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Unemployment, Trade Openness and Optimal Monetary Policy

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This paper studies how the monetary policymaker should conduct the monetary policy in a small open economy with labor market frictions.The welfare loss function shows that the monetary policymaker faces a stabilization trade-off between the PPI inflation and the unemployment rate gap.We find that the welfare gains from implementing the optimal monetary policy are small relative to the optimized simple rule.In addition, when conducting the optimized simple rule, the monetary policymaker should attach a greater weight to the unemployment rate gap when the economy is more open.

Key Words: Labor market frictions; Small open economy; Optimal monetary policy; Optimized simple policy rule.

JEL Classification Numbers: E24, E52, F16, F41.

1. INTRODUCTION

The widespread unemployment in the wake of the Covid-19 pandemic has caught the global attention, thus taking the topic on how to stabilize the global economy and alleviate the unemployment to the forefront of policy debates. However, it is not until recently that the standard New Keynesian model incorporates unemployment into the design of the optimal monetary policy (Thomas,2008; Faia,2009; Blanchard and Gali,2010; Ravenna and Walsh,2011;Campolmi and Faia,2015; Cacciatore and Ghironi, 2020; Kekre,2021), which is in sharp contrast to its adoption by many central banks as the backbone for monetary policy analysis.

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In open economies, a central bank should take the fluctuation in the exchange rate into account, thus the monetary policy prescription is different from its counterpart in a closed economy (Clarida et al., 2002; Gali and Monacelli, 2005,2016; Corsetti et al., 2011; Engel, 2011; Gong et al., 2016). In spite of the important role played by the exchange rate in shaping monetary policy prescription, the literature on monetary policy in the presence of unemployment and the fluctuation in the exchange rate is scarce.

To fill the gap in the literature, we extend Blanchard and Gali (2010) to analyze how trade openness affects the optimal monetary policy in a small open economy with labor market frictions, which are characterized as hiring cost. Following Blanchard and Gali (2010) , the hiring cost increases with labor market tightness, defined as the ratio of new hires to the size of unemployment pool. Our model's structure is similar to Gali and Monacelli (2005, 2016) except the labor market. To examine how the monetary policy is shaped by trade openness in the presence of labor market frictions, we take a second-order approximation to the household's utility function and solve for the optimal monetary policy numerically. Since the optimal monetary policy is not feasible in practice, we also consider the macroeconomic implications of an optimized simple Taylor-type monetary policy rule, under which the monetary policymaker adjusts the nominal interest rate in response to the volatility of the PPI inflation and that of the unemployment rate gap. The optimized simple rule is different from the standard Taylor rule in the sense that the monetary policymaker can optimally choose the response coefficients over a grid spanning the reasonable intervals.

We find that the welfare gains from implementing the optimal monetary policy are small relative to the optimized simple rule. In addition, under the optimized simple rule, the monetary policymaker attaches a greater weight to the unemployment rate gap when the degree of trade openness becomes larger. When the degree of trade openness goes up, the home household's expenditure share on domestic goods becomes lower, implying that the welfare loss from the fluctuation in PPI inflation becomes smaller. Accordingly, the monetary policymaker should pay more attention to the unemployment rate gap, when conducting the optimized simple rule.

Under the optimized simple rule, we also find that the degree of decrease in the nominal interest rate is smaller when the home country is more open following a positive productivity shock. In general, the monetary policy produces expenditure level and expenditure switching effects in open economies, with the former affecting domestic aggregate demand and the latter the relative demand for one country's goods. When one country is more open, the expenditure switching effect is more powerful such that the degree of adjustment of the nominal interest rate is smaller.

