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# Dynamic Effects of Federal Grants on Local Spending

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In a dynamic model of local government spending, this paper examines both long-run and short-run effects of permanent federal grant changes on local public investment and recurrent expenditures. It also utilizes the Judd approach to quantify the short-run effects of temporary (current and future) policy shocks. The interesting, perhaps surprising, findings are: (1) a permanent increase in the matching grants for investment and recurrent expenditures may accelerate or slow down public investment and (2) a current, temporary grant increase stimulates current public investment, but a temporary, future increase in the nonmatching grant reduces current investment and raises current recurrent expenditures.


## I. INTRODUCTION

The effects of intergovernmental grants have been studied extensively in both the theoretical and the empirical literature; see Wilde [11, 12], Gramlich [3], Gramlich and Galper [4], Inman [5], Mieszkowski and Oakland [9], and Rosen [10], among many others. Most of the studies have modeled local (including state, metropolitan, county, and town) government behavior in a static, utility maximization framework. The responses of local spending to federal grants are typically divided into the income effect and the price (substitution) effect. While Gramlich and Galper [4] offered, to our knowledge, the first and the only dynamic analysis to include local capital services in a general equilibrium model, their model specification is limited to a quadratic utility function; the properties of their dynamic model such as stability and comparative statics are not worked out, and the short-run effects versus the long-run effects of changes in federal grants are not examined.

It goes without saying that a dynamic approach to the effects of intergovernmental grants is well justified. First of all, local government spending is readily divided into recurrent expenditures and local public

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capital formation or investment. Local public investment often takes the tangible form of roads, buildings, streetlights, water supply, and highways; it also takes the intangible form of human capital development such as education financing, provision of health services, and maintenance of public security and order. Federal transfers to local governments are often provided through various grants tied to different items in recurrent expenditures and capital expenditures. In the United States, federal aid to state and local governments has been largely categorical grants designed to support closely specified programs in localities. Many of those categorical grants are related to local public investment. For example, in recent years, grants on highways accounted for about 11.6% of total federal aid to localities; grants on housing and education together accounted for about another 23.7%. How does one identify the effects of these grants on local public investment? Obviously, due to the time-to-build property of capital formation, the static framework used in most of the existing literature is not well suited to deal with this question. Only when studied in a dynamic model of local capital accumulation can the effects of federal grants on both local public investment and recurrent expenditures be identified.

This distinction is of empirical importance, too. A dynamic framework can shed light on the recent policy debate in the United States on the desirability of block grants versus categorical grants. Suppose that the federal government intends to stimulate local investment in response to the alarming deterioration in the nation's infrastructure. It is necessary for policymakers to have a clear idea about the dynamic effects of block or nonmatching grants and categorical or matching grants on local investment. As we will see later, while a matching grant for investment can lead to more local capital formation in the long run, it may even slow down local investment during the transitional period. On the other hand, a nonmatching grant unambiguously raises both the rate of investment in the short run and the capital stock in the long run.

While a dynamic model provides the necessary framework to study both long-run and short-run effects of federal grants on local recurrent expenditures and local investment, it also allows us to distinguish the effects of different grant changes, e.g., a permanent grant change versus a temporary grant change, a current grant change versus a future grant change. In this way, we can see more clearly how the effects of grants are closely related to the timing of grants. From this perspective, we can improve our empirical studies of the effects of intergovernmental grants by explicitly modeling the dynamic behavior of local public investment and by specifying the timing, duration, and expectation of the changes in federal grants.

Motivated by these considerations, this paper represents a formal attempt to model local government behavior within a dynamic framework. In Section II, a dynamic optimization model of a representative local government is set up, the stability of the dynamic system is analyzed, and the dynamic paths of recurrent expenditures and public investment are characterized. In Section III, we focus on the long-run effects of permanent changes in federal grant policies on local recurrent expenditures and capital formation. We show in particular how different grants affect public investment in the transition to the long-run equilibrium. In Section IV, instead of using phase diagrams to obtain qualitative results, we utilize the Judd [6–8] approach to quantify the short-run effects of temporary grant policy changes on local spending and investment. In addition to summarizing our results in Section V, we also point out directions of further research.

## II. THE MODEL

In this paper, local government expenditures are divided into two parts: recurrent expenditures, $e$, and public investment, $I$. The representative local government or community has continuously differentiable preferences defined on $e$ and the local public capital stock, $k$,

$$U(e, k) = u(e) + v(k), \tag{1}$$

with $u'(e) > 0$, $v'(k) > 0$, $u''(e) < 0$, and $v''(k) < 0$. Here $u(e)$ represents the utility from the services of recurrent expenditures and $v(k)$ the utility from the services of the public capital stock. This is the utility function used in Gramlich [3], Arrow and Kurz [1], Gramlich and Galper [4], and Barro [2] among others. The separability of the utility function is assumed for simplicity.

