ON THE DYNAMICS OF PRIVATIZATION

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ABSTRACT: In this paper, we answer two questions about how privatization should proceed. First, we assume an exogenously given time span of privatization and study how the rate of privatization is related to the initial total state capital, the adjustment cost of privatization, the efficiency difference between the private sector and the state sector, the income discount rate and the exogenous terminal time for privatization. Second, from the perspective of income maximization and adjustment cost minimization, we endogenize the choice of the time span of privatization and offer a solution to the optimal terminal time for the completion of the privatization process. *JEL Classification Numbers: E2, O2, P2, P5.*

INTRODUCTION

It is generally agreed that the most difficult task in transforming Central and Eastern European economies into market economies is the privatization of the state sector. While price libralization and currency convertibility may be achieved through the "shock therapy" or "big bang," the process of privatization may last for years (Lipton & Sachs, 1990a, b) or even decades (Kornai, 1990). The experience of privatization up to date in East Europe has shown a mix of the one-by-one (the gradualist or British-style privatization) approach and the systemic (the mass privatization) approach (Sachs, 1992), and, for most countries, the privatization processes seem to continue for many years to come.

It goes without saying that, in any former socialist state, the speed of privatization and the time span of privatization are determined by many factors other than purely economic considerations because privatization is not only necessary for economic efficiency, it is also a precondition for fundamental social and political changes. In this paper, we will limit our attention to the economic rationals for privatization and answer two questions about how privatization should proceed. First, we assume an exogenously given time span of privatization and study how the rate of privatization is related to the initial total state capital, the adjustment cost of privatization, the efficiency difference between the private sector and the state sector, the income discount rate and the exogenous terminal time for privatization. Second, from the perspective of income maximization and adjustment cost minimization, we endogenize the choice of the time span of privatization and offer a solution to the optimal terminal time for the completion of the privatization process. We will deal with the first problem in the second section and the second problem in the third section. Conclusions and extensions will be presented in the fourth section.

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THE BASIC DYNAMICS

We consider a typical socialist economy at the beginning of transition. The aggregate capital stock in this economy is given by $\bar{k}$, which consists of two parts: a dominant state sector k_s and a relatively small share of private capital k_p:

$$\bar{k} = k_s + k_p. \tag{1}$$

The income generated in the state sector is:

$$y_s = f(k_s), \tag{3}$$

and the income generated in the private sector is:

$$y_p = g(k_p). \tag{4}$$

In general, the private sector in a competitive environment is more efficient than the state sector. This is the most important reason why there should be privatization in this economy. The efficiency discrepancy between the private and state ownership can be most easily characterized by two simple, linear versions of the income functions:

$$y_s = \theta_s k_s \tag{3'}$$

$$y_p = \theta_p k_p \tag{4'}$$

where θ_i's (i, $= s$ and p) are all positive, but

$$\theta_p > \theta_s > 0. \tag{5}$$

This formulation of a linear technology in the capital stock has been quite popular in recent theory of endogenous growth, in particular, see Barro (1990) and Rebelo (1991). It can be justified in two ways. First, the capital stock can be understood to include both physical and human capital. In our case, the privatization process not only transforms the state capital stock to the private sector, it also involves the reallocation of workers (human capital) through firing, unemployment, retraining, and rehiring. With this broad definition of capital, the usual neoclassical production function with capital and labor as separate inputs can be approximated by the linear technologies in (3') and (4'). Second, following Barro (1990), we can also think that capital and labor enter the production process in certain fixed proportion as in the Leontief technology. Then a Cobb-Douglas production function can be simplified to be a linear function of the capital stock.

The transformation of state capital into private capital involves the cost of adjustment such as the overhaul and reorganization of the existing production method and management system. If we interpret the capital stock in the broad sense above, the privatization process involves reallocation of both physical capital and human capital. In this way, the adjustment cost or the privatization cost also includes the cost of unemployment, social safety net, and retraining. Following the usual assumption in the text-version investment theory such as Blanchard and Fischer (1989), the adjustment cost for investment is assumed to be increasing and convex in the new investment in the private sector. Let $h(\dot{k}_p)$ denote the adjustment cost. The function h(.)is assumed to be quadratic:

$$h(\dot{k}_p) = (\gamma \dot{k}_p^2/2), \gamma > 0, \tag{7}$$

where $\dot{k}_p$ is the incremental investment in the private sector or the rate of privatization. By equation (1), given the total capital in this economy, an increase in private capital comes from an equal amount of reduction in the state sector:

$$\dot{k}_p = -\dot{k}_s. \tag{8}$$

This economy intends to maximizes its net discounted income. In this section, we first consider the case where the time span of privatization, from now (time zero) to a future time T, is determined exogenously. That is to say, the choice of time span of privatization $[0, T]$ is not our concern and, by time T, k_p,(T) will account for all existing capital stock $\bar{k}$.

