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Currency misalignments, international trade in intermediate inputs, and inflation targeting

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ABSTRACT

In the literature on optimal monetary policy in open economies, the presence of local- currency pricing provides a rationale for targeting CPI inflation rather than PPI inflation. In this paper, we reexamine this conclusion by incorporating international trade in intermediate inputs into Engel (2011). We find that the cooperative monetary policymaker should target the final-goods output gaps, the PPI inflation rates at both stages of production, the currency misalignments at both stages of production, and the vertical relative price gaps. Welfare analysis shows that the monetary policymaker should target the weighted average intermediate-goods PPI (WPPI) inflation rather than CPI inflation for most combinations of price stickiness at both stages of production.

1. Introduction

The Covid-19 pandemic highlights the importance of global supply chains (Bonadio et al., 2020). As a matter of fact, economic globalization over the past several decades has been fueling the integration of the world through international trade in both final goods and intermediate inputs. It is noteworthy that the trade volume of intermediate inputs has been growing faster than that of final goods. A large body of literature on international trade explores the importance of vertical production and trade (Feenstra, 1998; Hummels et al., 2001; Yi, 2003, 2010; Koopman et al., 2014; Antras, 2016).[1] Countries integrated through vertical production and trade exhibit business cycle comovement increasingly (Huang and Liu, 2007; Giovanni and Levchenko, 2010; Johnson, 2014). Thus, the design of monetary policy in open economies needs to be adjusted to take vertical production and trade into account (Shi and Xu, 2007; Gong et al., 2016, 2020; Wei and Xie, 2020).

A widely accepted principle in economics is that violations of the law of one price are inefficient, but they are pervasive in reality. Many researchers attribute violations of the law of one price to local-currency pricing (LCP) or pricing to market (Engel, 1999; Bergin and Feenstra, 2001; Atkeson and Burstein, 2008). The distortion related to LCP, termed currency misalignments, becomes a separate source of inefficiency, to which the monetary policymakers should pay attention when

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[1] Vertical production and trade involve international trade in intermediate inputs, thus we use them interchangeably.

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conducting the monetary policy (Engel, 2011; Fujiwara and Wang, 2017; Chen et al., 2021). However, the existing literature on the monetary policy in a two-country New Keynesian model with LCP has not recognized the rising importance of vertical production and trade in shaping the design of monetary policy yet.

To fill the gap in the literature, we introduce international trade in intermediate inputs into Engel (2011) to examine how the monetary policy prescription changes. In our model, there is a two-stage production and trade structure. At the stage of intermediate-goods production, the firms in each country employ domestic labor to produce the differentiated intermediate inputs for the home and foreign final-goods producers, while at the stage of final-goods production, the firms in each country input domestic and imported intermediate inputs to produce the final consumption goods for the households in both countries.

Engel (2011) derives the welfare loss function of a cooperative monetary policymaker and finds that the optimal monetary policy should target CPI inflation, the output gap, and the currency misalignment. By comparison, in our model, the joint welfare loss function reveals that the cooperative monetary policymaker should target the final-goods output gaps, the PPI inflation rates at both stages of production, the currency misalignments at both stages of production, and the vertical relative price gaps.

Since we cannot give an analytical solution to the dynamic system describing the optimal monetary policy, we compute the welfare loss of the optimal monetary policy quantitatively instead and use it to evaluate two different Taylor-type monetary policy rules: the weighted average final-goods PPI inflation-based Taylor rule (CPIT) and the weighted average intermediate-goods PPI inflation-based Taylor rule (WPPIT). [2] We find that the monetary policymaker should target the weighted average intermediate-goods PPI (WPPI) inflation rather than CPI inflation for most combinations of price stickiness at both stages of production, which is in marked contrast to Engel (2011)'s conclusion that the monetary policymaker should target CPI inflation rather than PPI inflation.

In the standard two-country New Keynesian model with producer-currency pricing (PCP), the fluctuation in the nominal exchange rate can achieve nearly efficient allocations by adjusting the terms of trade in both countries. In addition, PPI inflation leads to the same relative price distortions as a result of the fact that the consumers in both countries face the same relative prices of goods produced in one country. It implies that eliminating PPI inflation can undo the relative price distortions in both countries simultaneously (Clarida et al., 2002). By contrast, the stabilization of CPI results in the inefficient allocations due to the restriction on the expenditure-switching effect of the nominal exchange rate. However, when exporters set prices in consumers' currency (LCP), the benefit of the expenditure-switching effect produced by the nominal exchange rate adjustment decreases significantly, while the currency misalignments distortion gradually plays a major role. Thus, it is desirable for the monetary policymaker to target CPI inflation to eliminate the staggered price distortion (Engel, 2011).

In our model, in addition to the distortions present in Engel (2011), the monetary policymaker also needs to tackle newly introduced distortions related to vertical production and trade, namely, the price dispersions and currency misalignment at the stage of intermediate-goods production, and the distortions from the sluggish adjustment of the relative prices of intermediate goods in terms of final goods. By targeting CPI inflation, the monetary policymaker cannot undo the newly introduced distortions. However, by targeting WPPI inflation, the monetary policymaker can kill two birds with one stone in the sense that both the newly introduced distortions and those present in Engel (2011) can be mitigated simultaneously. The reason is that the final-goods inflation rates are composed of the intermediate-goods inflation rates, not vice versa. In addition, in our model, CPI inflation is identical to the weighted average final-goods PPI inflation due to the fact that the expenditure shares of the households on domestic and imported final goods are assigned as the weights. Therefore, the monetary policymaker should target WPPI inflation rather than CPI inflation for most combinations of price stickiness at both stages of production.

Our paper is related to the literature on optimal monetary policy in open economies under LCP. Based on Obstfeld and Rogoff (2000,2002), Devereux and Engel (2003) examine the optimal monetary policy when exporters set prices in consumers' currency one period in advance, and conclude that the optimal monetary policy should fix the nominal exchange rate. Duarte and Obstfeld (2008) introduce nontraded goods into Devereux and Engel (2003) and find that the optimal monetary policy requires the flexible exchange rate even if the nominal exchange rate does not play the expenditure-switching role. Gong et al. (2017) find that Devereux and Engel (2003)'s conclusion depends on the assumption that the households in both countries have the same expenditure shares. When the expenditure shares between home and foreign households are different, the optimal monetary policy requires the flexible exchange rate.

In Devereux and Engel (2003), the fact that the prices being set one period in advance implies that inflation does not lead to the distortions related to price dispersion. Engel (2011) introduces local-currency pricing into Clarida et al. (2002) and finds that the optimal monetary policy should target consumer price inflation, the output gap, and the currency misalignment. In addition, Engel (2011) also provides a rationale that the cooperative monetary policymaker should target CPI inflation rather than PPI inflation, which stands in contrast to the conclusion arrived at in the model with PCP that the optimal monetary policy should target PPI inflation rather than CPI inflation (Clarida et al., 2002). Fujiwara and Wang (2017) generalize Engel (2011) to the noncooperative case to examine whether there exist welfare gains from monetary policy cooperation in the model with LCP. They find that the welfare gains are relatively small, though not negligible. In a model similar to

[2] We assign the expenditure shares of the households on domestic and imported final goods as weights of the weighted average final-goods PPI inflation-based Taylor rule so that it is identical to the CPI-based Taylor rule (CPIT).

Engel (2011) and Fujiwara and Wang (2017), Chen et al. (2021) explore whether the optimal monetary and fiscal policy combination can eliminate currency misalignment and achieve efficient allocations. They conclude that the optimal monetary and fiscal policy combination can achieve efficient allocations under the condition that the fiscal policy should be chosen cooperatively while the monetary policy non-cooperatively.

Our paper is also closely related to the literature on optimal monetary policy in open economies with international trade in intermediate inputs. In a two-country New Keynesian monetary model with vertical production and trade and a one-year period of price stickiness, Shi and Xu (2007) examine how the monetary policymakers make the optimal money supply rule in response to stage-specific productivity shocks. By introducing international trade in intermediate inputs into Clarida et al. (2002), Gong et al. (2016) find which monetary policy rules the cooperative monetary policymaker should follow depend on the degree of price stickiness at the stage of intermediate-goods production. Specifically, if the degree of price stickiness at the stage of intermediate-goods production is high, the monetary policymaker should follow the intermediate-goods PPI-based Taylor rule, whereas the CPI-based Taylor rule should be followed when the degree of price stickiness at the stage of intermediate-goods production is intermediate or low. In a small open New Keynesian monetary model with multiple stages of production, Wei and Xie (2020) find that the central bank should follow the monetary policy rule targeting separate PPI inflation at the different production stages, the output gap, and the real exchange rate.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 derives model's equilibrium. Section 4 discusses monetary policy design. Section 5 concludes.

2. The model

The model's structure is nearly identical to Gong et al. (2016). The world economy consists of two countries of equal size, home H and foreign F, each inhabited with a continuum of households of unit mass $[0,1]$. The representative household in each country derives utility from the consumption of both home and foreign final goods and incurs disutility from the provision of labor services to domestic intermediate-goods producers. In addition, the representative household in each country can trade in a complete set of state-contingent claims denominated in the home currency. Like Engel (2011), the representative household in each country exhibits a home bias in consumption.

Following Gong et al. (2016), the world economy is integrated by vertical production and trade. Specifically, the production in each country entails a sequence of stages. Without loss of generality, we focus on two stages, from intermediate goods to final goods. At the intermediate-goods production stage, a continuum of intermediate-goods producers in each country employ domestic labor to produce differentiated products, which are then used by final-goods producers worldwide as inputs. At the final-goods production stage, a continuum of final-goods producers in each country input domestic and imported intermediate goods to produce differentiated products, which are then consumed by households worldwide as consumption goods. Thus, besides consumption openness, we also introduce production openness, which provides an additional channel of openness to influence the monetary policy design.

Following Engel (2011), we allow for local-currency pricing (LCP) in the sense that the exporters at both stages of production set prices in the importers' currency rather than their own currency. Thus, the law of one price does not hold, and the exchange rate does not play the role in automatically adjusting the relative prices facing households and firms because imported goods prices do not respond to the fluctuation in the exchange rate. When prices are sticky, the fluctuation in the exchange rate can bring about the inefficient change in the prices of the same good sold in different countries. In what follows, foreign variables are marked with an asterisk, subscript f denotes final good, i intermediate good.

2.1. Households

The representative household $h \in [0,1]$in the home country maximizes

$$\mathbf{E}_0\sum_{t=0}^{\infty}\beta^t U(C_t(h),N_t(h)) = \mathbf{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{C_t(h)^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\varphi}}{1+\varphi}\right\}, \tag{1}$$

in which $\beta \in (0,1))$is the discount factor, σ denotes the coefficient of relative risk aversion, φ is the inverse of the Frisch elasticity of labor supply, $C_t(h)$is the consumption aggregate, and $N_t(h)$is labor services that the representative household h provides to domestic intermediate-goods producers. The consumption $C_t(h)$is a Cobb-Douglas composite of home and foreign final goods, $C_{Ht}(h)$and $C_{Ft}(h)$. Following Engel (2011), the home representative household's expenditure shares on domestic and imported final goods are $\frac{v}{2}$ and $1-\frac{v}{2}$, respectively, with $0 \leqslant v \leq 2$. Thus, when $\upsilon > 1$, home representative household has a home bias in consumption. The foreign representative household $h^* \in [0,1]$also has the same consumption aggregate, with expenditure share on domestic final goods being $\frac{v}{2}$. By contrast, in Clarida et al. (2002), the households in both countries exhibit no home bias in consumption.

In addition, $C_{Ht}(h) = \left(\int_0^1 C_{Ht}(h,j_f)^{\frac{\xi_f-1}{\xi_f}} dj_f\right)^{\frac{\xi_f}{\xi_f-1}}$ is a CES aggregate over a continuum of final goods produced in the home country. Similarly, $C_{Ft}(h) = \left(\int_0^1 C_{Ft}\left(h,j_f^*\right)^{\frac{\xi_f-1}{\xi_f}} dj_f^*\right)^{\frac{\xi_f}{\xi_f-1}}$ is an index of consumption of imported final goods. The expressions of

$C_{Ht}(h)$and $C_{Ft}(h)$imply that the elasticity of substitution among differentiated final goods in each country is ξ_f, which is assumed to be strictly greater than unity.

The home representative household h's budget constraint is given by

$$P_{ft}C_t(h) + \sum_{s_{t+1}\in\Omega_{t+1}} Z(s_{t+1}|s_t)D(h,s_{t+1}) = W_tN_t(h) + D(h,s_t) - T_t + \Gamma_t, \tag{2}$$

in which $P_{ft} = k^{-1}P_{Hft}^{\upsilon/2}P_{Fft}^{1-\upsilon/2}, k = (\upsilon/2)^{\upsilon/2}(1-\upsilon/2)^{1-\upsilon/2}$is home CPI,[3] $D(h,s_t)$ is the nominal payoffs on state-contingent claims for state $s_t, Z(s_{t+1}|s_t)$is the state-s_t price of a claim that yields one unit of home currency in state s_{t+1}, W_t is the nominal wage, T_t denotes lump-sum taxes, and Γ_t denotes aggregate profits accruing from ownership of home firms.

2.2. Firms

Different from Clarida et al. (2002) and Engel (2011), the production in each country involves a sequence of two stages: from intermediate goods to final goods. At the final-goods production stage, there are a continuum of final-goods producers in each country which input domestic and imported intermediate goods to produce differentiated final goods. A home representative final-goods producer $j_f \in [0,1]$produces consumption good j_f with a Cobb–Douglas production function given by

$$Y_{ft}(j_f) = \frac{A_{ft}Y_{Hit}^{\phi}(j_f)Y_{Fit}^{1-\phi}(j_f)}{\phi^{\phi}(1-\phi)^{1-\phi}}, \tag{3}$$

in which $Y_{Hit}(j_f) = \left[\int_0^1 Y_{Hit}(j_f,j_i)^{\frac{\xi_i-1}{\xi_i}}dj_i\right]^{\frac{\xi_i}{\xi_i-1}}$and $Y_{Fit}(j_f) = \left[\int_0^1 Y_{Fit}(j_f,j_i^*)^{\frac{\xi_i-1}{\xi_i}}dj_i^*\right]^{\frac{\xi_i}{\xi_i-1}}$are home and foreign composite intermediate goods respectively, $\xi_i > 1$ is the elasticity of substitution between differentiated intermediate goods, A_{ft} is a productivity shock which is common to all home final-goods producers, and ϕ measures the expenditure share of the firm j_f on home composite intermediate good.

A foreign representative final-goods producer $j_f^* \in [0,1]$produces consumption good j_f^* with a similar Cobb–Douglas production function, but the expenditure share on imported composite intermediate good is $1-\phi$ instead. Thus, as in Gong et al. (2016), when $\phi > 1/2$, final-goods producers in both countries exhibit home bias in production.

At the intermediate-goods production stage, there are a continuum of intermediate-goods producers in each country that input domestic labor to produce differentiated products. A home representative intermediate-goods producer $j_i \in [0,1]$inputs domestic labor to produce consumption good j_i with a linear production function given by

$$Y_{it}(j_i) = A_{it}N_t(j_i), \tag{4}$$

in which A_{it} is a productivity shock which is common to all home intermediate-goods producers.

Let $a_{ft} \equiv \log(A_{ft})$. We assume that a_{ft} follows the $AR(1)$process $a_{ft} = \rho_f a_{ft-1} + \varepsilon_{ft}$, in which $\varepsilon_{ft} \overset{i.i.d}{\sim} N\left(0,\sigma_f^2\right)$. Similarly, $a_{it} \equiv \log(A_{it})$follows the $AR(1)$process $a_{it} = \rho_i a_{it-1} + \varepsilon_{it}$ in which $\varepsilon_{it} \overset{i.i.d}{\sim} N(0,\sigma_i^2)$. In addition, ε_{it} is independent of ε_{ft}.

In Gong et al. (2016), the producers at both stages set export prices in their own currency (PCP) and the law of one price holds. In this paper, we follow Engel (2011) and assume LCP. Thus, firm j_f sets a price $P_{Hft}(j_f)$in its own currency when selling good j_f domestically, at the same time, it sets another price $P_{Hft}^*(j_f)$in the importers' currency when exporting j_f to the foreign country. By contrast, in Gong et al. (2016), firm j_f sets a single price $P_{Hft}(j_f)$in its own currency and the price facing foreign consumers changes with the fluctuation in the nominal exchange rate E_t and is given by $P_{Hft}(j_f)/E_t$. [4]

We assume that firms at both stages of production set prices in a staggered fashion, as in Calvo (1983). In each period, at the final-goods production stage, a home representative firm j_f sets prices in both countries with probability $1-\theta_f$,whereas it keeps the prices fixed with probability θ_f. Similarly, at the intermediate-goods production stage, a home representative firm j_i sets prices in both countries with probability $1-\theta_i$,whereas it keeps the prices unchanged with probability θ_i.

Thus, in the home country, when final-goods firm j_f is able to choose price in period t, it chooses $P_{Hft}^o(j_f)$to maximize

$$\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_f^{\tau-t}Q_{t,\tau}\left[(1+\tau_f)P_{Hft}^o(j_f) - MC_{f\tau|t}\right]Y_{Hf\tau|t}(j_f), \tag{5}$$

in which $Q_{t,\tau} = \beta^{\tau-t}(C_\tau/C_t)^{-\sigma}(P_{ft}/P_{f\tau})$is the stochastic discount factor, τ_f is a subsidy to home final-goods producers by the home government, $MC_{f\tau|t} = \frac{P_{Hi\tau}^{\phi}P_{Fi\tau}^{1-\phi}}{A_{f\tau}}$ and $Y_{Hf\tau|t}(j_f)$are respectively period-τ marginal cost and demand schedule for final-goods firm j_f that last reset its price in period t,[5] and $Y_{Hf\tau|t}(j_f)$has the following expression

[3] In the expression of $P_{ft}, P_{Hft} = \left(\int_0^1 P_{Hft}\left(j_f\right)^{1-\xi_f}dj_f\right)^{\frac{1}{1-\xi_f}}$ is home local final-goods PPI, and $P_{Fft} = \left(\int_0^1 P_{Fft}\left(j_f^*\right)^{1-\xi_f}dj_f^*\right)^{\frac{1}{1-\xi_f}}$ is home imported final-goods PPI.

[4] The nominal exchange rate E_t represents the home currency price of one unit of foreign currency.

[5] in the expression of the marginal cost, $P_{Hit} = \left[\int_0^1 P_{Hit}(j_i)^{1-\xi_i}dj_i\right]^{\frac{1}{1-\xi_i}}$ is home local intermediate-goods PPI, and $P_{Fit} = \left[\int_0^1 P_{Fit}(j_i^*)^{1-\xi_i}dj_i^*\right]^{\frac{1}{1-\xi_i}}$ is home imported intermediate-goods PPI.

$$Y_{Hf\tau|t}(j_f) = \frac{\upsilon}{2}\left(\frac{P^o_{Hft}(j_f)}{P_{Hf\tau}}\right)^{-\xi_f}\left(\frac{P_{Hf\tau}}{P_{f\tau}}\right)^{-1}C_\tau. \tag{6}$$

In equation (6), $C_\tau = \int_0^1 C_\tau(h)dh$ is home aggregate consumption which is also equal to home consumption per capita. The solution to the optimal price setting problem facing final-goods firm j_f in the home country is

$$P^o_{Hft}(j_f) = \frac{\xi_f}{(\xi_f-1)(1+\tau_f)}\frac{\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_f^{\tau-t}Q_{t,\tau}MC_{f\tau|t}Y_{Hf\tau|t}(j_f)}{\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_f^{\tau-t}Q_{t,\tau}Y_{Hf\tau|t}(j_f)}, \tag{7}$$

as in Huang and Liu (2005), Gong et al. (2016), $\frac{\xi_f}{(\xi_f-1)(1+\tau_f)}$ is an effective markup over the weighted average of final-goods firm j_f's current and expected future marginal costs in the periods during which its reset price $P^o_{Hft}(j_f)$keeps effective.

Meanwhile, in the foreign country, when final-goods firm j_f has the opportunity to reset the price in period t, it chooses $P^{*o}_{Hft}(j_f)$to maximize

$$\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_f^{\tau-t}Q_{t,\tau}\left[(1+\tau_f)E_\tau P^{*o}_{Hft}(j_f) - MC_{f\tau|t}\right]Y^*_{Hf\tau|t}(j_f), \tag{8}$$

in which the demand schedule is given by

$$Y^*_{Hf\tau|t}(j_f) = \left(1-\frac{\upsilon}{2}\right)\left(\frac{P^{*o}_{Hft}(j_f)}{P^*_{Hf\tau}}\right)^{-\xi_f}\left(\frac{P^*_{Hf\tau}}{P^*_{f\tau}}\right)^{-1}C^*_\tau. \tag{9}$$

In equation (9), $P^*_{Hf\tau} = \left(\int_0^1 P^*_{Hf\tau}(j_f)^{1-\xi_f}dj_f\right)^{\frac{1}{1-\xi_f}}$ is foreign imported final-goods PPI, $P^*_{f\tau} = k^{-1}P^{*1-\nu/2}_{Hf\tau}P^{*\nu/2}_{Ff\tau}, k = (\upsilon/2)^{\nu/2}(1-\upsilon/2)^{1-\nu/2}$is foreign CPI,[6] and $C^*_\tau = \int_0^1 C^*_\tau(h^*)dh^*$ is foreign aggregate consumption which is also equal to foreign consumption per capita. The solution for $P^{*o}_{Hft}(j_f)$is given by

$$P^{*o}_{Hft}(j_f) = \frac{\xi_f}{(\xi_f-1)(1+\tau_f)}\frac{\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_f^{\tau-t}Q_{t,\tau}MC_{f\tau|t}Y^*_{Hf\tau|t}(j_f)}{\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_f^{\tau-t}Q_{t,\tau}E_\tau Y^*_{Hf\tau|t}(j_f)}. \tag{10}$$

At the stage of intermediate-goods production, firm j_i sets a price $P_{Hit}(j_i)$in its own currency when it sells good j_i domestically. Simultaneously, it sets another price $P^*_{Hit}(j_i)$in the importer's currency when exporting good j_i to the foreign country.

In the home country, when it is the turn for firm j_i to reset price, its optimal choice is

$$P^o_{Hit}(j_i) = \frac{\xi_i}{(\xi_i-1)(1+\tau_i)}\frac{\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_i^{\tau-t}Q_{t,\tau}MC_{i\tau|t}Y_{Hi\tau|t}(j_i)}{\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_i^{\tau-t}Q_{t,\tau}Y_{Hi\tau|t}(j_i)}, \tag{11}$$

in which $\frac{\xi_i}{(\xi_i-1)(1+\tau_i)}$ is an effective markup, $MC_{i\tau|t} = \frac{W_\tau}{A_{i\tau}}$ and $Y_{Hi\tau|t}(j_i)$are respectively period-τ unit cost function and demand schedule facing firm j_i that last reset its price in period t. The demand schedule $Y_{Hi\tau|t}(j_i)$has the following expression

$$Y_{Hi\tau|t}(j_i) = \phi\left(\frac{P_{Hit}(j_i)}{P_{Hi\tau}}\right)^{-\xi_i}\frac{MC_{f\tau}}{P_{Hi\tau}}\int_0^1 Y_{f\tau}(j_f)dj_f. \tag{12}$$

Similarly, in the foreign country, firm j_i's optimal choice is

$$P^{*o}_{Hit}(j_i) = \frac{\xi_i}{(\xi_i-1)(1+\tau_i)}\frac{\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_i^{\tau-t}Q_{t,\tau}MC_{i\tau|t}Y^*_{Hi\tau|t}(j_i)}{\mathbf{E}_t\sum_{\tau=t}^{\infty}\theta_i^{\tau-t}Q_{t,\tau}E_\tau Y^*_{Hi\tau|t}(j_i)}, \tag{13}$$

in which the demand schedule $Y^*_{Hi\tau|t}(j_i)$is given by

[6] $P^*_{Ff\tau} = \left(\int_0^1 P^*_{Ff\tau}(j^*_f)^{1-\xi_f}dj^*_f\right)^{\frac{1}{1-\xi_f}}$ is foreign local final-goods PPI.

$$Y^*_{Hi\tau|t}(j_i) = (1-\phi)\left(\frac{P^*_{Hit}(j_i)}{P^*_{Hi\tau}}\right)^{-\xi_i}\frac{MC^*_{f\tau}}{P^*_{Hi\tau}}\int_0^1 Y^*_{f\tau}\left(j^*_f\right)dj^*_f.$$

Furthermore, we assume that all firms at both stages of production take input prices as given but have the monopoly power in their own product markets.

3. Equilibrium

Before we turn our attention to the equilibrium system, it is helpful to introduce several definitions. The relative prices of imported to local goods at final-goods and intermediate-goods production stages in the home country are defined respectively as $S_{ft} = \frac{P_{Fft}}{P_{Hft}}$ and $S_{it} = \frac{P_{Fit}}{P_{Hit}}$. Their foreign counterparts are respectively $S^*_{ft} = \frac{P^*_{Hft}}{P^*_{Fft}}$ and $S^*_{it} = \frac{P^*_{Hit}}{P^*_{Fit}}$. In PCP case, the relative price of imported to local goods is identical to the terms of trade. However, they are different in LCP case. For convenience, in what follows, we rename the relative price of imported to local goods at the same stage of production the horizontal relative price.