Our research is closely related to the literature on the monetary policy in the presence of labor market frictions. Thomas (2008) introduces the search and matching frictions into the standard New Keynesian monetary model to analyze the effect of labor market frictions on the optimal monetary policy. When the steady state is efficient and all wages are Nash-bargained in each period, Thomas (2008) finds that the optimal monetary policy requires zero inflation. By contrast, when nominal wage bargaining is staggered, the monetary policymaker has the incentive to deviate from price stability. The reason is that, in response to real shocks, the monetary policymaker can avoid inefficient unemployment volatility and dispersion in hiring rates by adjusting price inflation so that the real wages are close to their flexible-wage values. Faia (2009) examines the optimal monetary policy in a model with price adjustment costs and matching frictions. The author concludes that the optimal monetary policy deviates from price stability when the economy is hit by productivity and government expenditure shocks. In this case, search externalities produce a trade-off between unemployment and inflation, which requires the monetary policymaker to deviate from price stability to strike a balance between reducing price adjustment costs and boosting inefficiently low employment. Blanchard and Gali (2010) construct a highly tractable model by introducing labor market frictions in the form of hiring costs into a standard New Keynesian monetary model. The model allows for an analysis of the trade-off between inflation and unemployment facing a monetary policymaker. Ravenna and Walsh (2011) examine the optimal monetary policy in a model with unemployment and sticky prices and find that, in contrast to the conclusion found in the standard New Keynesian monetary model that the divine coincidence breaks down following cost-push shocks, price stability is nearly optimal if the shocks mirror random fluctuations in the relative bargaining power of workers and firms.

Campolmi and Faia (2015) explore the optimal exchange rate policy in a two-country model with labor market frictions. Since the monetary policymaker faces a trade-off between the insulating property of the float exchange rate and the destabilizing effects of the fluctuation in the exchange rate on job flows, the optimal monetary policy needs to respond to the fluctuation in the exchange rate. Cacciatore and Ghironi (2020) introduce labor market frictions into a two-country model with heterogeneous firms and endogenous producer entry to study how trade linkages shape the optimal monetary policy. They find that the monetary policymaker has less incentive to correct long-run distortions by keeping a positive inflation rate as a result of productivity gains through firm selection, implying that the optimal average inflation rate is lower when trade costs go down. In addition, the authors find that the optimal stabilization policy is inward looking as country-specific shocks produce more global effects through stronger trade linkages. Kekre (2021) examines the optimal monetary policy in a two-country currency union with labor market frictions. Due to the fact that the welfare loss caused by output volatility is greater when hiring costs are greater or labor market flows are lower, the optimal monetary policy should respond to smaller price and output distortions in the member country with more sclerotic labor market following asymmetric shocks. As a result, the welfare gains arise from adjusting the inflation target by putting greater weight on the member country with more sclerotic labor market in a currency union.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 derives model's constrained-efficient allocation. Section 4 analyzes the flexible-price equilibrium. Section 5 explores the sticky-price equilibrium. Section 6 discusses monetary policy design. Section 7 concludes.

2. THE MODEL

The model's structure is similar to Gali and Monacelli (2005, 2016) except the labor market. The world economy consists of a continuum of small open economies represented by the unit interval. All economies are symmetric in the sense that they have the same preferences, technology, and market structure. Since the measure of each economy is zero, one country's policy has no impact on the rest of the world. Thus it is reasonable to take world aggregates as exogenous. In addition, we follow Gali and Monacelli (2005, 2016) and assume that both domestic and international financial markets are complete, and the law of one price holds.

In view of the fact that the standard New Keynesian model has nothing to say on unemployment, in particular, the literature on the effect of unemployment on the design of the optimal monetary policy in open economies is scarce, we extend Gali and Monacelli (2005) by introducing the labor market frictions in the spirit of Blanchard and Gali (2010).

In what follows, world variables are marked with an asterisk, subscript H denotes the home country, F the rest of the world.

2.1. Households

In each country, the representative household has a continuum of members represented by the unit interval. The representative household in the home country maximizes

$$\mathbf{E}_0\sum_{t=0}^{\infty}\beta^t U\left(C_t, N_t\right) = \mathbf{E}_0\sum_{t=0}^{\infty}\beta^t \left\{\log C_t - \chi\frac{{N_t}^{1+\phi}}{1+\phi}\right\}, \tag{1}$$

in which $\beta \in (0,1)$ is the discount factor, ϕ is the inverse of the Frisch elasticity of labor supply, C_t is the consumption aggregate, N_t denotes the fraction of household members who are employed. To be specific, C_t is a Cobb-Douglas composite of home and imported goods, C_{Ht} and C_{Ft},