$$k_p(T) = \bar{k}. \tag{9}$$

If part of the existing capital stock is still owned by the state at time T, we can just write $k_p(\mathrm{T}) = \alpha\bar{k}$ and $0 < \alpha < 1$. But in this paper, for notational simplicity, we will assume a total privatization of the state sector, namely, $\alpha = 1$ by time T.

Since our focus is the privatization of the state sector, we ignore the part of private capital formation through new investment other than the privatized state capital. With this simplification, the net income during the period of privatization is:

$$y = \theta_p k_p + \theta_s k_s - h(\dot{k}_p) \text{ for } 0 < t \leq T,$$

and the net income after the completion of privatization is produced, by our assumption, only in the private sector:

$$y = \theta_p \bar{k} \text{ for } t > T.$$

Then, the economy's objective function can be formulated as to:

$$\text{Maximize } \int_0^T \left[\theta_p k_p + \theta_s k_s - (\gamma \dot{k}_p^2/2)\right] e^{-rt} dt + \theta_p \bar{k} e^{-rT}/r, \tag{10}$$

subject to:

$$\bar{k} = k_s + k_p \tag{1}$$

$$\dot{k}_p = -\dot{k}_s \tag{8}$$

$$k_p(T) = \bar{k} \tag{9}$$

$$k_p(0) = k_{p0}. \tag{11}$$

here r (> 0) is the income discount rate and T is given. The term $(\theta_p \bar{k} e^{-rT}/r)$ represents a kind of "salvage value" because it is the discounted income generated in the private sector from time T on. Since we exclude the possibility of new private investment other than the privatization of the existing state capital stock, we have to maintain the condition that $k_p(t) \le \bar{k}$ for all t in the interval [0, T]. In addition, we might as well impose the assumption of irreversibility in the investment process of the private sector, namely, $\dot{k}_p(t) \ge 0$.

Substituting equations (1), (2) and (8) into (10), we have:

$$\text{Maximize} \int_0^T \left[(\theta_p - \theta_s) k_p + \theta_s \bar{k} - (\gamma \dot{k}_p^2/2) e^{-rt} \right] dt + \theta_p \bar{k} e^{-rT}/r, \tag{12}$$

subject to:

$$k_p(T) = \bar{k} \tag{9}$$

$$k_p(0) = k_{p0} \tag{11}$$

and T is given.

The Euler equation for this problem is:

$$\partial y e^{-rt}/\partial k_p(t) = d[\partial y e^{-rt}/\partial \dot{k}_p(t)]dt,$$

namely,

$$\ddot{k}_p - r\dot{k}_p + (\theta_p - \theta_s)/\gamma = 0. \tag{13}$$

In equation (13), we make the change of variable $\dot{k}_p = z$, so that $\ddot{k}_p = \dot{z}$. Then equation (13) can be written as a first-order differential equation:

$$\dot{z} - rz + (\theta_p - \theta_s)/\gamma = 0.$$

Its solution is:

$$z = \dot{k}_p = (\theta_p - \theta_s)/r + c_1 e^{rt},$$

where c_1 is a constant of integration. Integration again yields:

$$k_p = (\theta_p - \theta_s)\gamma/r + c_1 e^{rt}/r + c_2, \tag{14}$$

where c_2 is another constant of integration. The boundary conditions:

$$k_p(0) = c_1/r + c_2 = k_{p0}, \text{ and}$$

$$k_p(T) = (\theta_p - \theta_s)T/\gamma r + c_1 e^{rt}/r + c_2 = \bar{k},$$

yield values for the constants of integration:

$$c_2 = k_{p0} - \{[(\bar{k} - k_{p0})r/(e^{rT} - 1)] - [(\theta_p - \theta_s)T/(e^{rT} - 1)\gamma]\}/r,$$

$$c_1 = [(\bar{k} - k_{p0})r/(e^{rT} - 1)] - [(\theta_p - \theta_s)T/(e^{rT} - 1)\gamma].$$