In the home country, final-goods market clearing condition is

$$\begin{aligned} Y_{ft} &\equiv \left(\int_0^1 Y_{Hft}(j_f)^{\frac{\xi_f-1}{\xi_f}}dj_f\right)^{\frac{\xi_f}{\xi_f-1}} + \left(\int_0^1 Y^*_{Hft}(j_f)^{\frac{\xi_f-1}{\xi_f}}dj_f\right)^{\frac{\xi_f}{\xi_f-1}} \equiv Y_{Hft} + Y^*_{Hft} = C_{Ht} + C^*_{Ht} \\ &= \frac{\upsilon}{2}\frac{P_{ft}}{P_{Hft}}C_t + \left(1-\frac{\upsilon}{2}\right)\frac{P^*_{ft}}{P^*_{Hft}}C^*_t = k^{-1}\left(\frac{\upsilon}{2}S_{ft}^{1-\frac{\upsilon}{2}}C_t + \left(1-\frac{\upsilon}{2}\right)S_{ft}^{*-\frac{\upsilon}{2}}C^*_t\right). \end{aligned} \tag{14}$$

Intermediate-goods market clearing condition is

$$\begin{aligned} Y_{it} &\equiv \left(\int_0^1 Y_{Hit}(j_i)^{\frac{\xi_i-1}{\xi_i}}dj_i\right)^{\frac{\xi_i}{\xi_i-1}} + \left(\int_0^1 Y^*_{Hit}(j_i)^{\frac{\xi_i-1}{\xi_i}}dj_i\right)^{\frac{\xi_i}{\xi_i-1}} \equiv Y_{Hit} + Y^*_{Hit} \\ &= \phi\frac{MC_{ft}}{P_{Hit}}C_{Ht}D_{Hft} + \phi\frac{MC_{ft}}{P_{Hit}}C^*_{Ht}D^*_{Hft} + (1-\phi)\frac{MC^*_{ft}}{P^*_{Hit}}C_{Ft}D_{Fft} + (1-\phi)\frac{MC^*_{ft}}{P^*_{Hit}}C^*_{Ft}D^*_{Fft} \\ &= \phi\frac{S_{it}^{1-\phi}}{A_{ft}}\left(C_{Ht}D_{Hft} + C^*_{Ht}D^*_{Hft}\right) + (1-\phi)\frac{S_{it}^{*-\phi}}{A^*_{ft}}\left(C_{Ft}D_{Fft} + C^*_{Ft}D^*_{Fft}\right). \end{aligned} \tag{15}$$

Note that, due to the presence of LCP, there are two price dispersion measures at the final-goods production stage in the home country stemming from Calvo-style price staggering, they are $D_{Hft} \equiv \int_0^1 \left(\frac{P_{Hft}(j_f)}{P_{Hft}}\right)^{-\xi_f} dj_f$, and $D_{Fft} = \int_0^1 \left(\frac{P_{Fft}(j^*_f)}{P_{Fft}}\right)^{-\xi_f} dj^*_f$, respectively. Their foreign counterparts are $D^*_{Hft} \equiv \int_0^1 \left(\frac{P^*_{Hft}(j_f)}{P^*_{Hft}}\right)^{-\xi_f} dj_f$ and $D^*_{Fft} \equiv \int_0^1 \left(\frac{P^*_{Fft}(j^*_f)}{P^*_{Fft}}\right)^{-\xi_f} dj^*_f$ respectively.

Home aggregate employment is determined by the outputs of home intermediate-goods producers:

$$\begin{aligned} N_t &= \int_0^1 N_t(j_i)dj_i = \frac{1}{A_{it}}\int_0^1\left[\int_0^1 Y_{Hit}(j_f, j_i)dj_f + \int_0^1 Y^*_{Hit}\left(j^*_f, j_i\right)dj^*_f\right]dj_i \\ &= \frac{\phi}{A_{it}}\frac{S_{it}^{1-\phi}}{A_{ft}}\left(C_{Ht}D_{Hft} + C^*_{Ht}D^*_{Hft}\right)D_{Hit} + \frac{(1-\phi)}{A_{it}}\frac{S_{it}^{*-\phi}}{A^*_{ft}}\left(C_{Ft}D_{Fft} + C^*_{Ft}D^*_{Fft}\right)D^*_{Hit}. \end{aligned} \tag{16}$$

Because the home representative intermediate-goods producer chooses a price in terms of home currency and another in terms of foreign currency, Calvo-style price setting generates price dispersion in both countries. In the home country, it is $D_{Hit} = \int_0^1 \left(\frac{P_{Hit}(j_i)}{P_{Hit}}\right)^{-\xi_I} dj_i$, while it is $D^*_{Hit} = \int_0^1 \left(\frac{P^*_{Hit}(j_i)}{P^*_{Hit}}\right)^{-\xi_I} dj_i$ in the foreign country.

When asset markets are complete, the households in both countries equalize the marginal utility of one unit of the nominal asset in all states of the world. Thus, we have a risk sharing condition from the stochastic Euler equations describing the intertemporal consumption choice facing both home and foreign households.

$$\left(\frac{C_t}{C^*_t}\right)^{\sigma} = \mathbb{Q}_t = \frac{E_t P^*_{Hft}}{P_{Hft}} S_{ft}^{*-\frac{\upsilon}{2}} S_{ft}^{\frac{\upsilon}{2}-1} \tag{17}$$

where $\mathbb{Q}_t = \frac{E_t P^*_{ft}}{P_{ft}}$ denotes the real exchange rate.[7] Equation (17)is the familiar condition which ensures that the price ratio is identical to the marginal rate of substitution between consumption goods in the two countries.

3.1. The steady state and the flexible-price equilibrium

In the steady state, there are still markup distortions due to the presence of monopoly power at both stages of production. To offset the markup distortions and achieve an efficient steady state, we assume that the governments can subsidize monopolistic producers at both stages of production by collecting lump-sum taxes. Since there is no difference between

[7] Note the presence of home bias in consumption leads to deviations from purchasing power parity.

PCP and LCP cases in the steady state, our model essentially degenerates into Gong et al. (2016). The readers can refer to Gong et al. (2016) for the steady state values of the main endogenous variables. By the same token, we can obtain the efficient flexible-price equilibrium which is also identical to Gong et al. (2016).

3.2. Equilibrium dynamics under sticky prices

In this section, we present the log-linear approximation to the sticky-price model around the steady state. As in Engel (2011) and Gong et al. (2016), we use a lower-case variable to denote the log deviation of the upper-case variable from it's corresponding steady state value.

Engel (2011) formally defines the currency misalignment as the average deviation of consumer prices in the foreign country from consumer prices in the home country. After we introduce international trade in intermediate inputs, the currency misalignment appears at both stages of production. To distinguish the currency misalignment at the stage of final-goods production from that at the stage of intermediate-goods production, we define the final-goods currency misalignment as the average deviation of final-goods prices in the foreign country from final-goods prices in the home country, which corresponds to the currency misalignment in Engel (2011). Furthermore, we define the intermediate-goods currency misalignment as the average deviation of intermediate-goods prices in the foreign country from intermediate-goods prices in the home country.

Specifically, we define the final-goods currency misalignment as:

$$m_{ft} \equiv \frac{1}{2}\left(e_t + p^*_{Hft} - p_{Hft} + e_t + p^*_{Fft} - p_{Fft}\right). \tag{18}$$

Similarly, we define the intermediate-goods currency misalignment as:

$$m_{it} \equiv \frac{1}{2}\left(e_t + p^*_{Hit} - p_{Hit} + e_t + p^*_{Fit} - p_{Fit}\right). \tag{19}$$

The distortions caused by the currency misalignments arise only when both LCP and sticky prices are present at the same time. In PCP or flexible-price models, the currency misalignments disappear, i.e. $m_{ft} = m_{it} = 0$.

As in Engel (2011), we define the difference of the horizontal relative prices at the stage of final-goods production as:

$$z_{ft} \equiv \frac{1}{2}\left(p_{Fft} - p_{Hft} - \left(p^*_{Fft} - p^*_{Hft}\right)\right). \tag{20}$$

Similarly, the difference of the horizontal relative prices at the stage of intermediate-goods production is defined as

$$z_{it} \equiv \frac{1}{2}\left(p_{Fit} - p_{Hit} - \left(p^*_{Fit} - p^*_{Hit}\right)\right). \tag{21}$$

From equations (20) and (21), we know that, in PCP case, the horizontal relative price in the home country is the opposite of the horizontal relative price in the foreign country, the difference of the horizontal relative prices vanishes.

Log-linearizing home CPI around the steady state yields

$$p_{ft} = \frac{\upsilon}{2} p_{Hft} + \left(1 - \frac{\upsilon}{2}\right) p_{Fft} = p_{Hft} + \left(1 - \frac{\upsilon}{2}\right) s_{ft} = p_{Fft} - \frac{\upsilon}{2} s_{ft}. \tag{22}$$

Thus, home CPI inflation π_{ft}, home local final-goods PPI inflation π_{Hft}, and home imported final-goods PPI inflation π_{Fft} have the following relation

$$\pi_{ft} = \pi_{Hft} + \left(1 - \frac{\upsilon}{2}\right) \triangle s_{ft} = \pi_{Fft} - \frac{\upsilon}{2} \triangle s_{ft}, \tag{23}$$

in which $\triangle s_{ft} = s_{ft} - s_{ft-1}$. Equation (23)implies that there is a gap between the home CPI inflation rate and the two PPI inflation rates at the stage of final-goods production, and the gap is proportional to the percentage change in the horizontal relative price at final-goods production stage in the home country, with the gap adjusted by the degree of home bias in consumption.

In Gali and Monacelli (2005) and Gong et al. (2016), the purchasing power parity does not hold due to the presence of home bias in consumption. In their models, the consumption real exchange rate is proportional to the terms of trade, with the coefficient of proportionality given by the degree of home bias in consumption.[8] By contrast, in LCP case, besides the horizontal relative prices at the final-goods production stage, the final-goods currency misalignment also plays a role in affecting the consumption real exchange rate. The consumption real exchange rate can be expressed as

$$q_{ft} = e_t + p^*_{ft} - p_{ft} = m_{ft} + \frac{(\upsilon - 1)}{2}\left(s_{ft} - s^*_{ft}\right). \tag{24}$$

[8] In Gong et al. (2016), the real exchange rate is proportional to the final-goods terms of trade.

When households in both countries have a home bias in consumption ($\upsilon > 1$), the increase in the horizontal relative price at home final-goods production stage results in the depreciation of the home consumption real exchange rate. By contrast, the increase in the horizontal relative price at the foreign final-goods production stage leads to the appreciation of the home consumption real exchange rate. In addition, the home consumption real exchange rate depreciates in response to an increase in the degree of the final-goods currency misalignment.

The home final-goods market clearing condition and its foreign counterpart are given by, respectively

$$y_{ft} = \frac{\upsilon}{2}c_t + \left(1 - \frac{\upsilon}{2}\right)c_t^* + \frac{\upsilon}{2}\left(1 - \frac{\upsilon}{2}\right)\left(s_{ft} - s_{ft}^*\right), \tag{25}$$

$$y_{ft}^* = \left(1 - \frac{\upsilon}{2}\right)c_t + \frac{\upsilon}{2}c_t^* - \frac{\upsilon}{2}\left(1 - \frac{\upsilon}{2}\right)\left(s_{ft} - s_{ft}^*\right). \tag{26}$$

In both countries, a rise in global consumption increases the final-goods output. Furthermore, the expenditure-switching mechanism of the horizontal relative prices takes effect to some degree and induces the households in both countries to buy the relatively cheaper goods. Thus, both the increase in the horizontal relative price of domestic final goods and the decrease in the horizontal relative price of the other country's final goods boost domestic final-goods output.

Similarly, the home intermediate-goods market clearing condition and its foreign counterpart are given by, respectively

$$\begin{aligned} y_{it} = \phi(1-\phi)\left(s_{it} - s_{it}^*\right) - \phi a_{ft} - (1-\phi)a_{ft}^* + \phi\left(y_{ft} + \tfrac{\upsilon}{2}d_{Hft} + (1 - \tfrac{\upsilon}{2})d_{Hft}^*\right) \\ +(1-\phi)\left(y_{ft}^* + (1 - \tfrac{\upsilon}{2})d_{Fft} + \tfrac{\upsilon}{2}d_{Fft}^*\right), \end{aligned} \tag{27}$$

$$\begin{aligned} y_{it}^* = -\phi(1-\phi)\left(s_{it} - s_{it}^*\right) - (1-\phi)a_{ft} - \phi a_{ft}^* + \phi\left(y_{ft}^* + (1 - \tfrac{\upsilon}{2})d_{Fft} + \tfrac{\upsilon}{2}d_{Fft}^*\right) \\ +(1-\phi)\left(y_{ft} + \tfrac{\upsilon}{2}d_{Hft} + (1 - \tfrac{\upsilon}{2})d_{Hft}^*\right). \end{aligned} \tag{28}$$

Due to the vertical production and trade, a rise in final-goods output in both countries increases intermediate-goods output in each country. In addition, the change in the horizontal relative price of intermediate goods induces the final-goods producers in both countries to buy the relatively cheaper intermediate goods. Thus, both the increase in the horizontal relative price of domestic intermediate goods and the decrease in the horizontal relative price of the other country's intermediate goods boost domestic intermediate-goods output. Note that, up to a first-order approximation, the price dispersions are equal to zero.

The first-order approximation to the home and foreign labor market clearing conditions gives rise to

$$n_t = -a_{it} + \phi d_{Hit} + (1-\phi)d_{Hit}^* + y_{it}, \tag{29}$$

$$n_t^* = -a_{it}^* + \phi d_{Fit}^* + (1-\phi)d_{Fit} + y_{it}^*. \tag{30}$$

Since intermediate-goods producers use domestic labor as the only production factor, the increase in the output of domestic intermediate goods boosts the employment in each country.

Taking log of both sides of equation (17)yields

$$\sigma\left(c_t - c_t^*\right) = m_{ft} + \frac{(\upsilon - 1)}{2}\left(s_{ft} - s_{ft}^*\right). \tag{31}$$

In LCP case, besides the horizontal relative prices at the final-goods production stage in both countries, the final-goods currency misalignment also affects home consumption relative to its foreign counterpart.

As in Engel (2011), for any variables x_t and x_t^*, we use $x_t^R \equiv \frac{x_t - x_t^*}{2}$ and $x_t^W \equiv \frac{x_t + x_t^*}{2}$ to denote relative and world values, respectively. From equations (18)and (19), we know that the final-goods currency misalignment is associated with the intermediate-goods currency misalignment according to the following equation

$$m_{ft} = m_{it} + \upsilon_{Ht}^R + \upsilon_{Ft}^R, \tag{32}$$

in which $V_{Ht} \equiv \frac{P_{Hit}}{P_{Hft}}$ is the relative price of home intermediate goods in terms of home final goods when they are traded domestically, while $V_{Ht}^* \equiv \frac{P_{Hit}^*}{P_{Hft}^*}$ is the relative price of home intermediate goods in terms of home final goods when they are exported to the foreign country. Similarly, $V_{Ft} \equiv \frac{P_{Fit}}{P_{Fft}}$ is the relative price of foreign intermediate goods in terms of foreign final goods when they are exported to the home country, while $V_{Ft}^* \equiv \frac{P_{Fit}^*}{P_{Fft}^*}$ is the relative price of foreign intermediate goods in terms of foreign final goods when they are traded domestically. For convenience, we rename the relative price of intermediate goods in terms of final goods the vertical relative price.

From equations (20) and (21), we know that the difference of the horizontal relative prices at the stage of final-goods production and its counterpart at the stage of intermediate-goods production are related by the following equation

$$z_{ft} = z_{it} + \upsilon^R_{Ht} - \upsilon^R_{Ft}. \tag{33}$$

In the remaining part of the section, we use the final-goods outputs, the currency misalignments, the differences of the horizontal relative prices, and the vertical relative prices to express the New Keynesian Phillips curves, the dynamic IS equations, and the welfare loss function.

From equations (20), (25) , (26), and (31), we express c_t, c^*_t, s_{ft}, and s^*_{ft} in terms of y_{ft}, y^*_{ft}, m_{ft}, and z_{ft},

$$c_t = \frac{(\upsilon - 1)}{D} y^R_{ft} + y^W_{ft} + \frac{\upsilon(2-\upsilon)}{2D} m_{ft}, \tag{34}$$

$$c^*_t = -\frac{(\upsilon - 1)}{D} y^R_{ft} + y^W_{ft} - \frac{\upsilon(2-\upsilon)}{2D} m_{ft}, \tag{35}$$

$$s_{ft} = \frac{2\sigma}{D} y^R_{ft} - \frac{(\upsilon - 1)}{D} m_{ft} + z_{ft}, \tag{36}$$

$$s^*_{ft} = -\frac{2\sigma}{D} y^R_{ft} + \frac{(\upsilon - 1)}{D} m_{ft} + z_{ft}, \tag{37}$$

in which $D = \sigma\upsilon(2-\upsilon) + (\upsilon-1)^2$. Generally speaking, if final-good prices in the foreign country are higher than those in the home country, the final-goods currency misalignment is positive. Ceteris paribus, the distortion from the final-goods currency misalignment increases home consumption, whereas it decreases foreign consumption. By contrast, the influence of the final-goods currency misalignment on the horizontal relative prices at the stage of final-goods production in both countries depends on the degree of home bias in consumption. When households in both countries display home bias in consumption ($\upsilon > 1$), other things being equal, the distortion from the final-goods currency misalignment tends to lower the horizontal relative price at the stage of final-goods production in the home country when it is positive, whereas the opposite is true in the foreign country. Unlike the final-goods currency misalignment, the difference of the horizontal relative prices at the stage of final-goods production increases the horizontal relative price at the stage of final-goods production in both countries when it is positive.

From the definitions of the horizontal and vertical relative prices, we know that

$$s_{it} = s_{ft} - \upsilon_{Ht} + \upsilon_{Ft}, \tag{38}$$

$$s^*_{it} = s^*_{ft} + \upsilon^*_{Ht} - \upsilon^*_{Ft}. \tag{39}$$

From equations $(27), (28), (36) - (39)$, we can express y_{it} and y^*_{it} in terms of $y_{ft}, y^*_{ft}, m_{ft}, \upsilon_{Ht}, \upsilon^*_{Ht}, \upsilon_{Ft}$, and υ^*_{Ft}, up to a first-order approximation,

$$\begin{aligned} y_{it} = y^W_{ft} &+ \left[\tfrac{4\sigma\phi(1-\phi)}{D} + (2\phi-1)\right] y^R_{ft} - \tfrac{2\phi(1-\phi)(\upsilon-1)}{D} m_{ft} \\ &+ 2\phi(1-\phi)\left(\upsilon^W_{Ft} - \upsilon^W_{Ht}\right) + (1-2\phi)a^R_{ft} - a^W_{ft}, \end{aligned} \tag{40}$$

$$\begin{aligned} y^*_{it} = y^W_{ft} &- \left[\tfrac{4\sigma\phi(1-\phi)}{D} + (2\phi-1)\right] y^R_{ft} + \tfrac{2\phi(1-\phi)(\upsilon-1)}{D} m_{ft} \\ &+ 2\phi(1-\phi)\left(\upsilon^W_{Ht} - \upsilon^W_{Ft}\right) - (1-2\phi)a^R_{ft} - a^W_{ft}. \end{aligned} \tag{41}$$

When households in both countries display home bias in consumption ($\upsilon > 1$), ceteris paribus, the increase in the distortion from the final-goods currency misalignment lowers the horizontal relative price at the stage of final-goods production in the home country, thus decreasing the horizontal relative price at the stage of intermediate-goods production in the home country simultaneously. It implies that the price of local goods relative to that of imported goods rises at the intermediate-goods production stage in the home country, which lowers the demand for home intermediate goods by home final-goods producers. In addition, the increase in the distortion from the final-goods currency misalignment raises the horizontal relative price at the stage of final-goods production in the foreign country, thus driving up the horizontal relative price at the stage of intermediate-goods production in the foreign country simultaneously. Unlike what happens in the home country, the price of imported goods relative to that of local goods rises at the intermediate-goods production stage in the foreign country, which also lowers the demand for home intermediate goods by foreign final-goods producers. Taken together, the increase in the distortion from the final-goods currency misalignment reduces the home intermediate-goods output. Similarly, we can analyze the effect of the increase in the distortion from the final-goods currency misalignment on the foreign intermediate-goods output.

By combining the log-linearized optimal price setting equation (7)with the evolution equation of the home aggregate price level of local final goods $P_{Hft} = \left[\theta_f P^{1-\xi_f}_{Hft-1} + (1-\theta_f) P^{o\,1-\xi_f}_{Hft}\right]^{\frac{1}{1-\xi_f}}$, we can derive the following New Keynesian Phillips curve to describe the motion of the local final-goods PPI inflation rate in the home country:

$$\pi_{Hft} = \delta_f\left\{\phi\left(\tilde{v}^W_{Ht} + \tilde{v}^R_{Ht}\right) + (1-\phi)\left(\tilde{v}^W_{Ft} + \tilde{v}^R_{Ft} + z_{ft} + \frac{2\sigma}{D}\tilde{y}^R_{ft} - \frac{(\nu-1)}{D}m_{ft}\right)\right\} + \beta\mathbf{E}_t\{\pi_{Hft+1}\},$$

in which $\delta_f \equiv (1-\theta_f)(1-\beta\theta_f)/\theta_f$, and a variable with a tilde denotes the deviation of the log value of the variable from the corresponding value in the flexible-price equilibrium.

Unlike Gong et al. (2016), at the stage of final-goods production, there are two home PPI inflation rates: the local final-goods PPI inflation rate and the imported final-goods PPI inflation rate. Similar to the derivation of equation (42), we can obtain the following New Keynesian Phillips curve to describe the motion of the imported final-goods PPI inflation rate:

$$\pi_{Fft} = \delta_f\left\{\phi\left(\tilde{v}^W_{Ft} - \tilde{v}^R_{Ft} - z_{ft}\right) + (1-\phi)\left(\tilde{v}^W_{Ht} - \tilde{v}^R_{Ht} - \frac{2\sigma}{D}\tilde{y}^R_{ft} + \frac{(\nu-1)}{D}m_{ft}\right) + m_{ft}\right\} + \beta\mathbf{E}_t\{\pi_{Fft+1}\}. \tag{43}$$

Similarly, there are also two New Keynesian Phillips curves at the stage of final-goods production in the foreign country, they are respectively:

$$\pi^*_{Hft} = \delta_f\left\{\phi\left(\tilde{v}^W_{Ht} + \tilde{v}^R_{Ht} - z_{ft}\right) + (1-\phi)\left(\tilde{v}^W_{Ft} + \tilde{v}^R_{Ft} + \frac{2\sigma}{D}\tilde{y}^R_{ft} - \frac{(\nu-1)}{D}m_{ft}\right) - m_{ft}\right\} + \beta\mathbf{E}_t\left\{\pi^*_{Hft+1}\right\}, \tag{44}$$

$$\pi^*_{Fft} = \delta_f\left\{\phi\left(\tilde{v}^W_{Ft} - \tilde{v}^R_{Ft}\right) + (1-\phi)\left(\tilde{v}^W_{Ht} - \tilde{v}^R_{Ht} + z_{ft} - \frac{2\sigma}{D}\tilde{y}^R_{ft} + \frac{(\nu-1)}{D}m_{ft}\right)\right\} + \beta\mathbf{E}_t\left\{\pi^*_{Fft+1}\right\}. \tag{45}$$

If final-goods producers only input domestic intermediate goods and there are no price stickiness and the productivity shock at the stage of intermediate-goods production, the New Keynesian Phillips curves in our model are the same as those in Engel (2011). We can verify this point by comparing the expression of π^*_{Hft} given by equation (44)with its counterpart in Engel (2011). When $\phi = 1$ and $p_{Hit} = w_t$, we can simplify equation (44)as $\pi^*_{Hft} = \delta_f(w_t - p_{Hft} - z_{ft} - m_{ft} - a_{ft}) + \beta\mathbf{E}_t\left\{\pi^*_{Hft+1}\right\}$ which is identical to equation $(B31)$in Appendix to Engel (2011). When we introduce international trade in intermediate inputs, besides the final-goods output gaps, the currency misalignment at the stage of final-goods production, and the difference of the horizontal relative prices at the stage of final-goods production which are present in the expressions of the real marginal cost gaps in Engel (2011), the vertical relative price gaps also influence the real marginal cost gaps.

As in Gong et al. (2016), when we introduce international trade in intermediate inputs, there is a distinction between the final-goods PPI inflation rate and the intermediate-goods PPI inflation rate, while it is impossible to discuss the distinction in Clarida et al. (2002), Gali and Monacelli (2005), and Engel (2011). At the stage of the intermediate-goods production, there are four New Keynesian Phillips curves describing how the home and foreign intermediate-goods PPI inflation rates evolve.

In the home country, the local and imported intermediate-goods PPI inflation rates are, respectively

$$\begin{aligned}\pi_{Hit} = {} & \delta_i\left\{\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} + \left(1-\tfrac{\nu}{2}\right)z_{it} + \left(\tfrac{1}{2}-\Gamma'\right)m_{it} + \left(\tfrac{1}{2}-\Gamma'\right)\left(\tilde{v}^R_{Ht} + \tilde{v}^R_{Ft}\right)\right\} \\ & + \delta_i\left\{2\varphi\phi(1-\phi)\left(\tilde{v}^W_{Ft} - \tilde{v}^W_{Ht}\right) - \tfrac{\nu}{2}\tilde{v}^R_{Ht} - \left(1-\tfrac{\nu}{2}\right)\tilde{v}^R_{Ft} - \tilde{v}^W_{Ht}\right\} + \beta\mathbf{E}_t\{\pi_{Hit+1}\},\end{aligned} \tag{46}$$

$$\begin{aligned}\pi_{Fit} = {} & \delta_i\left\{-\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} - \tfrac{\nu}{2}z_{it} + \left(\tfrac{1}{2}+\Gamma'\right)m_{it} + \left(\tfrac{1}{2}+\Gamma'\right)\left(\tilde{v}^R_{Ht} + \tilde{v}^R_{Ft}\right)\right\} \\ & + \delta_i\left\{-2\varphi\phi(1-\phi)\left(\tilde{v}^W_{Ft} - \tilde{v}^W_{Ht}\right) - \tfrac{\nu}{2}\tilde{v}^R_{Ht} - \left(1-\tfrac{\nu}{2}\right)\tilde{v}^R_{Ft} - \tilde{v}^W_{Ft}\right\} + \beta\mathbf{E}_t\{\pi_{Fit+1}\},\end{aligned} \tag{47}$$

in which $\delta_i \equiv (1-\theta_i)(1-\beta\theta_i)/\theta_i, \Pi' \equiv \frac{\sigma}{D} + \varphi\left(\frac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right), \Gamma' = \frac{(\nu-1)}{2D} + \frac{2\varphi\phi(1-\phi)(\nu-1)}{D}$.

In Engel (2011), the currency misalignment is a separate concern of monetary policy. In our model, there are two kinds of currency misalignment: the final-goods currency misalignment m_{ft} and the intermediate-goods currency misalignment m_{it}, both of which affect the real marginal cost gaps at their respective stage of production, and thus are the source of inefficient allocations.