$$C_t = C_{Ht}^{1-\upsilon} C_{Ft}^{\upsilon} \tag{2}$$

where υ is the home representative household's expenditure share on imported goods, thus it is appropriate to use υ to measure the degree of trade openness. C_{Ht} and C_{Ft} are CES functions over a continuum of goods with elasticity of substitution ε, which is assumed to be strictly greater than unity. N_t satisfies the constraint

$$0 \leq N_t \leq 1. \tag{3}$$

Following Blanchard and Gali (2010) , the model assumes full risk sharing within a large family and indivisible labor. The formulation is standard since at least Merz (1995). The representative household's intertemporal consumption choice is given by

$$\beta \mathbf{E}_t \left(\frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right) = R_t^{-1}, \tag{4}$$

in which $P_t = k^{-1} P_{Ht}^{1-\upsilon} P_{Ft}^{\upsilon}$ is the home CPI price level, R_t is the gross rate of return on the riskless one-period bond. [1]

Since the international financial markets are complete, we can obtain the risk-sharing condition from equation (4) and its foreign counterpart by assuming that the home and foreign countries are symmetric in the initial steady state,

$$C_t = C_t^* \mathbb{Q}_t \tag{5}$$

where $\mathbb{Q}_t = \frac{E_t P_t^*}{P_t}$ is the real exchange rate.[2] Equation (5) implies that the ratio of the prices of consumption goods is equal to the marginal rate of substitution between home and foreign consumption.

Define the terms of trade as $S_t = \frac{P_{Ft}}{P_{Ht}}$, thus the real exchange rate can be written as

$$\mathbb{Q}_t = S_t^{1-\upsilon}, \tag{6}$$

when deriving equation (6), we have used the equation $E_t P_t^* = P_{Ft}$.

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[1] $k = (1-\upsilon)^{1-\upsilon} \upsilon^{\upsilon}$.

[2] E_t is the nominal exchange rate which represents the home currency price of one unit of foreign currency.

2.2. Production

2.2.1. Technology

When we introduce labor market frictions into Gali and Monacelli (2005) , the firms need to set optimal prices and take part in wage bargaining at the same time. To avoid the interaction between these two decisions, we follow Blanchard and Gali (2010) and assume that there are two types of firms: final-goods firms and intermediate-goods firms. Final-goods firms set optimal prices, while intermediate-goods firms take part in wage bargaining.

In the home country, there are a continuum of monopolistically competitive firms indexed by $i \in [0,1]$, each of which produces a differentiated final good. All firms can use a common production function given by

$$Y_t^T(i) = X_t^T(i) \tag{7}$$

in which $X_t^T(i)$ is the intermediate good, the superscript T denotes the total amount.

There are a large number of identical, perfectly competitive intermediate-goods firms indexed by $j \in [0,1]$. The representative firm $j \in [0,1]$ produces intermediate good $j \in [0,1]$ by using the following production function

$$X_t^T(j) = A_t N_t(j) \tag{8}$$

where A_t is a productivity shock which is common to all home intermediate-goods firms.

The employment in firm j evolves according to

$$N_t(j) = (1-\delta) N_{t-1}(j) + H_t(j) \tag{9}$$

in which $\delta \in (0,1)$ is an exogenous job separation rate, and $H_t(j)$ is newly hired workers who start working once they are hired.

2.2.2. Labor market

We assume full job participation in the sense that all family members are either employed or unemployed but willing to work given the prevailing labor market conditions. At the beginning of period t, there is a pool of unemployed family members finding jobs. The size of the unemployed family members is U_t, which evolves according to

$$U_t = 1 - N_{t-1} + \delta N_{t-1} = 1 - (1-\delta) N_{t-1} \tag{10}$$

Among the unemployed family members, some of them can obtain job chances and start working once they are hired. Aggregate hiring $H_t = \int_0^1 H_t(j)\,dj$ evolves according to

$$H_t = N_t - (1-\delta)\,N_{t-1} \tag{11}$$

where $N_t \equiv \int_0^1 N_t(j)\,dj$ is aggregate employment.

For expositional convenience, we define the labor market tightness x_t as

$$x_t \equiv \frac{H_t}{U_t}. \tag{12}$$

Since only the unemployed family members can be hired at the beginning of each period, the value of the labor market tightness x_t is within the interval $[0,1]$. In what follows, we assume that the productivity shocks are small such that new hires are positive at all times given they are positive in the steady state. The labor market tightness can be viewed as the probability of being hired in period t, thus it is also called job finding rate in the literature.

