Should Public Capital Be Subsidized or Provided?*

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Abstract

In an endogenous-growth model, we consider alternative ways of providing public capital using distortionary taxes. We show that if the government provides the good, the resulting growth rate and welfare may or may not be higher than under laissez-faire. By contrast, if the government subsidizes private providers, not only are growth and welfare higher than under public provision, they are also unambiguously higher than under laissez-faire.

1 Introduction

The early literature on endogenous-growth models (Romer 1986 and Lucas 1988) showed that, when there are stocks that generate positive externalities (knowledge, human capital, infrastructure capital), the government can increase the economy's growth rate by intervening to internalize the externality. This literature assumed that the government had access to lump-sum taxes to finance the intervention. The more recent literature (Barro 1990) has looked at situations where the government uses distortionary taxes. However, this literature does not model the underlying rationale for public intervention. Rather, they assume that government spending is productive by including it as an argument in the aggregate production function.

If the reason for public intervention is an externality, the solution need not be government provision of the good; it could be a subsidy to private providers. The purpose of this paper is to examine alternative forms of providing these so-called "public capital" goods using distortionary taxes where the externality associated with public capital is explicitly taken into account. In section 2, we set up a two-capital endogenous-growth model in which type 1 capital has no externality but type 2 has a positive externality. In section 3, we analyze the laissez-faire equilibrium in a special version of the model, deriving the long-run rate of growth of output. By laissez-faire, we mean that all capital formation is done by private individuals and the government plays no role. In section 4, we analyze the equilibrium under two popular kinds of government intervention, namely (1) the government's taking over type 2 capital formation; and (2) subsidizing private type 2 capital formation. In section 5, we compare the welfare and growth rates of the various cases. We show that with public provision, welfare and the growth rate of output may or may not be higher than under laissez-faire, since the distortionary costs of taxation may outweigh the benefits of capturing the positive externality. With the subsidy, however, not only are welfare and the growth rate higher than with public provision (because the former requires less of the distortionary tax) but it is also unambiguously higher than under laissez-faire. Section 6 concludes.

2 The Model

Consider an economy with an infinitely-lived representative agent. His preferences are given by

$$\sum_{t=0}^{\infty} \rho^t u(c_t),$$

where ρ is the discount factor ($0 < \rho < 1$) and $u(c_t)$ is increasing and concave in c_t, and satisfies the Inada conditions.

The per-capita production function is given by

$$y_t = f(k_t^1,\ k_t^2)e(\hat{k}_t^2),$$

where k_t^1 is type 1 capital stock in a representative firm; k_t^2 is type 2 capital stock in the representative firm. The positive externality generated by type 2 capital is captured by an increasing function, $e(\hat{k}_t^2)$, where $\hat{k}_t^2$ is the *average* of type 2 capital stock in the economy. Accumulation of the two types of capital goods is given by

$$\begin{aligned} k_{t+1}^1 &= y_t + (1-\delta^1)k_t^1 - c_t - z_t \\ k_{t+1}^2 &= z_t + (1-\delta^2)k_t^2, \end{aligned} \tag{1}$$

where δ^1 and δ^2 are the rates of depreciation in type 1 and type 2 capital goods, respectively, and z_t is investment in type 2 capital.

This completes the basic setup. In the next two sections, we characterize the benchmark case (laissez-faire) and the two scenarios outlined in the introduction by solving some dynamic optimization problems. We will use a special example which delivers an explicit solution and makes transparent the comparison of long-run growth rates and welfare.

3 The Benchmark Case: Laissez-Faire

In this section, we analyze the laissez-faire equilibrium in a special version of the model. We adopt a utility function and production function that permit an explicit solution. Specifically, the pair consists of a log utility and the Cobb-Douglas production function, which is a special case of the pairs studied in Benhabib and Rustichini (1994). For pairs of utility and production functions that allow for explicit dynamics in a *continuous* time framework, see Xie (1991) and Xie (1994).