In the foreign country, the local and imported intermediate-goods PPI inflation rates are, respectively

$$\begin{aligned}\pi^*_{Fit} = {} & \delta_i\left\{-\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} + \left(1-\tfrac{\nu}{2}\right)z_{it} + \left(\Gamma'-\tfrac{1}{2}\right)m_{it} + \left(\Gamma'-\tfrac{1}{2}\right)\left(\tilde{v}^R_{Ht} + \tilde{v}^R_{Ft}\right)\right\} \\ & + \delta_i\left\{-2\varphi\phi(1-\phi)\left(\tilde{v}^W_{Ft} - \tilde{v}^W_{Ht}\right) + \left(1-\tfrac{\nu}{2}\right)\tilde{v}^R_{Ht} + \tfrac{\nu}{2}\tilde{v}^R_{Ft} - \tilde{v}^W_{Ft}\right\} + \beta\mathbf{E}_t\{\pi^*_{Fit+1}\},\end{aligned} \tag{48}$$

$$\begin{aligned}\pi^*_{Hit} = {} & \delta_i\left\{\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} - \tfrac{\nu}{2}z_{it} + \left(-\tfrac{1}{2}-\Gamma'\right)m_{it} + \left(-\tfrac{1}{2}-\Gamma'\right)\left(\tilde{v}^R_{Ht} + \tilde{v}^R_{Ft}\right)\right\} \\ & + \delta_i\left\{2\varphi\phi(1-\phi)\left(\tilde{v}^W_{Ft} - \tilde{v}^W_{Ht}\right) + \left(1-\tfrac{\nu}{2}\right)\tilde{v}^R_{Ht} + \tfrac{\nu}{2}\tilde{v}^R_{Ft} - \tilde{v}^W_{Ht}\right\} + \beta\mathbf{E}_t\{\pi^*_{Hit+1}\}.\end{aligned} \tag{49}$$

In PCP case, both the intermediate-goods currency misalignment and the difference of the horizontal relative prices at the stage of intermediate-goods production are zero. In addition, the vertical relative prices are the same in both countries. In this case, we can verify that equations (46)and (48)are identical to their counterparts in Gong et al. (2016).

As shown in the derivations of the New Keynesian Phillips curves, the vertical relative prices play an important role in our model. By the definitions, we can construct an identity to describe the motion of the relative value of the vertical relative prices of home goods sold in both countries

$$\tilde{\upsilon}^R_{Ht} = \tilde{\upsilon}^R_{Ht-1} + \frac{1}{2}\left(\pi_{Hit} - \pi_{Hft} - \pi^*_{Hit} + \pi^*_{Hft}\right). \tag{50}$$

Note that in the derivation of equation (50), we use the fact that, in the flexible-price equilibrium, the vertical relative prices of home goods sold in both countries are identical. Similarly, we can construct an identity to describe the motion of the relative value of the vertical relative prices of foreign goods sold in both countries, it is

$$\tilde{\upsilon}^R_{Ft} = \tilde{\upsilon}^R_{Ft-1} + \frac{1}{2}\left(\pi_{Fit} - \pi_{Fft} - \pi^*_{Fit} + \pi^*_{Fft}\right). \tag{51}$$

By the same token, there is an identity to describe the motion of the world value of the vertical relative prices of home goods sold in both countries, it is

$$\tilde{\upsilon}^W_{Ht} = \tilde{\upsilon}^W_{Ht-1} + \frac{1}{2}\left(\pi_{Hit} - \pi_{Hft} + \pi^*_{Hit} - \pi^*_{Hft}\right) - \Delta\bar{\upsilon}_t, \tag{52}$$

in which $\triangle\overline{\upsilon}_t = \overline{\upsilon}_t - \overline{\upsilon}_{t-1}$, and

$$\bar{\upsilon}_t = \bar{\upsilon}_{Ht} = \bar{\upsilon}^*_{Ht} = \Xi\left(a_{ft} - a^*_{ft}\right) + a_{ft} + \mathsf{F}\left(a^*_{it} - a_{it}\right), \tag{53}$$

where $\Xi \equiv \frac{\varphi(1-\phi)(1-2\Lambda)}{(1-2\phi)(1+2\varphi\Lambda)}$, $\mathsf{F} \equiv \frac{(1+\varphi)(1-\phi)}{(1+2\varphi\Lambda)}$. In the expressions of Ξ and F, $\Lambda \equiv 2\phi(1-\phi) + \upsilon(1-\frac{\upsilon}{2})(1-2\phi)^2 + (\upsilon-1)^2(1-2\phi)^2/2\sigma$.

Surely, there is an identity to describe the motion of the world value of the vertical relative prices of foreign goods sold in both countries, it is

$$\tilde{\upsilon}^W_{Ft} = \tilde{\upsilon}^W_{Ft-1} + \frac{1}{2}\left(\pi_{Fit} - \pi_{Fft} + \pi^*_{Fit} - \pi^*_{Fft}\right) - \Delta\bar{\upsilon}^*_t \tag{54}$$

in which $\triangle\overline{\upsilon}^*_t = \overline{\upsilon}^*_t - \overline{\upsilon}^*_{t-1}$, and

$$\bar{\upsilon}^*_t = \bar{\upsilon}_{Ft} = \bar{\upsilon}^*_{Ft} = \Xi\left(a^*_{ft} - a_{ft}\right) + a^*_{ft} + \mathsf{F}\left(a_{it} - a^*_{it}\right) \tag{55}$$

As is standard in the literature, we need the dynamic IS equations in both countries to describe the equilibrium dynamics. Log-linearizing the home representative household's stochastic Euler equation around the steady state, we obtain the dynamic IS equation for the home country:

$$\tilde{y}_{ft} = \mathbf{E}_t\{\tilde{y}_{ft+1}\} - \frac{2D}{\sigma(1+D)}\left(i_t - \mathbf{E}_t\{\pi_{Hft+1}\} - \overline{rr}_t\right), \tag{56}$$

in which $i_t \equiv \ln R_t$ is the home nominal interest rate, and

$$\overline{rr}_t = \frac{\sigma(D-1)}{2D}\mathbf{E}_t\left\{\Delta y^*_{ft+1}\right\} + \frac{\sigma(1+D)}{2D}\mathbf{E}_t\{\Delta\bar{y}_{ft+1}\} + \left(\frac{1}{2} - \frac{\nu-1}{D}\right)\mathbf{E}_t\{\Delta m_{ft+1}\} + \left(1 - \frac{\nu}{2}\right)\mathbf{E}_t\{\Delta z_{ft+1}\} + \rho \tag{57}$$

is the home real interest rate in the flexible-price equilibrium.

Following the same steps as deriving equation (56), we obtain the dynamic IS equation for the foreign country, it is

$$\tilde{y}^*_{ft} = \mathbf{E}_t\left\{\tilde{y}^*_{ft+1}\right\} - \frac{2D}{\sigma(1+D)}\left(i^*_t - \mathbf{E}_t\left\{\pi^*_{Fft+1}\right\} - \overline{rr}^*_t\right), \tag{58}$$

by the uncovered interest-parity condition, we know that the foreign nominal interest rate i^*_t is associated with the home nominal interest rate i_t according to the equation $i^*_t = i_t - \mathbf{E}_t\{\triangle e_{t+1}\}$. The foreign real interest rate in the flexible-price equilibrium $\overline{rr}^*_t$ is given by

$$\overline{rr}^*_t = \frac{\sigma(D-1)}{2D}\mathbf{E}_t\{\Delta y_{ft+1}\} + \frac{\sigma(1+D)}{2D}\mathbf{E}_t\left\{\Delta\bar{y}^*_{ft+1}\right\} + \left(\frac{\nu-1}{2D} - \frac{1}{2}\right)\mathbf{E}_t\{\Delta m_{ft+1}\} + \left(1 - \frac{\nu}{2}\right)\mathbf{E}_t\{\Delta z_{ft+1}\} + \rho. \tag{59}$$

The expressions of the home and foreign final-goods outputs in the flexible-price equilibrium can be found in Gong et al. (2016).

In the next section, we follow Engel (2011) to solve for the optimal monetary policy for a cooperative policymaker. The policymaker can directly choose the final-goods output gaps, the inflation rates at both stages of production, the currency misalignments at both stages of production, the differences of the horizontal relative prices at both stages of production, and the vertical relative price gaps, subject to the constraints. But for now, to close the equilibrium dynamics system, part of which consists of equations (42) – (59), we need two simple monetary policy rules to describe how the nominal interest rates in both countries evolve over time. In the next section, we will introduce how the policymakers conduct the monetary policy by choosing the nominal interest rate as a policy instrument in detail.

4. Monetary policy design

In PCP case, the optimal monetary policy in a standard two-country New Keynesian model can achieve the "divine coincidence" in the sense that it can close the output gap and simultaneously realize zero inflation at all times, when there are no cost-push shocks. [9] However, Gong et al. (2016) show that the same conclusion does not hold in a similar model with international trade in intermediate inputs. In LCP case, Devereux and Engel (2003) show that, when the nominal prices are set one period in advance, the optimal monetary policy cannot replicate the flexible-price allocation. Thus, it is quite natural to have the following result.

Proposition 1. In a model with currency misalignments and international trade in intermediate inputs, it is impossible to achieve the flexible-price equilibrium allocation.

Proof. See Appendix B1.

In Proposition 1, we assume that, by collecting a lump-sum tax, the governments subsidize the producers to eliminate the market power distortions at both stages of production. Thus, in the flexible-price equilibrium, the allocation is Pareto optimal, which means that the output gaps, the PPI inflation rates, and the relative price gaps are all equal to zero simultaneously. In PCP case with no international trade in intermediate inputs, when there are no cost-push shocks, the monetary policymaker faces no trade-off between closing the output gaps and stabilizing the PPI inflation. In our model, when the monetary policymaker chooses to stabilize the PPI inflation rates completely, the distortions from the relative values of the vertical relative price gaps $(\tilde{v}_{Ht}^R \text{and} \tilde{v}_{Ft}^R)$can be dismantled. However, the distortions from the currency misalignments at both stages of production and the world values of the vertical relative price gaps $(\tilde{v}_{Ht}^W \text{and} \tilde{v}_{Ft}^W)$cannot be dismantled.[10] Thus, in our model, both international trade in intermediate inputs and currency misalignments play the roles in breaking the "divine coincidence".

In Engel (2011), if the initial value of the relative price difference $z_0 = 0$, it follows that $z_t = 0$ in all periods. Similarly, we have.

Proposition 2. $z_{it} = z_{ft} = 0$ for all $t \geqslant 1$.

Proof. See Appendix B2.

Proposition 2 implies that $s_{ft} = -s_{ft}^*$ and $s_{it} = -s_{it}^*$ in all periods. In other words, households and firms in both countries face the same horizontal relative price at each stage of production. Unlike PCP case, the nominal exchange rate plays a limited role in adjusting the horizontal relative prices in LCP case.[11] In PCP case, the exchange rate depreciation deteriorates the terms of trade in the home country, thus, the expenditure-switching effect induces home households and firms to substitute domestic goods for imported goods. By contrast, the exchange rate depreciation improves the terms of trade in the foreign country, the expenditure-switching effect induces foreign households and firms to substitute imported goods for local goods. In LCP case, it is appropriate to use the horizontal relative prices rather than the terms of trade to describe the trade-offs facing the households and firms when choosing between domestic and imported goods. Due to the fact that the nominal exchange rate plays a limited role in adjusting the horizontal relative prices, the expenditure-switching effect cannot efficiently adjust the expenditure decisions made by households and firms.

According to equations (32)and (33),and Proposition 2, we have $v_{Ht}^R = v_{Ft}^R = \frac{m_{ft} - m_{it}}{2}$.In addition, we have $\overline{v}_{Ht}^R = \overline{v}_{Ft}^R = 0$ in the flexible-price equilibrium. Thus, the relative values of the vertical relative price gaps in both countries can be written as

$$\tilde{v}_{Ht}^R = \tilde{v}_{Ft}^R = \frac{m_{ft} - m_{it}}{2}. \tag{60}$$

Gong et al. (2016) find that, when international trade in intermediate inputs is introduced in a standard two-country New Keynesian monetary model, the vertical relative price gaps are an important source of welfare loss. When we introduce LCP in Gong et al. (2016), currency misalignments are distortionary and a separate source of welfare loss. How do currency misalignments interact with the vertical relative price gaps? There are two vertical relative price gaps in Gong et al. (2016), whereas there are four vertical relative price gaps when LCP is present. However, as shown in equation (60), the relative value of the vertical relative price gap in the home country is equal to its foreign counterpart, and they are affected by the currency misalignments at both stages of production. To be specific, the currency misalignment at the state of final-goods production increases the relative value of the vertical relative price gaps, while the currency misalignment at the state of intermediate-goods production decreases it. The currency misalignments at both stages of production are assigned equal weights to affect the relative value of the vertical relative price gaps.

[9] See Clarida et al. (2002), Gali and Monacelli (2005) and Engel (2011). Note that Engel (2011) discusses both PCP and LCP cases.

[10] As will be shown, the differences of the horizontal relative prices are zero at both stages of production when their initial values are zero.

[11] Note that, in PCP case, the horizontal relative price is just the terms of trade.

Since $z_{it} = z_{ft} = 0$ for all periods, we focus our attention on the currency misalignments m_{ft} and m_{it}, both of which are the source of welfare loss from the deviation from the law of one price. When prices are flexible, there is no difference between PCP and LCP cases. Thus, it seems that the distortions caused by the currency misalignments are influenced by the degree of price stickiness. As a matter of fact, we have.

Proposition 3. If the degrees of price stickiness at both stages of production are identical, i.e. $\theta_f = \theta_i$, then the currency misalignment at the stage of final-goods production is the same as its counterpart at the stage of intermediate-goods production, i.e. $m_{ft} = m_{it}$ for all $t \geqslant 1$.

Proof. See Appendix B3.

Proposition 3 implies that, when the degrees of price stickiness at both stages of production are identical, the distortions caused by the currency misalignments at both stages of production are also identical. Intuitively, when the degree of price stickiness at one stage of production is higher than that at the other stage of production, the distortion caused by the currency misalignment at the stage of production with a higher degree of price stickiness is also higher than that caused by the currency misalignment at the other stage of production. Thus, due to the presence of vertical production and trade, the monetary policymaker needs to make a trade-off between the distortion caused by the currency misalignment at one stage of production and that at the other stage of production, depending on which stage of production has a higher degree of price stickiness.

4.1. Optimal monetary policy

Following Engel (2011), we focus on the cooperative case and derive the welfare loss function for the cooperative monetary policymaker. After taking a second-order approximation to the joint utility function of home and foreign households, we obtain

$$\mathbf{W} = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbf{X}_t + t.i.p. + O\left(\|a\|^3\right), \tag{61}$$

where

$$\begin{aligned}
\mathbf{X}_t = {} & \tfrac{\xi_f}{2\delta_f}\left(\tfrac{v}{2}\pi_{Hft}^2 + \tfrac{2-v}{2}\pi_{Hft}^{*2} + \tfrac{2-v}{2}\pi_{Fft}^2 + \tfrac{v}{2}\pi_{Fft}^{*2}\right) + (\sigma+\varphi)\left(\tilde{y}_{ft}^W\right)^2 + \tfrac{v(2-v)}{4D}m_{ft}^2 \\
& + \tfrac{\xi_i}{2\delta_i}\left(\phi\pi_{Hit}^2 + (1-\phi)\pi_{Hit}^{*2} + (1-\phi)\pi_{Fit}^2 + \phi\pi_{Fit}^{*2}\right) + (1+\varphi)\Omega_m^2 m_{it}^2 + 2(1+\varphi)\Gamma_y\Omega_v\bar{y}_{ft}^R\left(\tilde{v}_{Ft}^W - \tilde{v}_{Ht}^W\right) \\
& + \left[(1+\varphi)\Omega_y^2 - (1-\sigma)\tfrac{(v-1)^2}{D}\right]\left(\tilde{y}_{ft}^R\right)^2 + (1+\varphi)\Omega_v^2\left(\tilde{v}_{Ft}^W - \tilde{v}_{Ht}^W\right)^2 + 2(1+\varphi)\Omega_y\Omega_m\tilde{y}_{ft}^R m_{it} \\
& + \left[2(1+\varphi)\Gamma_y\Omega_y - 2(1-\sigma)\tfrac{(v-1)^2}{D}\right]\bar{y}_{ft}^R\tilde{y}_{ft}^R - 2(1+\varphi)\Gamma_y\Omega_m\bar{y}_{ft}^R m_{it} + (1+\varphi)\Omega_m^2\left(\tilde{v}_{Ht}^R + \tilde{v}_{Ft}^R\right)^2 \\
& + 2(1+\varphi)\Omega_y\Omega_v\tilde{y}_{ft}^R\left(\tilde{v}_{Ft}^W - \tilde{v}_{Ht}^W\right) - 2(1+\varphi)\Omega_y\Omega_m\tilde{y}_{ft}^R\left(\tilde{v}_{Ht}^R + \tilde{v}_{Ft}^R\right) - 2(1+\varphi)\Omega_m\Omega_v m_{it}\left(\tilde{v}_{Ft}^W - \tilde{v}_{Ht}^W\right) \\
& - 2(1+\varphi)\Omega_m\Omega_v\left(\tilde{v}_{Ft}^W - \tilde{v}_{Ht}^W\right)\left(\tilde{v}_{Ht}^R + \tilde{v}_{Ft}^R\right) + 2(1+\varphi)\Omega_m^2 m_{it}\left(\tilde{v}_{Ht}^R + \tilde{v}_{Ft}^R\right) - 2(1+\varphi)\Gamma_y\Omega_m\bar{y}_{ft}^R\left(\tilde{v}_{Ht}^R + \tilde{v}_{Ft}^R\right)
\end{aligned}$$

$t.i.p.$ stands for the terms independent of policy and $O\left(\|a\|^3\right)$ collects all terms of third or higher order. In the expression of $\mathbf{X}_t$, $\Omega_y = \left(\frac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right)$, $\Omega_m = \phi(1-\phi)\frac{2(v-1)}{D}$, $\Omega_v = 2\phi(1-\phi)$, $\Gamma_y = \frac{1}{1+\varphi}(1-2\phi)\frac{\sigma-D}{D}$.

When international trade in intermediate inputs is absent, and there is no nominal stickiness at the stage of intermediate-goods production, we can show that $\mathbf{X}_t$ is identical to its counterpart in Engel (2011). In this case, the expenditure share of the final-goods producer on domestic composite intermediate goods is one and composite intermediate goods should be interpreted as labor. When $\phi = 1$, we know that $\Omega_y = 1, \Omega_m = 0, \Omega_v = 0$, and $\Gamma_y = -\frac{\sigma-D}{(1+\varphi)D}$. In addition, the fact that there is no nominal stickiness at the stage of intermediate-goods production implies that there are no price dispersion terms associated with intermediate-goods production. After calculation, we know that $\mathbf{X}_t$ is identical to its counterpart in Engel (2011).[12]

In PCP case, there is no currency misalignments, i.e. $m_{ft} = m_{it} = 0$. In addition, we have $v_{Ht} = v_{Ht}^* = v_t$, $v_{Ft} = v_{Ft}^* = v_t^*$. Thus in the expression of $\mathbf{X}_t$, $\tilde{v}_{Ht}^W = \tilde{v}_t$, $\tilde{v}_{Ft}^W = \tilde{v}_t^*$, and $\tilde{v}_{Ht}^R = \tilde{v}_{Ft}^R = 0$. After simple calculation, we can arrive at that $\mathbf{X}_t$ is identical to its counterpart in Gong et al. (2016). [13]

Thus, the expected period welfare loss function is

[12] Please refer to Appendix A for the proof.

[13] Note that, when deriving the joint loss function of home and foreign households, Gong et al. (2016) do not take a second-order approximation to the first-order terms. We conclude that $\mathbf{X}_t$ is identical to its counterpart in Gong et al. (2016) in the sense that the first-order terms are approximated up to the second-order terms.

$$
\begin{aligned}
\mathbf{L} = {} & \tfrac{\xi_f}{2\delta_f}\left(\tfrac{\nu}{2}\,var(\pi_{Hft}) + \tfrac{2-\nu}{2}\,var\left(\pi^*_{Hft}\right) + \tfrac{2-\nu}{2}\,var(\pi_{Fft}) + \tfrac{\nu}{2}\,var\left(\pi^*_{Fft}\right)\right) + (\sigma+\varphi)\,var\left(\tilde{y}^W_{ft}\right) \\
& + \tfrac{\xi_i}{2\delta_i}\left(\phi\,var(\pi_{Hit}) + (1-\phi)\,var(\pi^*_{Hit}) + (1-\phi)\,var(\pi_{Fit}) + \phi\,var(\pi^*_{Fit})\right) + \tfrac{\nu(2-\nu)}{4D}\,var(m_{ft}) \\
& + \left((1+\varphi)\Omega_y^2 - (1-\sigma)\tfrac{(\nu-1)^2}{D}\right) var\left(\tilde{y}^R_{ft}\right) + (1+\varphi)\Omega_m^2\,var(m_{it}) + 2(1+\varphi)\Gamma_y\Omega_\upsilon cov\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}, \bar{y}^R_{ft}\right) \\
& + \left(2(1+\varphi)\Gamma_y\Omega_y - 2(1-\sigma)\tfrac{(\nu-1)^2}{D}\right) cov\left(\bar{y}^R_{ft}, \tilde{y}^R_{ft}\right) - 2(1+\varphi)\Omega_m\Omega_\upsilon cov\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}, \tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) \\
& + 2(1+\varphi)\Omega_m\Omega_y cov\left(\tilde{y}^R_{ft}, m_{it}\right) - 2(1+\varphi)\Gamma_y\Omega_m cov\left(m_{it}, \bar{y}^R_{ft}\right) - 2(1+\varphi)\Omega_m\Omega_\upsilon cov\left(m_{it}, \tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) \\
& + (1+\varphi)\Omega_\upsilon^2\,var\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) + 2(1+\varphi)\Omega_m^2 cov\left(m_{it}, \tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) - 2(1+\varphi)\Gamma_y\Omega_m cov\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}, \bar{y}^R_{ft}\right) \\
& + 2(1+\varphi)\Omega_y\Omega_\upsilon cov\left(\tilde{y}^R_{ft}, \tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - 2(1+\varphi)\Omega_y\Omega_m cov\left(\tilde{y}^R_{ft}, \tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) + (1+\varphi)\Omega_m^2\,var\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right)
\end{aligned}
$$

Comparing with Engel (2011), we know that, besides the variables present in Engel (2011), the cooperative monetary policymaker should also care about such variables which are specific to the vertical production and trade structure as the intermediate-goods PPI inflation rates, the vertical relative prices, and the intermediate-goods currency misalignment.

When the monetary policymaker is able to commit, with full credibility, to the policy rule at time zero, she chooses $\tilde{y}^R_{ft}, \tilde{y}^W_{ft}, m_{ft}, m_{it}, \tilde{\upsilon}^W_{Ht}, \tilde{\upsilon}^W_{Ft}, \tilde{\upsilon}^R_{Ht}, \tilde{\upsilon}^R_{Ft}, \pi_{Hft}, \pi^*_{Hft}, \pi_{Fft}, \pi^*_{Fft}, \pi_{Hit}, \pi^*_{Hit}, \pi_{Fit}, \pi^*_{Fit}$ to minimize the welfare loss function subject to the sequence of equilibrium dynamics given by equations (42) – (55). In Appendix A, we give the first-order conditions for the optimization problem facing the monetary policymaker.

Given zero initial values of the vertical relative prices and the Lagrange multipliers corresponding to equations (42) – (49), the first-order conditions, together with equations (42) - (55), constitute a dynamic system describing the optimal monetary policy.

Unlike Engel (2011), we cannot give an analytical solution to the dynamic system describing the optimal monetary policy. Thus, we compute the welfare loss of the optimal monetary policy according to the expected period welfare loss function after the parameterization and the shocks of the model are given. In addition, we can use the welfare loss under the optimal monetary policy as a benchmark to assess various monetary policy rules, i.e. rank these rules according to welfare losses. Thus, in practice, the monetary policymaker should choose the monetary policy rule whose welfare loss is closest to that of the optimal monetary policy.

4.2. Quantitative Analysis

When we introduce LCP in Gong et al. (2016), the prices of imported goods in both countries are sticky, whereas they change flexibly with the fluctuation of the nominal exchange rate in Gong et al. (2016). In Gong et al. (2016), it is natural to consider three different Taylor-type monetary policy rules: the CPI-based Taylor rule, the final-goods PPI-based Taylor rule, and the intermediate-goods PPI-based Taylor rule. Here, when the prices of local and imported goods in both countries are sticky, it is reasonable to have the nominal interest rate respond to the weighted average of local and imported goods inflation rates. At the stage of final-goods production, we choose the expenditure shares of the representative household on domestic and imported final goods in both countries as weights of the final-goods PPI inflation rates. Similarly, at the stage of intermediate-goods production, we choose the expenditure shares of the final-goods producers on domestic and imported intermediate goods in both countries as weights of the intermediate-goods PPI inflation rates. Thus, in our model, we consider two different Taylor-type monetary policy rules: the weighted average final-goods PPI inflation-based Taylor rule (W-FPPIT) and the weighted average intermediate-goods PPI inflation-based Taylor rule (W-IPPIT). In addition, as is standard in the literature, the nominal interest rate also responds to the output gap. To be specific, W-FPPIT is described as

$$
i_t = \rho + \phi_\pi\left(\frac{\upsilon}{2}\pi_{Hft} + \left(1-\frac{\upsilon}{2}\right)\pi_{Fft}\right) + \phi_y\tilde{y}_{ft}, i^*_t = \rho + \phi^*_\pi\left(\frac{\upsilon}{2}\pi^*_{Fft} + \left(1-\frac{\upsilon}{2}\right)\pi^*_{Hft}\right) + \phi^*_y\tilde{y}^*_{ft}. \tag{62}
$$

Similarly, W-IPPIT is specified as

$$
i_t = \rho + \phi_\pi(\phi\pi_{Hit} + (1-\phi)\pi_{Fit}) + \phi_y\tilde{y}_{ft}, i^*_t = \rho + \phi^*_\pi(\phi\pi^*_{Fit} + (1-\phi)\pi^*_{Hit}) + \phi^*_y\tilde{y}^*_{ft}. \tag{63}
$$

Note that the weighted average inflation under W-FPPIT is just the CPI inflation. Thus, by choosing the expenditure shares of the representative household on domestic and imported final goods in both countries as weights of the final-goods PPI inflation rates, W-FPPIT is identical to the CPI-based Taylor rule (CPIT). In what follows, to make a comparison with Engel (2011), we use WPPIT to represent W-IPPIT, and CPIT to represent W-FPPIT.

4.2.1. Parameterization

In order to quantitatively analyze the optimal monetary policy and Taylor-type monetary policy rules, we calibrate the model at a quarterly frequency. The parameters and their values are reported in Table 1. We set the subjective discount factor, β, to 0.99, which implies that the annual real interest rate is 4% in the steady state. The coefficient of relative risk aver-

Table 1
Parameter values in the benchmark case.

