$, the equilibrium interest rate is unique on $(0, \beta)$. If $r > \beta$, both $\Gamma(r)$ and $\Psi(r)$ in the expression for savings in the risk-free asset, $D(r) = \Gamma(r) - \Psi(r)$, are positive, which contradicts the equilibrium condition $D(r) = 0$. Since $\Gamma(r) - \Psi(r) < 0$ (> 0) when $r = 0$ $(r = \beta)$, the continuity of the expression for savings implies that there exists at least one interest rate $r^* \in (0, \beta)$ such that $D(r^*) = 0$. To establish the conditions under which this equilibrium is unique, we take the derivative:

$$\frac{\partial D(r)}{\partial r} = \gamma\frac{\sigma_y^2}{(r+b+\rho)^2}\left(\frac{1}{2} - \frac{r+b}{r+b+\rho}\right) + \frac{\delta\psi}{(r+b)^2}$$

and note a sufficient condition for this derivative to be positive for any $r > 0$ is

$$\frac{1}{2} - \frac{r+b}{r+b+\rho} > 0 \Leftrightarrow r < \rho - b.$$

Therefore, if $\rho > \beta$, there is only one equilibrium in $(0, \beta)$. ■

3 Online Appendix C: Solving the Model with Rare Events

We have so far assumed that the income process takes a diffusive form. In this section, we consider how the SOC affects the equilibrium in the presence of rare macroeconomic disasters. Barro (2009)

and Barro and Ursua (2012) showed that the presence of rare macroeconomic disasters has the potential to explain an array of macro-asset pricing puzzles. To model rare disasters (e.g., financial crisis), here we assume that the labor income process facing the agents in our model economy not only includes a diffusion component but also includes a Poisson jump process.[3] For simplicity, suppose that the event of an income reduction is a Poisson process with parameter λ and labor income is reduced by a constant amount φ at random points in time.[4] The dynamics of income is then described by

$$dy_t = (\mu - \rho y_t)\, dt + \sigma_y dB_t - \varphi dq, \tag{8}$$

where B_t reflects "normal" economic fluctuations and q is a pure jump process. That is, $dq = 1$ when the jump happens and $dq = 0$ otherwise. It is worth noting that we can use this Poisson process to model financial crises. When a financial crisis happens, the mean income of the economy is dropped by φ percent. The reason that we consider jumps with constant size is to keep our heterogeneous-agent incomplete markets model tractable and deliver new insights about the general equilibrium implications of SOC in the presence of rare events.[5]

Under our REU specification, following the same procedure as before, the HJB can be written as:

$$\delta f(J_t) = \max_{c_t} \left\{ \delta f(c_t) + f'(J_t) \left(\partial J \cdot E[ds_t] - \frac{1}{2}\gamma \left[\partial J \cdot \Sigma \cdot (\partial J)^T \right] \right) + \lambda \left(f(J(w_t, y_t - \varphi)) - f(J(w_t, y_t)) \right) \right\}, \tag{9}$$

where $f(J_t)$ and γ have the same definition as before, $s_t = \begin{bmatrix} w_t & y_t \end{bmatrix}^T$, $ds_t = \begin{bmatrix} dw_t & dy_t \end{bmatrix}^T$, $\partial J = \begin{bmatrix} J_w & J_y \end{bmatrix}$, and $\Sigma = \sigma_y^2$. We can now solve (9) and obtain the consumption rule. The following proposition summarizes the solution:

Proposition 1 *The consumption function is*

$$c_t = r \left[w_t + \frac{1}{r+\rho} y_t + \frac{1}{r(r+\rho)} \mu \right] + \Psi - \Gamma - \Lambda. \tag{10}$$

During the normal time in which $dq = 0$, the saving function is

$$d_t^n = x_t - \Psi + \Gamma + \Lambda, \tag{11}$$

whereas during the crisis in which $dq = 1$, the saving function becomes

$$d_t^r = x_t - \varphi - \Psi + \Gamma + \Lambda, \tag{12}$$

[3] Barro (2009) and Barro and Ursua (2012) considered the similar output jump process within a discrete-time Lucas-type asset pricing setting.

[4] Note that here we can also interpret φ as a fraction of the mean of income because we have assumed that the mean level of income ($\overline{y}$) is 1.