To fix ideas, we specify in detail the functional forms in this example.

$$\begin{aligned} u(c_t) &= \ln(c_t) \\ f(k_t^1,\ k_t^2) &= A(k_t^1)^\alpha (k_t^2)^\beta \\ e(\hat{k}_t^2) &= (\hat{k}_t^2)^{1-\alpha-\beta}, \end{aligned}$$

where $\alpha \in (0,1)$, $\beta \in (0,1)$ and $\alpha + \beta < 1$. The functional form for the externality term is specified in such a way that long-run growth is possible (see Lucas 1988 and Romer 1990).

In order to have an explicit solution, we also need to impose that $\delta^1 = \delta^2 = 1$. The assumption of 100% depreciation of both capital goods is not realistic and should be abandoned when it comes to simulation. For the theoretical purpose here, the assumption helps us to draw qualitative conclusions.

Under laissez-faire, formation of both type 1 and type 2 capital is done by private individuals. The government plays no role. As explained in Kehoe, Levine and Romer (1992), the competitive equilibrium allocation is the result of the following optimization problem:

$$\max \sum_{t=0}^{\infty} \rho^t \ln(c_t)$$

$$\begin{aligned} \text{subject to} \quad k_{t+1}^1 &= A(k_t^1)^\alpha (k_t^2)^\beta (\hat{k}_t^2)^{1-\alpha-\beta} - c_t - z_t \\ k_{t+1}^2 &= z_t. \end{aligned}$$

To solve this problem, we write down the Lagrangian,

$$\begin{aligned} \mathcal{L} = \textstyle\sum_{t=0}^{\infty} \rho^t \{ & \ln(c_t) + \lambda_t [A(k_t^1)^\alpha (k_t^2)^\beta (\hat{k}_t^2)^{1-\alpha-\beta} - c_t - z_t - k_{t+1}^1] \\ & + \mu_t [z_t - k_{t+1}^2] \}. \end{aligned}$$

The first-order conditions are:

$$1/c_t = \lambda_t \tag{2}$$

$$\lambda_t = \mu_t \tag{3}$$

$$\lambda_t = \rho \lambda_{t+1} \alpha y_{t+1} / k^1_{t+1} \tag{4}$$

$$\mu_t = \rho \lambda_{t+1} \beta y_{t+1} / k^2_{t+1} \tag{5}$$

$$k^1_{t+1} = A(k^1_t)^\alpha (k^2_t)^{1-\alpha} - c_t - k^2_{t+1}. \tag{6}$$

The transversality conditions are: $\rho^t \lambda_t k^1_{t+1} \to 0$ and $\rho^t \mu_t k^2_{t+1} \to 0$ as $t \to \infty$. Note that in equation (6), the equilibrium condition $k^2_t = \hat{k}^2_t$ has been substituted in.

We guess that the solution to the above equations has the following form:

$k^1_{t+1} = a y_t$, $k^2_{t+1} = b y_t$ and $c_t = (1 - a - b) y_t$ with a and b constant.

It is straightforward to verify that when $a = \rho\alpha$, $b = \rho\beta$, the guess above satisfies all the first-order conditions and the transversality conditions and therefore is the solution. To find the rate of output growth, we calculate that

$$\begin{aligned} y_{t+1} &= A(k^1_{t+1})^\alpha (k^2_{t+1})^{1-\alpha} \\ &= A(\rho\alpha y_t)^\alpha (\rho\beta y_t)^{1-\alpha} \\ &= \rho A \alpha^\alpha \beta^{1-\alpha} y_t. \end{aligned} \tag{7}$$

Thus the rate of output growth in the benchmark case is

$$g_0 = \rho A \alpha^\alpha \beta^{1-\alpha} - 1. \tag{8}$$

Note that $k^1_{t+1} = \rho\alpha y_t$ and $k^2_{t+1} = \rho\beta y_t$. It is clear that an increase in α raises next period type 1 capital for any given current output; an increase in β raises next period type 2 capital for any given current output. However, from the growth rate formula (8), g_0 increases in β while the effect of an increase in

α is ambiguous. The explanation for this is that β does not appear in the production function at the equilibrium whereas α has an ambiguous effect on output since $y_{t+1} = A(k^1_{t+1})^\alpha(k^2_{t+1})^{1-\alpha}$. Equation (8) also says that the growth rate is increasing in ρ and A. This is intuitive because an increase in ρ means that individuals discount future utility to a lesser extent and thus would save more and the economy would grow faster; an increase in A means that productivity is higher and therefore the growth rate is higher.