Description	Parameter	Value
Discount factor	β	0.99
Coefficient of relative risk aversion	σ	2
Frisch elasticity of labor supply	φ^{-1}	3/4
Home bias in consumption	v	1.5
Home bias in production	ϕ	0.67
Elasticity of substitution between final goods	ξ_f	6
Elasticity of substitution between intermediate goods	ξ_i	6
Nominal contract duration in final-goods sector	θ_f	0.75
Nominal contract duration in intermediate-goods sector	θ_i	0.75

sion, σ, usually takes the value from 1 to 5. We set it to 2 in the benchmark case. There is a wide discrepancy between micro and macro estimates of the Frisch elasticity of labor supply φ^{-1}, we follow Chetty et al. (2012) and set it to $3/4$. [14] We follow Engel (2011) and set the parameter describing home bias in consumption, v, to 1.5, which means that households in both countries put a consumption weight of $3/4$ on local final goods. The parameter describing home bias in production, ϕ, is set to 0.67, which is in line with Barbiero et al. (2019). We assume that the markups at both stages of production are the same in the steady state and set the corresponding elasticities of substitution between differentiated varieties, ξ_f and ξ_i, to 6, which implies that the steady-state markup is 20%. The probability that a firm cannot adjust the price at the stage of final-goods production is equal to its counterpart at the stage of intermediate-goods production, and we set both θ_f and θ_i to 0.75, implying that the average duration of the nominal contracts lasts four quarters.

In our model, there are four productivity shocks. These shocks are assumed to be uncorrelated across production stages and across countries. We set $\rho_f = \rho_i = \rho_f^* = \rho_i^* = 0.95$ and $var(\varepsilon_{ft}) = var(\varepsilon_{it}) = var\left(\varepsilon_{ft}^*\right) = var(\varepsilon_{it}^*) = 0.02^2$ to capture the persistence and variance of the shocks. In addition, following Taylor (1993)'s initial calibration, we set $\phi_\pi = \phi_\pi^* = 1.5, \phi_y = \phi_y^* = 0.125$.[15]

4.2.2. Welfare comparison of alternative rules

In the standard two-country New Keynesian monetary model with producer-currency pricing, the fluctuation in the nominal exchange rate plays a pivotal role in changing the price of imported goods relative to local goods (Clarida et al., 2002). The law of one price implies that the consumers in both countries face the same relative prices of goods produced in one country. Consequently, producer price inflation results in the same relative price distortions when prices are sticky in the currency of the producer. Thus stabilizing PPI inflation in one country can get rid of the relative price distortions in both countries simultaneously. By contrast, the stabilization of CPI inflation involves limiting the fluctuation in the nominal exchange rate to some degree, thus leading to inefficient allocations due to the fact that the expenditure-switching effect of the nominal exchange rate adjustment is impaired.

When the exporters set prices in the consumers' currency, the law of one price does not hold anymore in the presence of price stickiness, which leads to currency misalignments (Engel, 2011; Fujiwara and Wang, 2017; Chen et al., 2021). Thus the optimal monetary policy needs to trade off currency misalignments against inflation rates and output gaps. Since the fluctuation in the nominal exchange rate affects instantaneously the currency misalignments, rather than adjusting the relative prices under LCP, the monetary policymaker should keep the nominal exchange rate less volatile and target CPI inflation, rather than PPI inflation, to eliminate the distortions stemming from price stickiness (Engel, 2011).

By contrast, after introducing the vertical production and trade structure in Engel (2011), we find that, for most combinations of price stickiness at both stages of production, the monetary policymaker should target WPPI inflation rather than CPI inflation, thus changing the monetary policy prescription in the LCP model. When the world is buffeted by all the productivity shocks, Fig. 1 depicts the welfare losses in three cases: the optimal monetary policy; WPPIT; CPIT. As shown in Fig. 1, the welfare loss from implementing the optimal monetary policy is always lower than those from targeting WPPI inflation and CPI inflation, and the welfare loss from targeting WPPI inflation is lower than that from targeting CPI inflation for most combinations of price stickiness at both stages of production when the degrees of price stickiness at both stages of production range from 0.25 to 0.75, which cover all the reasonable parameter values used in the literature (Huang and Liu, 2005; Gong et al., 2016; Wei and Xie, 2020).

To be specific, when the degree of price stickiness at the intermediate-goods production stage varies from intermediate to high, the welfare loss from targeting WPPI inflation is lower than that from targeting CPI inflation, irrespective of the degree of price stickiness at the final-goods production stage. Accordingly, the monetary policymaker should implement WPPIT rather than CPIT. By contrast, when the degree of price stickiness at the intermediate-goods production stage varies from low to intermediate, the results of welfare comparison between targeting WPPI inflation and CPI inflation are mixed. However, when the degree of price stickiness at the intermediate-goods production stage is low, the welfare loss from targeting

[14] Note that Engel (2011) assumes a linear disutility from labor, which means that the Frisch elasticity of labor supply is infinite.

[15] Note that $\rho = -\log\beta = 0.0101$ is chosen to make the rules are consistent with a zero inflation steady state.

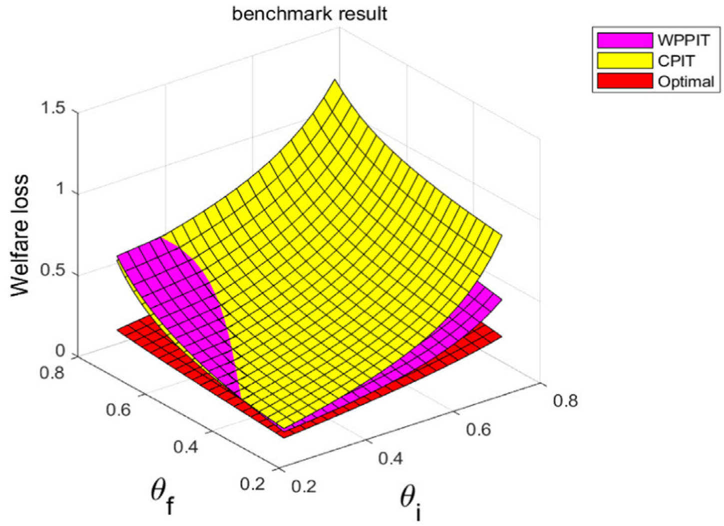

Fig. 1. The welfare losses in three cases: the optimal monetary policy; WPPIT; CPIT.

WPPI inflation is greater than that from targeting CPI inflation in most cases, especially when the degree of price stickiness at the final-goods production stage is higher than its counterpart at the intermediate-goods production stage.

Table 2 reports the quantitative welfare losses in three cases: the optimal monetary policy; WPPIT; CPIT. The last column of Table 2 also reports the quantitative welfare gains from targeting WPPI inflation relative to targeting CPI inflation. Without loss of generality, we let 0.75, 0.5, and 0.25 denote high, intermediate, and low degrees of price stickiness, respectively. The presence of price stickiness at both stages of production implies that there are 9 combinations of price stickiness.

As shown in Table 2, the welfare gain from targeting WPPI inflation relative to targeting CPI inflation reaches 0.2942% of the steady state consumption in the benchmark case. According to the literature on international monetary policy cooperation, the welfare gain of 0.2942% of the steady state consumption is sizable and should not be neglected (Obstfeld and Rogoff, 2002; Pappa, 2004; Liu and Pappa, 2008; Rabitsch, 2012; Fujiwara and Wang, 2017).

In addition, when the degree of price stickiness at the intermediate-goods production stage is intermediate or high, the welfare gain from targeting WPPI inflation relative to targeting CPI inflation decreases with the increase in the degree of price stickiness at the final-goods production stage. When the degree of price stickiness at the intermediate-goods production stage is low, there are welfare gains from targeting WPPI inflation relative to targeting CPI inflation, if the degree of price stickiness at the final-goods production stage is also low. However, when the degree of price stickiness at the intermediate-goods production stage is low, but at the same time the degree of price stickiness at the final-goods production stage is higher than its counterpart at the intermediate-goods production stage, there are welfare gains from targeting CPI inflation relative to targeting WPPI inflation. In this case, the welfare gain from targeting CPI inflation relative to targeting WPPI inflation is relatively small.

4.2.3. Impulse responses

To gain the intuition behind the welfare results, we compare the impulse responses of the main variables of the model to a positive productivity shock hitting the home final-goods production sector. Fig. 2 depicts the impulse responses of the main variables of the model in three cases: the optimal monetary policy; WPPIT; CPIT, when the home final-goods production sector is buffeted by a one-standard-deviation positive productivity shock. As shown in Fig. 2, WPPIT outperforms CPIT in terms of stabilizing the inflation rates in both countries when the degrees of price stickiness at both stages of production are high. Since the fluctuation in the inflation rates plays an important role in affecting the welfare loss, there are welfare gains from targeting WPPI inflation relative to targeting CPI inflation. Why?.

In general, there are five types of distortion preventing the world economy from achieving the efficient allocations: monopolistic competition, price dispersion, the horizontal relative price distortion, the vertical relative price distortion, and currency misalignment. The distortion related to monopolistic competition is eliminated by subsidies to producers at both stages of production which are raised by the governments in a lump-sum fashion. In addition, from equations (50) – (52),and (54),the distortion stemming from the vertical relative price can be alleviated if the monetary policymaker stabilizes the inflation rates at both stages of production.

According to Eqs. (36) to (39), we have

$$\widetilde{s}_{ft} = \frac{2\sigma}{D}\widetilde{y}^{R}_{ft} - \frac{(\upsilon-1)}{D}m_{ft}, \tag{64}$$

Table 2
Welfare losses and welfare gains from targeting WPPI inflation relative to targeting CPI inflation.

	Optimal	WPPIT	CPIT	Welfare gain
$\mathbf{L}(\theta_i = 0.75, \theta_f = 0.25)$	0.2712	0.4989	0.8934	0.3945
$\mathbf{L}(\theta_i = 0.75, \theta_f = 0.5)$	0.3572	0.6300	0.9868	0.3568
$\mathbf{L}(\theta_i = 0.75, \theta_f = 0.75)$	0.4770	0.9895	1.2836	0.2942
$\mathbf{L}(\theta_i = 0.5, \theta_f = 0.25)$	0.1545	0.2138	0.3234	0.1096
$\mathbf{L}(\theta_i = 0.5, \theta_f = 0.5)$	0.2166	0.3508	0.4229	0.0721
$\mathbf{L}(\theta_i = 0.5, \theta_f = 0.75)$	0.2832	0.7267	0.7825	0.0557
$\mathbf{L}(\theta_i = 0.25, \theta_f = 0.25)$	0.0854	0.1238	0.1441	0.0203
$\mathbf{L}(\theta_i = 0.25, \theta_f = 0.5)$	0.1253	0.2594	0.2372	−0.0222
$\mathbf{L}(\theta_i = 0.25, \theta_f = 0.75)$	0.1788	0.6340	0.6064	−0.0276

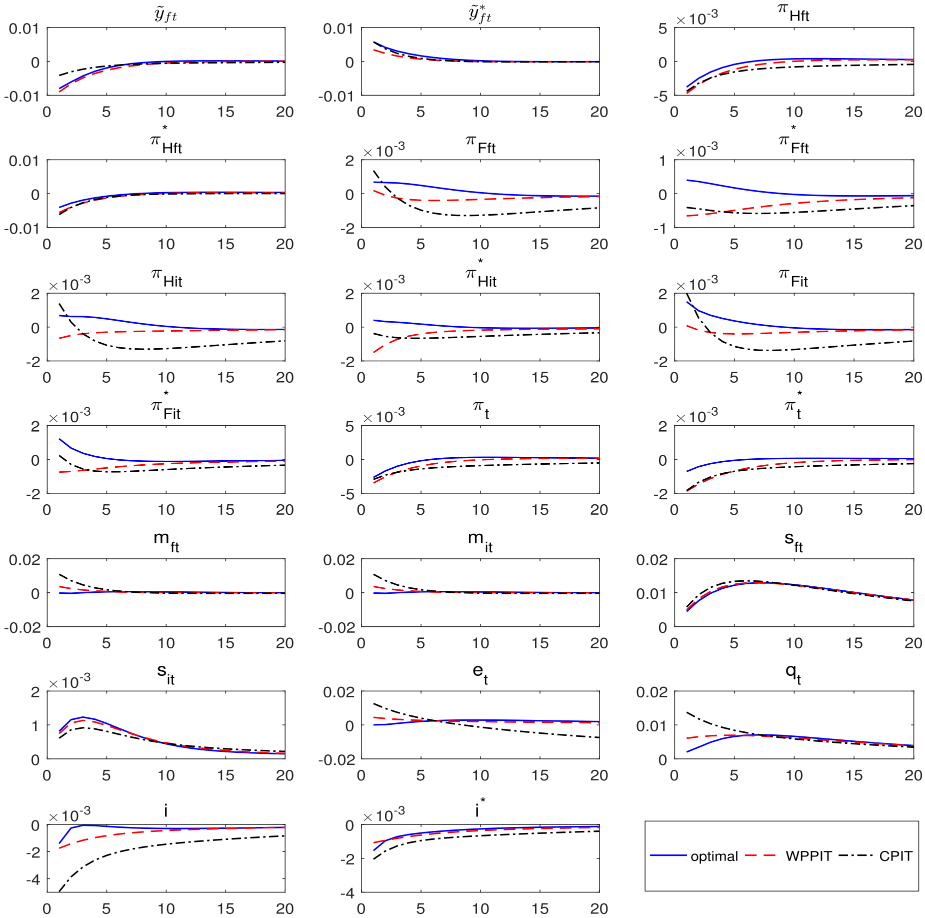

Fig. 2. Impulse responses to a positive productivity shock hitting the home final-goods production sector under alternative monetary policy regimes.

$$\tilde{s}^*_{ft} = -\frac{2\sigma}{D}\tilde{y}^R_{ft} + \frac{(\upsilon - 1)}{D}m_{ft} \tag{65}$$

$$\tilde{s}_{it} = \tilde{s}_{ft} - \tilde{\upsilon}_{Ht} + \tilde{\upsilon}_{Ft}, \tag{66}$$

$$\tilde{s}^*_{it} = \tilde{s}^*_{ft} + \tilde{\upsilon}^*_{Ht} - \tilde{\upsilon}^*_{Ft}. \tag{67}$$

Eqs. (64)–(67) imply that the inefficiencies caused by the horizontal relative price distortion are determined by output gaps, the vertically relative price distortions, and the final-goods currency misalignment. Therefore, when choosing the monetary policy to minimize the joint welfare loss, the monetary policymaker should attach great importance to output gaps, price dispersions, and currency misalignments.

The monetary policymaker trades off among the distortions to try to replicate the flexible-price equilibrium. In response to a positive productivity shock hitting the home final-goods production sector, the monetary policymaker implements the expansionary monetary policy in both countries to boost the global demands for the increased final-goods supply in the home county. Furthermore, the monetary policymaker also desires to change the horizontal relative prices at the final-goods production stage in both countries, with the purpose of directing more demands towards the country experiencing productivity improvement. Under PCP, the monetary policymaker can achieve it by depreciating the nominal exchange rate. By contrast, under LCP, the fluctuation in the nominal exchange rate only has a limited effect on the horizontal relative prices, while it aggravates the currency-misalignment distortion. Thus, compared with the PCP case, the optimal monetary policy under LCP involves the stabilization of the nominal exchange rate, with the result that the currency misalignments at both stages of production are nearly eliminated completely.

Due to the fact that the expenditure-switching effect of the nominal exchange rate is crippled in the LCP case, the degrees of monetary expansion in both countries are different from those in PCP case. Though the monetary policymaker wants to stabilize the nominal exchange rate to alleviate the inefficiencies caused by the currency misalignments, she also wants to produce the depreciation of the nominal exchange rate to adjust the horizontal relative prices at both stages. To achieve the desired depreciation, the degree of the decrease in the nominal interest rate in the home country is slightly lower than its foreign counterpart.

The productivity improvement at the stage of final-goods production in the home country tends to drive down the home local final-goods PPI inflation rate π_{Hft} and the foreign imported final-goods PPI inflation rate π^*_{Hft} by lowering the marginal cost facing home final-goods producers. But, meanwhile, the expansionary monetary policy in both countries has a tendency to drive up π_{Hft} and π^*_{Hft} sluggishly. In the benchmark case, the marginal cost effect dominates the monetary expansion effect so that both π_{Hft} and π^*_{Hft} fall. By contrast, the expansionary monetary policy in both countries drives up the home imported final-goods PPI inflation rate π_{Fft} and the foreign local final-goods PPI inflation rate π^*_{Fft}. Our analysis implies an increase in the horizontal relative price at the stage of final-goods production in the home country.

The drop in π_{Hft} and π^*_{Hft} gains the upper hand over the rise in π_{Fft} and π^*_{Fft} so that the CPI inflation rates π_t and π^*_t fall in both countries. But the home bias in consumption implies that the degree of the decrease in the home CPI level is larger than that in the foreign CPI level. Thus, as in Engle (2011) and Fujiwara and Wang (2017), the optimal monetary policy requires the real exchange rate depreciation.

In the flexible-price equilibrium, the productivity improvement at the stage of final-goods production in the home country has different influences on final-goods output in both countries. The drop in the price of home final goods relative to foreign final goods generates the income effect and the substitution effect in both countries. Both effects move in the same direction such that home consumption rises to meet the increased potential final-goods output. Accordingly, home final-goods output rises. By contrast, in the foreign country, the income effect and the substitution effect move in opposite directions, with the latter being larger than the former in the benchmark model. Unlike what happens in the home country, foreign final-goods output falls.

In the sticky-price equilibrium, the expansionary monetary policy increases the final-goods outputs in both countries. Due to the fact that the optimal monetary policy cannot eliminate the above-mentioned distortions, the home final-goods output in the sticky-price equilibrium falls short of its counterpart in the flexible-price equilibrium, leading to a negative final-goods output gap in the home country. By contrast, the increased final-goods output in the sticky-price equilibrium together with the decreased final-goods output in the flexible-price equilibrium imply a positive final-goods output gap in the foreign country.

The expansionary monetary policy increases the consumption, thus lowering the marginal utility of consumption obtained by providing an additional unit of labor. Given the labor supply, the nominal wages rise to compensate for the utility loss of the real wage in both countries. The increases in the nominal wage tend to drive up the intermediate-goods PPI inflation rates in both countries. In addition, the increase in the foreign final-goods output requires more input of local and imported intermediate goods, which, together with the rising wages, drive up the local and imported intermediate-goods PPI inflation rates π^*_{Fit} and π^*_{Hit} further. The situation is different in the home country. Though final-goods output goes up, the productivity improvement at the stage of final-goods production lowers the demands for local and imported intermediate goods, which tends to drive down the local and imported intermediate-goods PPI inflation rates π_{Hit} and π_{Fit}. But, in our benchmark model, the nominal wage effect dominates the productivity improvement effect so that π_{Hit} and π_{Fit} rise under the optimal monetary policy regime. In the presence of home bias in production, the dampening effect of the productivity improvement on the home intermediate-goods price is larger than that on the imported intermediate-goods price, thus the horizontal relative price at the stage of intermediate-goods production in the home country rises.

Unlike Engel (2011), in the presence of international trade in intermediate inputs, the monetary policymaker should target WPPI inflation rather than CPI inflation, when the degrees of price stickiness at both stages of production are high. The foremost reason is that the monetary policymaker cannot alleviate the newly introduced distortions related to the vertical production and trade by implementing CPIT. The newly introduced distortions include price dispersions and currency misalignment at the stage of intermediate-goods production, and the vertical relative price distortions. By contrast, the monetary policymaker can kill two birds with one stone by implementing WPPIT in the sense that both the newly introduced distortions and price dispersions and currency misalignment at the stage of final-goods production can be alleviated simultaneously, because the final-goods inflation rates are composed of the intermediate-goods inflation rates, not vice versa. In addition, due to the fact that the CPI inflation rates are composed of the final-goods inflation rates, the monetary policymaker can also stabilize the CPI inflation rates by implementing WPPIT to some degree.

In view of the fact that the violation of the law of one price is pervasive and the global supply chain is increasingly important in the sense that more output is produced as intermediate inputs rather than final goods, our monetary policy prescription that the monetary policymaker should target WPPI inflation rather than CPI inflation, when the degrees of price stickiness at both stages of production are high, makes a meaningful addition to the literature on monetary policy in the open economy.

As shown in Fig. 1, when the degree of price stickiness at the intermediate-goods production stage varies from intermediate to high, the monetary policymaker should target WPPI inflation rather than CPI inflation, irrespective of the degree of price stickiness at the final-goods production stage. However, when the degree of price stickiness at the intermediate-goods production stage is low, the monetary policymaker should target CPI inflation rather than WPPI infaltion in most cases, especially when the degree of price stickiness at the final-goods production stage is larger than its counterpart at the intermediate-goods production stage. It means that, in this case, our conclusion is the same as that drawn by Engel (2011) by and large. The reason is that the newly introduced distortions related to the vertical production and trade are less important than those present in Engel (2011) in this case. In addition, it is hard for the monetary policymaker to target WPPI inflation, when the intermediate-goods inflation rates are more volatile than the final-goods inflation rates.

4.2.4. Alternative monetary policy rules

In the above analysis, we follow Taylor (1993)'s initial calibration, and set $\phi_\pi = \phi_\pi^* = 1.5, \phi_y = \phi_y^* = 0.125$.Admittedly, we only show that our conclusion is true for this specific coefficient calibration. It is natural to ask whether our conclusion changes when the coefficients $\phi_\pi, \phi_\pi^*, \phi_y$,and ϕ_y^* are optimally chosen by the monetary policymaker implementing WPPIT or CPIT. That is, the coefficients $\phi_\pi, \phi_\pi^*, \phi_y$,and ϕ_y^* are chosen, for each calibration, to minimize the welfare loss. The choices are completed numerically by searching over a grid spanning the intervals ϕ_π and $\phi_\pi^* \in [1.5, 5]$, and $\phi_y$and $\phi_y^* \in [0, 2]$. The optimal coefficients vary according to the combination of the degrees of price stickiness at both stages of production.

Fig. 3 depicts the welfare losses in three cases: the optimal monetary policy; the optimal WPPIT; the optimal CPIT. As shown in Fig. 3, obviously, the welfare loss from implementing the optimal monetary policy is always lower than those from implementing the optimal WPPIT and the optimal CPIT. The welfare loss comparison between implementing the optimal WPPIT and the optimal CPIT reveals that the monetary policymaker should target WPPI inflation rather than CPI inflation for most combinations of price stickiness at both stages of production. In particular, when the degree of price stickiness at the intermediate-goods production stage is higher than or equal to its counterpart at the final-goods production stage, the optimal WPPIT outperforms the optimal CPIT. Otherwise, when the degree of price stickiness at the intermediate-goods production stage is lower than its counterpart at the final-goods production stage, we cannot give a conclusive result. In this case, the optimal CPIT outperforms the optimal WPPIT, when the degree of price stickiness at the final-goods production stage is high. Thus, roughly speaking, our conclusion drawn in the benchmark case keeps unchanged when the monetary policymaker chooses the optimal monetary policy rules.

Besides the WPPIT and the CPIT, the vertical production and trade structure also permits us to consider a Taylor rule targeting a weighted intermediate goods inflation and final goods inflation (WIFT). To be specific, the WIFT is described as

$$i_t = \rho + \phi_\pi(\omega\pi_{it} + (1-\omega)\pi_{ft}) + \phi_y\widetilde{y}_{ft}, i_t^* = \rho + \phi_\pi^*\left(\omega\pi_{it}^* + (1-\omega)\pi_{ft}^*\right) + \phi_y^*\widetilde{y}_{ft}^* \tag{68}$$

in which

$$\pi_{it} = \phi\pi_{Hit} + (1-\phi)\pi_{Fit}, \pi_{it}^* = \phi\pi_{Fit}^* + (1-\phi)\pi_{Hit}^*$$

$$\pi_{ft} = \frac{v}{2}\pi_{Hft} + \left(1-\frac{v}{2}\right)\pi_{Fft}, \pi_{ft}^* = \frac{v}{2}\pi_{Fft}^* + \left(1-\frac{v}{2}\right)\pi_{Hft}^*$$

Fig. 4 depicts the welfare losses in three cases: the optimal WIFT; the optimal WPPIT; the optimal CPIT.[16] As shown in Fig. 4, the welfare loss from implementing the optimal WIFT is always lower than those from implementing the optimal CPIT and the optimal WPPIT. As for the welfare loss comparison between implementing the optimal WPPIT and the optimal CPIT, we can draw the same conclusion as that obtained in Fig. 3.

Huang and Liu (2005) examine what inflation rate a central bank should target in a closed-economy New Keynesian model in which there are two stages of production, and find that a simple hybrid rule under which the interest rate responds to inflation rates at both stages of production outperforms any other rule. We can arrive at a similar conclusion in a two-country New Keynesian model with local-currency pricing.

Fig. 5 shows how the weight of inflation in the optimal simple WIFT changes when the degree of price stickiness changes. As shown in the left and middle panels of Fig. 5, the relative weight of intermediate-goods inflation to final-goods inflation rises, when the degree of relative price stickiness of the intermediate-goods production stage to the final-goods production stage goes up. The right panel of Fig. 5 shows that the weight of final-goods inflation rises as the degree of price stickiness at

[16] Note that the weight ω is also chosen optimally.

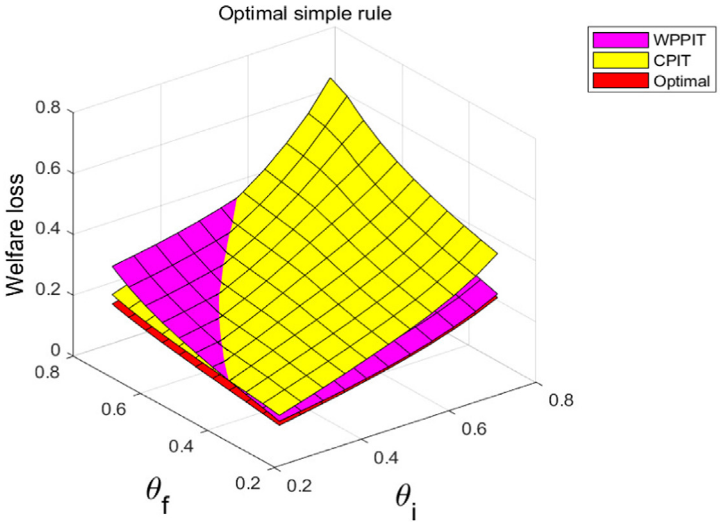

Fig. 3. The welfare losses in three cases: the optimal monetary policy; the optimal simple WPPIT; the optimal simple CPIT.