At each time period, the local government collects tax revenues $T$ from its jurisdiction. It also receives the following grants from the federal government: a nonmatching grant $g$, a matching grant for local public investment $\alpha I$ $(1 > \alpha \geq 0)$, and a matching grant for local recurrent expenditures $\beta e$ $(1 > \beta \geq 0)$. Thus the budget constraint for the local government is

$$e + I = T + g + \alpha I + \beta e. \tag{2}$$

The accumulation of local public capital is given as

$$\dot{k} = I - \delta k, \tag{3}$$

where $\delta$ is the depreciation rate of the local capital stock.

The local government tries to maximize a discounted stream of utility with a positive time discount rate $\rho$,

$$\int_0^{\infty} [u(e) + v(k)]\exp(-\rho t)\,dt, \tag{4}$$

subject to constraints (2) and (3). The initial public capital stock is given by $k_0$.

This is perhaps the simplest dynamic specification of intergovernmental grants and local spending. In this setup, three essential aspects of local government finance are not considered.[2] First, we have assumed away the externality of local public investment on private production as in the models by Arrow and Kurz [1] and Barro [2]. Including private capital accumulation and production in this model is straightforward, but it will make our dynamic analysis, especially the short-run analysis, either much more complicated or intractable. If we consider the dynamics of both local government and private sector independently, we must study this extended model as a differential game played by the local government on one side and the private sector on the other. If we follow the Barro [2] model and consider public investment as an externality to private production, we need to consider the private sector's optimization first and model the reaction function of the private sector as a constraint on the optimization problem of the local government in a dynamic Stackelberg game.

Second, due to the absence of private production in our model, we have taken the nongrant revenues or local own revenues for the typical local government as exogenous. This is another serious limitation of our model. It is clear that local government revenues are closely linked to local production. If public capital is an input to private production in the form of a positive externality, more public capital will attract more business and more business ultimately generates an expanded tax base for the local government. But, in our simple model, the effects of public investment on tax policies and revenues of the local government are ignored.

Third, it is a well-known fact that, at least in the United States, states consider federal grants to be in many cases a nuisance. In fact, the federal government typically decides the amount of the grant ($\alpha$ and $\beta$ in the model) and the amount of recurrent expenditures and public investment through mandatory spending programs, thereby determining the size of the state's net spending on the whole project. This means that the state may have to settle for a nonoptimal spending level, one which in general will be above what the state would like to pursue. The implication is that

[2] I thank two anonymous referees for pointing out the limitations of the model and for suggesting possible extensions.

other spending projects may be crowded out by federal matching grants on mandatory projects. By treating matching grants very much like an "investment credit," our model is somewhat limited by the fact that the local government is assumed to have complete control over the amount of spending while in reality the decision is often of a second-best nature.

Returning to the analysis of the model, we substitute $I$ from (2) into (3):

$$\dot{k} = (1-\alpha)^{-1}[T + g - (1-\beta)e] - \delta k. \tag{5}$$

Thus the model consisting of the objective function in (4) and the dynamic constraint in (5) is analogous to those with an infinitely lived representative agent who can consume now or invest. The expressions $(1-\alpha)$ and $(1-\beta)$ are simply "prices" for investment and consumption, and the nonmatching grant $g$ is simply a change in income. From this perspective, our model is essentially an extension of the dynamic analysis of optimal consumption and investment from a representative consumer to a representative local government. Here the control variable is recurrent expenditures $e$, and the state variable is the stock of public capital $k$. The dynamic paths of local own revenues and federal grants are exogenously given.

To solve this optimization problem, we first define the current-value Hamiltonian function,

$$H(e,k,\lambda) = u(e) + v(k) + \lambda\{(1-\alpha)^{-1}[T + g - (1-\beta)e] - \delta k\}, \tag{6}$$

where $\lambda$ is the current marginal utility of an extra unit of public capital.

The necessary conditions for an optimum are

$$u'(e)/\lambda = (1-\beta)/(1-\alpha), \tag{7}$$

$$v'(k) - \lambda(\delta + \rho) = -\dot{\lambda}, \tag{8}$$

$$\dot{k} = (1-\alpha)^{-1}[T + g - (1-\beta)e] - \delta k, \tag{5}$$

and the transversality condition is

$$\lim_{t\to\infty} \lambda(t)k(t)\exp(-\rho t) = 0. \tag{9}$$

We interpret these conditions as follows. Equation (7) says that the marginal rate of substitution between recurrent expenditures and public investment equals their price ratio. Equation (8) is the Euler equation trading off current and future public investment. Equation (5) again is the dynamic budget constraint for the local government.