Therefore the optimal time path for capital formation in the private sector is:

$$k_p(t) = (\theta_p - \theta_s)t/r\gamma + k_{p0} + \{[(\bar{k} - k_{p0})r/(e^{rT} - 1)] - [(\theta_p - \theta_s)T/(e^{rT} - 1)\gamma]\}(e^{rT} - 1)/r. \tag{15}$$

Since $k_p(t) = \bar{k} - k_s(t)$, equation (15) also describes the divestiture of the state sector. Differentiating $k_p(t)$ with respect to time t in equation (15):

$$\dot{k}_p(t) = (\theta_p - \theta_s)/r\gamma + \{[(\bar{k} - k_{p0})r/(e^{rT} - 1)] - [(\theta_p - \theta_s)T/((e^{rT} - 1)\gamma)]\}e^{rt}. \tag{16}$$

Equation (16) is the optimal dynamic path for the private sector investment or the rate of privatization, which has the following properties:

Proposition 1: If $(rT) > (rT)^2$ for $0 < t < T$, the privatization rate is positively related to the efficiency difference between the private-sector capital and the state-sector capital; if $(rT) < (rT)^2$ for $0 < t < T$, the privatization rate is negatively related to the efficiency difference.

Proof:

$$\begin{aligned} dk_p(t)/d(\theta_p - \theta_s) &= 1/(r\gamma) - Te^{rt}/(e^{rT} - 1)\gamma \\ &= (e^{rT} - 1 - rTe^{rt})/(e^{rT} - 1)r\gamma. \end{aligned}$$

The denominator is always positive as e^{rt} is always larger than one and $rT > 0$. But the sign of the numerator is ambiguous. To see this, note that $e^{rT} = 1 + rT + (rT)^2/2! + ..., e^{rT} = 1 + rt + (rt)^2/2! + ...$; thus $e^{rT} - 1 - rTe^{rt} = (rT)^2/2! - rT(rT)^2/2! + (rT)^3/3! - r(rT)^3/3! + ...$ This expression is positive if $(rT) > (rT)^2$, and it is negative if $(rT < (rt)^2$. Let us illustrate this proposition with the following example. Suppose that the income discount rate, r, is 12 percent. If the privatization process is required to finish in 15 years or $T = 15$, then rT is 1.5. Then, when t is less than 10.351 years, the privatization rate is positively related to the efficiency difference $\theta_p - \theta_s$; but when $t > 10.351$, $(rt)^2$ is greater than 1.5 and the privatization rate changes its direction with respect to the efficiency difference $(\theta_p - \theta_s)$.

From intuition, it seems that, if the private sector is much more efficient than the state sector, the privatization should proceed faster during the period of $[0, T]$. But Proposition 1 only partly confirms this conjecture. In particular, if the time discount rate is large and the period of privatization lasts long (namely, a large T), it is likely that the pace of privatization will slow down as t gradually approaches T. Thus Proposition 1 provides some useful information on the time path of the privatization rate during the period $[0, T]$. At the beginning of the privatization process, as t is much smaller than T and the privatization rate is positively linked to the efficiency deference between the private sector and the state sector. With time going, t is increasing and, after t reaches certain value, $(rt)^2$ can be greater than the value rT, the privatization rate will be inversely related to the efficiency difference $(\theta_p - \theta_s)$.

Proposition 2: If $(rT) > (rT)^2$ for $0 < t < T$, the privatization rate is a decreasing function of the adjustment cost γ; if $(rT) < (rT)^2$ for $0 < t < T$, the privatization rate is an increasing function of the adjustment cost.

The proof follows Proposition 1 because we have:

$$d\dot{k}_p(t)/d\gamma = -(\theta_p - \theta_s)(e^{rT} - 1 - rTe^{rT})/(e^{rT} - 1)r\gamma^2.$$

In the expression above, since $(\theta p - \theta s)$ is positive, the numerator will be negative if $(rT) > (rT)^2$ for $0 < t < T$. In this case, a high adjustment cost will lower the privatization rate. Since this case fits more to the initial stage of the transformation process, this proposition seems to suggest that the initial privatization should proceed slowly if the adjustment cost or the privatization cost is very high. But, when $(rT) < (rT)^2$ for $0 < t < T$, privatization will have progressed for a time and the adjustment cost, discounted at the rate r, will become small and, accordingly, the pace of privatization will be speeded up.