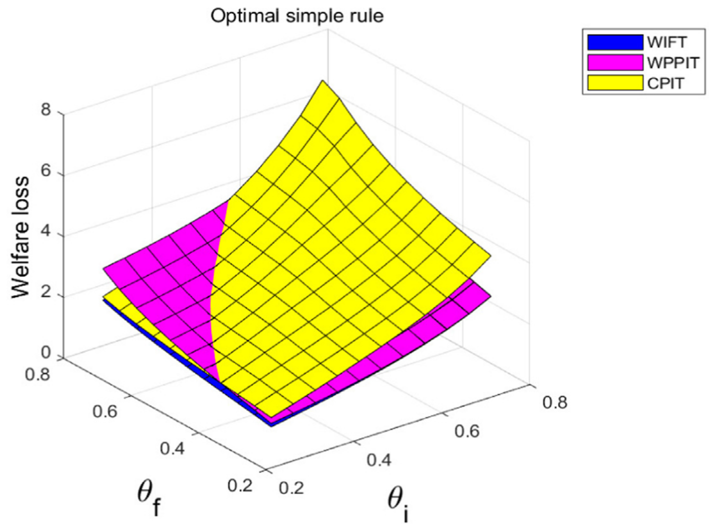

Fig. 4. The welfare losses in three cases: the optimal simple WIFT; the optimal simple WPPIT; the optimal simple CPIT.

the stage of final-goods production increases. The results reported in Fig. 5 imply that the monetary policymaker in each country should give greater weight to inflation at the stage in which prices are more sticky.

4.3. A comparison between LCP and PCP

Gong et al. (2016) examine the monetary policy in a two-country New Keynesian model with international trade in intermediate inputs under PCP. To evaluate various monetary policy rules, Gong et al. (2016) use the welfare loss under the optimal monetary policy as a benchmark to rank three alternative Taylor-type monetary rules: a CPI-based Taylor rule; a final-goods PPI-based Taylor rule; and an intermediate-goods PPI-based Taylor rule. In this case, Gong et al. (2016) find that a cooperative monetary policymaker should target the intermediate-goods price inflation rates, the final-goods price inflation rates, the final-goods output gaps, and the vertical relative price gaps, the last of which is specific to the vertical production and trade structure.

By introducing LCP into Clarida et al. (2002), Engel (2011) examines the difference in optimal monetary policy between PCP and LCP, and finds that the optimal monetary policy should target consumer price inflation, the output gap, and the currency misalignment. In particular, the distortion related to LCP, currency misalignments, becomes a separate source of inefficiency.

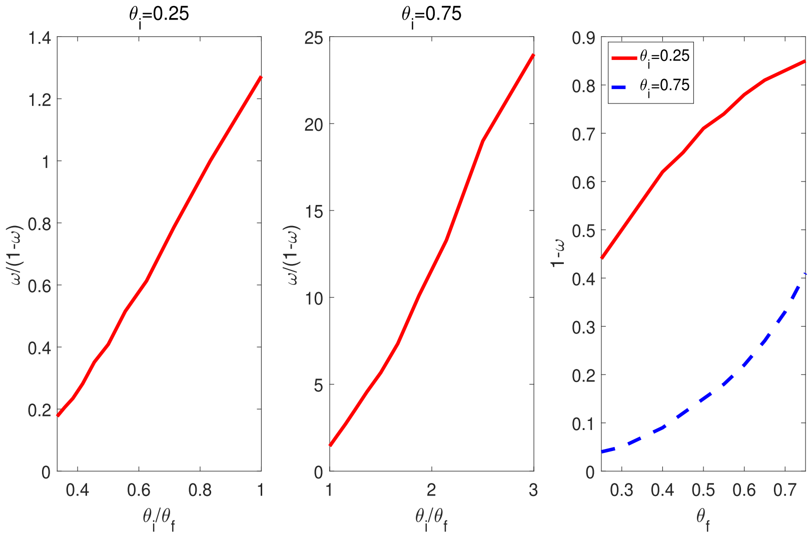

Fig. 5. How the weight of inflation in the optimal simple WIFT changes when the degree of price stickiness changes.

In this paper, we introduce international trade in intermediate inputs into Engel (2011) to examine the implications of the interaction between currency misalignments and the vertical production and trade structure for the monetary policy prescription. As in Engel (2011), the presence of currency misalignments distorts the horizontal relative prices at the stage of final-goods production and relative consumption. From equations (64), (65), (34),and (35), we know

$$\widetilde{s}_{ft} = \frac{2\sigma}{D}\tilde{y}_{ft}^R - \frac{(\upsilon-1)}{D}m_{ft}, \tag{64}$$

$$\widetilde{s}_{ft}^* = -\frac{2\sigma}{D}\tilde{y}_{ft}^R + \frac{(\upsilon-1)}{D}m_{ft} \tag{65}$$

$$\widetilde{c}_t^R = \frac{(\upsilon-1)}{D}\tilde{y}_{ft}^R + \frac{\upsilon(2-\upsilon)}{2D}m_{ft}. \tag{69}$$

Thus, under PCP $(m_{ft}=0)$, the horizontal relative prices at the stage of final-goods production and relative consumption are efficient, if both home and foreign final-goods output gaps are zero. By contrast, under LCP $(m_{ft}\neq 0)$, the horizontal relative prices at the stage of final-goods production and relative consumption differ from their efficient levels, even if both home and foreign final-goods output gaps are eliminated. Equation (69)implies that, under LCP, there exists a misallocation of consumption between home and foreign countries, even if distortions from home and foreign final-goods output gaps are eliminated.

After we introduce international trade in intermediate inputs into Engel (2011), there are currency misalignments at both stages of production. In addition, the vertical relative price gaps also become a source of welfare loss. From equations (66)and (67), we know that

$$\widetilde{s}_{it} = \widetilde{s}_{ft} - \tilde{\upsilon}_{Ht} + \tilde{\upsilon}_{Ft}, \tag{66}$$

$$\widetilde{s}_{it}^* = \widetilde{s}_{ft}^* + \tilde{\upsilon}_{Ht}^* - \tilde{\upsilon}_{Ft}^*. \tag{67}$$

Equations (66)and (67)imply that, under PCP $(m_{ft}=0)$, the horizontal relative prices at the stage of intermediate-goods production are inefficient, even if both home and foreign final-goods output gaps are eliminated. The reason is that the vertical relative price gaps distort the horizontal relative prices at the stage of intermediate-goods production. Under LCP $(m_{ft}\neq 0)$,compared to what happens under PCP, the distortions from the horizontal relative prices at the stage of intermediate-goods production are aggravated. The reason is that, in addition to the vertical relative price gaps, currency misalignment at the stage of final-goods production also distorts the horizontal relative prices at the stage of intermediate-goods production via the horizontal relative prices at the stage of final-goods production in this case.

Currency misalignment at the stage of final-goods production distorts relative consumption. By contrast, Currency misalignments at both stages of production, together with the vertical relative price gaps, distort production. To see how this happens, we make a difference between foreign and home final-goods marginal costs,

$$e_t + mc_{ft}^* - mc_{ft} = m_{it} - (1-2\phi)s_{it} - a_{ft}^* + a_{ft}. \tag{70}$$

Under PCP $(m_{ft} = m_{it} = 0)$, the difference between foreign and home final-goods marginal costs depends on productivity shocks at the stage of final-goods production and the horizontal relative price at the stage of intermediate-goods production. Note that the horizontal relative price at the stage of intermediate-goods production is determined by the vertical relative prices which are not efficient in the presence of international trade in intermediate inputs. As a result, the difference between foreign and home final-goods marginal costs is not efficient anymore. It implies that relative production is not efficient in comparison with the case with no international trade in intermediate inputs.

By contrast, under LCP $(m_{ft} \neq 0, m_{it} \neq 0)$, in addition to the factors affecting the difference between foreign and home final-goods marginal costs under PCP, currency misalignments at both stages of production also play a role in affecting the difference. To be specific, currency misalignment at the stage of intermediate-goods production affects the difference directly, while currency misalignment at the stage of final-goods production affects it via the horizontal relative price at the stage of intermediate-goods production. Due to the fact that currency misalignments at both stages of production and the vertical relative prices are inefficient, relative production is distorted further compared to what happens under PCP.

To better understand the role of LCP in the presence of international trade in intermediate inputs, we compare dynamics under PCP with that under LCP. Fig. 6 depicts the impulse responses of the main variables to a positive productivity shock hitting the home final-goods production sector for PCP and LCP cases. One main difference between PCP and LCP is that the expenditure-switching effect of the nominal exchange rate is crippled in LCP case, while it functions effectively in PCP case. When a positive productivity shock hitting the home final-goods production sector occurs, the cooperative monetary policymaker implements the expansionary monetary policy in both countries to boost the global demands for the increased final-goods supply in the home county. Under PCP, the monetary policymaker depreciates the nominal exchange rate to direct more demands towards the home country. By contrast, under LCP, the nominal exchange rate only plays a limited role in affecting the horizontal relative prices in both countries. On the contrary, the fluctuation in the nominal exchange rate aggravates the currency-misalignment distortion. Therefore, the degree of the depreciation in the nominal exchange rate is lower under LCP than that under PCP. In the presence of price stickiness at both stages of production, the real exchange rate also displays the same pattern as the nominal exchange rate. That is, the degree of the depreciation in the real exchange rate is lower under LCP than that under PCP.

As explained previously, in the flexible-price equilibrium, home final-goods output rises following a positive productivity shock hitting the home final-goods production sector, whereas foreign final-goods output falls. In the sticky-price equilibrium, under PCP, even if the expenditure-switching effect of the nominal exchange rate directs the global final-goods demands towards the home country, the optimal monetary policy cannot eliminate the distortions completely. Accordingly, the increase in the home final-goods output in the sticky-price equilibrium falls short of its counterpart in the flexible-price equilibrium, implying a negative home final-goods output gap. By contrast, though the expenditure-switching effect of the nominal exchange rate directs the global final-goods demands away from the foreign country, the effect of the expansionary monetary policy on foreign final-goods output exceeds the expenditure-switching effect of the nominal exchange rate, with the result that the foreign final-goods output goes up. The increased foreign final-goods output in the sticky-price equilibrium and the decreased final-goods output in the flexible-price equilibrium bring about a positive foreign final-goods output gap.

Under LCP, the expenditure-switching effect of the nominal exchange rate is impaired. It means that the home final-goods output is lower than that under PCP, while the opposite is true for the foreign country. Therefore, the output gap is larger under LCP than that under PCP in each country.

After a positive productivity shock occurs at the stage of final-goods production in the home country, the home local final-goods PPI inflation rate π_{Hft} declines. However, weaker demand for home local final goods under LCP implies that the degree of reduction in π_{Hft} is greater under LCP than that under PCP. The demand of home households for imported goods increases in both PCP and LCP cases, driving up the home imported final-goods PPI inflation rate π_{Fft}. The fact that the degree of the depreciation in the nominal exchange rate is greater under PCP than that under LCP implies that the degree of increase in π_{Fft} is greater under PCP than that under LCP. Our analysis implies that the horizontal relative price at the stage of final-goods production s_{ft} increases in both PCP and LCP cases. Since the degree of the depreciation in the nominal exchange rate is greater, the degree of increase in s_{ft} is greater under PCP than that under LCP. The fact that the home CPI inflation rate π_t is composed of π_{Hft} and π_{Fft}, and the home bias in consumption implies that π_{Hft} has a greater effect on π_t than π_{Fft},thus the responses of π_t are similar to those of π_{Hft}.

The rise in home final-goods output causes the home final-goods producers to increase the demands for local and imported intermediate goods, thus driving up their prices. Since the home final-goods producers need to produce more output under PCP than that under LCP, the local and imported intermediate-goods PPI inflation rates π_{Hit} and π_{Fit} in the home country are higher under PCP than their corresponding values under LCP. Similar to what happens at the stage of final-goods production, the degree of increase in s_{it} is greater under PCP than that under LCP, resulting from a greater degree of the depreciation in the nominal exchange rate. In the flexible-price equilibrium, the home vertical relative price goes up, after a positive productivity shock occurs at the stage of final-goods production in the home country.[17] Since the optimal monetary policy cannot eliminate the distortions completely in both PCP and LCP cases, the home vertical relative price gap is negative.

[17] Please refer to the Appendix to Gong et al. (2016) to find the proof.

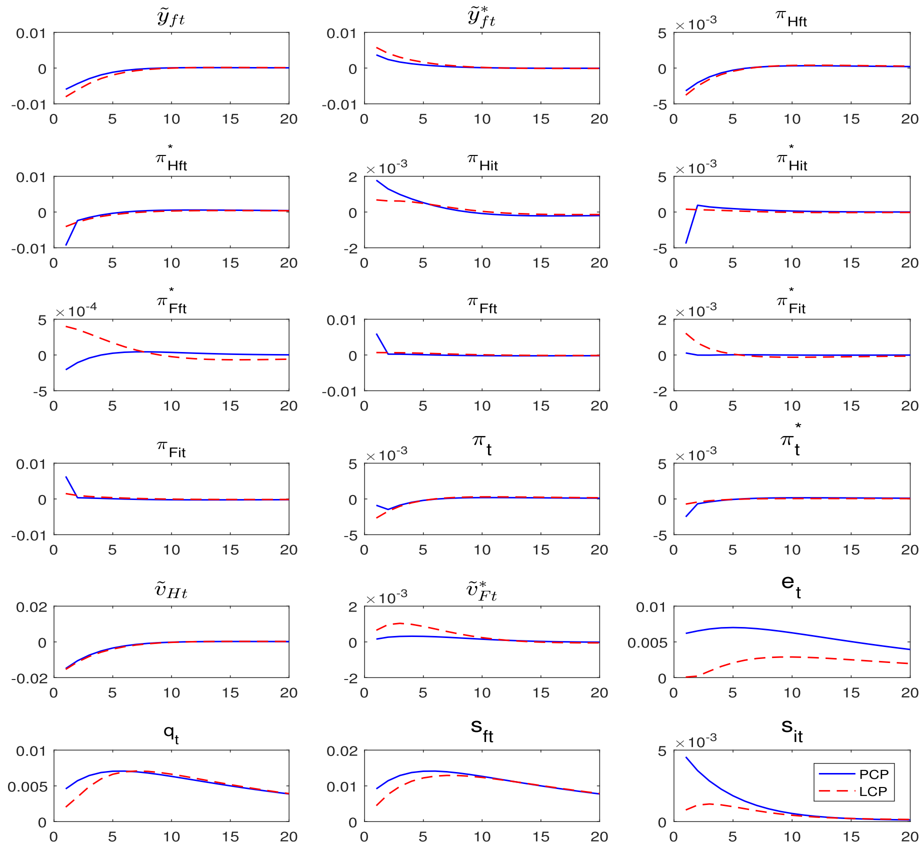

Fig. 6. Impulse responses to a positive productivity shock hitting the home final-goods production sector under PCP and LCP.

The previous quantitative analysis of the responses of π_{Hit} and π_{Hft} shows that the home vertical relative price gap under PCP is almost the same as that under LCP.

The productivity improvement at the stage of final-goods production in the home country tends to drive down the foreign imported final-goods PPI inflation rate π^*_{Hft},while the expansionary monetary policy in both countries has the opposite effects on π^*_{Hft}.Our quantitative analysis shows that the former effect is larger than the latter, with the result that π^*_{Hft} decreases. A greater degree of the depreciation in the nominal exchange rate leads to the degree of reduction in π^*_{Hft} being greater under PCP than that under LCP. By contrast, the expansionary monetary policy in both countries tends to drive up the foreign local final-goods PPI inflation rate π^*_{Fft}, but the expenditure-switching effect of the nominal exchange rate depresses π^*_{Fft}. Our quantitative analysis shows that the latter effect is greater than the former, and π^*_{Fft} decreases under PCP. On the contrary, because the expenditure-switching effect of the nominal exchange rate has a limited role in adjusting the horizontal relative prices under LCP, the former effect is greater than the latter, resulting in an increase in π^*_{Fft}.

In the sticky-price equilibrium, foreign final-goods output rises, thus increasing the demands for local and imported intermediate inputs. The fact that the foreign final-goods output is higher under LCP than that under PCP implies that the local and imported intermediate-goods PPI inflation rates π^*_{Fit} and π^*_{Hit} are higher under LCP than their corresponding values under PCP. In particular, since the degree of the depreciation in the nominal exchange rate is greater under PCP than that under LCP, π^*_{Hit} decreases initially under PCP when the shock occurs. The previous analysis shows that foreign CPI inflation rate π^*_t decreases under PCP, while it is hard to determine whether π^*_t goes up or down under LCP. Our quantitative analysis shows that π^*_t goes down under LCP.

In the flexible-price equilibrium, the foreign vertical relative price goes down, after a positive productivity shock occurs at the stage of final-goods production in the home country.[18] Our quantitative analysis of the responses of π^*_{Hit} and π^*_{Fit} shows that the foreign vertical relative price gap is positive in both PCP and LCP cases, and it is greater under LCP than that under PCP.

[18] Please refer to Appendix to Gong et al. (2016) for the proof.

4.4. Sensitivity analysis

In this section, we perform the sensitivity analysis to show that our conclusion is robust to two key parameters: the degree of trade openness at the stage of intermediate-goods production and its counterpart at the stage of final-goods production. In the benchmark model, we set the degree of trade openness at the stage of intermediate-goods production $1-\phi$ to 0.33 and its counterpart at the stage of final-goods production $1-\frac{\nu}{2}$ to 0.25, respectively. To examine whether our conclusion depends on the calibrated values, we calculate the respective welfare losses from targeting WPPI inflation and CPI inflation and the corresponding welfare gain from targeting WPPI inflation rather than CPI inflation when $1-\phi$ and $1-\frac{\nu}{2}$ vary within reasonable ranges. In Fig. 7, the red line with crosses represents the welfare loss from targeting CPI inflation, while the blue line with stars represents the welfare loss from targeting WPPI inflation.

As shown in Fig. 7, at each stage of production, when the degree of trade openness increases from 0.05 to 0.45, the welfare loss from targeting WPPI inflation is always lower than that from targeting CPI inflation, thus our conclusion that WPPIT outperforms CPIT, when the degrees of price stickiness at both stages of production are high, is robust to the degrees of trade openness at both stages of production. In Fig. 7, the black line with circles represents the welfare gain from targeting WPPI inflation rather than CPI inflation. The observation of Fig. 7 reveals that the welfare gain from targeting WPPI inflation rather than CPI inflation increases with the degree of trade openness at the stage of intermediate-goods production, while it decreases with the degree of trade openness at the stage of final-goods production.

The main difference between Engel (2011) and our paper is international trade in intermediate inputs. If international trade in intermediate inputs is absent, the expenditure share of the final-goods producer on domestic composite intermediate goods is one, and there is no nominal stickiness at the stage of intermediate-goods production, our model is identical to Engel (2011), in which the monetary policymaker should target CPI inflation. [19]With the decrease in the degree of trade openness at the stage of intermediate-goods production, Engel (2011)'s mechanism tends to be stronger, and thus the welfare gain from targeting WPPI inflation rather than CPI inflation becomes smaller. Equivalently, with the increase in the degree of trade openness at the stage of intermediate-goods production, our model's mechanism tends to be stronger, and thus the welfare gain from targeting WPPI inflation rather than CPI inflation becomes larger.

After introducing local-currency pricing and home bias in consumption into Clarida et al. (2002), Engel (2011) concludes that the monetary policymaker should target CPI inflation rather than PPI inflation as in Clarida et al. (2002). However, when the degree of trade openness becomes smaller, the welfare gain from targeting CPI inflation rather than PPI inflation tends to be smaller as a result of the fact that PPI increasingly approaches CPI. By contrast, when the economy is more open, the distortion related to currency misalignments matters, and thus the welfare gain from targeting CPI inflation rather than PPI inflation becomes larger. Likely, in our model, with the degree of trade openness at the stage of intermediate-goods production being set to the calibrated value, Engel (2011)'s mechanism tends to be stronger when the degree of trade openness at the stage of final-goods production becomes larger. It implies that the welfare gain from targeting WPPI inflation rather than CPI inflation becomes smaller.

5. Conclusion

This paper examines the implications of international trade in intermediate inputs for the monetary policy in a two-country New Keynesian model with local-currency pricing *à la* Engel (2011). The welfare loss function shows that the cooperative monetary policymaker should pay attention to the output gaps, the PPI inflation rates at both stages of production, the currency misalignments at both stages of production, and the vertical relative price gaps.

The main conclusion in Engel (2011) is that the monetary policymaker should target CPI inflation rather than PPI inflation. Unlike Engel (2011), we cannot give an analytical solution to the dynamic system describing the optimal monetary policy. Instead, we compute the welfare loss of the optimal monetary policy quantitatively, and use it to evaluate two different Taylor-type monetary policy rules: the weighted average final-goods PPI inflation-based Taylor rule (CPIT) and the weighted average intermediate-goods PPI inflation-based Taylor rule (WPPIT). Different from Engel (2011), we find that the monetary policymaker should target WPPI inflation rather than CPI inflation for most combinations of price stickiness at both stages of production.

To be specific, when the degree of price stickiness at the intermediate-goods production stage varies from intermediate to high, the monetary policymaker should target WPPI inflation rather than CPI inflation. By contrast, when the degree of price stickiness at the intermediate-goods production stage varies from low to intermediate, the results of welfare comparison between targeting WPPI inflation and CPI inflation are mixed. Thus it is hard to determine which inflation rates the monetary policymaker should target. However, when the degree of price stickiness at the intermediate-goods production stage is low, the welfare loss from targeting WPPI inflation is greater than that from targeting CPI inflation in most cases, especially when the degree of price stickiness at the final-goods production stage is higher than its counterpart at the intermediate-goods production stage. It means that, when the degree of price stickiness at the intermediate-goods production stage is low, the monetary policymaker should target CPI inflation rather than WPPI inflation in most cases.

[19] In this case, the composite intermediate goods should be interpreted as labor.

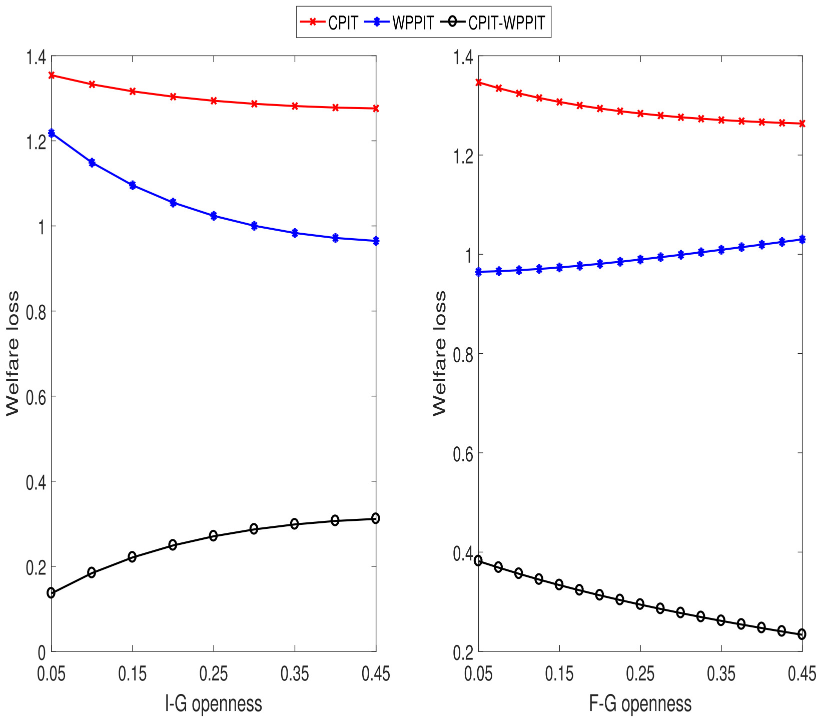

Fig. 7. Sensitivity analysis.

The reason for targeting WPPI inflation rather than CPI inflation for most combinations of price stickiness at both stages of production is that the monetary policymaker cannot reduce the newly introduced distortions related to the vertical production and trade, price dispersion and currency misalignment at the stage of intermediate-goods production, and the vertical relative price distortions, by implementing CPIT. However, due to the fact that the CPI inflation rates are composed of the final-goods inflation rates, which then are composed of the intermediate-goods inflation rates, not vice versa, the monetary policymaker can alleviate the newly introduced distortions and those present in Engel (2011) simultaneously by implementing WPPIT.

The reason for targeting CPI inflation rather than WPPI inflation in most cases, when the degree of price stickiness at the intermediate-goods production stage is low, is that the newly introduced distortions related to the vertical production and trade are less important than those present in Engel (2011) in this case. In addition, it is hard for the monetary policymaker to target WPPI inflation, when the intermediate-goods inflation rates are more volatile than the final-goods inflation rates.

In addition, we also consider whether our conclusion is still true when the monetary policymaker chooses the response coefficients of the nominal interest rates optimally. Roughly speaking, our conclusion drawn previously keeps unchanged when the monetary policymaker chooses the optimal monetary policy rules. By contrast, when the monetary policymaker adopts a Taylor rule targeting a weighted intermediate goods inflation and final goods inflation(WIFT). we find that the welfare loss from implementing the optimal WIFT is always lower than those from implementing the optimal CPIT and the optimal WPPIT.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

1. Derivation of the DIS equations

From the final-goods market clearing conditions in both countries

$$y_{ft} = \frac{v}{2}c_t + \left(1-\frac{v}{2}\right)c_t^* + \frac{v}{2}\left(1-\frac{v}{2}\right)\left(s_{ft} - s_{ft}^*\right), \tag{A1}$$

$$y_{ft}^* = \left(1-\frac{v}{2}\right)c_t + \frac{v}{2}c_t^* - \frac{v}{2}\left(1-\frac{v}{2}\right)\left(s_{ft} - s_{ft}^*\right), \tag{A2}$$

and the risk-sharing condition

$$\sigma\left(c_t - c_t^*\right) = m_{ft} + \frac{(\upsilon-1)}{2}\left(s_{ft} - s_{ft}^*\right), \tag{A3}$$

we can express c_t and c_t^* in terms of y_{ft}^R, y_{ft}^W, and m_{ft}

$$c_t = \frac{v-1}{D}y_{ft}^R + y_{ft}^W + \frac{v(2-v)}{2D}m_{ft}, \tag{A4}$$

$$c_t^* = -\frac{v-1}{D}y_{ft}^R + y_{ft}^W - \frac{v(2-v)}{2D}m_{ft}, \tag{A5}$$

in which $D = \sigma v(2-v) + (\upsilon-1)^2$.