[5] For models with stochastic jump size, see Barro and Ursua (2012). It is worth noting that given that rare disasters are absent in short samples, it is difficult to quantify the probability and size distribution of the disasters.

where $x_t = \rho\left(y_t - \overline{y}\right)/\left(r+\rho\right)$, $\Psi = \psi\left(\delta/r - 1\right)$, $\Gamma = r\gamma\sigma_s^2/2$, *and*

$$\Lambda = \frac{\psi\lambda}{r}\left(\exp\left(\frac{r}{\psi}\frac{\varphi}{r+\rho}\right) - 1\right). \tag{13}$$

Proof. Since the Poisson process has constant jump size, we only apply robustness into the diffusion process. Under our REU specification, the HJB can be written as:

$$\delta f\left(J_t\right) = \max_{c_t}\left\{\delta f\left(c_t\right) + f'\left(J_t\right)\left(\partial J \cdot E\left[ds_t\right] - \frac{1}{2}\widetilde{\gamma}\left[\partial J \cdot \Sigma \cdot \left(\partial J\right)^T\right]\right) + \lambda\left(f\left(J\left(w_t, y_t - \varphi\right)\right) - f\left(J\left(w_t, y_t\right)\right)\right)\right\},$$

where $f\left(J_t\right) = \left(-\psi\right)\exp\left(-J_t/\psi\right)$, $f'\left(J_t\right) = \exp\left(-J_t/\psi\right)$, $\widetilde{\gamma} = \gamma + \vartheta/\psi$, $s_t = \begin{bmatrix} w_t & y_t \end{bmatrix}^T$, $ds_t = \begin{bmatrix} dw_t & dy_t \end{bmatrix}^T$, $\partial J = \begin{bmatrix} J_w & J_y \end{bmatrix}$, and $\Sigma = \sigma_y^2$. We then guess that $J_t = \alpha_0 + \alpha_1 w_t + \alpha_2 y_t$. The HJB equation reduces to

$$\delta f\left(J_t\right) = \max_{c_t}\left\{\begin{array}{c} \delta f\left(c_t\right) + f'\left(J_t\right)\left(\begin{array}{c} \begin{bmatrix} J_w & J_y \end{bmatrix} \cdot \begin{bmatrix} dw_t & dy_t \end{bmatrix}^T \\ -\frac{1}{2}\gamma\left[\begin{bmatrix} J_w & J_y \end{bmatrix} \cdot \sigma_y^2 \cdot \begin{bmatrix} J_w & J_y \end{bmatrix}^T\right] \end{array}\right) \\ +\lambda\left(f\left(J\left(w_t, y_t - \varphi\right)\right) - f\left(J\left(w_t, y_t\right)\right)\right) \end{array}\right\},$$

The FOC for consumption is:

$$\delta\exp\left(-\frac{c_t}{\psi}\right) - \exp\left(-\frac{J_t}{\psi}\right)\alpha_1 = 0,$$

which implies that

$$c_t = -\psi\ln\left(\frac{\alpha_1}{\delta}\right) + \alpha_0 + \alpha_1 w_t + \alpha_2 y_t. \tag{14}$$

Substituting this expression for c back to the Bellman equation and matching the coefficients, we have:

$$\begin{aligned} \delta\left(-\psi\right) = -\psi\alpha_1 + &\left(\begin{array}{c} \alpha_1\left(rw_t + y_t - \left[-\psi\ln\left(\frac{\alpha_1}{\delta}\right) + \left(\alpha_0 + \alpha_1 w_t + \alpha_2 y_t\right)\right]\right) \\ +\alpha_2\left(\left(\mu - \rho y_t\right) - \varphi\lambda\right) - \frac{1}{2}\gamma J_y^2\sigma_y^2 \end{array}\right) \\ &+ \lambda\left(\left(-\psi\right)\exp\left(\alpha_2\varphi/\psi\right) - \left(-\psi\right)\right), \end{aligned}$$

where $dw_t = \left(rw_t + y_t - c_t\right)dt$. Matching the coefficients in the w_t, y_t, and constant terms yields:

$$\begin{aligned} \alpha_1 &= r, \\ \alpha_2 &= \frac{r}{r+\rho}, \\ \alpha_0 &= \psi\left(\frac{\delta - r}{r}\right) + \psi\ln\left(\frac{r}{\delta}\right) + \frac{1}{r+\rho}\left(\mu - \varphi\lambda\right) - \frac{1}{2}r\gamma\frac{1}{\left(r+\rho\right)^2}\sigma_y^2 \\ &\quad - \frac{\psi\lambda}{r}\left(\exp\left(\alpha_2\varphi/\psi\right) - 1\right) \end{aligned}$$