Proposition 3: The larger the initial state capital stock $k_{s0}(= \bar{k} - k_{p0})$, the faster the privatization rate:

$$d\dot{k}_p(t)/d(\bar{k} - k_{p0}) = re^{rt}/(e^{rT} - 1) > 0.$$

This proposition is what we have expected. If most of the capital is in the hand of the state, the time limit set exogenously will exert pressure upon the privatization process and the privatization rate will be increasing. At the same time, the discounted efficiency gain is also large when the inefficiency of the state ownership is got rid of quickly. On the other hand, when the existing private sector in the economy is already very significant, the time horizon of privatization, $[0, T]$, does not demand fast speed of privatization and it is better for the economy to get privatized slowly. This proposition applies quite well to the case of Hungary and China, where, at the initial stage of transformation, private ownership and market elements are significant compared to countries like Poland and Russia, and, thus, the privatization process has taken place at a relatively slower pace.

Proposition 4: An increase in the the time span of privatization is likely to reduce the privatization rate.

Proof:

$$dk_p(t)/dT = -[(k-k_{p0})re^{r(t+T)}/(e^{rT}-1)]^2\gamma$$
$$-(\theta_p-\theta_s)e^{rt}(e^{rt}-1-rTe^{rT})/r(e^{rT}-1)^2.$$

In the expression above, the first term on the right hand side is always negative as $(k - k_{p0})$ is positive; the second term is negative or positive depending on the condition whether (rT) is greater or smaller than $(rT)^2$ for $0 < t < T$. But we should note the value of $e^{r(t+T)}$ in the first term will be much larger than the value of e^{rt} in the second term, and the negative effect of an increase in T seems to dominate. That is to say, a lengthening of the time span of privatization is likely to slow down the privatization rate. The intuition also supports this conclusion. For a given amount of state capital stock, more time available for the privatization process can only reduce or at most does not affect the rate of privatization.

Proposition 5: If $(\bar{k} - k_p^0)r\gamma > (\theta_p - \theta_s)T$, the privatization rate accelerates during the time span $[0, T]$; on the other hand, if $(\bar{k} - k_p^0)r\gamma < (\theta_p - \theta_s)T$, the privatization rate decelerates from time zero to T.

Proof:

$$d\dot{k}_p(t)/dt = \{[(\bar{k}-k_{p0})r/(e^{rT}-1)] - [(\theta_p-\theta_s)T/(e^{rT}-1)\gamma]\}re^{rt}.$$

The term in the braces is positive if $(\bar{k} - k_{p0})r\gamma > (\theta_p - \theta_s)T$, and it is negative if $(\bar{k} - k_{p0})r\gamma < (\theta_p - \theta_s)T$. If the term in the braces is positive, the privatization rate will rise with time. If the term is negative, the private investment rate or the rate of privatization will keep decreasing.

The condition whether $(\bar{k} - k_{p0})r\gamma > (\theta_p - \theta_s)T$ or $(\bar{k} - k_{p0})r\gamma < (\theta_p - \theta_s)T$ leads us to study the requirements for an accelerating privatization process and a decelerating privatization process. If the initial state capital stock, $k_{s0} = (k - k_{p0})$, is large, if the income discount rate, r, is large, and if the adjustment cost, γ, is also large, then privatization will become faster and faster during $[0, T]$. On the other hand, if the efficiency difference $(\theta_p - \theta_s)$ is very significant and the terminal time T is remote from today, the privatization rate will be decreasing from the present to T.

To understand this proposition, we offer some economic intuition here. For the accelerating case, the driving forces are high adjustment cost, high income discount rate, and very small initial private capital stock compared to the total capital stock or a large initial state capital stock. Since the income discount rate and the adjustment cost are high, it is advantageous to privatize at a small scale initially and then to gradually increase the scale. This is reasonable because a high income discount rate often leads to the preference of the status quo over the future while the discounted future income and cost appear to be worth much less than the present ones. So with a high income discount rate, the privatization will be accelerated throughout the time span [0. *T*]. In addition, the accelerating process is likely to happen if the efficiency difference between the private sector and the state sector is small. This is well justified because rapid privatization at the beginning brings about small efficiency gain but large adjustment cost; thus it is worthwhile to postpone the large scale privatization and the discounted adjustment cost will become small. On the contrary, if the efficiency difference is great and if the gain from privatization outweighs the adjustment cost today, the economy should privatize at large scale today and in the near future. For the role of the terminal time for privatization, *T*, a small *T* naturally speeds up thep-rivatization process while a large *T* provides plenty of time for gradualist approach to privatization.