From equations (A4) and (A5), we have

$$c_t^R = \frac{v-1}{D}y_{ft}^R + \frac{v(2-v)}{2D}m_{ft} \tag{A6}$$

$$c_t^W = y_{ft}^W \tag{A7}$$

Solving for the horizontal relative prices at the stage of final-goods production in both countries, we obtain

$$s_{ft} = \frac{2\sigma}{D}y_{ft}^R - \frac{(\upsilon-1)}{D}m_{ft} + z_{ft}, \tag{A8}$$

$$s_{ft}^* = -\frac{2\sigma}{D}y_{ft}^R + \frac{(\upsilon-1)}{D}m_{ft} + z_{ft}, \tag{A9}$$

Loglinearizing the Euler equation yields

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbf{E}_t\pi_{ft+1}\right) \tag{A10}$$

Substituting equation (A4) into equation (A10) and using $\pi_{ft} = \pi_{Hft} + \left(1-\frac{v}{2}\right)\Delta s_{ft}$,[20] we have

$$\begin{aligned}&\frac{v-1}{D}y_{ft}^R + y_{ft}^W + \frac{v(2-v)}{2D}m_{ft}\\ &\quad = \mathbf{E}_t\left\{\frac{v-1}{D}y_{ft+1}^R + y_{ft+1}^W + \frac{v(2-v)}{2D}m_{ft+1}\right\} - \frac{1}{\sigma}\left(i_t - \mathbf{E}_t\pi_{Hft+1}\right) + \frac{2-v}{2\sigma}\mathbf{E}_t\Delta s_{ft+1}\end{aligned} \tag{A11}$$

Substituting equation (A8) into equation (A11) and rearranging the resulting equation, we can obtain

$$y_{ft}^R = y_{ft+1}^R + D\mathbf{E}_t\Delta y_{ft+1}^W + \frac{D+1-v}{2\sigma}\mathbf{E}_t\Delta m_{ft+1} + \frac{D(2-v)}{2\sigma}\mathbf{E}_t\Delta z_{ft+1} - \frac{D}{\sigma}\left(i_t - \mathbf{E}_t\pi_{Hft+1}\right) \tag{A12}$$

Since $x_t^R = \frac{x_t - x_t^*}{2}, x_t^W = \frac{x_t + x_t^*}{2}, \Delta x_t = x_t - x_{t-1}$,the above equation can be written as

$$\begin{aligned}\frac{\tilde{y}_{ft} - \tilde{y}_{ft}^*}{2} &= \mathbf{E}_t\left\{\frac{\tilde{y}_{ft+1} - \tilde{y}_{ft+1}^*}{2}\right\} + \mathbf{E}_t\left\{\frac{\Delta\bar{y}_{ft+1} - \Delta\bar{y}_{ft+1}^*}{2}\right\} + D\mathbf{E}_t\left\{\frac{\Delta y_{ft+1} + \Delta y_{ft+1}^*}{2}\right\}\\ &\quad + \frac{D+1-v}{2\sigma}\mathbf{E}_t\Delta m_{ft+1} + \frac{D(2-v)}{2\sigma}\mathbf{E}_t\Delta z_{ft+1} - \frac{D}{\sigma}\left(i_t - \mathbf{E}_t\pi_{Hft+1}\right)\end{aligned} \tag{A13}$$

Rearranging equation (A13), we have

$$\tilde{y}_{ft} = \mathbf{E}_t\tilde{y}_{ft+1} - \frac{2D}{\sigma(1+D)}\left(i_t - \mathbf{E}_t\pi_{Hft+1} - \overline{rr}_t\right) \tag{A14}$$

in which

[20] In which $\Delta s_{ft} = s_{ft} - s_{ft-1}$.

$$\overline{rr}_t = \frac{\sigma(D-1)}{2D}\mathbf{E}_t\Delta y^*_{ft+1} + \frac{\sigma(1+D)}{2D}\mathbf{E}_t\Delta\bar{y}_{ft+1} + \left(\frac{1}{2} - \frac{\nu-1}{2D}\right)\mathbf{E}_t\Delta m_{ft+1} + \left(1-\frac{\nu}{2}\right)\mathbf{E}_t\Delta z_{ft+1}$$

Similarly, the foreign DIS equation is

$$\tilde{y}^*_{ft} = \tilde{y}^*_{ft+1} - \frac{2D}{\sigma(1+D)}\left(i^*_t - \mathbf{E}_t\pi^*_{Fft+1} - \overline{rr}^*_t\right), \tag{A15}$$

in which

$$\overline{rr}^*_t = \frac{\sigma(D-1)}{2D}\mathbf{E}_t\Delta y_{ft+1} + \frac{\sigma(1+D)}{2D}\mathbf{E}_t\Delta\bar{y}^*_{ft+1} + \left(\frac{\nu-1}{2D} - \frac{1}{2}\right)\mathbf{E}_t\Delta m_{ft+1} + \left(1-\frac{\nu}{2}\right)\mathbf{E}_t\Delta z_{ft+1}.$$

2. Derivation of the New-Keynesian Phillips Curves

2.1 Derivation of the home New-Keynesian Phillips Curves at the stage of intermediate-goods production

2.1.1 Solving for p^0_{Hit} and p^{*0}_{Hit}

Solving the optimal pricing problem facing home intermediate-goods firm $j_i \in [0,1]$,

$$\max_{P^0_{Hit}(j_i),P^{*0}_{Hit}(j_i)} \mathbf{E}_t\sum_{j=0}^{\infty}\theta_i^j Q_{t,t+j}\left\{\begin{array}{c}(1+\tau_i)P^0_{Hit}(j_i)Y_{Hi,t+j}(j_i) + (1+\tau_i)P^{*0}_{Hit}(j_i)Y^*_{Hi,t+j}(j_i)\\ -MC_{it+j}\left(Y_{Hi,t+j}(j_i) + Y^*_{Hi,t+j}(j_i)\right)\end{array}\right\}$$

in which

$$Y_{Hi,t+j}(j_i) = \left(\frac{P^0_{Hit}(j_i)}{P_{Hit+j}}\right)^{-\xi_i} Y_{Hi,t+j} = \phi\left(\frac{P^0_{Hit}(j_i)}{P_{Hit+j}}\right)^{-\xi_i}\frac{MC_{f,t+j}}{P_{Hit+j}}\int_0^1 Y_{f,t+j}(j_f)dj_f,$$

$$Y^*_{Hi,t+j}(j_i) = \left(\frac{P^{*0}_{Hit}(j_i)}{P^*_{Hit+j}}\right)^{-\xi_i} Y^*_{Hi,t+j} = (1-\phi)\left(\frac{P^{*0}_{Hit}(j_i)}{P^*_{Hit+j}}\right)^{-\xi_i}\frac{MC^*_{f,t+j}}{P^*_{Hit+j}}\int_0^1 Y^*_{f,t+j}\left(j^*_f\right)dj^*_f,$$

$$MC_{it+j} = \frac{W_{t+j}}{A_{it+j}}.$$

The first-order conditions are,

$$\mathbf{E}_t\sum_{j=0}^{\infty}\theta_i^j Q_{t,t+j}Y_{Hi,t+j}(j_i)\left[(1+\tau_i)(1-\xi_i)P^0_{Hit}(j_i) + \xi_i MC_{it+j}\right] = 0,$$

$$\mathbf{E}_t\sum_{j=0}^{\infty}\theta_i^j Q_{t,t+j}Y^*_{Hi,t+j}(j_i)\left[(1+\tau_i)(1-\xi_i)E_{t+j}P^{*0}_{Hit}(j_i) + \xi_i MC_{it+j}\right] = 0,$$

in which E_{t+j} is the nominal exchange rate at time $t+j$.

Thus, we can solve for $P^0_{Hit}(j_i)$ and $P^{*0}_{Hit}(j_i)$

$$P^0_{Hit}(j_i) = \frac{\xi_i}{(\xi_i-1)(1+\tau_i)}\frac{\mathbf{E}_t\sum_{j=0}^{\infty}\theta_i^j Q_{t,t+j}Y_{Hi,t+j}(j_i)MC_{it+j}}{\mathbf{E}_t\sum_{j=0}^{\infty}\theta_i^j Q_{t,t+j}Y_{Hi,t+j}(j_i)} \tag{A16}$$

$$P^{*0}_{Hit}(j_i) = \frac{\xi_i}{(\xi_i-1)(1+\tau_i)}\frac{\mathbf{E}_t\sum_{j=0}^{\infty}\theta_i^j Q_{t,t+j}Y^*_{Hi,t+j}(j_i)MC_{it+j}}{\mathbf{E}_t\sum_{j=0}^{\infty}\theta_i^j Q_{t,t+j}E_{t+j}Y^*_{Hi,t+j}(j_i)} \tag{A17}$$

Using the expressions $P_{Hit} = \left[\int_0^1 P_{Hit}(j_i)^{1-\xi_i}dj_i\right]^{\frac{1}{1-\xi_i}}, P^*_{Hit} = \left[\int_0^1 P^*_{Hit}(j_i)^{1-\xi_i}dj_i\right]^{\frac{1}{1-\xi_i}}$,and the fact that all home intermediate-goods firms resetting prices will choose an identical price, we have

$$P_{Hit} = \left[\theta P^{1-\xi_i}_{Hit-1} + (1-\theta)\left(P^0_{Hit}\right)^{1-\xi_i}\right]^{\frac{1}{1-\xi_i}}$$

$$P^*_{Hit} = \left[\theta P^{*1-\xi_i}_{Hit-1} + (1-\theta)\left(P^{*0}_{Hit}\right)^{1-\xi_i}\right]^{\frac{1}{1-\xi_i}}$$

A first-order Taylor expansion of equations ($A16$)and ($A17$), around the zero inflation steady state yields

$$p^0_{Hit} = (1-\beta\theta_i)\sum_{j=0}^{\infty}(\beta\theta_i)^j \mathbf{E}_t\{mc_{Hit+j}\}$$

and

$$p^{*0}_{Hit} = (1-\beta\theta_i)\sum_{j=0}^{\infty}(\beta\theta_i)^j \mathbf{E}_t\{mc_{Hit+j} - e_{t+j}\}.$$

2.1.2 Solving for π_{Hit} and π^*_{Hit}

Following the standard procedures adopted in the literature, we can obtain

$$\pi_{Hit} = \beta \mathbf{E}_t \pi_{Hit+1} + \delta_i mc^r_{Hit}, \tag{A18}$$

$$\pi^*_{Hit} = \beta \mathbf{E}_t \pi^*_{Hit+1} + \delta_i mc^{r*}_{Hit} \tag{A19}$$

where $\delta_i = \frac{(1-\beta)(1-\beta\theta_i)}{\theta_i}$, $mc^r_{Hit} = mc_{it+j} - p_{Hit} = w_t - p_{Hit} - a_{it}$, and $mc^{r*}_{Hit} = mc_{it+j} - p^*_{Hit} - e_{t+j}$.

Using $\frac{W_t}{P_{ft}} = C^\sigma_t N^\varphi_t$ and $P_{ft} = \kappa^{-1} P^{\frac{\nu}{2}}_{Hft} P^{1-\frac{\nu}{2}}_{Fft} = \kappa^{-1} P_{Hft} S^{1-\frac{\nu}{2}}_{ft}$, we have

$$\frac{W_t}{P_{Hft}} = \kappa^{-1} S^{1-\frac{\nu}{2}}_{ft} C^\sigma_t N^\varphi_t$$

Log-linearizing the above equation yields

$$w_t - p_{Hft} = \sigma c_t + \varphi n_t + \left(1 - \frac{\nu}{2}\right) s_{ft}$$

which implies that

$$\begin{aligned} w_t - p_{Hit} &= p_{Hft} - p_{Hit} + \sigma c_t + \varphi n_t + \left(1 - \tfrac{\nu}{2}\right) s_{ft} \\ &= \sigma c_t + \varphi n_t + \left(1 - \tfrac{\nu}{2}\right) s_{ft} - \upsilon_{Ht}. \end{aligned}$$

Log-linearizing labor market clearing conditions yields

$$\begin{aligned} n_t = {} & \phi(1-\phi)\left(\upsilon_{Ft} - \upsilon_{Ht} - \upsilon^*_{Ht} + \upsilon^*_{Ft}\right) + \left(\tfrac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right) y^R_{ft} + y^W_{ft} \\ & -\phi(1-\phi)\tfrac{2(\nu-1)}{D} m_{ft} - \phi a_{ft} - (1-\phi) a^*_{ft} - a_{it} \end{aligned} \tag{A20}$$

and

$$\begin{aligned} n^*_t = {} & -\phi(1-\phi)\left(\upsilon_{Ft} - \upsilon_{Ht} - \upsilon^*_{Ht} + \upsilon^*_{Ft}\right) + \left(-\tfrac{4\sigma\phi(1-\phi)}{D} + 1 - 2\phi\right) y^R_{ft} \\ & + y^W_{ft} + \phi(1-\phi)\tfrac{2(\nu-1)}{D} m_{ft} - (1-\phi) a_{ft} - \phi a^*_{ft} - a^*_{it} \end{aligned} \tag{A21}$$

Therefore

$$\begin{aligned} mc^r_{Hit} &= w_t - p_{Hit} - a_{it} \\ &= \sigma c_t + \varphi n_t + \left(1 - \tfrac{\nu}{2}\right) s_{ft} - \upsilon_{Ht} - a_{it} \end{aligned}$$

Substituting equations ($A4$), ($A20$), and ($A8$)into the above expression

$$\begin{aligned} mc^r_{Hit} = {} & \sigma\left(\tfrac{\nu-1}{D} y^R_{ft} + y^W_{ft} + \tfrac{\nu(2-\nu)}{2D} m_{ft}\right) \\ & + \varphi\phi(1-\phi)\left(\upsilon^*_{Ht} - \upsilon_{Ht} + \upsilon^*_{Ft} - \upsilon_{Ft}\right) + \varphi\left(\tfrac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right) y^R_{ft} + \varphi y^W_{ft} \\ & - \varphi\phi(1-\phi)\tfrac{2(\nu-1)}{D} m_{ft} - \varphi\left[\phi a_{ft} + (1-\phi) a^*_{ft} + a_{it}\right] \\ & + \left(1 - \tfrac{\nu}{2}\right)\left(\tfrac{2\sigma}{D} y^R_{ft} + z_{ft} - \tfrac{\nu-1}{D} m_{ft}\right) - \upsilon_{Ht} - a_{it} \end{aligned}$$

Using the definitions of m_{ft} and z_{ft}, we have

$$m_{ft} = m_{it} + \upsilon^R_{Ht} + \upsilon^R_{Ft}, \tag{A22}$$

$$z_{ft} = z_{it} + \upsilon^R_{Ht} - \upsilon^R_{Ft}. \tag{A23}$$

After some algebra, we can obtain

$$\begin{aligned} mc^r_{Hit} = &\left[\tfrac{\sigma}{D} + \varphi\left(\tfrac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right)\right]\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} + \left(1-\tfrac{v}{2}\right)z_{it} \\ &+\left(\tfrac{1}{2} - \tfrac{v-1}{2D} - \varphi\phi(1-\phi)\tfrac{2(v-1)}{D}\right)m_{it} + \left(\tfrac{1}{2} - \tfrac{v-1}{2D} - \varphi\phi(1-\phi)\tfrac{2(v-1)}{D}\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\ &+2\varphi\phi(1-\phi)\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - \tfrac{v}{2}\tilde{\upsilon}^R_{Ht} - \left(1-\tfrac{v}{2}\right)\tilde{\upsilon}^R_{Ft} - \tilde{\upsilon}^W_{Ht} \end{aligned} \tag{A24}$$

Substituting equation (A24) into equation (A18), we have

$$\begin{aligned} \pi_{Hit} = &\delta_i\left\{\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} + \left(1-\tfrac{v}{2}\right)z_{it} + \left(\tfrac{1}{2} - \Gamma'\right)m_{it} + \left(\tfrac{1}{2} - \Gamma'\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right)\right\} \\ &+\delta_i\left\{2\varphi\phi(1-\phi)\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - \tfrac{v}{2}\tilde{\upsilon}^R_{Ht} - \left(1-\tfrac{v}{2}\right)\tilde{\upsilon}^R_{Ft} - \tilde{\upsilon}^W_{Ht}\right\} + \beta E_t\{\pi_{Hit+1}\} \end{aligned} \tag{A25}$$

in which

$$\Pi' \equiv \frac{\sigma}{D} + \varphi\left(\frac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right), \Gamma' = \frac{(v-1)}{2D} + \frac{2\varphi\phi(1-\phi)(v-1)}{D}.$$

Equation (A25) is the New-Keynesian Phillips Curve for the home intermediate-goods firm selling goods in the domestic market.

Similarly, we can get the expression of mc^{r*}_{Hit}, which is given by

$$\begin{aligned} mc^{r*}_{Hit} &= mc_{Hit} - p^*_{Hit} - e_{t+j} = mc^r_{Hit} + \tilde{\upsilon}_{Ht} - \tilde{\upsilon}_{Ft} - m_{ft} - z_{ft} \\ &= \left[\tfrac{\sigma}{D} + \varphi\left(\tfrac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right)\right]\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} - \tfrac{v}{2}z_{it} \\ &\quad +\left(-\tfrac{1}{2} - \tfrac{v-1}{2D} - \varphi\phi(1-\phi)\tfrac{2(v-1)}{D}\right)m_{it} \\ &\quad +\left(-\tfrac{1}{2} - \tfrac{v-1}{2D} - \varphi\phi(1-\phi)\tfrac{2(v-1)}{D}\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\ &\quad +2\varphi\phi(1-\phi)\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) + \left(1-\tfrac{v}{2}\right)\tilde{\upsilon}^R_{Ht} + \tfrac{v}{2}\tilde{\upsilon}^R_{Ft} - \tilde{\upsilon}^W_{Ht} \end{aligned} \tag{A26}$$

Substituting equation (A26) into equation (A19), we have

$$\begin{aligned} \pi^*_{Hit} = &\delta_i\left\{\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} - \tfrac{v}{2}z_{it} + \left(-\tfrac{1}{2} - \Gamma'\right)m_{it} + \left(-\tfrac{1}{2} - \Gamma'\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right)\right\} \\ &+\delta_i\left\{2\varphi\phi(1-\phi)\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) + \left(1-\tfrac{v}{2}\right)\tilde{\upsilon}^R_{Ht} + \tfrac{v}{2}\tilde{\upsilon}^R_{Ft} - \tilde{\upsilon}^W_{Ht}\right\} + \beta E_t\left\{\pi^*_{Hit+1}\right\} \end{aligned} \tag{A27}$$

which is the New-Keynesian Phillips Curve for the home intermediate-goods firm selling goods in the foreign market.

2.2 Derivation of the foreign New-Keynesian Phillips Curves at the stage of intermediate-goods production

Following the similar procedures, we can obtain the expressions of mc^{r*}_{Fit} and mc^r_{Fit}

$$\begin{aligned} mc^{r*}_{Fit} &= mc^*_{Fit} - p^*_{Fit} = w^*_t - a^*_{it} - p^*_{Fit} = \sigma c^*_t + \varphi n^*_t + \left(1-\tfrac{v}{2}\right)s^*_{ft} - \upsilon^*_{Ft} - a^*_{it} \\ &= \left[\varphi\left(-\tfrac{4\sigma\phi(1-\phi)}{D} + 1 - 2\phi\right) - \tfrac{\sigma}{D}\right]\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} + \left(1-\tfrac{v}{2}\right)z_{it} \\ &\quad +\left(\tfrac{v-1}{2D} - \tfrac{1}{2} + 2\varphi\phi(1-\phi)\tfrac{(v-1)}{D}\right)m_{it} - 2\varphi\phi(1-\phi)\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) \\ &\quad +\left(\tfrac{v-1}{2D} - \tfrac{1}{2} + 2\varphi\phi(1-\phi)\tfrac{(v-1)}{D}\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) + \left(1-\tfrac{v}{2}\right)\tilde{\upsilon}^R_{Ht} + \tfrac{v}{2}\tilde{\upsilon}^R_{Ft} - \tilde{\upsilon}^W_{Ft} \end{aligned}$$

$$\begin{aligned} mc^r_{Fit} &= mc^*_{Fit} - p_{Fit} + e_t = w^*_t - a^*_{it} - p^*_{Fit} + e_t = mc^{r*}_{Fit} + \upsilon^*_{Ft} - \upsilon^*_{Ht} + m_{ft} - z_{ft} \\ &= \left[\varphi\left(-\tfrac{4\sigma\phi(1-\phi)}{D} + 1 - 2\phi\right) - \tfrac{\sigma}{D}\right]\tilde{y}^R_{ft} + \left(\tfrac{v-1}{2D} + \tfrac{1}{2} + 2\varphi\phi(1-\phi)\tfrac{(v-1)}{D}\right)m_{it} \\ &\quad +(\sigma+\varphi)\tilde{y}^W_{ft} - \tfrac{v}{2}z_{it} + \left(\tfrac{v-1}{2D} + \tfrac{1}{2} + 2\varphi\phi(1-\phi)\tfrac{(v-1)}{D}\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\ &\quad -2\varphi\phi(1-\phi)\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - \tfrac{v}{2}\tilde{\upsilon}^R_{Ht} - \left(1-\tfrac{v}{2}\right)\tilde{\upsilon}^R_{Ft} - \tilde{\upsilon}^W_{Ft} \end{aligned}$$

Thus, the New-Keynesian Phillips Curves for the foreign intermediate-goods firm selling goods in the local and home markets are, respectively

$$\begin{aligned} \pi^*_{Fit} = &\delta_i\left\{-\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} + \left(1-\tfrac{v}{2}\right)z_{it} + \left(\Gamma' - \tfrac{1}{2}\right)m_{it} + \left(\Gamma' - \tfrac{1}{2}\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right)\right\} \\ &+\delta_i\left\{-2\varphi\phi(1-\phi)\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) + \left(1-\tfrac{v}{2}\right)\tilde{\upsilon}^R_{Ht} + \tfrac{v}{2}\tilde{\upsilon}^R_{Ft} - \tilde{\upsilon}^W_{Ft}\right\} + \beta E_t\left\{\pi^*_{Fit+1}\right\}, \end{aligned} \tag{A28}$$

$$\begin{aligned} \pi_{Fit} = &\delta_i\left\{-\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} - \tfrac{v}{2}z_{it} + \left(\tfrac{1}{2} + \Gamma'\right)m_{it} + \left(\tfrac{1}{2} + \Gamma'\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right)\right\} \\ &+\delta_i\left\{-2\varphi\phi(1-\phi)\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - \tfrac{v}{2}\tilde{\upsilon}^R_{Ht} - \left(1-\tfrac{v}{2}\right)\tilde{\upsilon}^R_{Ft} - \tilde{\upsilon}^W_{Ft}\right\} + \beta E_t\{\pi_{Fit+1}\}. \end{aligned} \tag{A29}$$

2.3 Derivation of the home New-Keynesian Phillips Curves at the stage of final-goods production

Similarly, we can obtain the expressions of mc^r_{Hft} and mc^{r*}_{Hft}

$$
\begin{aligned}
mc^r_{Hft} &= mc_{Hft} - p_{Hft} = \phi p_{Hit} + (1-\phi)p_{Fit} - a_{ft} - p_{Hft} \\
&= \phi(v_{Ht} + p_{Hft}) + (1-\phi)(v_{Ft} + p_{Fft}) - a_{ft} - p_{Hft} \\
&= \phi\tilde{v}_{Ht} + (1-\phi)\tilde{v}_{Ft} + (1-\phi)\tilde{s}_{ft} \\
&= \phi\tilde{v}_{Ht} + (1-\phi)\tilde{v}_{Ft} + (1-\phi)\left(\tfrac{2\sigma}{D}\tilde{y}^R_{ft} + z_{ft} - \tfrac{v-1}{D}m_{ft}\right) \\
&= \phi(\tilde{v}^W_{Ht} + \tilde{v}^R_{Ht}) + (1-\phi)(\tilde{v}^W_{Ft} + \tilde{v}^R_{Ft}) + (1-\phi)\tfrac{2\sigma}{D}\tilde{y}^R_{ft} + (1-\phi)z_{ft} - (1-\phi)\tfrac{v-1}{D}m_{ft}
\end{aligned}
$$

$$
\begin{aligned}
mc^{r*}_{Hft} &= mc^r_{Hft} - (m_{ft} + z_{ft}) \\
&= \phi(\tilde{v}^W_{Ht} + \tilde{v}^R_{Ht} - z_{ft}) + (1-\phi)\left(\tilde{v}^W_{Ft} + \tilde{v}^R_{Ft} + \tfrac{2\sigma}{D}\tilde{y}^R_{ft} - \tfrac{(v-1)}{D}m_{ft}\right) - m_{ft}
\end{aligned}
$$

Thus, the New-Keynesian Phillips Curves for the home final-goods firm selling goods in the local and foreign markets are, respectively

$$
\pi_{Hft} = \delta_f\left\{\phi(\tilde{v}^W_{Ht} + \tilde{v}^R_{Ht}) + (1-\phi)\left(\tilde{v}^W_{Ft} + \tilde{v}^R_{Ft} + z_{ft} + \frac{2\sigma}{D}\tilde{y}^R_{ft} - \frac{(v-1)}{D}m_{ft}\right)\right\} + \beta E_t\{\pi_{Hft+1}\}, \tag{A30}
$$

$$
\pi^*_{Hft} = \delta_f\left\{\phi(\tilde{v}^W_{Ht} + \tilde{v}^R_{Ht} - z_{ft}) + (1-\phi)\left(\tilde{v}^W_{Ft} + \tilde{v}^R_{Ft} + \frac{2\sigma}{D}\tilde{y}^R_{ft} - \frac{(v-1)}{D}m_{ft}\right) - m_{ft}\right\} + \beta E_t\left\{\pi^*_{Hft+1}\right\}. \tag{A31}
$$

2.4 Derivation of the foreign New-Keynesian Phillips Curves at the stage of final-goods production

Following the similar procedures, we have

$$
\begin{aligned}
mc^r_{Fft} &= mc^*_{Fft} - p_{Fft} + e_t = mc^*_{Fft} - p^*_{Fft} + p^*_{Fft} - p_{Fft} + e_t \\
&= mc^*_{Fft} - p^*_{Fft} + m_t - z_t = mc^{r*}_{Fft} + m_t - z_t
\end{aligned}
$$

$$
\begin{aligned}
mc^{r*}_{Fft} &= mc^*_{Fft} - p^*_{Fft} \\
&= \phi p^*_{Fit} + (1-\phi)p^*_{Hit} - a^*_{ft} - p^*_{Fft} \\
&= \phi\tilde{v}^*_{Ft} + (1-\phi)\tilde{v}^*_{Ht} + (1-\phi)\tilde{s}^*_{ft}
\end{aligned}
$$