Substituting these expressions into (14) yields

$$c_t = r\left[w_t + \frac{1}{r+\rho}y_t + \frac{1}{r\left(r+\rho\right)}\left(\mu - \varphi\lambda\right)\right] + \psi\left(\frac{\delta - r}{r}\right) - \frac{1}{2}r\gamma\frac{\sigma_y^2}{\left(r+\rho\right)^2} - \frac{\psi\lambda}{r}\left(\exp\left(\frac{r}{\psi}\frac{\varphi}{r+\rho}\right) - 1\right),$$

where the last term is the capitalized value of the expected future declines to labor income, capitalized at a higher rate than the risk-free rate reflecting the agents' aversions to risk, uncertainty, and intertemporal substitution. In this case, the agents determine the certainty equivalent value by computing the present value of future labor income.

Using the consumption function, the expression for individual saving can be written as:

$$\begin{aligned} d_t &= rw_t + y_t - c_t \\ &= rw_t + y_t - \left\{ \begin{array}{c} r\left[w_t + \frac{1}{r+\rho}y_t + \frac{1}{r(r+\rho)}\mu\right] + \psi\left(\frac{\delta - r}{r}\right) \\ -\frac{1}{2}r\gamma\frac{\sigma_y^2}{(r+\rho)^2} - \frac{\psi\lambda}{r}\left(\exp\left(\frac{r}{\psi}\frac{\varphi}{r+\rho}\right) - 1\right) \end{array} \right\} \\ &= x_t + \Gamma - \Psi + \Lambda, \end{aligned}$$

respectively, where $x_t = \rho\left(y_t - \overline{y}\right)/\left(r+\rho\right)$ is the demand for savings "for a rainy day",

$$\Lambda = \frac{\psi\lambda}{r}\left(\exp\left(\frac{r}{\psi}\frac{\varphi}{r+\rho}\right) - 1\right).$$

The model with rare events has the potential to further drive down the risk-free rate. In this model,

$$\text{var}\left(dy_t\right) = \sigma_y^2 dt + \lambda\varphi^2 dt.$$

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4 Online Appendix D: Solving the Model with an Alternative Income Specification

Following Wang (2004), we assume that labor income has two distinct components:

$$y_t = y_{1,t} + y_{2,t},$$

where

$$dy_{1,t} = \left(\mu_1 - \rho_1 y_{1,t}\right)dt + \sigma_1 dB_{1,t}, \tag{15}$$

$$dy_{2,t} = \left(\mu_2 - \rho_2 y_{2,t}\right)dt + \sigma_2 dB_{2,t}, \tag{16}$$

where we assume that the two innovations, $dB_{1,t}$ and $dB_{2,t}$, are independent. Without loss of generality, we assume that $\rho_1 < \rho_2$. That is, the first income component is more persistent than the second component.

Given the alternative income process, the HJB can be written as:

$$\sup_{c_t}\left[-\frac{1}{\gamma}\exp\left(-\gamma c_t\right)-\beta J\left(w_t,y_{1,t},y_{2,t}\right)+\mathcal{D}J\left(w_t,y_{1,t},y_{2,t}\right)\right], \tag{17}$$

where the third term is the adjustment to the expected continuation value:

$$\begin{aligned}\mathcal{D}J\left(w_t,y_{1,t},y_{2,t}\right)=&J_w\left(rw_t+y_{1,t}+y_{2,t}-c_t\right)+J_{y_1}\left(\mu_1-\rho_1 y_{1,t}\right)+\frac{1}{2}J_{y_1y_1}\sigma_1^2\\&+J_{y_2}\left(\mu_2-\rho_2 y_{2,t}\right)+\frac{1}{2}J_{y_2y_2}\sigma_2^2.\end{aligned} \tag{18}$$

Performing the indicated optimization yields the first-order condition for c_t:

$$c_t=-\frac{1}{\gamma}\ln\left(J_w\right). \tag{19}$$

Substituting (19) back into (17) to arrive at the PDE:

$$0=\left[\begin{array}{c}-\frac{J_w}{\gamma}-\delta J\left(w_t,y_{1,t},y_{2,t}\right)+J_w\left(rw_t+y_{1,t}+y_{2,t}+\frac{1}{\gamma}\ln\left(J_w\right)\right)+J_{y_1}\left(\mu_1-\rho_1 y_{1,t}\right)+\frac{1}{2}J_{y_1y_1}\sigma_1^2\\+J_{y_2}\left(\mu_2-\rho_2 y_{2,t}\right)+\frac{1}{2}J_{y_2y_2}\sigma_2^2.\end{array}\right].$$