Solving $\lambda$ from Eq. (7) and substituting the solution into (8), we obtain a complete system of dynamic equations in terms of $e$ and $k$:

$$\dot{e} = \left[-v'(k)(1-\beta)(1-\alpha)^{-1}/u''(e)\right] + \left[u'(e)(\delta+\rho)/u''(e)\right], \quad (10)$$

$$\dot{k} = (1-\alpha)^{-1}[T + g - (1-\beta)e] - \delta k \quad (5)$$

Now it can be shown that the dynamic system is saddle-point stable in the neighborhood of the steady-state values of $e$ and $k$. Let $e^*$ and $k^*$ denote the steady-state values of $e$ and $k$, respectively. Linearizing the system around $e^*$ and $k^*$,

$$\begin{bmatrix} \dot{e} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} (\delta+\rho) & -v''(k^*)(1-\beta)(1-\alpha)^{-1}/u''(e^*) \\ -(1-\beta)(1-\alpha)^{-1} & -\delta \end{bmatrix} \times \begin{bmatrix} e - e^* \\ k - k^* \end{bmatrix}. \quad (11)$$

Let $J$ be the constant Jacobian matrix of the equilibrium equations in (11) and let $\Delta$ be the determinant of $J$. It is simple to see that $\Delta$ is negative from (11):

$$\Delta = -\delta(\delta+\rho) - \left\{v''(k^*)\left[(1-\beta)(1-\alpha)^{-1}\right]^2/u''(e^*)\right\} < 0. \quad (12)$$

Since the product of the two characteristic roots for (11) equals $\Delta$, a negative $\Delta$ means that one root is negative and one root positive. Thus the dynamic system is saddle-point stable.

Let $\mu$ be the positive characteristic root and $\omega$ the negative root:

$$\mu = \left[\rho + (\rho^2 - 4\Delta)^{1/2}\right]/2, \quad \text{and} \quad \omega = \left[\rho - (\rho^2 - 4\Delta)^{1/2}\right]/2. \quad (13)$$

The perfect-foresight convergent path is given by

$$k(t) = k^* - (k^* - k_0)\exp(\omega t), \quad (14)$$

$$e(t) = e^* + (\omega - \delta)(1-\alpha)(1-\beta)^{-1}(k(t) - k^*). \quad (15)$$

In (14) and (15), the capital stock and recurrent expenditures converge to the steady state $k^*$ and $e^*$ in the long run, because $\exp(\omega t)$ approaches zero for sufficiently large time $t$.

## III. PERMANENT POLICY SHOCKS AND LONG-RUN EFFECTS

To examine the long-run effects of different federal grants on local recurrent expenditures and the capital stock, it is necessary to assume that all policy shocks in this section are permanent in nature, for temporary policy shocks cannot affect the long-run equilibrium values of $e$ and $k$. Let the state prior to the policy shock be a steady state. The steady-state equations are $\dot{e} = 0$ and $\dot{k} = 0$:

$$-v'(k^*)(1-\beta)(1-\alpha)^{-1} + u'(e^*)(\delta+\rho) = 0, \tag{16}$$

$$(1-\alpha)^{-1}[T + g - (1-\beta)e^*] - \delta k^* = 0. \tag{17}$$

If permanent policy changes happen to the three grant parameters $\alpha, \beta$, and $g$, they will alter the equilibrium values and their effects on the equilibrium values of $e$ and $k$ can be derived from the total differentiation of Eqs. (16) and (17):

$$\begin{bmatrix} (\delta+\rho)u''(e^*) & -v''(k^*)(1-\beta)(1-\alpha)^{-1} \\ -(1-\beta)(1-\alpha)^{-1} & -\delta \end{bmatrix} \begin{bmatrix} de^* \\ dk^* \end{bmatrix} = \begin{bmatrix} -v'(k^*)(1-\alpha)^{-1}\,d\beta + v'(k^*)(1-\beta)(1-\alpha)^{-2}\,d\alpha \\ -(1-\alpha)^{-1}e^* d\beta - [T+g-(1-\beta)e^*] \\ \times(1-\alpha)^{-2}\,d\alpha - (1-\alpha)^{-1}\,dg \end{bmatrix}. \tag{18}$$

First, a permanent increase in the matching grant for public investment raises the long-run public capital stock but has an ambiguous effect on recurrent expenditures; furthermore, it may speed up or slow down investment along the unique perfect foresight path. To see the former, apply Cramer's rule to (18),

$$dk^*/d\alpha = \{-(\delta+\rho)[T+g-(1-\beta)e^*](1-\alpha)^{-2}u''(e^*) + (1-\beta)^2(1-\alpha)^3 v'(k^*)\}/\Delta u''(e^*),$$

which is positive, but

$$de^*/d\alpha = \{-v''(k^*)[T+g-(1-\beta)e^*](1-\beta)(1-\alpha)^3 - v'(k^*)\delta(1-\beta)(1-\alpha)^2\}/\Delta u''(e^*),$$

which has an ambiguous sign because the first term in parentheses is positive while the second term is negative. The intuition is the same as in the static model: an increase in the matching grant for investment lowers the relative price of investment $(1-\alpha)/(1-\beta)$. Hence, the local government tends to substitute investment for recurrent spending. At the same time, a higher $\alpha$ means more budget revenue for the local government, and the income effect works in the opposite direction.