Under laissez-faire, since the positive externality from type 2 capital stock is not internalized, private individuals will invest in this type of capital *less* than the socially optimal amount. This is one of the popular arguments for government action. In the next section, we study the costs and benefits of different types of government intervention to internalize the externality.

4 Costs and Benefits of Government Intervention

In the last section, we reiterated the conventional wisdom that laissez-faire leads to under-investment in the presence of positive externality. The popular actions that the government takes in this circumstance are: (1) Take over type 2 capital formation, providing it publicly; and (2) Subsidize type 2 capital formation by the private sector. When lump-sum taxes are available to the government, actions (1) and (2) can both restore the social optimum. In this case, it is straightforward to derive that in equilibrium, $k^1_{t+1} = \rho\alpha y_t$, $k^2_{t+1} = \rho(1-\alpha)y_t$ and the rate of output growth is $\rho A\alpha^\alpha(1-\alpha)^{1-\alpha} - 1$.

But what if lump-sum taxes are not available and the government has to use *distortionary* taxes? Several issues arise. First, given the tax distortions, is it worthwhile for the government to take any action? Second, which of the two actions – public provision or a subsidy – is more desirable from the social-welfare point of view?

We now analyze the two actions. To simplify matters, we limit our analysis to constant tax/subsidy rates.

4.1 Action 1: Public Capital Formation by Output Tax

The setup is as follows. The government announces that a tax rate τ will be levied on output and all the tax proceeds spent on type 2 capital formation for public use. Private individuals then respond optimally to the announced government policy and decide how much to consume and how much to save for type 1 capital investment. Finally, the government takes the individuals' response as given to maximize the representative individual's welfare.

To proceed, let us write down the individual's optimization problem:

$$\max \sum_{t=0}^{\infty} \rho^t \ln(c_t)$$

$$\text{subject to} \quad k_{t+1}^1 = A(k_t^1)^\alpha (\tilde{k}_t^2)^{1-\alpha}(1-\tau) - c_t$$

where $\tilde{k}_t^2$ and τ are controlled by the government and are taken as given by the individual. The Lagrangian in this case is,

$$\mathcal{L} = \sum_{t=0}^{\infty} \rho^t \left\{ \ln(c_t) + \gamma_t \left[A(k_t^1)^\alpha (\tilde{k}_t^2)^{1-\alpha}(1-\tau) - c_t - k_{t+1}^1 \right] \right\}. \tag{9}$$

The first-order conditions are:

$$1/c_t = \gamma_t \tag{10}$$

$$\gamma_t = \rho\gamma_{t+1}\alpha(1-\tau)y_{t+1}/k^1_{t+1} \tag{11}$$

The transversality condition is: $\rho^t\gamma_t k^1_{t+1} \to 0$ as $t \to \infty$. Note that all government tax revenue is assumed to be spent on type 2 capital formation. Thus we have: $\tilde{k}^2_{t+1} = \tau y_t$.

It is easy to verify that the individual's optimal response is the following:

$$k^1_{t+1} = \rho\alpha(1-\tau)y_t \tag{12}$$

$$c_t = [1-\rho\alpha(1-\tau)-\tau]\, y_t \tag{13}$$

The growth rate of output is thus:

$$g(\tau) = A\rho^\alpha\alpha^\alpha(1-\tau)^\alpha\tau^{1-\alpha} - 1. \tag{14}$$