In the next step, conjecture that the value function is of the form:

$$J\left(w_t,y_{1,t},y_{2,t}\right)=-\frac{1}{\alpha_1}\exp\left(-\alpha_0-\alpha_1 w_t-\alpha_2 y_{1,t}-\alpha_3 y_{2,t}\right),$$

where α_0, α_1, α_2, and α_3 are constants to be determined. Substituting these results into (17) and matching the coefficients in the w_t, $y_{1,t}$, $y_{2,t}$, and constant terms yields:

$$\begin{aligned}&\alpha_1=r\gamma,\alpha_2=\frac{r\gamma}{r+\rho_1},\alpha_3=\frac{r\gamma}{r+\rho_2},\\&\alpha_0=\frac{\delta-r}{r}+\gamma\left[\frac{\mu_1}{r+\rho_1}+\frac{\mu_2}{r+\rho_2}-\frac{1}{2}r\gamma\left(\frac{\sigma_1}{r+\rho_1}+\frac{\sigma_2}{r+\rho_2}\right)^2\right].\end{aligned}$$

Substituting these back into the first-order condition (19) yields the consumption function in the main text.

5 Online Appendix E: Solving the Internal Habit Formation Model

Given the optimization problem:

$$V\left(J_t\right)=\max_{\left(c_t,\alpha_t\right)}\left\{\left(1-e^{-\beta\Delta t}\right)V\left(c_t-\theta z_t\right)+e^{-\beta\Delta t}V\left(\mathbb{C}\mathbb{E}_t\left[J_{t+\Delta t}\right]\right)\right\}, \tag{20}$$

subject to:

$$dw_t = (rw_t + y_t - c_t)\,dt + \alpha_t\,(\pi dt + \sigma_e dB_{e,t})\,, \tag{21}$$

$$dy_t = (\mu - \rho y_t)\,dt + \sigma_y dB_t, \tag{22}$$

$$dh_t = \kappa\,(c_t - h_t)\,dt, \tag{23}$$

where z_t is the stock of habit and $V\,(c_t - \gamma z_t) = (-\psi)\exp\left(-\left(c_t - \theta z_t\right)/\psi\right)$, we first guess that $J_t = -\alpha_0 - \alpha_1 w_t - \alpha_2 y_t - \alpha_3 z_t$, and then write $J_{t+\Delta t}$ as:

$$\begin{aligned} J_{t+\Delta t} &\equiv J\left(w_{t+\Delta t}, y_{t+\Delta t}, z_{t+\Delta t}\right) = -\alpha_0 - \alpha_1 w_{t+\Delta t} - \alpha_2 y_{t+\Delta t} - \alpha_3 z_{t+\Delta t} \\ &\approx -\alpha_0 - \left[\alpha_1 w_t + \alpha_1\left(rw_t + y_t - c_t + \alpha_t \pi\right)\Delta t + \alpha_1 \sigma_e \alpha_t \Delta B_{e,t}\right] \\ &\quad - \left[\alpha_2 y_t + \alpha_2\left(\mu - \rho y_t\right)\Delta t + \alpha_2 \rho_{ye}\sigma_y \Delta B_{e,t} + \alpha_2\sqrt{1-\rho_{ye}^2}\sigma_y \Delta B_{i,t}\right] - \left[\alpha_3 z_t + \kappa\left(c_t - z_t\right)\Delta t\right] \end{aligned}$$

Using the above expression for $J_{t+\Delta t}$ and assume that the time interval Δt goes to infinitesimal dt, we can evaluate the certainty equivalent of $J_{t+\Delta t}$ as follows:

$$\begin{aligned} &\exp\left(-\gamma \mathbb{CE}_t\right) = \mathbb{E}_t\left[\exp\left(-\gamma J_{t+\Delta t}\right)\right] \\ &= \exp\left(-\gamma \mathbb{E}_t\left[-\alpha_1 w_{t+dt} - \alpha_2 y_{t+dt} - \alpha_3 z_{t+dt}\right] + \frac{1}{2}\gamma^2 \operatorname{var}_t\left[-\alpha_1 w_{t+dt} - \alpha_2 y_{t+dt} - \alpha_3 z_{t+dt}\right] + \gamma\alpha_0\right) \\ &= \exp\left(\gamma\alpha_0 - \gamma\left(\partial J\right)^T \cdot \left(s_t + \mathbb{E}_t\left[ds_t\right]\right) + \frac{\gamma^2}{2}\left[\left(\partial J\right)^T \cdot \Sigma \cdot \partial J\right] dt\right) \\ &= \exp\left(-\gamma J_t\right)\exp\left(-\gamma\left(\partial J\right)^T \cdot \mathbb{E}_t\left[ds_t\right] + \frac{\gamma^2}{2}\left[\left(\partial J\right)^T \cdot \Sigma \cdot \partial J\right] dt\right), \end{aligned}$$

where $s_t = \begin{bmatrix} w_t & y_t & z_t \end{bmatrix}^T$, $ds_t = \begin{bmatrix} dw_t & dy_t & dz_t \end{bmatrix}^T$, $\partial J = \begin{bmatrix} J_w & J_y & J_h \end{bmatrix}^T$, and $\Sigma = \begin{bmatrix} \alpha_t^2\sigma_e^2 & \rho_{ye}\sigma_y\alpha_t\sigma_e & 0 \\ \rho_{ye}\sigma_y\alpha_t\sigma_e & \sigma_y^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Simplifying the above expression yields:

$$\mathbb{CE}_t\left[J_{t+dt}\right] = J_t + \left(\left(\partial J\right)^T \cdot \mathbb{E}_t\left[ds_t\right] - \frac{\gamma}{2}\left[\left(\partial J\right)^T \cdot \Sigma \cdot \partial J\right] dt\right). \tag{24}$$

Substituting the expression of $\mathbb{CE}_t$ into (20), we can obtain the following HJB equation:

$$\beta V\left(J_t\right) = \max_{\{c_t, \alpha_t\}}\left\{\beta V\left(c_t - \theta z_t\right) + \mathcal{D}V\left(s_t\right)\right\}, \tag{25}$$

where

$$\begin{aligned} \mathcal{D}V\left(J_t\right) &= V'\left(J_t\right)\left(\left(\partial J\right)^T \cdot \mathbb{E}_t\left[ds_t\right] - \frac{\gamma}{2}\left[\left(\partial J\right)^T \cdot \Sigma \cdot \partial J\right]\right) \\ &= V'\left(J_t\right)\begin{bmatrix} -\alpha_1\left(rw_t + y_t - c_t + \alpha_t\pi\right) - \alpha_2\left(\mu - \rho y_t\right) - \alpha_3\kappa\left(c_t - z_t\right) \\ -\frac{\gamma}{2}\left(\alpha_1^2\sigma_e^2\alpha_t^2 + 2\rho_{ey}\alpha_1\alpha_2\sigma_e\alpha_t\sigma_y + \alpha_2^2\sigma_y^2\right) \end{bmatrix} \end{aligned}$$

The first-order condition (FOC) for consumption (c_t) is then:

$$\beta V_c\left(c_t-\theta h_t\right)+\left(\alpha_1-\alpha_3\kappa\right)V'\left(J_t\right)=0,\ \text{or}\ \frac{\psi\left(\alpha_1-\alpha_3\kappa\right)}{\beta}V'\left(J_t\right)=V\left(c_t-\gamma z_t\right),$$

which implies that

$$c_t=-\psi\ln\left(-\frac{\alpha_1}{\beta}\right)-\alpha_0-\alpha_1 w_t-\alpha_2 y_t-\left(\alpha_3-\theta\right)z_t. \tag{26}$$

The FOC for portfolio choice (α_t) is:

$$\alpha_t=-\frac{\pi}{\alpha_1\gamma\sigma_e^2}-\frac{\alpha_2\rho_{ye}\sigma_y}{\alpha_1\sigma_e}. \tag{27}$$

Substituting the FOCs, (26) and (27), back into (20), we have:

$$-\beta\psi=\psi\left(\alpha_1-\alpha_3\kappa\right)+\left[\begin{array}{c}-\alpha_1\left(rw_t+y_t+\alpha_t\pi\right)+\left(\alpha_1-\alpha_3\kappa\right)\left(-\psi\ln\left(-\frac{\alpha_1}{\beta}\right)-\alpha_0-\alpha_1 w_t-\alpha_2 y_t-\left(\alpha_3-\gamma\right)z_t\right)\\ -\alpha_2\left(\mu-\rho y_t\right)-\alpha_3\kappa\left(-z_t\right)-\frac{\gamma}{2}\left(\alpha_1^2\sigma_e^2\alpha_t^2+2\rho_{ey}\alpha_1\alpha_2\sigma_e\alpha_t\sigma_y+\alpha_2^2\sigma_y^2\right)\end{array}\right]$$