Home intermediate-goods firms need to pay hiring costs to have the job vacancies filled. The hiring costs can be expressed in terms of the CES bundle of the domestic final goods. Specifically, the intermediate-goods firm $j \in [0,1]$ pays the hiring cost $G_t H_t(j)$,in which G_t, the cost per hire, is independent of $H_t(j)$ and taken as given by each individual intermediate-goods firm.

Following Blanchard and Gali (2010) , we assume that

$$G_t = A_t B x_t^{\alpha} \tag{13}$$

in which the parameter $\alpha \geq 0$ and B is a positive constant. Without loss of generality, we can define $g_t = B x_t^{\alpha}$, thus $G_t = A_t g_t$. Equation (13) implies that G_t increases with labor market tightness in spite of the fact that it is taken as given by each individual intermediate-goods firm.

The unemployment rate is just the fraction of the family members who remain jobless after the intermediate-goods firms pay the hiring cost and provide job chances to all unemployed family members. Since we assume full job participation, we can define the unemployment rate u_t as

$$u_t \equiv U_t - H_t = 1 - N_t \tag{14}$$

2.3. Equilibrium

Since the rest of the world has a symmetric consumption structure, the export demand for the final goods produced in the home country EX_t can be expressed as

$$EX_t = \nu \left(\frac{P_{Ht}}{E_t P_t^*} \right)^{-1} Y_t^* = \nu S_t Y_t^* \tag{15}$$

in which E_t is the nominal exchange rate representing the home country's price of one unit of foreign currency, Y_t^* denotes the world net final-goods output which is what is left after the hiring costs are deducted.

Aggregating the consumption over all countries, we can obtain a world market clearing condition

$$C_t^* = Y_t^* \tag{16}$$

The final-goods market clearing condition in the home country is

$$\begin{aligned} Y_t(i) &= C_{Ht}(i) + EX_t(i) \\ &= \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon} (C_{Ht} + EX_t) \\ &= \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon} \left[(1-\nu) \frac{P_t}{P_{Ht}} C_t + \nu S_t Y_t^* \right]. \end{aligned} \tag{17}$$

Substituting equation (17) into the expression of the net final-goods output in the home country $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$, we have

$$Y_t = (1-\nu) S_t^{\nu} C_t + \nu S_t Y_t^*. \tag{18}$$

From the risk sharing condition (5), the expression of the real exchange rate (6), and the world market clearing condition (16), we have

$$S_t = \left(\frac{C_t}{Y_t^*} \right)^{\frac{1}{1-\nu}}. \tag{19}$$

Substituting equation (19) into equation (18) and then rearranging the resulting equation, we have

$$C_t = Y_t^{1-\nu} (Y_t^*)^{\nu} \tag{20}$$

3. THE CONSTRAINED-EFFICIENT ALLOCATION

We can get the constrained-efficient allocation by maximizing the representative household's utility subject to the technological constraints and labor market frictions. Due to the fact that preferences and technology are symmetric, the same amount of goods is produced and consumed in the constrained-efficient allocation. In addition, when solving the optimization problem, the social planner internalizes the effect of changes in labor market tightness on hiring costs and keeps full job participation.

Since $C_t = Y_t^{1-\nu}\left(Y_t^*\right)^{\nu}$ and $Y_t = A_t\left(N_t - Bx_t^{\alpha}H_t\right) = A_t\left[N_t - B\frac{(N_t-(1-\delta)N_{t-1})^{1+\alpha}}{(1-(1-\delta)N_{t-1})^{\alpha}}\right]$, the social planner chooses Y_t and N_t to maximize

$$\mathbf{E}_0\sum_{t=0}^{\infty}\beta^t\left[\log\left(Y_t^{1-\nu}\left(Y_t^*\right)^{\nu}\right) - \chi\frac{N_t^{1+\phi}}{1+\phi}\right] \tag{21}$$

subject to equation (3) and the resource constraint

$$Y_t \leq A_t\left[N_t - B\frac{\left(N_t-(1-\delta)\,N_{t-1}\right)^{1+\alpha}}{\left(1-(1-\delta)\,N_{t-1}\right)^{\alpha}}\right] \tag{22}$$

The optimality condition for the social planner's problem is

$$\frac{\chi}{1-\nu}Y_tN_t^{\phi} = A_t - (1+\alpha)\,A_tBx_t^{\alpha} + \beta\,(1-\delta)\,\mathbf{E}_t\left\{A_{t+1}B\left[(1+\alpha)\,x_{t+1}^{\alpha} - \alpha x_{t+1}^{\alpha+1}\right]\frac{Y_t}{Y_{t+1}}\right\} \tag{23}$$

which holds with equality if $N_t < 1$.