For investment, take the time derivative of $k(t)$ in (14),

$$\dot{k}(t) = \omega(k - k^*), \tag{19}$$

while in (19) an increase in $\alpha$ raises $k^*$, which in turn accelerates investment if $k < k^*$ (since $\omega < 0$); a higher $\alpha$ may make $\omega$ less negative, which results in a slower capital accumulation. In fact,

$$d\omega/d\alpha = \left(\rho^2 - 4\Delta\right)^{-1/2} d\Delta/d\alpha.$$

So $d\Delta/d\alpha$ and $d\omega/d\alpha$ have the same sign. From (12),

$$\begin{aligned} d\Delta/d\alpha = & -(1-\beta)^2(1-\alpha)^{-2}(u''(e^*))^{-1} v'''(k^*)(dk^*/d\alpha) \\ & +(1-\beta)^2(1-\alpha)^{-2}(u''(e^*))^{-2} v''(k^*)u'''(e^*)(de^*/d\alpha) \\ & -\left[(1-\beta)^2(1-\alpha)^{-3} v''(k^*)/u''(e^*)\right]. \end{aligned}$$

It is easy to provide specific examples to illustrate that $d\Delta/d\alpha$ is positive. In this case, $d\omega/d\alpha$ will be positive, and an increase in the matching grant for investment may lead to slower investment, even though a higher investment matching grant will eventually raise the long-run public capital stock. It should be emphasized here that a less-negative eigenvalue $\omega$ has a significant impact on the rate of convergence from the initial capital stock $k_0$ to the steady-state capital $k^*$ because $\exp(-\omega t)$ approaches zero much more slowly for a less-negative $\omega$ in Eq. (14):

$$k(t) = k^* - (k^* - k_0)\exp(\omega t).$$

With a less negative eigenvalue $\omega$, the time it takes to get to the steady state may actually increase even though the matching grant for public investment eventually raises the long-run stock of public capital. This result also has implications for empirical studies on the stimulating effects of government grants on local spending. Take the highway construction as an example. If the long-run demand for the highway system in a certain state can be estimated as a function of various federal grants, especially the highway grant, in addition to other exogenous factors, it should not be surprising to find that, while the steady-state stock of highways increases with highway grant, the annual highway construction may not show a significant upward jump because the construction may be spread over a longer time period.

Second, an increase in the matching grant for recurrent expenditures raises recurrent spending; its effect on the equilibrium stock of public capital is ambiguous. To see this, we apply Cramer's rule again in (18),

$$de^*/d\beta = \left[v'(k^*)\delta(1-\alpha)^{-1} - v''(k^*)(1-\beta)(1-\alpha)^{-2} e^*\right]/u''(e^*)\Delta > 0,$$

$$dk^*/d\beta = \left[-(\delta+\rho)(1-\alpha)^{-1} u''(e^*)e^* - v'(k^*)(1-\beta)(1-\alpha)^{-2}\right]/u''(e^*)\Delta,$$

where $dk^*/d\beta$ does not have a definite sign.

The economic intuition is also similar to the static case. A rise in the matching grant for recurrent expenditures reduces the price for recurrent expenditures $(1-\beta)$ and raises the relative price for public investment $(1-\alpha)/(1-\beta)$. Therefore, the substitution effect of this matching grant gives rise to more recurrent spending and less public capital stock in the long run. But a higher matching grant of any kind always implies more income for the local government. Since both recurrent expenditures and local public capital are normal goods in our model, the income effect of the matching grant for recurrent expenditures will increase the long-run stock of public capital.

Third, an increase in the nonmatching grant results in more capital stock and more recurrent expenditures. In fact, from (18),

$$de^*/dg = -v''(k^*)(1-\beta)(1+\alpha)^2(1-\alpha)^{-2}/\Delta u''(e^*) > 0,$$

$$dk^*/dg = -(\delta+\rho)\delta/\Delta > 0.$$

It can also be shown that the rise in the nonmatching grant accelerates the rate of public investment. Substituting $k(t)$ in (14) into (18), we have

$$\dot{k}(t) = -\omega \exp(\omega t)(k^* - k_0).$$

The response of public investment to the nonmatching grant is given by (note that $\omega < 0$)

$$d\dot{k}(t)/dg = -\omega \exp(\omega t) dk^*/dg > 0.$$

The effects of a nonmatching grant on public capital and recurrent spending are what we expect, for they are the results of a pure income effect. However, the positive effect on the rate of public investment is a bit surprising when compared to the ambiguous effect of the matching grant for investment on the rate of public investment. Technically, the reason is that the negative characteristic root $\omega$ is independent of the nonmatching grant $g$. As usual in dynamic analysis, characteristic roots are often complicated functions of various parameters in a dynamic system. It is often misleading to make an assertion based only on intuition. Perhaps our analysis in this section provides another illustration of this kind of complexity.