To find the optimal tax rate, we first calculate the individual's welfare as a function of τ, $W(\tau)$. This can be done explicitly because equation (13) says that consumption starts from $c_0(\tau)$ and grows at a constant rate $g(\tau)$, where $c_0(\tau) = [1-\rho\alpha(1-\tau)-\tau]\, y_0$. In fact, we have:

$$W(\tau) = \Gamma + \frac{\ln c_0(\tau)}{1-\rho} + \frac{\rho}{(1-\rho)^2}\left[\alpha\ln(1-\tau) + (1-\alpha)\ln\tau\right] \tag{15}$$

where Γ is independent of τ. Setting $W'(\tau) = 0$, we get the optimal tax rate:

$$\tau^*_1 = \rho(1-\alpha) \tag{16}$$

and the resulting growth rate:

$$g_1 = \rho A\alpha^\alpha(1-\alpha)^{1-\alpha}\left[1-\rho(1-\alpha)\right]^\alpha - 1 \tag{17}$$

It is worth noting that the optimal tax rate derived here is consistent with the one found in Glomm and Ravikumar (1994). Similar to Barro (1990), Glomm and Ravikumar (1994) *assume* that infrastructure is provided by the government and then ask what the optimal tax rate should be. We in this paper raise the question of whether infrastructure *should be* provided by the government. The answer to this question is deferred to section 5 where we compare growth rates and welfare in various cases.

4.2 Action 2: Investment Subsidy Financed by Output Tax

This is the case in which the government intervenes indirectly through incentive schemes instead of directly taking over type 2 capital formation. We represent the government's action by two constant rates s and τ, where s is the subsidy rate to private type 2 capital formation and τ is the output tax rate. We assume that the government budget is balanced in each period. To ensure a balanced budget, the two rates must satisfy a simple relation: $\tau = \rho\beta s/(1 + \rho\beta s)$. This says that when the subsidy rate s is zero (the laissez-faire case), the tax rate needed to finance the subsidy is obviously zero; when s increases without bound, τ increases and approaches unity. We will verify later that this relation does guarantee a balanced budget.

The above discussion means that the government's action can be represented by the subsidy rate s alone. When s is given, the required τ is determined by $\rho\beta s/(1 + \rho\beta s)$. With this in mind, we raise two questions. First, will welfare increase when we move from $s = 0$ (the laissez-faire case) to a slightly positive s? In other words, does intervention pay? Second, what is the optimal subsidy rate s^* that leads to the highest welfare among all possible subsidy rates?

To answer these questions, we form and solve the representative individual's optimization problem. Each individual takes s and τ as given. He also takes other individuals' actions as given. Since we assume there are an infinite number of individuals, no individual has control over $\{\hat{k}_t^2\}_{t=0}^{t=\infty}$, the average of type 2 capital in the economy. The individual's optimization problem is as follows:

$$\max \sum_{t=0}^{\infty} \rho^t \ln(c_t)$$

$$\begin{aligned} \text{subject to} \quad k_{t+1}^1 &= A(k_t^1)^\alpha (k_t^2)^\beta (\hat{k}_t^2)^{1-\alpha-\beta}(1-\tau) - c_t - z_t \\ k_{t+1}^2 &= z_t(1+s), \end{aligned}$$

where c_t and z_t (investment on type 2 capital) are the control variables; k_t^1 and k_t^2 are the state variables.

The optimal decision by the representative agent can be summarized by the following results:

$$\begin{aligned} k_{t+1}^1 &= \rho\alpha(1-\tau)y_t \\ k_{t+1}^2 &= \rho\beta(1+s)(1-\tau)y_t \\ z_t &= \rho\beta(1-\tau)y_t. \end{aligned}$$

It is straightforward to verify that for s and τ that satisfy $\tau = \rho\beta s/(1+\rho\beta s)$, the government budget is always balanced:

$$sz_t - \tau y_t = s\rho\beta[1 - \rho\beta s/(1+\rho\beta s)]y_t - [\rho\beta s/(1+\rho\beta s)]y_t \equiv 0.$$