The left-hand side of equation (23) is the marginal rate of substitution between labor and consumption, while the right-hand side is the marginal rate of transformation. When the labor market frictions are present, the marginal rate of transformation has two components. The first is the net output produced by hiring one additional worker, namely the output left after the hiring costs are deducted. The second is the benefit brought by incurring less hiring costs in the next period.

When there are no labor market frictions, namely, the hiring cost is zero, we have $Y_t = A_tN_t$. Thus equation (23) becomes

$$\frac{\chi}{1-\nu}N_t^{\phi+1} = 1. \tag{24}$$

It implies that the constrained-efficient employment is independent of the productivity shock. This result follows from the fact that the income effect of the productivity shock on labor supply cancels out the substitution effect on labor supply.

By contrast, when labor market frictions are present such that the hiring cost is not zero, as in Blanchard and Gali (2010) , the constrained-efficient allocation involves a constant job finding rate x^e,which is given by the solution to

$$\frac{\chi}{1-\nu}\left(1-\delta Bx^{\alpha}\right)N\left(x\right)^{1+\phi}=1-\left(1+\alpha\right)Bx^{\alpha}+\beta\left(1-\delta\right)\left(1+\alpha\right)Bx^{\alpha}-\alpha\beta\left(1-\delta\right)Bx^{1+\alpha} \tag{25}$$

where $N\left(x\right)=\frac{x}{\delta+(1-\delta)x}$ is the level of employment when x is taken as given. Thus the constrained-efficient unemployment rate is

$$u=\frac{\delta\left(1-x^e\right)}{\delta\left(1-x^e\right)+x^e}. \tag{26}$$

It implies that $Y_t^e=A_tN^e\left[1-B\frac{(1-(1-\delta))(N^e)^{\alpha}}{(1-(1-\delta)N^e)^{\alpha}}\right]$ and $C_t=\left(Y_t^e\right)^{1-\upsilon}\left(Y_t^*\right)^{\nu}$.

In our model, the marginal rate of substitution and the marginal rate of transformation increase in the same proportion when the productivity changes such that the constrained-efficient employment is independent of the productivity shock.

4. THE FLEXIBLE-PRICE EQUILIBRIUM

4.1. Real wage rigidity

In much of the literature on labor market frictions, the wage is determined by the Nash bargaining between workers and employers. However, Blanchard and Gali (2010) find that the equilibrium unemployment rate under Nash-bargained wage is independent of the productivity shock, which follows from the offsetting income and substitution effects. When a productivity shock occurs, the Nash-bargained wage makes a one-for-one response to the change in productivity with the result that the employment and unemployment rates are constant.

As emphasized by Blanchard and Gali (2010) , this neutrality result is different from the Shimer puzzle, which means that the movement in the unemployment rate is small in response to productivity shock when the labor market features the DMP frictions. Shimer (2005) assumes that the marginal rate of substitution is constant, whereas the marginal rate of substitution moves one-for-one with productivity in Blanchard and Gali (2010) .

The large fluctuation in the Nash-bargained wage is in conflict with empirical evidence. In what follows, we follow Blanchard and Gali (2010) and introduce real wage rigidity to generate the small fluctuation in the wage and the large fluctuation in the unemployment rate. [3]To be specific, the

[3]Also see Shimer (2005), Hall (2005), and Gertler and Trigari (2009).

real wage W_t has the following form

$$W_t = \Theta A_t^{1-\gamma} \tag{27}$$

in which Θ is a positive constant, and the parameter $\gamma \in [0,1]$ governs the degree of real wage rigidity.[4] When $\gamma = 0$, the real wage is completely flexible. By contrast, when $\gamma = 1$, the real wage is completely rigid.