## IV. TEMPORARY POLICY SHOCKS AND SHORT-RUN EFFECTS

Many grant policies are temporary and even project-specific in their nature. During the course of time, grant policies also change frequently. While these temporary shocks cannot influence the long-run equilibrium values of recurrent expenditures and the stock of public capital, they are significant elements in shaping the short-run behavior of local government spending. Very often in dynamic economic analysis, the study of the short-run effect of a temporary shock uses phase diagrams to obtain some qualitative results. Here we will follow the approach developed in a series of papers by Judd [6–8] to quantify the short-run effects of temporary changes in federal government grants on local government spending.

Suppose that at time $t = 0$, the stock of public capital and recurrent spending are at the steady-state level corresponding to the grant parameters $\alpha, \beta$, and $\bar{g}$. Also at time $t = 0$, federal grant policies change as

$$\alpha' = \alpha + \varepsilon h_\alpha(t), \tag{20a}$$

$$\beta' = \beta + \varepsilon h_\beta(t), \tag{20b}$$

$$g' = \bar{g} + \varepsilon g(t), \tag{20c}$$

where $\varepsilon$ is a parameter. Functions $h_\alpha(t), h_\beta(t)$, and $g(t)$ describe the intertemporal policy changes in a magnitude-free fashion since $\varepsilon$ can represent different magnitudes of changes. For example, a change in the matching grant for investment during time period $T_1 < t < T_2$ can be represented by setting $h_\alpha(t)$ to be one for $T_1 < t < T_2$ and zero otherwise.

Substituting $\alpha', \beta'$, and $g'$ for $\alpha, \beta$, and $g$ in Eqs. (5) and (10), respectively;

$$\begin{aligned}\dot{e} = {} & -v'(k)(1-\beta-\varepsilon h_\beta)(1-\alpha-\varepsilon h_\alpha(t))^{-1}/u''(e) \\ & +u'(e)(\delta+\rho)/u''(e),\end{aligned} \tag{21a}$$

$$\dot{k} = (1-\alpha-\varepsilon h_\alpha(t))^{-1}\left[T+\bar{g}+\varepsilon g(t)-(1-\beta-\varepsilon h_\beta)e\right]-\delta k. \tag{21b}$$

The solutions for $k$ and $e$ depend on both $t$ and $\varepsilon$. We write the solutions as $k(t, \varepsilon)$ and $e(t, \varepsilon)$. Since $\varepsilon = 0$ implies that the system remains at the initial position, the effects of a grant policy change can be seen from the impact on the paths of $e$ and $k$ as $\varepsilon$ shifts from zero to a small positive or negative value. Formally, we define the initial impact of $\varepsilon$ on $e$ and $k$ here:

$$e_\varepsilon(t) = \partial e(t,0)/\partial\varepsilon, \qquad k_\varepsilon(t) = \partial k(t,0)/\partial\varepsilon,$$
$$\dot{e}_\varepsilon(t) = \partial(\partial e(t,0)/\partial\varepsilon)/\partial t, \qquad \dot{k}_\varepsilon(t) = \partial(\partial k(t,0)/\partial\varepsilon)/\partial t.$$

Differentiation of Eqs. (21a) and (21b) evaluated at $\varepsilon = 0$ yields a linear differential equation in variables $e_\varepsilon$ and $k_\varepsilon$,

$$\begin{bmatrix} \dot{e}_\varepsilon \\ \dot{k}_\varepsilon \end{bmatrix} = J \begin{bmatrix} e_\varepsilon \\ k_\varepsilon \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}, \tag{22}$$

where

$$\begin{aligned} w_1(t) &= \left[-(1-\beta)(1-\alpha)^{-2} v'(k^*) h_\alpha(t)/u''(e^*)\right] \\ &\quad + \left[(1-\alpha)^{-1} v'(k^*) h_\beta(t)/u''(e^*)\right], \\ w_2(t) &= [T + \bar{g} - (1-\beta)e^*](1-\alpha)^{-2} h_\alpha(t) \\ &\quad + (1-\alpha)^{-1} e^* h_\beta(t) + (1-\alpha)^{-1} g(t), \end{aligned}$$

and $J$ is the Jacobian matrix in (11).