THE CHOICE OF THE TIME SPAN OF PRIVATIZATION

Should privatization be proceeded gradually or in a "big-bang"? Our model in the last section totally avoided this problem by assuming an exogenously determined terminal time for privatization, *T*. But, by focusing on the problem of income maximization or adjustment cost minimization, our model can shed light on this problem. Of course, what has happened in practice is far more complex than our model specified in this paper. Political and social adjustments are often closely linked to privatization, and they often demand rapid privatization to facilitate political and social transitions from communist dictatorship to democracy because state ownership is the economic foundation of communist dictatorship. Furthermore, the success of economic transition as a whole depends on the speed of privatization. Therefore, conclusions derived from our model have to be viewed together with social, political and other economic factors.

Recall our optimization problem in the last section:

$$\text{Maximize} \int_0^T \left[(\theta_p - \theta_s) k_p + \theta_s \bar{k} - \gamma (\dot{k}_p^2/2) \right] e^{-rt} dt + \theta_p \bar{k} e^{-rT}/r,$$

subject to:

$$k_p(T) = \bar{k} \tag{9}$$

$$k_p(0) = k_{p0}. \tag{11}$$

In the last section, the terminal time *T* is exogenously given. Now we hope to choose the privatization rate as well as the terminal time *T* optimally. This modification does not change the Euler condition for the optimal choice of privatization rate $\dot{k}_p$ and we still have the first-order condition:

$$\ddot{k}_p - r\dot{k}_p + (\theta_p - \theta_s)/\gamma = 0.$$

But the boundary condition for the optimal terminal time T requires that, at time $t = T$,

$$\theta_p\bar{k} - \gamma(\dot{k}_p(T))^2/2 + \gamma(\dot{k}_p(T))^2 - \theta_p\bar{k} = 0; \tag{18}$$

which is the same as requiring that the optimal terminal time should be chosen such that the amount of investment in the private sector is zero at time $t = T$:

$$\dot{k}_p(T) = 0. \tag{19}$$

Given the initial and the terminal conditions (9) and (11), we can solve for the optimal rate of privatization as before:

$$\begin{aligned} k_p(t) = {} & (\theta_p - \theta_s)t/r\gamma + k_{p0} + \{[(\bar{k} - k_{p0})r/(e^{rT} - 1)] \\ & -[(\theta_p - \theta_s)T/(e^{rT} - 1)\gamma]\}e^{rt}, \end{aligned} \tag{15}$$

and

$$\begin{aligned} \dot{k}_p(t) = {} & (\theta_p - \theta_s)/r\gamma + \{[\bar{k}(-k_{p0})r/(e^{rT} - 1)] \\ & -[(\theta_p - \theta_s)T/(e^{rT} - 1)\gamma]\}e^{rt}. \end{aligned} \tag{16}$$

With the optimal rate of privatization given by equation (16), the optimal terminal time in equation (19) can be determined by setting time t to T in equation (16) and letting the whole expression equal zero:

$$\begin{aligned} \dot{k}_p(T) = {} & (\theta_p - \theta_s)/r\gamma + \{[(\bar{k} - k_{p0})r/(e^{rT} - 1)] \\ & -[(\theta_p - \theta_s)T/(e^{rT} - 1)\gamma]\}e^{rT} = 0. \end{aligned} \tag{20}$$

Rearranging equation (20), we have:

Proposition 6: The optimal terminal time T is given implicitly in the following equation:

$$e^{rT}[(\theta_p - \theta_s)(1 - rT) + (\bar{k} - k_{p0})r^2\gamma] = (\theta_p - \theta_s). \tag{21}$$

It is obvious that, for equation (21) to hold, the term, $[(\theta_p - \theta_s)(1 - rT) + (\bar{k} - k_{p0})r^2\gamma]$, has to be positive:

$$[(\theta_p - \theta_s)(1 - rT) + (\bar{k} - k_{p0})r^2\gamma] > 0. \tag{22}$$

Also, for equation (20) to hold, it needs:

$$(\bar{k} - k_{p0})r\gamma - (\theta_p - \theta_s)T < 0. \tag{23}$$

From now on, in our model, we will choose the proper unit for the capital stock and make condition (23) satisfied. In passing, we note that we already used both inequality (23) and its opposite in Proposition 5 of the last section. There we took the terminal time as exogenously given, but, here, the determination of the optimal terminal time is precisely our task.