Given the optimal decision of the representative agent, we can easily calculate his welfare as a function of the subsidy rate s. To proceed, we note that,

$$c_t = y_t - k_{t+1}^1 - k_{t+1}^2 = (1 - \rho\alpha - \rho\beta) y_t / (1 + \rho\beta s),$$

and

$$\begin{aligned} y_{t+1} &= A(k_{t+1}^1)^{\alpha}(k_{t+1}^2)^{1-\alpha} \\ &= \rho A \alpha^{\alpha} \beta^{1-\alpha} (1+s)^{1-\alpha} y_t / (1 + \rho\beta s) \\ &\equiv [1 + g(s)] y_t, \end{aligned}$$

where we use the notation $g(s)$ to denote the growth rate of output when the subsidy rate is s. Since c_t is proportional to y_t, the growth rate of consumption is equal to $g(s)$. Hence the consumption series starts with $c_0(s) = (1 - \rho\alpha - \rho\beta) y_0 / (1 + \rho\beta s)$ and grows at the rate $g(s)$. As a result, welfare as a function of s can be calculated:

$$W(s) = J + \frac{\rho(1-\alpha)}{(1-\rho)^2} \ln(1+s) - \frac{1}{(1-\rho)^2} \ln(1 + \rho\beta s), \tag{18}$$

where J is a constant and is independent of the subsidy rate. We now answer the questions raised above in two propositions.

Proposition 1: *Welfare increases when s increases from zero (the laissez-faire case) to a slightly positive rate. That is, $W'(0) > 0$.*

Proof. From equation (18), we obtain

$$W'(s) = \frac{\rho(1-\alpha-\beta) - [1 - \rho(1-\alpha)]\rho\beta s}{(1-\rho)^2 (1+\rho\beta s)(1+s)}. \tag{19}$$

Thus, $W'(0) = \rho(1-\alpha-\beta)/(1-\rho)^2 > 0$. ■

Proposition 2: *The optimal rate of subsidy is given by*

$$s^* = \frac{1-\alpha-\beta}{\beta[1-\rho(1-\alpha)]}. \tag{20}$$

Proof. From equation (19), we see that,

$$W'(s) = \begin{cases} + & 0 \leq s < s^* \\ 0 & s = s^* \\ - & s > s^* \end{cases}$$

where s^* is given in equation (20). Thus $W(s)$ is maximized at s^*. The corresponding output tax rate required to finance the subsidy is given by

$$\tau_2^* = \frac{\rho(1-\alpha-\beta)}{1-\rho\beta}$$

which is seen to lie in the open interval $(0, 1)$. ∎

Propositions 1 and 2 have two implications for policy based on welfare considerations. First, the government can improve upon laissez-faire by subsidizing public capital formation. Second, government intervention should not be overdone; when the subsidy rate is greater than s^*, welfare starts to decline.

When the government subsidizes type 2 capital formation at the rate s^* and finances it by an output tax at the rate τ_2^*, the equilibrium levels of capital formation are given by

$$k_{t+1}^1 = \rho\alpha \left[\frac{1-\rho(1-\alpha)}{1-\rho\beta}\right] y_t \tag{21}$$

$$k_{t+1}^2 = \rho(1-\alpha)y_t, \tag{22}$$

and the resulting growth rate of output is:

$$g_2 = \rho A \alpha^\alpha (1-\alpha)^{1-\alpha} \left[\frac{1-\rho(1-\alpha)}{1-\rho\beta}\right]^\alpha - 1. \tag{23}$$

Equation (23) will be used for comparison with other growth rates.

5 Comparison of Growth Rates and Welfare

In this section, we provide two propositions. Proposition 3 compares growth rates and Proposition 4 compares welfare.

PROPOSITION 3: *Among the growth rates in the laissez-faire case and in the subsequent cases with government intervention, we have*

$$g_1 < g_2$$
$$g_0 < g_2$$

Proof. From equations (17) and (23), we see that $g_1 < g_2$. To show that $g_0 < g_2$, let us look at the ratio of the two growth factors:

$$\frac{1+g_2}{1+g_0} = \left[\frac{1-\alpha}{\beta}\right]^{1-\alpha} \left[\frac{1-\rho(1-\alpha)}{1-\rho\beta}\right]^{\alpha}. \tag{24}$$