As in Judd [8], the Laplace transform can be used to solve Eq. (22). For sufficiently large positive $s$, the Laplace transform of a function $f(t)$ $(t > 0)$ is another function $F(s)$, where

$$F(s) = \int_0^\infty f(t) \exp(-st)\, dt.$$

Naturally, let $E_\varepsilon(s), K_\varepsilon(s), H_\alpha(s), H_\beta(s), G(s)$, and $W(s)$ be the Laplace transforms of $e_\varepsilon(t), k_\varepsilon(t), h_\alpha(t), h_\beta(t), g(t)$, and $w(t)$, respectively. Then

$$\begin{bmatrix} E_\varepsilon \\ K_\varepsilon \end{bmatrix} = (s\Lambda - J)^{-1} \begin{bmatrix} W_1(s) + e_\varepsilon(0) \\ W_2(s) \end{bmatrix}, \tag{23a}$$

where $\Lambda$ is the identity matrix. Write out $(s\Lambda - J)^{-1}$ explicitly in (23a):

$$\begin{bmatrix} E_\varepsilon \\ K_\varepsilon \end{bmatrix} = [(s-\mu)(s-\omega)]^{-1} \times \begin{bmatrix} s+\delta & -(1-\beta)(1-\alpha)^{-1}v''/u'' \\ -(1-\beta)(1-\alpha)^{-1} & s-\delta-\rho \end{bmatrix} \times \begin{bmatrix} W_1(s) + e_\varepsilon(0) \\ W_2(s) \end{bmatrix}. \tag{23b}$$

For a temporary shock at present or in the future, the steady-state values of recurrent expenditures and the capital stock remain the same. Therefore,

$$W_1(s) = -(1-\beta)(1-\alpha)^{-2}v'(k^*)H_\alpha(s)/u''(e^*) + (1-\alpha)^{-1}v'(k^*)H_\beta(s)/u''(e^*),$$

$$W_2(s) = [T + \bar{g} - (1-\beta)e^*](1-\alpha)^{-2}H_\alpha(s) + (1-\alpha)^{-1}e^*H_\beta(s) + (1-\alpha)^{-1}G(s).$$

In Eq. (23b), $e_\varepsilon(0)$ represents the initial jump in recurrent expenditures corresponding to grant policy changes. As usual in dynamic analysis, this jump is necessary to assure the convergence of the variables along the perfect foresight path. To determine $e_\varepsilon(0)$, we note that the existence of a saddle-point equilibrium in our model implies a bounded, steady-state capital stock for any $\varepsilon$. Therefore, $K_\varepsilon(s)$ must be finite for all $s > 0$, even for $s = \mu$ (the positive eigenvalue of the dynamic system). However, when $s = \mu$, the matrix $(s\Lambda - J)$ is singular and the denominator in the inverse matrix is zero. To have a bounded $K_\varepsilon(\mu)$, implicitly, the numerator on the right-hand side of (23b) must be zero (see the Appendix in Judd [8] for technical details). That is to say,

$$(\mu + \delta)[W_1(\mu) + e_\varepsilon(0)] - (1-\beta)(1-\alpha)^{-1}W_2(\mu)v''/u'' = 0,$$

or

$$
\begin{aligned}
e_\varepsilon(0) &= (1-\beta)(1-\alpha)^{-1}(\mu+\alpha)^{-1}W_2(\mu)v''/u'' - W_1(\mu) \\
&= \left[(1-\beta)(1-\alpha)^{-1}(\mu+\alpha)^{-1}v''/u''\right] \\
&\quad \times\Big\{[T+\bar{g}-(1-\beta)e^*](1-\alpha)^{-2} \\
&\qquad +(1-\beta)(1-\alpha)^{-2}v'(k^*)/u''(e^*)\Big\}H_\alpha(\mu) \quad \text{(effect of } \alpha) \\
&\quad +\Big\{\left[-(1-\alpha)^{-2}v'(k^*)/u''(e^*)\right] \\
&\qquad +(1-\alpha)^{-2}(1-\beta)(\mu+\alpha)^{-1}v''/u''\Big\}H_\beta(\mu) \quad \text{(effect of } \beta) \\
&\quad +\left[(1-\beta)(1-\alpha)^{-2}(\mu+\alpha)^{-1}v''/u''\right]G(\mu)\Big\}\text{(effect of } g). \qquad (24)
\end{aligned}
$$

Equation (24) presents that impact of temporary grant policy changes on the initial recurrent expenditures. First, any future increase in the matching grant for the recurrent expenditures ($H_\beta(\mu)$) will stimulate recurrent expenditures today. From (24),

$$
\begin{aligned}
de_\varepsilon(0)/dH_\beta(\mu) = \Big\{&\left[-(1-\alpha)^{-1}v'(k^*)/u''(e^*)\right] \\
&+\left[(1-\alpha)^{-2}(1-\beta)(\mu+\alpha)^{-1}v''/u''\right]\Big\} > 0.
\end{aligned}
$$

For example, let the change in the matching grant for recurrent expenditures take the time path

$$
h_\beta(t)=0 \quad \text{for } t<A, \qquad h_\beta(t)=1 \quad \text{for } A\le t\le A+i, \qquad h_\beta(t)=0 \quad \text{for } t>A+i.
$$