With the help of condition (23), we can analyze the responses of optimal terminal time T with respect to various parameters in our model. A total differentiation of equation (21) yields:

$$
\begin{aligned}
re^{rT}[(\bar{k} - k_{p0})r^2\gamma - (\theta_p - \theta_s)rT]\,dT &= [rTe^{rT} - e^{rT} + 1]\,d(\theta_p - \theta_s) \\
&- e^{rT}r^2\gamma d(\bar{k} - k_{p0}) - e^{rT}(\bar{k} - k_{p0})r^2 d\gamma \\
&- [e^{rT}(\theta_p - \theta_s)rT^2 + (\bar{k} - k_{p0})(r^2\gamma Te^{rT} + 2e^{rT}r\gamma)]\,dr.
\end{aligned} \tag{24}
$$

Condition (23) implies that the coefficient for dT on the left side of equation (24) is negative. Therefore,

Proposition 7: The larger the initial state capital stock, the longer the optimal time span for the privatization process.

This proposition can be easily shown from equation (24) (note $k_{s0} = (\bar{k} - k_{p0})$):

$$dT/d(\bar{k} - k_{p0}) = -r^2\gamma/[(\bar{k} - k_{p0})r^2\gamma - (\theta_p - \theta_s)rT] > 0. \tag{25}$$

In other words, if the initial private capital stock is large, the optimal time for privatization will be short; and vice versa:

$$dT/dk_{p0} = r^2\gamma/[(\bar{k} - k_{p0})r^2\gamma - (\theta_p - \theta_s)rT] < 0.$$

This proposition implies that, other things equal, the economy with a large state sector needs more time to privatize than the economy with a small state sector. From the consideration of long-run income maximization, it also suggests that the exogenously determined time span, which we considered in the last section, cannot achieve long-run income maximization at least from the perspective of narrowly defined cost and benefit of privatization in our model. The adoption of the time span for privatization to country specifics is further required by the next proposition.

Proposition 8: Both a high privatization cost and a high income discount rate increase the optimal terminal time of privatization.

This can be seen from equation (24):

$$dT/d\gamma = -r^2 / [\,(\bar{k}-k_{p0})\,r^2\gamma - (\theta_p-\theta_s)\,rT] > 0.$$

$$dT/dr = -[e^{rT}(\theta_p-\theta_s)\,rT^2 + (\bar{k}-k_{p0})\,(r^2\gamma Te^{rT} + 2e^{rT}r\gamma)\,] / [\,(\bar{k}-k_{p0})\,r^2\gamma - (\theta_p-\theta_s)\,rT] > 0. \tag{26}$$

Thus with different adjustment costs and income discount rates, different economies should choose different time horizons of privatization. The adjustment cost not only slows down the pace of privatization directly as we argued in the last section, it reinforces this effect indirectly through a longer time for privatization. The role of the income discount rate can be interpreted in two senses. For the case of a small economy, if we take the income discount rate as the interest rate of the world capital market, then a high world interest rate will increase the time span for privatization. If we take the income discount rate as the subjective time discount rate,then an economy with a high time preference will privatize longer than an economy with a low time preference.

Proposition 9: If $(rT) > (rT)^2$ for $0 < t < T$, the larger the efficiency difference between the private sector and the state sector, the shorter the time span for privatization; if $(rT) < (rt)^2$ for $0 < t < T$, the larger the efficiency difference, the longer the time span for privatization.

To show this proposition, note that, from equation (24),

$$dT/d(\theta_p-\theta_s) = (\,[rTe^{rT} - e^{rT} + 1] / [\,(k-k_{p0})\,r^2\gamma - (\theta_p-\theta_s)\,rT]\,). \tag{27}$$

As shown in Proposition 1, the term $[rTe^{rT} - e^{rT} + 1]$ is positive or negative depending on whether $(rT) < (rt)^2$ or $(rT) > (rt)^2$ for $0 < t < T$. This proposition indicates that the efficiency difference between the private and the state sectors can have two effects on the optimal terminal time. On one hand, when the efficiency difference is large, income maximization demands rapid privatization in a short time span. But, on the other hand, fast privatization incurs more adjustment cost. Hence the optimal terminal time will be determined by balancing the efficiency gain and adjustment loss at the margin.