Define an auxiliary function $\Phi(x)$ by

$$\Phi(x) = x^{1-\alpha}[1-\rho x]^{\alpha}.$$

It is straightforward to show that $\Phi(x)$ is increasing in x when $0 < x < (1-\alpha)/\rho$. Note that $0 < \beta < 1-\alpha < (1-\alpha)/\rho$. It must be true that $\Phi(\beta) < \Phi(1-\alpha)$. In other words,

$$\frac{1+g_2}{1+g_0} = \frac{\Phi(1-\alpha)}{\Phi(\beta)} > 1 \blacksquare$$

The intuition behind this proposition is as follows. To begin with, it is natural that g_2 is greater than g_1 because in taking action 2, the government needs less of the distortionary tax to finance the investment subsidy than in taking action 1 where all of public capital formation has to be financed. The result that g_2 is always greater than g_0 says that the benefit from an investment subsidy to correct for the externality is always greater than the cost of financing the subsidy through an output tax. The reason for this stems from Proposition 1, which stated that starting from zero, a small subsidy will improve welfare (it will also increase the growth rate). But a subsidy of zero is the laissez-faire equilibrium. Thus, we can always improve welfare and the growth rate (relative to the laissez-faire case) by increasing the subsidy from zero to its optimal amount.

What Proposition 3 leaves out is the comparison between g_1 and g_0. We now tackle this problem. From equations (8) and (17), we obtain

$$\frac{1+g_1}{1+g_0} = \left[\frac{1-\alpha}{\beta}\right]^{1-\alpha} [1-\rho(1-\alpha)]^{\alpha}. \tag{25}$$

The first term on the right-hand side, which is greater than 1, captures the benefit of internalizing the positive externality; the second term, which is less than 1, captures the cost of the distortionary output tax that is imposed in order to finance the public capital formation.

When α approaches zero, $(1+g_1)/(1+g_0)$ approaches $1/\beta$, which is greater than 1. The net effect of public capital formation is clearly favorable to long-run growth. This occurs because with a small α, type 1 capital is unimportant. The tax distortion on type 1 capital formation is thus not very costly. The benefit from internalizing the positive externality dominates the outcome.

When $(1-\alpha)$ is close to β, the first term is close to 1 and the second term is close to $[1-\rho\beta]^{1-\beta}$. Therefore the cost dominates the benefit. When $(1-\alpha)$ is close to β, $1-\alpha-\beta$ is close to zero. The externality in the production function is hardly significant and hence not worth internalizing.

The greater is ρ, the less people discount the future. The accumulation of both types of capital, and type 2 capital in particular, will be faster. This requires a greater tax burden and the cost from tax distortion becomes more severe. Therefore, the net benefit from public capital formation tends to be lower with a greater ρ. This intuition is confirmed in equation (25).

PROPOSITION 4: *Among the representative individual's welfare achieved in the laissez-faire case and in the subsequent cases with government intervention, we have:*

$$\begin{aligned} W_0 &< W_2 \\ W_1 &< W_2 \end{aligned}$$

PROOF: $W_0 < W_2$ was established in Propositions 1 and 2. As for W_1 and W_2, they can be calculated as follows. First, note that from (13) and (16), we have

$$c_1(0) = \{1 - \rho\alpha\,[1-\rho(1-\alpha)] - \rho(1-\alpha)\}\, y(0) \tag{26}$$

and from equations (21) and (22), we have

$$c_2(0) = \left\{1 - \rho\alpha\left[\frac{1-\rho(1-\alpha)}{1-\rho\beta}\right] - \rho(1-\alpha)\right\} y(0) \tag{27}$$