4.2. Optimization problem of the intermediate-goods firms

The home intermediate-goods firms choose N_{t+k} to solve the following profit maximization problem

$$\max \mathbf{E}_t \sum_{k=0}^{\infty} \beta^k \frac{C_t}{C_{t+k}} \left[\frac{P^I_{Ht+k}}{P_{t+k}} A_{t+k} N_{t+k} - W_{t+k} N_{t+k} - G_{t+k} \left(N_{t+k} - (1-\delta) N_{t+k-1} \right) \right] \tag{28}$$

in which P^I_{Ht+k} is the price of intermediate goods.

The optimality condition is

$$\frac{P^I_{Ht}}{P_t} A_t = W_t + G_t - \beta (1-\delta) \mathbf{E}_t \left[\frac{C_t}{C_{t+1}} G_{t+1} \right]. \tag{29}$$

The left-hand side of equation (29) denotes the real value of marginal product of labor, while the right-hand side represents the real marginal cost when the hiring cost is present.

From equation (29) , we can obtain the real marginal cost for home final-goods firms

$$MC_t = \frac{P^I_{Ht}}{P_{Ht}} = S_t^{\nu} \left\{ \Theta A_t^{-\gamma} + B x_t^{\alpha} - \beta (1-\delta) \mathbf{E}_t \left[\frac{C_t}{C_{t+1}} \frac{A_{t+1}}{A_t} B x_{t+1}^{\alpha} \right] \right\}. \tag{30}$$

Equation (30) implies that both labor market frictions and real wage rigidity affect the real marginal cost for final-goods firms.

5. THE STICKY-PRICE EQUILIBRIUM

5.1. Introducing Nominal Rigidity

We introduce final-goods price rigidity in a staggered fashion, as in Calvo (1983) . In each period, a home representative final-goods firm i sets price with probability $1-\theta$ whereas it keeps the prices fixed with probability θ.

[4]In line with Blanchard and Gali (2010) , $\Theta = \left(\frac{1}{\mathcal{M}} - (1-\beta(1-\delta)) B x^{\alpha} \right) A^{\gamma}$.

Thus when the home final-goods firm i has the opportunity to reset price in period t, it chooses P^o_{Ht} to maximize

$$\mathbf{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[(1+\tau) P^o_{Ht} - P_{Ht+k} MC_{t+k} \right] Y_{Ht+k|t}(i) \tag{31}$$

in which $\Lambda_{t,t+k} = \beta^k \frac{C_t}{C_{t+k}} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor, τ is a subsidy to home final-goods firms from home government, and $Y_{Ht+k|t}(i) = \left(\frac{P^o_{Ht}}{P_{Ht+k}} \right)^{-\varepsilon} Y_{Ht+k|t}$ is the demand schedule for the home representative final-goods firm i that last reset its price in period t.

The solution to the optimal price setting problem facing the home representative final-goods firm i is

$$P^0_{Ht} = \frac{\varepsilon}{(\varepsilon - 1)(1+\tau)} \frac{\mathbf{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \frac{P_t}{P_{t+k}} Y_{Ht+k}(i) P_{Ht+k} MC_{t+k}}{\mathbf{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \frac{P_t}{P_{t+k}} Y_{Ht+k}(i)} \tag{32}$$

in which $\frac{\varepsilon}{(\varepsilon-1)(1+\tau)}$ is an effective markup over the weighted average of the final-goods firm i's current and expected future marginal costs in the periods during which its reset price P^o_{Ht} keeps effective.

Therefore, the aggregate price level satisfies

$$P_{Ht} = \left((1-\theta) \left(P^0_{Ht} \right)^{1-\varepsilon} + \theta P^{1-\varepsilon}_{Ht-1} \right)^{\frac{1}{1-\varepsilon}} \tag{33}$$