Matching the coefficients on the w_t terms yields:

$$\alpha_1=\alpha_3\kappa-r \tag{28}$$

Matching the coefficients on the y_t terms yields:

$$\alpha_2=\frac{\alpha_1}{r+\rho}=-\frac{r}{r+\rho}\left(1-\frac{\theta\kappa}{r+\kappa}\right) \tag{29}$$

Matching the coefficients on the z_t terms yields:

$$\alpha_3=\frac{r\theta}{r+\kappa} \tag{30}$$

Solving (28), (29), and (30) yields:

$$\alpha_1=-r\left(1-\frac{\theta\kappa}{r+\kappa}\right),\alpha_2=-\frac{r}{r+\rho}\left(1-\frac{\theta\kappa}{r+\kappa}\right),\alpha_3=\frac{r\theta}{r+\kappa}. \tag{31}$$

Matching the constant coefficients yields:

$$\begin{aligned}\alpha_0=&\psi\left(1-\frac{\beta}{r}\right)-\psi\ln\left(\frac{r}{\beta}\left(1-\frac{\theta\kappa}{r+\kappa}\right)\right)-\frac{\mu}{r+\rho}\left(1-\frac{\theta\kappa}{r+\kappa}\right)-\frac{\pi^2}{2r\gamma\sigma_e^2}\\&+\frac{1}{r+\rho}\left(1-\frac{\theta\kappa}{r+\kappa}\right)\frac{\pi\rho_{ye}\sigma_y}{\sigma_e}+\frac{1}{2}\left(1-\rho_{ey}^2\right)\left(1-\frac{\theta\kappa}{r+\kappa}\right)^2\frac{r\gamma\sigma_y^2}{\left(r+\rho\right)^2}.\end{aligned} \tag{32}$$

Substituting these coefficients back into (26) and (27), we obtain the following optimal consumption-portfolio rules under habit formation:

$$\begin{aligned} c_t^* = & r\left(1-\frac{\theta\kappa}{r+\kappa}\right)w_t + \frac{r}{r+\rho}\left(1-\frac{\theta\kappa}{r+\kappa}\right)y_t + \frac{\theta\kappa}{r+\kappa}z_t \\ & + \frac{\mu}{r+\rho}\left(1-\frac{\theta\kappa}{r+\kappa}\right) + \frac{\pi^2}{2r\gamma\sigma_e^2} - \frac{1}{r+\rho}\left(1-\frac{\theta\kappa}{r+\kappa}\right)\frac{\pi\rho_{ye}\sigma_y}{\sigma_e} \\ & - \psi\left(1-\frac{\beta}{r}\right) - \frac{1}{2}\left(1-\frac{\theta\kappa}{r+\kappa}\right)^2 r\gamma\frac{\left(1-\rho_{ey}^2\right)\sigma_y^2}{\left(r+\rho\right)^2}, \end{aligned} \tag{33}$$

and

$$\alpha_t = \left(1-\frac{\theta\kappa}{r+\kappa}\right)^{-1}\frac{\pi}{r\gamma\sigma_e^2} - \frac{\rho_{ye}\sigma_y}{\left(r+\rho\right)\sigma_e}. \tag{34}$$

Define the 'excess consumption' $e_t = c_t^* - z_t$, the mean of the stationary distribution is:

$$\overline{e} = \frac{1}{\left(1-\gamma\right)\kappa}\left(\psi\left(r-\beta\right) + \frac{1}{2}\left(1-\frac{\theta\kappa}{r+\kappa}\right)^2 r^2\frac{\left(1-\rho_{ey}^2\right)\gamma\sigma_y^2}{\left(r+\rho\right)^2}\right).$$