After some algebra, we can obtain

$$
\begin{aligned}
mc^{r*}_{Fft} &= \phi\tilde{v}^*_{Ft} + (1-\phi)\tilde{v}^*_{Ht} - (1-\phi)\tfrac{2\sigma}{D}\tilde{y}^R_{ft} + (1-\phi)z_{ft} + (1-\phi)\tfrac{v-1}{D}m_{ft} \\
&= \phi(\tilde{v}^W_{Ft} - \tilde{v}^R_{Ft}) + (1-\phi)\left(\tilde{v}^W_{Ht} - \tilde{v}^R_{Ht} + z_{ft} - \tfrac{2\sigma}{D}\tilde{y}^R_{ft} + \tfrac{(v-1)}{D}m_{ft}\right)
\end{aligned}
$$

$$
\begin{aligned}
mc^r_{Fft} &= mc^{r*}_{Fft} + m_{ft} - z_{ft} \\
&= \phi\tilde{v}^*_{Ft} + (1-\phi)\tilde{v}^*_{Ht} - (1-\phi)\tfrac{2\sigma}{D}\tilde{y}^R_{ft} - \phi z_{ft} + \left(1 + (1-\phi)\tfrac{v-1}{D}\right)m_{ft} \\
&= \phi(\tilde{v}^W_{Ft} - \tilde{v}^R_{Ft}) + (1-\phi)(\tilde{v}^W_{Ht} - \tilde{v}^R_{Ht}) - (1-\phi)\tfrac{2\sigma}{D}\tilde{y}^*_{ft} - \phi z_{ft} + \left(1 + (1-\phi)\tfrac{v-1}{D}\right)m_{ft}
\end{aligned}
$$

Thus, the New-Keynesian Phillips Curves for the foreign final-goods firm selling goods in the local and home markets are, respectively

$$
\pi^*_{Fft} = \delta_f\left\{\phi(\tilde{v}^W_{Ft} - \tilde{v}^R_{Ft}) + (1-\phi)\left(\tilde{v}^W_{Ht} - \tilde{v}^R_{Ht} + z_{ft} - \frac{2\sigma}{D}\tilde{y}^R_{ft} + \frac{(v-1)}{D}m_{ft}\right)\right\} + \beta E_t\left\{\pi^*_{Fft+1}\right\} \tag{A32}
$$

$$
\pi_{Fft} = \delta_f\left\{\phi(\tilde{v}^W_{Ft} - \tilde{v}^R_{Ft} - z_{ft}) + (1-\phi)\left(\tilde{v}^W_{Ht} - \tilde{v}^R_{Ht} - \frac{2\sigma}{D}\tilde{y}^R_{ft} + \frac{(v-1)}{D}m_{ft}\right) + m_{ft}\right\} + \beta E_t\{\pi_{Fft+1}\} \tag{A33}
$$

3. Derivation of the welfare loss function

Following Engel (2011), after taking a second-order log approximation around the non-stochastic steady state, the period utility function of the cooperative monetary policymaker can be written as

$$
v_t = c_t + c^*_t - n_t - n^*_t + \frac{1-\sigma}{2}(c^2_t + c^{*2}_t) - \frac{1+\varphi}{2}(n^2_t + n^{*2}_t) + o(||a^3||) \tag{A34}
$$

When the prices are flexible, the period utility function given by equation (A34) is maximized

$$
v^{\max}_t = \overline{c}_t + \overline{c}^*_t - \overline{n}_t - \overline{n}^*_t + \frac{1-\sigma}{2}(\overline{c}^2_t + \overline{c}^{*2}_t) - \frac{1+\varphi}{2}(\overline{n}^2_t + \overline{n}^{*2}_t) + o(||a^3||) \tag{A35}
$$

If we define $\widetilde{x}_t = x_t - \overline{x}_t$, then

$$\begin{aligned} v_t - v_t^{\max} = 2\tilde{c}_t^W - 2\tilde{n}_t^W + (1-\sigma)\left[\left(\tilde{c}_t^R\right)^2 + \left(\tilde{c}_t^W\right)^2\right] - (1+\varphi)\left[\left(\tilde{n}_t^R\right)^2 + \left(\tilde{n}_t^W\right)^2\right] \\ +2(1-\sigma)\left[\bar{c}_t^R\tilde{c}_t^R + \bar{c}_t^W\tilde{c}_t^W\right] - 2(1+\varphi)\left[\bar{n}_t^R\tilde{n}_t^R + \bar{n}_t^W\tilde{n}_t^W\right] \end{aligned} \tag{A36}$$

According to Engel (2011), we need a second-order approximation to the term $2\tilde{c}_t^W - 2\tilde{n}_t^W$, while the first-order approximations to the rest of the terms.

Taking the second-order approximation to home and foreign final-goods market clearing conditions, we get, respectively

$$y_{ft} = \frac{v}{2}c_t + \left(1-\frac{v}{2}\right)c_t^* + \frac{v}{2}\left(1-\frac{v}{2}\right)s_{ft} - \left(1-\frac{v}{2}\right)\frac{v}{2}s_{ft}^* + f_1, \tag{A37}$$

$$y_{ft}^* = \left(1-\frac{v}{2}\right)c_t + \frac{v}{2}c_t^* - \frac{v}{2}\left(1-\frac{v}{2}\right)s_{ft} + \left(1-\frac{v}{2}\right)\frac{v}{2}s_{ft}^* + f_2, \tag{A38}$$

in which

$$f_1 \equiv \frac{1}{2}\left[\frac{v}{2}\left(c_t + \left(1-\frac{v}{2}\right)s_{ft}\right)^2 + \left(1-\frac{v}{2}\right)\left(c_t^* - \frac{v}{2}s_{ft}^*\right)^2\right] - \frac{1}{2}y_{ft}^2,$$

$$f_2 \equiv \frac{1}{2}\left[\left(1-\frac{v}{2}\right)\left(c_t - \frac{v}{2}s_{ft}\right)^2 + \frac{v}{2}\left(c_t^* + \left(1-\frac{v}{2}\right)s_{ft}^*\right)^2\right] - \frac{1}{2}y_{ft}^{*2}.$$

From (A37) and (A38), we have

$$y_{ft} + y_{ft}^* = c_t + c_t^* + f_1 + f_2 \tag{A39}$$

After rearranging equation (A39), we obtain

$$2c_t^W = 2y_{ft}^W - f_1 - f_2 \tag{A40}$$

Subtracting the counterparts in the flexible-price equilibrium from both sides of equation (A40), we have

$$2\tilde{c}_t^W = 2\tilde{y}_{ft}^W - \left(f_1 - \bar{f}_1\right) - \left(f_2 - \bar{f}_2\right) \tag{A41}$$

Taking the second-order approximation to home and foreign labor market clearing conditions, we get, respectively

$$n_t = \phi y_{Hit} + (1-\phi)y_{Hit}^* + \phi d_{Hit} + (1-\phi)d_{Hit}^* + \frac{v}{2}d_{Hft} + \frac{2-v}{2}d_{Hft}^* - a_{it} \tag{A42}$$

$$n_t^* = (1-\phi)y_{Fit} + \phi y_{Fit}^* + \phi d_{Fit} + (1-\phi)d_{Fit}^* + \frac{2-v}{2}d_{Fft} + \frac{v}{2}d_{Fft}^* - a_{it}^* \tag{A43}$$

Adding both sides of equation (A42) and (A43), we obtain

$$\begin{aligned} n_t + n_t^* = \phi y_{Hit} + (1-\phi)y_{Hit}^* - a_{it} + (1-\phi)y_{Fit} + \phi y_{Fit}^* \\ + \tfrac{v}{2}d_{Hft} + \tfrac{2-v}{2}d_{Hft}^* + \tfrac{2-v}{2}d_{Fft} + \tfrac{v}{2}d_{Fft}^* - a_{it}^* \\ = y_{ft} + y_{ft}^* - a_{it} - a_{it}^* + \phi d_{Hit} + (1-\phi)d_{Hit}^* + \phi d_{Fit} + (1-\phi)d_{Fit}^* \\ + \tfrac{v}{2}d_{Hft} + \tfrac{2-v}{2}d_{Hft}^* + \tfrac{2-v}{2}d_{Fft} + \tfrac{v}{2}d_{Fft}^*, \end{aligned} \tag{A44}$$

which is equivalent to

$$\begin{aligned} 2n_t^W = 2y_{ft}^W - a_{it} - a_{it}^* + \phi d_{Hit} + (1-\phi)d_{Hit}^* + \phi d_{Fit} + (1-\phi)d_{Fit}^* \\ + \tfrac{v}{2}d_{Hft} + \tfrac{2-v}{2}d_{Hft}^* + \tfrac{2-v}{2}d_{Fft} + \tfrac{v}{2}d_{Fft}^* \end{aligned} \tag{A45}$$

Subtracting the counterparts in the flexible-price equilibrium from both sides of equation (A45), we have

$$\begin{aligned} 2\tilde{n}_t^W = 2\tilde{y}_{ft}^W + \phi d_{Hit} + (1-\phi)d_{Hit}^* + \phi d_{Fit} + (1-\phi)d_{Fit}^* \\ + \tfrac{v}{2}d_{Hft} + \tfrac{2-v}{2}d_{Hft}^* + \tfrac{2-v}{2}d_{Fft} + \tfrac{v}{2}d_{Fft}^* \end{aligned} \tag{A46}$$

From (A41) and (A46), the second-order approximation to the term $2\tilde{c}_t^W - 2\tilde{n}_t^W$ is

$$\begin{aligned} 2\tilde{c}_t^W - 2\tilde{n}_t^W = -\left(f_1 - \bar{f}_1\right) - \left(f_2 - \bar{f}_2\right) + \phi d_{Hit} + (1-\phi)d_{Hit}^* + \phi d_{Fit} + (1-\phi)d_{Fit}^* \\ + \tfrac{v}{2}d_{Hft} + \tfrac{2-v}{2}d_{Hft}^* + \tfrac{2-v}{2}d_{Fft} + \tfrac{v}{2}d_{Fft}^*. \end{aligned} \tag{A47}$$

Substituting equation (A47) into equation (A36), we have

$$\begin{aligned} v_t - v_t^{\max} = & -\left(f_1 - \bar{f}_1\right) - \left(f_2 - \bar{f}_2\right) + (1-\sigma)\left[\left(\tilde{c}_t^R\right)^2 + \left(\tilde{c}_t^W\right)^2\right] - (1+\varphi)\left[\left(\tilde{n}_t^R\right)^2 + \left(\tilde{n}_t^W\right)^2\right] \\ & + 2(1-\sigma)\left[\bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W\right] - 2(1+\varphi)\left[\bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W\right] \\ & - \left[\phi d_{Hit} + (1-\phi) d_{Hit}^* + \phi d_{Fit} + (1-\phi) d_{Fit}^*\right] \\ & - \left[\tfrac{v}{2} d_{Hft} + \tfrac{2-v}{2} d_{Hft}^* + \tfrac{2-v}{2} d_{Fft} + \tfrac{v}{2} d_{Fft}^*\right], \end{aligned} \tag{A48}$$

which includes only the terms involving squares and products. The first-order approximation to equation (A48) is sufficient.

Using $(A4), (A5), (A8)$and $(A9)$and the expressions of f_1 and f_2, we have

$$\begin{aligned} f_1 - \bar{f}_1 + f_2 - \bar{f}_2 = & \, v(2-v)\tfrac{(v-1)^2(\sigma-1)^2}{D^2}\left(\left(\tilde{y}_{ft}^R\right)^2 + 2\bar{y}_{ft}^R \tilde{y}_{ft}^R\right) + \tfrac{v(2-v)}{4D^2} m_{ft}^2 \\ & + \tfrac{v(2-v)}{4} z_{ft}^2 + \tfrac{(v-1)(1-\sigma)v(2-v)}{D^2} m_{ft} y_{ft}^R \end{aligned} \tag{A49}$$

From equations (A6) and (A7), we have, in the flexible-price equilibrium

$$\bar{c}_t^R = \frac{v-1}{D}\bar{y}_{ft}^R, \tag{A50}$$

$$\bar{c}_t^W = \bar{y}_{ft}^W \tag{A51}$$

Subtracting the counterparts in the flexible-price equilibrium from both sides of equation (A6), we have

$$\tilde{c}_t^R = \frac{v-1}{D}\tilde{y}_{ft}^R + \frac{v(2-v)}{2D} m_{ft}. \tag{A52}$$

Similarly, we have

$$\tilde{c}_t^W = \tilde{y}_{ft}^W. \tag{A53}$$

Log-linearizing home and foreign labor market clearing conditions yields, respectively

$$\begin{aligned} n_t = & \, \phi(1-\phi)\left(v_{Ft} - v_{Ht} - v_{Ht}^* + v_{Ft}^*\right) + \left(\tfrac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right) y_{ft}^R + y_{ft}^W \\ & - \phi(1-\phi)\tfrac{2(v-1)}{D} m_{ft} - \phi a_{ft} - (1-\phi) a_{ft}^* - a_{it} \end{aligned} \tag{A54}$$

and

$$\begin{aligned} n_t^* = & -\phi(1-\phi)\left(v_{Ft} - v_{Ht} - v_{Ht}^* + v_{Ft}^*\right) + \left(-\tfrac{4\sigma\phi(1-\phi)}{D} + 1 - 2\phi\right) y_{ft}^R \\ & + y_{ft}^W + \phi(1-\phi)\tfrac{2(v-1)}{D} m_{ft} - (1-\phi) a_{ft} - \phi a_{ft}^* - a_{it}^* \end{aligned} \tag{A55}$$

From equation (A54) and (A55) and their counterparts in the flexible-price equilibrium, we obtain

$$\tilde{n}_t^W = \tilde{y}_{ft}^W, \tag{A56}$$

$$\tilde{n}_t^R = \Omega_y \tilde{y}_{ft}^R - \Omega_m m_{it} - \Omega_v \tilde{v}_{Ht}^W + \Omega_v \tilde{v}_{Ft}^W - \Omega_m \tilde{v}_{Ht}^R - \Omega_m \tilde{v}_{Ft}^R \tag{A57}$$

in which

$$\Omega_y = \frac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1, \Omega_m = \phi(1-\phi)\frac{2(v-1)}{D}, \Omega_v = 2\phi(1-\phi)$$

From the flexible-price equilibrium, we have

$$\bar{n}_t^W = \frac{1-\sigma}{1+\varphi}\bar{y}_{ft}^W \tag{A58}$$

$$\bar{n}_t^R = \Gamma_y \bar{y}_{ft}^R, \tag{A59}$$

in which $\Gamma_y = \frac{(1-2\phi)(\sigma-D)}{(1+\varphi)D}$.

Substituting equations $(A49) - (A53), (A56) - (A59)$into (A48), we have

$$
\begin{aligned}
\upsilon_t - \upsilon_t^{\max} = -\nu(2-\nu)\tfrac{(\nu-1)^2(\sigma-1)^2}{D^2}\left(\left(\tilde{y}_{ft}^R\right)^2 + 2\bar{y}_{ft}^R\tilde{y}_{ft}^R\right) - 2(1-\sigma)\bar{y}_{ft}^W\tilde{y}_{ft}^W \\
-\tfrac{\nu(2-\nu)}{4D^2}m_{ft}^2 - \tfrac{(\nu-1)(1-\sigma)\nu(2-\nu)}{D^2}m_{ft}y_t^R \\
+(1-\sigma)\left[\left(\tfrac{\nu-1}{D}\tilde{y}_{ft}^R + \tfrac{\nu(2-\nu)}{2D}m_{ft}\right)^2 + \left(\tilde{y}_{ft}^W\right)^2\right] \\
-(1+\varphi)\left[\left(\Omega_y\tilde{y}_{ft}^R - \Omega_m m_{it} + \Omega_\upsilon\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right) - \Omega_m\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right)\right)^2 + \left(\tilde{y}_{ft}^W\right)^2\right] \\
+2(1-\sigma)\left[\tfrac{\nu-1}{D}\bar{y}_{ft}^R\left(\tfrac{\nu-1}{D}\tilde{y}_{ft}^R + \tfrac{\nu(2-\nu)}{2D}m_{ft}\right) + \bar{y}_{ft}^W\tilde{y}_{ft}^W\right] \\
-2(1+\varphi)\left[\Gamma_y\bar{y}_{ft}^R\left(\Omega_y\tilde{y}_{ft}^R - \Omega_m m_{it} + \Omega_\upsilon\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right) - \Omega_m\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right)\right)\right] \\
-\left[\phi d_{Hit} + (1-\phi)d_{Hit}^* + \phi d_{Fit} + (1-\phi)d_{Fit}^*\right] \\
-\left[\tfrac{\nu}{2}d_{Hft} + \tfrac{2-\nu}{2}d_{Hft}^* + \tfrac{2-\nu}{2}d_{Fft} + \tfrac{\nu}{2}d_{Fft}^*\right].
\end{aligned}
\tag{A60}
$$

After some algebra, equation (A60) can be written as

$$
\begin{aligned}
\upsilon_t - \upsilon_t^{\max} = -\tfrac{\xi_f}{2\delta_f}\left(\tfrac{\nu}{2}\pi_{Hft}^2 + \tfrac{2-\nu}{2}\pi_{Hft}^{2*} + \tfrac{2-\nu}{2}\pi_{Fft}^2 + \tfrac{\nu}{2}\pi_{Fft}^{2*}\right) \\
-\tfrac{\xi_i}{2\delta_i}\left(\phi\pi_{Hit}^2 + (1-\phi)\pi_{Hit}^{2*} + (1-\phi)\pi_{Fit}^2 + \phi\pi_{Fit}^{2*}\right) \\
+\left[(1-\sigma)\tfrac{(\nu-1)^2}{D} - (1+\varphi)\Omega_y^2\right]\left(\tilde{y}_{ft}^R\right)^2 - (\sigma+\varphi)\left(\tilde{y}_{ft}^W\right)^2 \\
+\left[2(1-\sigma)\tfrac{(\nu-1)^2}{D} - 2(1+\varphi)\Gamma_y\Omega_y\right]\bar{y}_{ft}^R\tilde{y}_{ft}^R - \tfrac{\nu(2-\nu)}{4D}m_{ft}^2 \\
-(1+\varphi)\Omega_m^2 m_{it}^2 + 2(1+\varphi)\Omega_y\Omega_m\tilde{y}_{ft}^R m_{it} + 2(1+\varphi)\Gamma_y\Omega_m\bar{y}_{ft}^R m_{it} \\
-(1+\varphi)\Omega_\upsilon^2\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right)^2 - (1+\varphi)\Omega_m^2\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right)^2 \\
-2(1+\varphi)\Omega_y\Omega_\upsilon\tilde{y}_{ft}^R\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right) + 2(1+\varphi)\Omega_y\Omega_m\tilde{y}_{ft}^R\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right) \\
+2(1+\varphi)\Omega_m\Omega_\upsilon m_{it}\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right) - 2(1+\varphi)\Omega_m^2 m_{it}\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right) \\
-2(1+\varphi)\Gamma_y\Omega_\upsilon\bar{y}_{ft}^R\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right) + 2(1+\varphi)\Gamma_y\Omega_m\bar{y}_{ft}^R\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right) \\
+2(1+\varphi)\Omega_m\Omega_\upsilon\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right)\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right)
\end{aligned}
\tag{A61}
$$

Thus, the welfare loss function is given by

$$
\mathbf{W} = \mathbf{E}_0\sum\beta^t\mathbf{X}_t + o\left(||x^3||\right) + t.i.p. \tag{A62}
$$

in which

$$
\begin{aligned}
\mathbf{X}_t = \tfrac{\xi_f}{2\delta_f}\left(\tfrac{\nu}{2}\pi_{Hft}^2 + \tfrac{2-\nu}{2}\pi_{Hft}^{2*} + \tfrac{2-\nu}{2}\pi_{Fft}^2 + \tfrac{\nu}{2}\pi_{Fft}^{2*}\right) \\
+\tfrac{\xi_i}{2\delta_i}\left(\phi\pi_{Hit}^2 + (1-\phi)\pi_{Hit}^{2*} + (1-\phi)\pi_{Fit}^2 + \phi\pi_{Fit}^{2*}\right) \\
+\left[(1+\varphi)\Omega_y^2 - (1-\sigma)\tfrac{(\nu-1)^2}{D}\right]\left(\tilde{y}_{ft}^R\right)^2 + (\sigma+\varphi)\left(\tilde{y}_{ft}^W\right)^2 \\
+\left[2(1+\varphi)\Gamma_y\Omega_y - 2(1-\sigma)\tfrac{(\nu-1)^2}{D}\right]\bar{y}_{ft}^R\tilde{y}_{ft}^R + \tfrac{\nu(2-\nu)}{4D}m_{ft}^2 \\
+(1+\varphi)\Omega_m^2 m_{it}^2 + 2(1+\varphi)\Omega_y\Omega_m\tilde{y}_{ft}^R m_{it} - 2(1+\varphi)\Gamma_y\Omega_m\bar{y}_{ft}^R m_{it} \\
+(1+\varphi)\Omega_\upsilon^2\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right)^2 + (1+\varphi)\Omega_m^2\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right)^2 \\
+2(1+\varphi)\Omega_y\Omega_\upsilon\tilde{y}_{ft}^R\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right) - 2(1+\varphi)\Omega_y\Omega_m\tilde{y}_{ft}^R\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right) \\
-2(1+\varphi)\Omega_m\Omega_\upsilon m_{it}\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right) + 2(1+\varphi)\Omega_m^2 m_{it}\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right) \\
+2(1+\varphi)\Gamma_y\Omega_\upsilon\bar{y}_{ft}^R\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right) - 2(1+\varphi)\Gamma_y\Omega_m\bar{y}_{ft}^R\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right) \\
-2(1+\varphi)\Omega_m\Omega_\upsilon\left(\tilde{\upsilon}_{Ft}^W - \tilde{\upsilon}_{Ht}^W\right)\left(\tilde{\upsilon}_{Ht}^R + \tilde{\upsilon}_{Ft}^R\right)
\end{aligned}
\tag{A63}
$$

$t.i.p.$ stands for the terms independent of policy and $o\left(||x^3||\right)$collects all terms of third or higher order.
The expected period welfare loss function is

$$
\begin{aligned}
\mathbf{L} = {} & \tfrac{\xi_f}{2\delta_f}\left(\tfrac{\nu}{2}\,var(\pi_{Hft}) + \tfrac{2-\nu}{2}\,var\left(\pi^*_{Hft}\right) + \tfrac{2-\nu}{2}\,var(\pi_{Fft}) + \tfrac{\nu}{2}\,var\left(\pi^*_{Fft}\right)\right) \\
& + \tfrac{\xi_i}{2\delta_i}\left(\phi\, var(\pi_{Hit}) + (1-\phi)\, var(\pi^*_{Hit}) + (1-\phi)\, var(\pi_{Fit}) + \phi\, var(\pi^*_{Fit})\right) \\
& + \left((1+\varphi)\Omega_y^2 - (1-\sigma)\tfrac{(\nu-1)^2}{D}\right) var\left(\tilde{y}^R_{ft}\right) + (\sigma+\varphi)\, var\left(\tilde{y}^W_{ft}\right) \\
& + \left(2(1+\varphi)\Gamma_y\Omega_y - 2(1-\sigma)\tfrac{(\nu-1)^2}{D}\right) cov\left(\bar{y}^R_{ft}, \tilde{y}^R_{ft}\right) + \tfrac{\nu(2-\nu)}{4D}\, var(m_{ft}) \\
& + (1+\varphi)\Omega_m^2\, var(m_{it}) + 2(1+\varphi)\Omega_m\Omega_y cov\left(\tilde{y}^R_{ft}, m_{it}\right) - 2(1+\varphi)\Gamma_y\Omega_m cov\left(m_{it}, \bar{y}^R_{ft}\right) \\
& + (1+\varphi)\Omega_\nu^2\, var(\tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht}) + (1+\varphi)\Omega_m^2\, var(\tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft}) \\
& 2(1+\varphi)\Omega_y\Omega_\nu cov\left(\tilde{y}^R_{ft}, \tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht}\right) - 2(1+\varphi)\Omega_y\Omega_m cov\left(\tilde{y}^R_{ft}, \tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft}\right) \\
& - 2(1+\varphi)\Omega_m\Omega_\nu cov(m_{it}, \tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht}) + 2(1+\varphi)\Omega_m^2 cov(m_{it}, \tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft}) \\
& + 2(1+\varphi)\Gamma_y\Omega_\nu cov\left(\tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht}, \bar{y}^R_{ft}\right) - 2(1+\varphi)\Gamma_y\Omega_m cov\left(\tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft}, \bar{y}^R_{ft}\right) \\
& - 2(1+\varphi)\Omega_m\Omega_\nu cov(\tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft}, \tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht})
\end{aligned} \tag{A64}
$$