5.2. Log-linearized equilibrium dynamics

In this section, we use lower case variables with hats to denote log deviations of the corresponding upper case variables from their steady state values. By log-linearizing the optimal price setting equation (32) and the aggregate price level equation (33) around the zero inflation steady state, we can get the following New Keynesian Phillips curve to describe the motion of home final-goods inflation rate

$$\pi_{Ht} = \beta \mathbf{E}_t \pi_{Ht+1} + \lambda \widehat{mc}_t \tag{34}$$

in which $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

Log-linearization of equation (30) around the zero inflation steady state gives us the following equation

$$\widehat{mc}_t = \alpha g \mathcal{M}\hat{x}_t - \beta(1-\delta) g\mathcal{M}\left[\hat{c}_t - \hat{a}_t - (\hat{c}_{t+1} - \hat{a}_{t+1}) + \alpha\hat{x}_{t+1}\right] - \gamma\mathcal{M}\Theta\hat{a}_t + \nu\hat{s}_t \tag{35}$$

where $\mathcal{M} = \frac{\varepsilon}{\varepsilon-1}$ is the gross markup.

From equations (10), (11), (12), and (14), we can express labor market tightness as a function of current and lagged employment

$$\delta\hat{x}_t = \hat{n}_t - (1-\delta)(1-x)\hat{n}_{t-1}. \tag{36}$$

Log-linearizing equation (20) around the zero inflation steady state, we have

$$\hat{c}_t = (1-\nu)\hat{y}_t + \nu\hat{y}_t^*. \tag{37}$$

From the resource constraint $Y_t = A_t(N_t - Bx_t^\alpha H_t)$ and aggregate hiring equation (11), we have

$$\hat{y}_t = \hat{a}_t + \frac{1-g}{1-\delta g}\hat{n}_t - \frac{g\alpha\delta}{1-\delta g}\hat{x}_t + \frac{(1-\delta)g}{1-\delta g}\hat{n}_{t-1} \tag{38}$$

Substituting equation (38) into equation (37), we can get

$$\hat{c}_t = (1-\nu)\hat{a}_t + \frac{(1-\nu)(1-g)}{1-\delta g}\hat{n}_t - \frac{(1-\nu)g\alpha\delta}{1-\delta g}\hat{x}_t + \frac{(1-\nu)(1-\delta)g}{1-\delta g}\hat{n}_{t-1} + \nu\hat{y}_t^*. \tag{39}$$

Log-linearizing the home representative household's stochastic Euler equation around the steady state, we can obtain the dynamic IS equation for the home country

$$\hat{c}_t = \mathbf{E}_t\hat{c}_{t+1} - (i_t - \mathbf{E}_t\pi_{t+1}) \tag{40}$$

in which $i_t \equiv \ln R_t$ is the home nominal interest rate.

From the expression for home CPI price level $P_t = k^{-1}P_{Ht}^{1-\upsilon}P_{Ft}^{\upsilon}$ and that for the terms of trade $S_t = \frac{P_{Ft}}{P_{Ht}}$, we have

$$\pi_t = \pi_{Ht} + \nu\Delta\hat{s}_t \tag{41}$$

From the risk-sharing condition, we can get

$$\hat{s}_t = \frac{\hat{c}_t}{1-\nu} - \frac{\hat{y}_t^*}{1-\nu}. \tag{42}$$

Equations (34) – (36) and (39) – (42), together with an equation to describe how the home nominal interest rate is determined and a process to describe how the productivity shock evolves, characterize the equilibrium dynamics system.

5.3. Unemployment and Inflation

Before we turn our attention to analyze how trade openness affects the optimal monetary policy in a small open economy model with labor market frictions, we derive a new version of New Keynesian Phillips curve to describe the relationship between the unemployment rate and the inflation rate.

From equation (14) , we have

$$\hat{n}_t = -\frac{\hat{u}_t}{1-u}. \tag{43}$$

Substituting equations (20) , (36) , (38) , (42), and (43) into equation (35) ,we can obtain a new expression for $\widehat{mc}_t$. Thus equation (34) can be rewritten as

$$\pi_{Ht} = \beta \mathbf{E}_t \pi_{Ht+1} - \kappa_0 \hat{u}_t + \kappa_l \hat{u}_{t-1} + \kappa_f \hat{u}_{t+1} + \lambda \Gamma_{a0} \hat{a}_t + \lambda \Gamma_{af} \hat{a}_{t+1} + \lambda \Gamma_{y0} \hat{y}_t^* + \lambda \Gamma_{yf} \hat{y}_{t+1}^* \tag{44}$$