In the stationary equilibrium in which $e_t = \overline{e}$, it follows that

$$\begin{aligned} c_t^* = & rw_t + \frac{r}{r+\rho}y_t - \frac{\theta\kappa}{r+\kappa}\left(1-\frac{\theta\kappa}{r+\kappa}\right)^{-1}\overline{e} \\ & + \frac{\mu}{r+\rho} + \frac{\pi^2}{2r\gamma\sigma_e^2}\left(1-\frac{\theta\kappa}{r+\kappa}\right)^{-1} - \frac{1}{r+\rho}\frac{\pi\rho_{ye}\sigma_y}{\sigma_e} \\ & \left(1-\frac{\theta\kappa}{r+\kappa}\right)^{-1}\left[-\psi\left(1-\frac{\beta}{r}\right) - \frac{1}{2}\left(1-\frac{\theta\kappa}{r+\kappa}\right)^2 r\gamma\frac{\left(1-\rho_{ey}^2\right)\sigma_y^2}{\left(r+\rho\right)^2}\right]. \end{aligned}$$

Using the above optimal consumption-portfolio rules, the individual saving function can be written as:

$$\begin{aligned} d_t^* &= rw_t + y_t - c_t^* + \pi\alpha^* \\ &= \frac{\rho}{r+\rho}\left(y_t - \frac{\mu}{\rho}\right) + \frac{\gamma\kappa}{r+\kappa}\left(1-\frac{\gamma\kappa}{r+\kappa}\right)^{-1}\overline{e} \\ &+ \left(1-\frac{\gamma\kappa}{r+\kappa}\right)^{-1}\left[\psi\left(1-\frac{\beta}{r}\right) + \frac{1}{2}\left(1-\frac{\gamma\kappa}{r+\kappa}\right)^2 r\gamma\frac{\left(1-\rho_{ey}^2\right)\sigma_y^2}{\left(r+\rho\right)^2}\right] + \frac{\pi^2}{2r\gamma\sigma_e^2}\left(1-\frac{\gamma\kappa}{r+\kappa}\right)^{-1} \\ &= \frac{\rho}{r+\rho}\left(y_t - \frac{\mu}{\rho}\right) + \left(1-\frac{\theta\kappa}{r+\kappa}\right)^{-1}\left(1+\frac{r\theta}{\left(1-\gamma\right)\left(r+\kappa\right)}\right)\left[\begin{array}{c} \psi\left(1-\frac{\beta}{r}\right) \\ +\frac{1}{2}\left(1-\frac{\gamma\kappa}{r+\kappa}\right)^2 r\gamma\frac{\left(1-\rho_{ey}^2\right)\sigma_y^2}{\left(r+\rho\right)^2} \end{array}\right] \\ &+ \frac{\pi^2}{2r\gamma\sigma_e^2}\left(1-\frac{\theta\kappa}{r+\kappa}\right)^{-1} \\ &= x_t + \left(1-\frac{\theta\kappa}{r+\kappa}\right)^{-1}\left(1+\frac{r\theta}{\left(1-\gamma\right)\left(r+\kappa\right)}\right)\left(-\Psi\left(r\right) + \Gamma\left(r\right)\right) + \Pi, \end{aligned}$$

where $x_t = \rho\left(y_t - \overline{y}\right)/\left(r+\rho\right)$,

$$\begin{aligned}
\Psi\left(r\right) &\equiv \psi\left(1-\frac{\beta}{r}\right), \\
\Gamma\left(r\right) &\equiv \left(1-\frac{\theta\kappa}{r+\kappa}\right)^2 \frac{r\gamma}{2}\frac{\left(1-\rho_{ey}^2\right)\sigma_y^2}{\left(r+\rho\right)^2}, \\
\Pi\left(r\right) &\equiv \left(1-\frac{\theta\kappa}{r+\kappa}\right)^{-1} \frac{\pi^2}{2r\gamma\sigma_e^2}.
\end{aligned}$$

From the above expressions of $\Psi\left(r\right)$ and $\Gamma\left(r\right)$, it is clear that the two HF parameters, θ and κ, reduce the precautionary saving demand, $\Gamma\left(r\right)$, while they have no effect on the $\Psi\left(r\right)$. Using the equilibrium condition on aggregate saving in the risk-free asset:

$$D\left(r^*\right) = \Psi\left(r^*\right) - \Gamma\left(r^*\right) = 0,$$

it is straightforward to show that

$$\frac{dr^*}{d\theta} = -\frac{\partial D/\partial\theta}{\partial D/\partial r^*} > 0 \text{ and } \frac{dr^*}{d\kappa} = -\frac{\partial D/\partial\kappa}{\partial D/\partial r^*} > 0.$$

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