After we have qualitatively analyzed the effects of various parameters on the optimal terminal time T in our model, we need to go back to the optimal investment equation (16). Now since the terminal time is endogenously determined, some of our results obtained in the last section cannot apply here quantitatively. In particular, Propositions 1 to 3 should be re-examined because all parameters in our model not only affect the optimal rate of investment or privatization directly, they also impact on the optimal terminal time T, which in turn influences the optimal rate of privatization in equation (16). As the preparations for these re-examinations, we note that Proposition 4 still holds if we change the derivative $d\dot{k}_p(t)\,dT$ into $\partial \dot{k}_p(t)/\partial T$. The reason for this is the same as before: more time available for the completion of the privatization process does not affect, or, at most, slows down, the rate of privatization; that is to say,

$$\partial \dot{k}_p(t)/\partial T \leq 0. \tag{28}$$

In the following, we can see that Propositions 1 to 3 can be extended qualitatively from the case of exogenous terminal time to the case of endogenous terminal time. We present them here without detailed arguments.

First, with Proposition 7 and condition (28), we have

Proposition 3': The optimal rate of privatization is always increasing in the initial proportion of the state capital stock.

$$\begin{aligned} d\dot{k}_p(t)/d(\bar{k}-k_{p0}) &= \partial \dot{k}_p(t)/\partial(\bar{k}-k_{p0}) + [\partial \dot{k}_p(t)/\partial T]\,dT/d(\bar{k}-k_{p0}) \\ &= [(re^{rt}/(e^{rT})-1)] - [\partial \dot{k}_p(t)/\partial T]\,r^2\gamma/[(\bar{k}-k_{p0})r^2\gamma - (\theta_p-\theta_s)rT] > 0. \end{aligned}$$

Second, Proposition 8 and condition (28) together give rise to:

Proposition 2': While the direct adjustment cost may increase or decrease the rate of privatization, it always lengthens the optimal time span of privatization, which in turn slows down the privatization rate.

$$d\dot{k}_p(t)/d\gamma = \partial \dot{k}_p(t)/\partial\gamma + [\partial \dot{k}_p(t)/\partial T]\,dT/d\gamma.$$

Here $\partial \dot{k}_p(t)/\partial\gamma$ has an ambiguous sign as shown in Proposition 2, but the second term on the right is always negative. Thus, in general, $d\dot{k}_p(t)/d\gamma$ does not have a definite sign.

Finally, with Proposition 8 and condition (28),

Proposition 1': The efficiency difference between the private sector and the public sector has an ambiguous effect on the rate of privatization.

$$d\dot{k}_p(t)/d(\theta_p-\theta_s) = \partial \dot{k}_p(t)/\partial(\theta_p-\theta_s) + [\partial \dot{k}_p(t)/\partial T]\,dT/d(\theta_p-\theta_s),$$

which has an ambiguous sign because both terms on the right side can be negative or positive.

CONCLUDING REMARKS

In this paper, we have considered the time path of privatization and the optimal time span of privatization from the perspective of adjustment cost minimization or income maximization. We have found that:

1. the rate of privatization is negatively related to the amount of the initial private capital stock and positively related to the total existing capital stock. That is to say, the rate is positively related to the existing state capital stock. In addition, the optimal terminal time is positively related to the existing state capital stock;
2. the efficiency difference between the private sector and state sector has ambiguous effects on the rate of privatization and the optimal terminal time of privatization;
3. the adjustment cost may speed up or slow down the rate of privatization and, it unambiguously increases the optimal terminal time of privatization;
4. the rate of privatization is time-dependent. For certain parameters of the initial state capital, efficiency difference, adjustment cost and the time discount rate, the rate of privatization can be accelerating and decelerating.

We can take a more general approach to the dynamics of privatization instead of specializing our case to the linear technology and quadratic adjustment cost. As our results from the simple model have suggested, the complicated relations between the rate and time span of privatization on the one hand and various parameters on the other can be seen most clearly through an explicit analytical solution to our problem.

The adjustment in the labor market cannot be seen directly in our model. But we want to re-emphasize that it can be modeled if the capital stock is broadly defined as the combination of physical and human capital. In this way, the reallocation cost of labor force from the state sector to the private sector can be easily included into the adjustment cost of investment in our model.

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