Then $H_\beta(\mu) = i\ \exp(-\mu A)$ and today's recurrent expenditures increase by (assuming that other grant policies remain the same)

$$
\begin{aligned}
e_\varepsilon(0) = \Big\{&\left[-(1-\alpha)^{-1}v'(k^*)/u''(e^*)\right] \\
&+\left[(1-\alpha)^{-2}(1-\beta)(\mu+\alpha)^{-1}v''/u''\right]\Big\}i\exp(-\mu A).
\end{aligned}
$$

We can explain this result as follows. Anticipating a temporary future increase in the matching grant for recurrent expenditures, the local government will have more income in the future than it has now. To smooth its spending, the local government devotes more current resources to recurrent expenditures. This income effect is further reinforced by the substitution effect that recurrent expenditures will become less costly than public investment as a result of the rise in the matching grant for recurrent spending.

Next, the impact on today's recurrent expenditures of a future rise in the nonmatching grant ($G(\mu)$) is also positive:

$$de_\varepsilon(0)/dG(\mu) = \left[(1-\beta)(1-\alpha)^{-2}(\mu+\alpha)^{-1}v''/u''\right] > 0.$$

If the nonmatching grant follows the same time path given above for the matching grant on recurrent expenditures, then $g(t) = 0$ for $t < A$, $g(t) = 1$ for $A \le t \le A + i$, $g(t) = 0$ for $t > A + i$, and $G(\mu) = i \exp(-\mu A)$. Its effect on the initial recurrent expenditures is

$$e_\varepsilon(0) = \left[(1-\beta)(1-\alpha)^{-2}(\mu+\alpha)^{-1}v''/u''\right] i \exp(-\mu A).$$

The reason for this result is simple. A rise in a future nonmatching grant will increase future income relative to current income. Then an obvious response from the local government would be to increase its current expenditures.

But, from (24), a rise in the investment grant in the future ($H_\alpha(\mu)$) has an ambiguous effect on today's recurrent expenditures,

$$\begin{aligned} de_\varepsilon(0)/dH_\alpha(\mu) = {} & \left[(1-\beta)(1-\alpha)^{-1}(\mu+\alpha)^{-1}v''/u''\right] \\ & \times\Big\{[T+\bar{g}-(1-\beta)e^*](1-\alpha)^{-2} \\ & \quad +\left[(1-\beta)(1-\alpha)^{-2}v'(k^*)/u''(e^*)\right]\Big\}, \end{aligned}$$

which does not have a definite sign because the first term in parentheses is positive and the second term is negative. This is understandable because, while a future rise in the investment grant leads to more future income for the local government, the investment grant also increases the relative price of recurrent expenditures; the substitution effect tends to reduce current recurrent expenditures.

To find out the impact of grant policy changes on current public investment, we substitute (24) into (22) and set $t = 0$ (also note that $k_\varepsilon(0)$ is zero because the initial capital stock is given and cannot jump),

$$\dot{k}_\varepsilon(0) = -(1-\beta)(1-\alpha)^{-1}e_\varepsilon(0) + w_2(0), \qquad (25)$$

where

$$w_2(0) = [T + \bar{g} - (1-\beta)e^*](1-\alpha)^{-2} h_\alpha(0) + (1-\alpha)^{-1} e^* h_\beta(0) + (1-\alpha)^{-1} g(0).$$

From (25), we note that all extra grants today ($t = 0$), i.e., positive $h_\alpha(0)$, $h_\beta(0)$, and $g(0)$, always increase public investment today. These effects are given by the three positive terms of $w_2(0)$ in (25). When these three kinds of grants change at $t = 0$, namely, $h_\alpha(0) = h_\beta(0) = g(0) = 1$, and when there is no change in future grant policies, current public investment will go up by

$$\dot{k}_\varepsilon(0) = [T + \bar{g} - (1-\beta)e^*](1-\alpha)^{-2} + (1-\alpha)^{-1}(e^* + 1) > 0.$$

We can provide some economic intuition for this positive association between a current rise in all federal grants and a current increase in local public investment. As in the typical intertemporal model of consumption and investment, the representative local government in our model tries to smooth recurrent expenditures over time. Therefore, a momentary increase in current federal grants in any form, while future grants remain unchanged, will only increase current public investment.