Also, the growth rates are given explicitly in (17) and (23). With this information, the computation of W_1 and W_2 is straightforward and we find

$$\mathrm{sgn}(W_2 - W_1) = \mathrm{sgn}\Psi(\beta),$$

where $\Psi(\beta) = (1-\rho)\ln(1-\rho\alpha-\rho\beta) - (1-\rho)\ln(1-\rho\alpha) - [1-\rho(1-\alpha)]\ln(1-\rho\beta)$. We now show that $\Psi(\beta) > 0$, recalling that $\beta \in (0, 1-\alpha)$. First, we see that $\lim \Psi(\beta) = 0$ as $\beta \to 0+$. Second, we calculate $\Psi'(\beta)$ and find

$$\mathrm{sgn}\Psi'(\beta) = \mathrm{sgn}\,(1-\alpha-\beta)\,.$$

Thus, $\Psi'(\beta) > 0$ for any $\beta \in (0,\, 1-\alpha)$. ■

While the intuition behind $W_1 < W_2$ is the same as that for the comparison of the two growth rates, note that the initial consumption under action 1 is *higher* than under action 2. Thus, it must be the case that the two consumption paths cross, and that the higher initial consumption under action 1 is dominated by the higher long-run consumption under action 2. As for the comparison between W_0 and W_1, we again have an ambiguous result. When the private return to type 2 capital is small (β close to zero), simple calculation shows that W_0 is less than W_1. Thus direct provision of type 2 capital is better than laissez-faire. When β approaches $1-\alpha$, we find that

$$\begin{aligned}
\mathrm{sgn}(W_1 - W_0) &= \mathrm{sgn}\left[\Omega(\alpha,\rho)\right] \\
\text{where, } \Omega(\alpha,\rho) &= \rho\ln[1-\rho(1-\alpha)]^{\alpha} + (1-\rho)\ln\frac{(1-\rho\alpha)[1-\rho(1-\alpha)]}{1-\rho} \\
&\leq \ln\left\{\rho[1-\rho(1-\alpha)]^{\alpha} + (1-\rho\alpha)[1-\rho(1-\alpha)]\right\} \\
&< \ln\left\{\rho[1-\alpha\rho(1-\alpha)] + (1-\rho\alpha)[1-\rho(1-\alpha)]\right\} \\
&= 0
\end{aligned}$$

in which the first inequality is due to the fact that $\ln(\cdot)$ is concave; the second inequality is because $(1-x)^{\alpha} < 1-\alpha x$ for $\alpha \in (0,1)$ and $x \in (0,1)$. Therefore, when β approaches $1-\alpha$, W_1 is less than W_0. This result is intuitive because in this case, the externality is so small that the benefit from internalizing the externality is negligible compared to the cost of the tax distortion.

6 Conclusion

In this paper, we attempted to combine two ideas arising from the literature on endogenous growth. One is that there are positive externalities associated with stocks, which if internalized can increase the economy's long-run growth rate. The other is that in order to intervene and internalize these externalities, governments have to resort to distortionary taxes. Using a simple model, we showed that the manner in which the government intervenes makes a big difference to whether the intervention is beneficial or not. Specifically, we showed that if the government provides the public capital stock, the resulting growth rate and welfare may not be superior to those when the government does nothing (laissez-faire). By contrast, if the government subsidizes private provision of public capital, the long-run growth rate and welfare will always dominate both the public-provision and laissez-faire cases.

While the model used to derive these results was highly simplified, the basic messages are quite robust. The normative lesson is that governments should always consider the option of subsidization before public provision when intervening to correct an externality. Even under the extreme assumption that the public sector is as efficient as the private sector, the costs of financing public programs through distortionary taxes may outweigh the benefits of internalizing the positive externality. The positive lesson is that government spending could be growth-enhancing in one country but growth-impeding in another, because of the relative importance of distortionary taxation and the externality being internalized. In fact, this idea may be part of the explanation of why empirical estimates of the Barro-type endogenous growth model have produced a wide range of results (see, for example, Aschauer 1989, King and Rebelo 1990, Devarajan, Swaroop and Zou 1993 and Easterly and Rebelo 1993).

Needless to say, the model in this paper can be enriched in several ways. For instance, it could be extended to include congestion effects in the use of the public capital stock. A wider array of instruments could be considered. For example, if the public capital stock is knowledge, then the government could consider patent policy as another option. Finally, some of the assumptions about functional forms could be relaxed. Simulation analysis would then permit us to characterize not just the long-run growth rate, but the transitional dynamics as well.

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