4. The optimal monetary policy

When the monetary policymaker is able to commit, with full credibility, to the policy rule at time zero, she chooses $\tilde{y}^R_{ft}, \tilde{y}^W_{ft}, m_{ft}, m_{it}, \tilde{\nu}^R_{Ht}, \tilde{\nu}^R_{Ft}, \tilde{\nu}^W_{Ht}, \tilde{\nu}^W_{Ft}, \pi_{Hft}, \pi^*_{Hft}, \pi_{Fft}, \pi^*_{Fft}, \pi_{Hit}, \pi^*_{Hit}, \pi_{Fit}, \pi^*_{Fit}$ to minimize the welfare loss function subject to the sequence of equilibrium dynamics given by the following equations:

$$\pi_{Hft} = \delta_f\left\{\phi(\tilde{\nu}^W_{Ht} + \tilde{\nu}^R_{Ht}) + (1-\phi)\left(\tilde{\nu}^W_{Ft} + \tilde{\nu}^R_{Ft} + z_{ft} + \frac{2\sigma}{D}\tilde{y}^R_{ft} - \frac{(\nu-1)}{D}m_{ft}\right)\right\} + \beta E_t\{\pi_{Hft+1}\}, \tag{A65}$$

$$\pi_{Fft} = \delta_f\left\{\phi(\tilde{\nu}^W_{Ft} - \tilde{\nu}^R_{Ft} - z_{ft}) + (1-\phi)\left(\tilde{\nu}^W_{Ht} - \tilde{\nu}^R_{Ht} - \frac{2\sigma}{D}\tilde{y}^R_{ft} + \frac{(\nu-1)}{D}m_{ft}\right) + m_{ft}\right\} + \beta E_t\{\pi_{Fft+1}\}, \tag{A66}$$

$$\pi^*_{Hft} = \delta_f\left\{\phi(\tilde{\nu}^W_{Ht} + \tilde{\nu}^R_{Ht} - z_{ft}) + (1-\phi)\left(\tilde{\nu}^W_{Ft} + \tilde{\nu}^R_{Ft} + \frac{2\sigma}{D}\tilde{y}^R_{ft} - \frac{(\nu-1)}{D}m_{ft}\right) - m_{ft}\right\} + \beta E_t\left\{\pi^*_{Hft+1}\right\}, \tag{A67}$$

$$\pi^*_{Fft} = \delta_f\left\{\phi(\tilde{\nu}^W_{Ft} - \tilde{\nu}^R_{Ft}) + (1-\phi)\left(\tilde{\nu}^W_{Ht} - \tilde{\nu}^R_{Ht} + z_{ft} - \frac{2\sigma}{D}\tilde{y}^R_{ft} + \frac{(\nu-1)}{D}m_{ft}\right)\right\} + \beta E_t\left\{\pi^*_{Fft+1}\right\}, \tag{A68}$$

$$
\begin{aligned}
\pi_{Hit} = {} & \delta_i\left\{\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} + \left(1-\tfrac{\nu}{2}\right)z_{it} + \left(\tfrac{1}{2}-\Gamma'\right)m_{it} + \left(\tfrac{1}{2}-\Gamma'\right)(\tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft})\right\} \\
& + \delta_i\left\{2\varphi\phi(1-\phi)(\tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht}) - \tfrac{\nu}{2}\tilde{\nu}^R_{Ht} - \left(1-\tfrac{\nu}{2}\right)\tilde{\nu}^R_{Ft} - \tilde{\nu}^W_{Ht}\right\} + \beta E_t\{\pi_{Hit+1}\},
\end{aligned} \tag{A69}
$$

$$
\begin{aligned}
\pi_{Fit} = {} & \delta_i\left\{-\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} - \tfrac{\nu}{2}z_{it} + \left(\tfrac{1}{2}+\Gamma'\right)m_{it} + \left(\tfrac{1}{2}+\Gamma'\right)(\tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft})\right\} \\
& + \delta_i\left\{-2\varphi\phi(1-\phi)(\tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht}) - \tfrac{\nu}{2}\tilde{\nu}^R_{Ht} - \left(1-\tfrac{\nu}{2}\right)\tilde{\nu}^R_{Ft} - \tilde{\nu}^W_{Ft}\right\} + \beta E_t\{\pi_{Fit+1}\},
\end{aligned} \tag{A70}
$$

$$
\begin{aligned}
\pi^*_{Fit} = {} & \delta_i\left\{-\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} + \left(1-\tfrac{\nu}{2}\right)z_{it} + \left(\Gamma'-\tfrac{1}{2}\right)m_{it} + \left(\Gamma'-\tfrac{1}{2}\right)(\tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft})\right\} \\
& + \delta_i\left\{-2\varphi\phi(1-\phi)(\tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht}) + \left(1-\tfrac{\nu}{2}\right)\tilde{\nu}^R_{Ht} + \tfrac{\nu}{2}\tilde{\nu}^R_{Ft} - \tilde{\nu}^W_{Ft}\right\} + \beta E_t\{\pi^*_{Fit+1}\},
\end{aligned} \tag{A71}
$$

$$
\begin{aligned}
\pi^*_{Hit} = {} & \delta_i\left\{\Pi'\tilde{y}^R_{ft} + (\sigma+\varphi)\tilde{y}^W_{ft} - \tfrac{\nu}{2}z_{it} + \left(-\tfrac{1}{2}-\Gamma'\right)m_{it} + \left(-\tfrac{1}{2}-\Gamma'\right)(\tilde{\nu}^R_{Ht} + \tilde{\nu}^R_{Ft})\right\} \\
& + \delta_i\left\{2\varphi\phi(1-\phi)(\tilde{\nu}^W_{Ft} - \tilde{\nu}^W_{Ht}) + \left(1-\tfrac{\nu}{2}\right)\tilde{\nu}^R_{Ht} + \tfrac{\nu}{2}\tilde{\nu}^R_{Ft} - \tilde{\nu}^W_{Ht}\right\} + \beta E_t\{\pi^*_{Hit+1}\},
\end{aligned} \tag{A72}
$$

$$\tilde{\nu}^R_{Ht} = \tilde{\nu}^R_{Ht-1} + \frac{1}{2}\left(\pi_{Hit} - \pi_{Hft} - \pi^*_{Hit} + \pi^*_{Hft}\right), \tag{A73}$$

$$\tilde{\nu}^R_{Ft} = \tilde{\nu}^R_{Ft-1} + \frac{1}{2}\left(\pi_{Fit} - \pi_{Fft} - \pi^*_{Fit} + \pi^*_{Fft}\right), \tag{A74}$$

$$\tilde{\nu}^W_{Ht} = \tilde{\nu}^W_{Ht-1} + \frac{1}{2}\left(\pi_{Hit} - \pi_{Hft} + \pi^*_{Hit} - \pi^*_{Hft}\right) - \Delta\bar{\nu}_t, \tag{A75}$$

$$\bar{\nu}_t = \bar{\nu}_{Ht} = \bar{\nu}^*_{Ht} = \Xi\left(a_{ft} - a^*_{ft}\right) + a_{ft} + \mathrm{F}(a^*_{it} - a_{it}), \tag{A76}$$

$$\tilde{\upsilon}^W_{Ft} = \tilde{\upsilon}^W_{Ft-1} + \frac{1}{2}\left(\pi_{Fit} - \pi_{Fft} + \pi^*_{Fit} - \pi^*_{Fft}\right) - \Delta\bar{\upsilon}^*_t, \tag{A77}$$

$$\bar{\upsilon}^*_t = \bar{\upsilon}_{Ft} = \bar{\upsilon}^*_{Ft} = \Xi\left(a^*_{ft} - a_{ft}\right) + a^*_{ft} + \mathrm{F}\left(a_{it} - a^*_{it}\right). \tag{A78}$$

The first-order conditions are

$$\begin{aligned} 0 = {} & 2D_1\tilde{y}^R_{ft} - 2(1+\varphi)\Omega_m\Omega_y m_{it} + 2(1+\varphi)\Omega_y\Omega_\upsilon\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - 2(1+\varphi)\Omega_y\Omega_m\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) - D_2\bar{y}^R_{ft} \\ & + \tfrac{2\sigma(1-\phi)}{D}\delta_f(\eta_{1t} - \eta_{2t} + \eta_{3t} - \eta_{4t}) + \Pi'\delta_i(\eta_{5t} - \eta_{6t} - \eta_{7t} - \eta_{8t}) \end{aligned} \tag{A79}$$

$$0 = 2\tilde{y}^W_{ft} + \delta_i(\eta_{5t} + \eta_{6t} + \eta_{7t} + \eta_{8t}) \tag{A80}$$

$$0 = \frac{\nu(2-\nu)}{2D}m_{ft} + \delta_f\left(\frac{(1-\phi)(\upsilon-1)}{D}(\eta_{4t} - \eta_{1t}) + \left(\frac{(1-\phi)(\upsilon-1)}{D} + 1\right)(\eta_{2t} - \eta_{3t})\right) \tag{A81}$$

$$\begin{aligned} 0 = {} & 2(1+\varphi)\Omega^2_m m_{it} - 2(1+\varphi)\Omega_m\Omega_y\tilde{y}^R_{ft} - 2(1+\varphi)\Omega_m\Omega_\upsilon\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) + 2(1+\varphi)\Omega^2_m\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\ & -2(1+\varphi)\Gamma_y\Omega_m\bar{y}^R_{ft} + \delta_i\left(\left(\tfrac{1}{2} - \Gamma'\right)(\eta_{5t} - \eta_{7t}) + \left(\tfrac{1}{2} + \Gamma'\right)(\eta_{6t} - \eta_{8t})\right) \end{aligned} \tag{A82}$$

$$\begin{aligned} 0 = {} & -2(1+\varphi)\Omega_y\Omega_m\tilde{y}^R_{ft} + 2(1+\varphi)\Omega^2_m m_{it} - 2(1+\varphi)\Omega_m\Omega_\upsilon\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - 2(1+\varphi)\Gamma_y\Omega_m\bar{y}^R_{ft} \\ & +\delta_f(\phi(\eta_{1t} + \eta_{3t}) - (1+\phi)(\eta_{2t} + \eta_{4t})) + \eta_{9t} - \beta\mathbf{E}_t\eta_{9t+1} \\ & \delta_i\left(\left(\tfrac{1-\upsilon}{2} - \Gamma'\right)(\eta_{5t} - \eta_{8t}) + \left(\tfrac{1-\upsilon}{2} + \Gamma'\right)(\eta_{6t} + \eta_{7t})\right) \end{aligned} \tag{A83}$$

$$\begin{aligned} 0 = {} & 8(1+\varphi)\Omega^2_m\tilde{\upsilon}^R_{Ft} - 2(1+\varphi)\Omega_y\Omega_m\tilde{y}^R_{ft} + 2(1+\varphi)\Omega^2_m m_{it} - 2(1+\varphi)\Omega_m\Omega_\upsilon\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) \\ & -2(1+\varphi)\Gamma_y\Omega_m\bar{y}^R_{ft} + \delta_i\left(\left(\tfrac{\upsilon-1}{2} - \Gamma'\right)(\eta_{5t} + \eta_{8t}) + \left(\tfrac{\upsilon-1}{2} + \Gamma'\right)(\eta_{6t} + \eta_{7t})\right) \\ & +\delta_f((1+\phi)(\eta_{1t} + \eta_{3t}) - \phi(\eta_{2t} + \eta_{4t})) + \eta_{10t} - \beta\mathbf{E}_t\eta_{10t+1} \end{aligned} \tag{A84}$$

$$\begin{aligned} 0 = {} & -2(1+\varphi)\Omega^2_\upsilon\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - 2(1+\varphi)\Omega_y\Omega_\upsilon\tilde{y}^R_{ft} + 2(1+\varphi)\Omega_m\Omega_\upsilon m_{it} + 2(1+\varphi)\Omega_m\Omega_\upsilon\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\ & -2(1+\varphi)\Gamma_y\Omega_\upsilon\bar{y}^R_{ft} + \delta_i(-(1+2\varphi\phi(1-\phi))(\eta_{5t} + \eta_{8t}) + 2\varphi\phi(1-\phi)(\eta_{6t} + \eta_{7t})) \\ & +\delta_f(\phi(\eta_{1t} + \eta_{3t}) - (1+\phi)(\eta_{2t} + \eta_{4t})) + \eta_{11t} - \beta\mathbf{E}_t\eta_{11t+1} \end{aligned} \tag{A85}$$

$$\begin{aligned} 0 = {} & 2(1+\varphi)\Omega^2_\upsilon\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) + 2(1+\varphi)\Omega_y\Omega_\upsilon\tilde{y}^R_{ft} - 2(1+\varphi)\Omega_m\Omega_\upsilon m_{it} - 2(1+\varphi)\Omega_m\Omega_\upsilon\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\ & +2(1+\varphi)\Gamma_y\Omega_\upsilon\bar{y}^R_{ft} + \delta_i(-(1+2\varphi\phi(1-\phi))(\eta_{6t} + \eta_{7t}) + 2\varphi\phi(1-\phi)(\eta_{5t} + \eta_{8t})) \\ & +\delta_f((1+\phi)(\eta_{1t} + \eta_{3t}) + \phi(\eta_{2t} + \eta_{4t})) + \eta_{12t} - \beta\mathbf{E}_t\eta_{12t+1} \end{aligned} \tag{A86}$$

$$\frac{\xi_f}{\lambda_f}\frac{\nu}{2}\pi_{Hft} - \eta_{1t} + \eta_{1t-1} + \frac{\eta_{9t}}{2} + \frac{\eta_{11t}}{2} = 0 \tag{A87}$$

$$\frac{\xi_f}{\lambda_f}\frac{2-\nu}{2}\pi^*_{Hft} - \eta_{3t} + \eta_{3t-1} - \frac{\eta_{9t}}{2} + \frac{\eta_{11t}}{2} = 0 \tag{A88}$$

$$\frac{\xi_f}{\lambda_f}\frac{2-\nu}{2}\pi_{Fft} - \eta_{2t} + \eta_{2t-1} + \frac{\eta_{10t}}{2} + \frac{\eta_{12t}}{2} = 0 \tag{A89}$$

$$\frac{\xi_f}{\lambda_f}\frac{\nu}{2}\pi^*_{Fft} - \eta_{4t} + \eta_{4t-1} - \frac{\eta_{10t}}{2} + \frac{\eta_{12t}}{2} = 0 \tag{A90}$$

$$\frac{\xi_i}{\lambda_i}\phi\pi_{Hit} - \eta_{5t} + \eta_{5t-1} - \frac{\eta_{9t}}{2} - \frac{\eta_{11t}}{2} = 0 \tag{A91}$$

$$\frac{\xi_i}{\lambda_i}(1-\phi)\pi^*_{Hit} - \eta_{8t} + \eta_{8t-1} + \frac{\eta_{9t}}{2} - \frac{\eta_{11t}}{2} = 0 \tag{A92}$$

$$\frac{\xi_i}{\lambda_i}(1-\phi)\pi_{Fit} - \eta_{6t} + \eta_{6t-1} - \frac{\eta_{10t}}{2} - \frac{\eta_{12t}}{2} = 0 \tag{A93}$$

$$\frac{\xi_i}{\lambda_i}\phi\pi^*_{Hit} - \eta_{7t} + \eta_{7t-1} + \frac{\eta_{10t}}{2} - \frac{\eta_{12t}}{2} = 0 \tag{A94}$$

in which $\eta_{1t}, \eta_{2t}, \eta_{3t}, \eta_{4t}, \eta_{5t}$,$\eta_{6t}, \eta_{7t}, \eta_{8t}, \eta_{9t}, \eta_{10t}, \eta_{11t}$, and η_{12t} are the Lagrange multipliers corresponding to equations $(A65) - (A75)$, and $(A77)$respectively. In addition, $D_1 = (1+\varphi)\Omega_y^2 - \frac{(1-\sigma)(\nu-1)^2}{D}, D_2 = 2(1-\sigma)\frac{(\nu-1)^2}{D} - 2(1+\varphi)\Gamma_y\Omega_y$. Given initial values of $\widetilde{\upsilon}_{H-1}, \widetilde{\upsilon}_{F-1}, \widetilde{\upsilon}^*_{H-1}$, and $\widetilde{\upsilon}^*_{F-1}$ and $\eta_{1,-1} = \eta_{2,-1} = \eta_{3,-1} = \eta_{4,-1} = \eta_{5,-1} = \eta_{6,-1} = \eta_{7,-1} = \eta_{8,-1} = 0$, the above first-order conditions, together with equations $(A65) - (A78)$, constitute a dynamic system describing the optimal monetary policy.

5.The welfare loss function is identical to its counterpart in Engel (2011), when international trade in intermediate inputs is absent (i.e. $\phi = 1$), and there is no any nominal stickiness at the stage of intermediate-goods production.

Proof: The welfare loss function is

$$\mathbf{W} = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbf{X}_t + t.i.p. + O\left(\|a\|^3\right), \tag{A95}$$

where

$$\begin{aligned}
\mathbf{X}_t = {} & \tfrac{\xi_f}{2\delta_f}\left(\tfrac{\nu}{2}\pi^2_{Hft} + \tfrac{2-\nu}{2}\pi^{2*}_{Hft} + \tfrac{2-\nu}{2}\pi^2_{Fft} + \tfrac{\nu}{2}\pi^{2*}_{Fft}\right) \\
& + \tfrac{\xi_i}{2\delta_i}\left(\phi\pi^2_{Hit} + (1-\phi)\pi^{2*}_{Hit} + (1-\phi)\pi^2_{Fit} + \phi\pi^{2*}_{Fit}\right) \\
& + \left[(1+\varphi)\Omega_y^2 - (1-\sigma)\tfrac{(\nu-1)^2}{D}\right]\left(\tilde{y}^R_{ft}\right)^2 + (\sigma+\varphi)\left(\tilde{y}^W_{ft}\right)^2 \\
& + \left[2(1+\varphi)\Gamma_y\Omega_y - 2(1-\sigma)\tfrac{(\nu-1)^2}{D}\right]\bar{y}^R_{ft}\tilde{y}^R_{ft} + \tfrac{\nu(2-\nu)}{4D}m^2_{ft} \\
& + (1+\varphi)\Omega_m^2 m^2_{it} + 2(1+\varphi)\Omega_y\Omega_m\tilde{y}^R_{ft}m_{it} - 2(1+\varphi)\Gamma_y\Omega_m\bar{y}^R_{ft}m_{it} \\
& + (1+\varphi)\Omega_\upsilon^2\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right)^2 + (1+\varphi)\Omega_m^2\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right)^2 \\
& + 2(1+\varphi)\Omega_y\Omega_\upsilon\tilde{y}^R_{ft}\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - 2(1+\varphi)\Omega_y\Omega_m\tilde{y}^R_{ft}\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\
& - 2(1+\varphi)\Omega_m\Omega_\upsilon m_{it}\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) + 2(1+\varphi)\Omega_m^2 m_{it}\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\
& + 2(1+\varphi)\Gamma_y\Omega_\upsilon\bar{y}^R_{ft}\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right) - 2(1+\varphi)\Gamma_y\Omega_m\bar{y}^R_{ft}\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right) \\
& - 2(1+\varphi)\Omega_m\Omega_\upsilon\left(\tilde{\upsilon}^W_{Ft} - \tilde{\upsilon}^W_{Ht}\right)\left(\tilde{\upsilon}^R_{Ht} + \tilde{\upsilon}^R_{Ft}\right)
\end{aligned} \tag{A63}$$

in which $t.i.p.$stands for the terms independent of policy and $O\left(\|a\|^3\right)$collects all terms of third or higher order. In the expression of $\mathbf{X}_t, \Omega_y = \left(\frac{4\sigma\phi(1-\phi)}{D} + 2\phi - 1\right), \Omega_m = \phi(1-\phi)\frac{2(\nu-1)}{D}, \Omega_\upsilon = 2\phi(1-\phi), \Gamma_y = \frac{1}{1+\varphi}(1-2\phi)\frac{\sigma-D}{D}$.

When there is no international trade in intermediate inputs (i.e. $\phi = 1$), we have $\Omega_y = 1, \Omega_m = 0, \Omega_\upsilon = 0$, and $\Gamma_y = -\frac{\sigma-D}{(1+\varphi)D}$. In addition, the fact that there is no any nominal stickiness at the stage of intermediate-goods production implies that there are no price dispersion terms associated with intermediate-goods production. Thus, $\mathbf{X}_t$ can be simplified as

$$\begin{aligned}
\mathbf{X}_t = {} & \tfrac{\xi_f}{2\delta_f}\left(\tfrac{\nu}{2}\pi^2_{Hft} + \tfrac{2-\nu}{2}\pi^{*2}_{Hft} + \tfrac{2-\nu}{2}\pi^2_{Fft} + \tfrac{\nu}{2}\pi^{*2}_{Fft}\right) + (\sigma+\varphi)\left(\tilde{y}^W_{ft}\right)^2 + \tfrac{\nu(2-\nu)}{4D}m^2_{ft} \\
& + \left[(1+\varphi) - (1-\sigma)\tfrac{(\nu-1)^2}{D}\right]\left(\tilde{y}^R_{ft}\right)^2 + \left[2(1+\varphi)\left(-\tfrac{\sigma-D}{(1+\varphi)D}\right) - 2(1-\sigma)\tfrac{(\nu-1)^2}{D}\right]\bar{y}^R_{ft}\tilde{y}^R_{ft}
\end{aligned}$$

in which $D = \sigma\nu(2-\nu) + (\nu-1)^2$.

Note that $2(1+\varphi)\left(-\frac{\sigma-D}{(1+\varphi)D}\right) - 2(1-\sigma)\frac{(\nu-1)^2}{D} = 0$ and $(1+\varphi) - (1-\sigma)\frac{(\nu-1)^2}{D} = \left(\varphi + \frac{\sigma}{D}\right)$, thus $\mathbf{X}_t$ can be written as

$$\begin{aligned}
\mathbf{X}_t = {} & \left[\varphi + \tfrac{\sigma}{D}\right]\left(\tilde{y}^R_{ft}\right)^2 + (\sigma+\varphi)\left(\tilde{y}^W_{ft}\right)^2 + \tfrac{\nu(2-\nu)}{4D}m^2_{ft} \\
& + \tfrac{\xi_f}{2\delta_f}\left(\tfrac{\nu}{2}\pi^2_{Hft} + \tfrac{2-\nu}{2}\pi^{*2}_{Hft} + \tfrac{2-\nu}{2}\pi^2_{Fft} + \tfrac{\nu}{2}\pi^{*2}_{Fft}\right)
\end{aligned}$$

which is identical to equation $(C45)$ in Appendix C to Engel (2011).[21]

Appendix B

B1. Proof of Proposition 1

We prove the proposition by contradiction. Suppose that there were a monetary policy which can achieve implement the flexible-price equilibrium allocation, then $\tilde{y}^R_{ft} = \tilde{y}^W_{ft} = \tilde{\upsilon}^R_{Ht} = \tilde{\upsilon}^R_{Ft} = \tilde{\upsilon}^W_{Ft} = \tilde{\upsilon}^W_{Ht} = 0$, for all t. In the flexible-price equilibrium, we know that $m_{ft} = m_{it} = z_{ft} = z_{it} = 0$, for all t. It follows that, from equations $(42) - (49), \pi_{Hft} = \pi_{Fft} = \pi^*_{Hft} = \pi^*_{Fft} = \pi_{Hit} = \pi_{Fit} = \pi^*_{Fit} = \pi^*_{Hit} = 0$, for all t. However, from equations (52)and (54), we know that $\pi_{Hit} - \pi_{Hft} + \pi^*_{Hit} - \pi^*_{Hft} = 2\Delta\bar{\upsilon}_t = 2\left(\Xi\left(\Delta a_{ft} - \Delta a^*_{ft}\right) + \Delta a_{ft} + \mathrm{F}\left(\Delta a^*_{it} - \Delta a_{it}\right)\right)$and

[21] Note that we use φ to denote the inverse of the Frisch elasticity of labor supply, while Engel (2011) *uses* ϕ to represent it.

$\pi_{Fit} - \pi_{Fft} + \pi^*_{Fit} - \pi^*_{Fft} = 2\Delta\bar{v}^*_t = 2\left(\Xi\left(\Delta a^*_{ft} - \Delta a_{ft}\right) + \Delta a^*_{ft} + \mathsf{F}\left(\Delta a_{it} - \Delta a^*_{it}\right)\right)$,which contradict $\pi_{Hft} = \pi_{Fft} = \pi^*_{Hft} = \pi^*_{Fft} = \pi_{Hit} = \pi_{Fit} = \pi^*_{Fit} = \pi^*_{Hit} = 0$, for all t.

B2. Proof of Proposition 2

We first show that, if $z_{it} = 0$, then $z_{it+1} = 0$. From equations (46) – (49), we know that, if $z_{it} = 0$, then $(\pi_{Fit} - \pi_{Hit}) - (\pi^*_{Fit} - \pi^*_{Hit}) = \beta\mathbf{E}_t\left[(\pi_{Fit+1} - \pi_{Hit+1}) - (\pi^*_{Fit+1} - \pi^*_{Hit+1})\right]$. It implies that $(\pi_{Fit} - \pi_{Hit}) - (\pi^*_{Fit} - \pi^*_{Hit}) = 0$ for all $t \geqslant 1$. Thus, we have $(p_{Fit+1} - p_{Hit+1}) - (p^*_{Fit+1} - p^*_{Hit+1}) = (p_{Fit} - p_{Hit}) - (p^*_{Fit} - p^*_{Hit})$. It means that, if $z_{it} = 0$, then $z_{it+1} = 0$. By induction, if $z_{i0} = 0$, then $z_{it} = 0$ for all $t \geqslant 1$.But from the expression of z_{it},we know that in the steady state, $z_{i0} = 0$. Following the same steps, we can similarly prove that $z_{ft} = 0$ for all $t \geqslant 1$.

B3. Proof of Proposition 3

From equation (32), we know that $m_{ft} = m_{it} + v^R_{Ht} + v^R_{Ft}$. Thus, it is enough to show that $v^R_{Ht} = v^R_{Ft} = 0$ for all $t \geqslant 1$. However, from equation (33)and Proposition 2, we know that $v^R_{Ht} = v^R_{Ft}$ for all $t \geqslant 1$. Thus, it is sufficient to show that $v^R_{Ht} = 0$ for all $t \geqslant 1$. Now we show that, if $v^R_{Ht} = 0$, then $v^R_{Ht+1} = 0$. From equations (42), (44), (46), and (49), we have that, if $\theta_f = \theta_i$, and $v^R_{Ht} = 0$, then $(\pi_{Hit} - \pi_{Hft}) - \left(\pi^*_{Hit} - \pi^*_{Hft}\right) = \beta\mathbf{E}_t\left[(\pi_{Hit+1} - \pi_{Hft+1}) - \left(\pi^*_{Hit+1} - \pi^*_{Hft+1}\right)\right]$. Note that this equation makes use of the conclusion that, if $v^R_{Ht} = 0$, then $v^R_{Ht} = v^R_{Ft} = 0$ and $m_{ft} = m_{it}$. From $(\pi_{Hit} - \pi_{Hft}) - \left(\pi^*_{Hit} - \pi^*_{Hft}\right) = \beta\mathbf{E}_t\left[(\pi_{Hit+1} - \pi_{Hft+1}) - \left(\pi^*_{Hit+1} - \pi^*_{Hft+1}\right)\right]$, we have $(\pi_{Hit} - \pi_{Hft}) - \left(\pi^*_{Hit} - \pi^*_{Hft}\right) = 0$ for all $t \geqslant 1$. It implies that $(p_{Hit+1} - p_{Hft+1}) - \left(p^*_{Hit+1} - p^*_{Hft+1}\right) = (p_{Hit} - p_{Hft}) - \left(p^*_{Hit} - p^*_{Hft}\right)$. Equivalently, $v^R_{Ht+1} = v^R_{Ht}$. Thus, if $v^R_{Ht} = 0$, then $v^R_{Ht+1} = 0$. By induction, if $v^R_{H0} = 0$, then $v^R_{Ht} = 0$ for all $t \geqslant 1$. But in the initial steady state, we have $v^R_{H0} = 0$, Thus, we complete the proof of the Proposition.

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