The impact of future changes in federal grants on local public investment can also be seen from (25). As the coefficient for $e_\varepsilon(0)$, i.e., $[-(1-\beta)(1-\alpha)^{-1}]$, is negative in (25), the effects on current public investment of any future increase in federal grants are just the opposite of the effects on current recurrent expenditures. Thus, the combination of Eqs. (24) and (25) indicates a negative impact on current investment from any future increase in the nonmatching grant $G(\mu)$:

$$d\dot{k}_\varepsilon(0)/dG(\mu) = \left[-(1-\beta)^2(1-\alpha)^{-3}(\mu+\alpha)^{-1} v''/u''\right] < 0.$$

The effect of future matching grant for recurrent spending $H_\beta(\mu)$ on current investment is also negative:

$$d\dot{k}_\varepsilon(0)/dH_\beta(\mu) = \left[(1-\beta)(1-\alpha)^{-2} v'(k^*)/u''(e^*)\right] - \left[(1-\alpha)^{-3}(1-\beta)^2(\mu+\alpha)^{-1} v''/u''\right] < 0.$$

In addition, a change in a future investment grant ($H_\alpha(\mu)$) has an ambiguous effect on current investment:

$$d\dot{k}_\varepsilon(0)/dH_\varepsilon(\mu) = \left[-(1-\beta)^2(1-\alpha)^{-4}(\mu+\alpha)^{-1}v''/u''\right] \times \left\{\left[T+\bar{g}-(1-\beta)e^*\right] + \left[(1-\beta)v'(k^*)/u''(e^*)\right]\right\}.$$

These findings can be interpreted as follows. At time $t = 0$, the total revenue for the local government is given if there is no grant change today. When a higher nonmatching grant is expected in the future, today's recurrent expenditures will be increased by the local government as a way to smooth its consumption. With fixed local revenue today, initial public investment must be cut. Similarly, a future rise in the matching grant for recurrent expenditures favors recurrent spending over current public investment in terms of both the income effect and the substitution effect; therefore, current investment is sacrificed as a result of an expected increase in the matching grant for recurrent spending. Finally, anticipating a future rise in the matching grant for investment, local government may raise its recurrent expenditures since it is going to have more future income. But the investment grant also lowers the opportunity cost of investment and makes recurrent expenditures relatively more expensive. Furthermore, since it takes time to build the stock of public capital, the expectation of a higher investment grant may also stimulate current investment. Hence, with these two offsetting effects, a future rise in the investment grant gives rise to an ambiguous impact on current public investment.

## V. CONCLUDING REMARKS

In a dynamic model of local government spending, this paper has examined both long-run and short-run effects of permanent federal grant changes on local public investment and recurrent expenditures. It has also utilized the Judd approach to quantify the short-run effects of temporary (current and future) policy shocks. We summarize our main findings here: (1) a permanent increase in the nonmatching grant leads to faster public investment, larger long-run capital stock, and greater long-run recurrent expenditures; (2) a permanent increase in the matching grants for investment and the recurrent expenditures may speed up or slow down local investment; (3) a temporary grant increase at present, no matter what form the federal grants take, stimulates current public investment; (4) a temporary, future increase in the nonmatching grant reduces current investment and raises current recurrent expenditures; (5) a temporary, future increase in the matching grant for recurrent expenditure leads to less current public investment and more current recurrent expenditures;

but (6) a temporary, future increase in the matching grant for investment has an ambiguous impact on current public investment.

Two interesting observations and comparisons regarding these findings should be emphasized. First, for a temporary change in a federal grant, whether it is present or future is crucial for predicting its effect on current local investment: while a current change does not have any effect on current recurrent expenditures and its full impact falls on current investment, an anticipated rise in both the nonmatching grant and the matching grant for recurrent expenditures reduces current investment. Also, the duration of a federal grant change is a significant factor in determining the responses of local governments. A permanent rise in the nonmatching grant leads to more public investment in the short run and more stock of public capital in the long run, but a temporary, future rise in the nonmatching grant reduces the short-run public investment.

Our theoretical findings also shed light on how to test the effects of intergovernmental grants in empirical studies. The distinction between public investment and recurrent spending is, of course, important in dealing with some general statistical tests on the effects of intergovernmental grants. In more specific areas such as highway construction, urban housing services, and community education, the dynamic behavior of local governments should be modeled explicitly in order to capture the time-to-build characteristic of local public investment. In addition, the timing of grants, the duration of grants, and the role of expectations should be considered as well. As our model has suggested, the effects of a current grant change on current investment are very different from the ones of a future grant change. It is not very difficult to test this difference empirically if we have the time series of grants. To give an example, we can test the following simultaneous equations about local public investment and local recurrent expenditures,

$$\dot{k}(t) = f(\alpha_t, \alpha_{t+1}, g_t, g_{t+1}, \beta_t, \beta_{t+1}, \theta),$$

$$e(t) = h(\alpha_t, \alpha_{t+1}, g_t, g_{t+1}, \beta_t, \beta_{t+1}, \theta),$$

where subscripts $t$ and $(t + 1)$ refer to current grants and expected grants, respectively, and $\theta$ represents other exogenous factors.

Even though our dynamic approach to intergovernmental grants has extended the usual static approach in many ways, our model in this paper is still oversimplified and suffers from several limitations as we have already pointed out. Future research should expand this model to include the role of public capital in private production, to endogenize local own revenues, and to model the private sector, local governments, and the federal government in a full general-equilibrium framework.

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