in which $\kappa_0 \equiv \frac{\lambda h_0}{1-u}, \kappa_l \equiv -\frac{\lambda h_l}{1-u}, \kappa_f \equiv -\frac{\lambda h_f}{1-u}$,and

$$h_0 = \frac{\alpha g \mathcal{M}}{\delta} + F\left(\xi_2 - \xi_0 + \xi_1\right) + \frac{\nu}{1-\nu} \mathcal{M} \xi_3 \xi_0,$$

$$h_l = -g \mathcal{M} \xi_2 - F \xi_1 + \frac{\nu}{1-\nu} \mathcal{M} \xi_3 \xi_1, h_f = F\left(\xi_0 - \frac{\alpha}{\delta}\right),$$

$$F = \beta\left(1-\delta\right) g \mathcal{M}, \xi_0 = \frac{\left(1-\nu\right)\left(1-g-g\alpha\right)}{1-\delta g}, \xi_1 = \frac{\left(1-\nu\right) g\left(1-\delta\right)\left[\alpha\left(1-x\right)+1\right]}{1-\delta g},$$

$$\xi_2 = \alpha \frac{\left(1-\delta\right)\left(1-x\right)}{\delta}, \xi_3 = \Theta + \alpha g - \beta\left(1-\delta\right) g,$$

$$\Gamma_{a0} = F\nu - \gamma \mathcal{M} \Theta + \nu \mathcal{M} \xi_3, \Gamma_{af} = -F\nu, \Gamma_{y0} = -F\nu - \mathcal{M} \nu \xi_3, \Gamma_{yf} = F\nu.$$

6. MONETARY POLICY DESIGN

6.1. Optimal monetary policy

In line with Blanchard and Gali (2010) , we assume that the unemployment rate fluctuates around a steady state value which corresponds to that of the constrained efficient allocation. After taking a second-order approximation to the utility function of the home representative household, we obtain

$$\mathbf{W} = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbf{X}_t + t.i.p + o\left(||a||^3\right) \tag{45}$$

where $t.i.p.$ stands for the terms independent of policy, $O\left(\| a \|^3\right)$ collects all terms of third or higher order, and

$$\mathbf{X}_t = \frac{(1-\nu)\,\varepsilon}{2\lambda}\pi_{Ht}^2 + \frac{1+\phi}{2}\chi\,(1-u)^{\phi-1}\,\hat{u}_t^2$$

When the small open economy degenerates into a closed economy $(\nu = 0)$, equation (45) is identical to its counterpart in Blanchard and Gali (2010).

Thus, the expected period welfare loss function is

$$\mathbf{L} = \frac{(1-\nu)\,\varepsilon}{2\lambda} var\left(\pi_{Ht}\right) + \frac{1+\phi}{2}\chi\,(1-u)^{\phi-1}\, var\left(\hat{u}_t\right)$$

It is evident that both home final-goods inflation and the fluctuation in unemployment lead to welfare loss. In addition, the increase in the degree of trade openness lowers the effect of home PPI inflation on the welfare loss.

When the monetary policymaker is able to commit, with full credibility, to the policy rule at time zero, she chooses π_{Ht} and $\hat{u}_t$ to minimize equation (45) subject to the sequence of equilibrium constraints given by equation (44),for $t = 0, 1, 2...$The first order conditions are

$$\frac{(1-\nu)\,\varepsilon}{\lambda}\pi_{Ht} - \lambda_t + \lambda_{t-1} = 0 \tag{46}$$

$$\chi\,(1+\phi)\,N^{\phi-1}\hat{u}_t - \kappa_0\lambda_t + \beta\kappa_l\lambda_{t+1} + \frac{1}{\beta}\kappa_f\lambda_{t-1} = 0 \tag{47}$$

for $t = 0, 1, 2...$,where λ_t is the Lagrange multiplier satisfying $\lambda_{-1} = 0$.

6.2. Calibration

We list the baseline parameter values used in the simulation in Table 1. The calibration of the model is presented such that each period corresponds to a quarter. The parameter value of the discount factor is set 0.99 with the result that the annual interest rate is about 4% in the steady state. There is disagreement among micro and macro economists as to the value of the Frisch elasticity of labor supply (Chetty, et al., 2011), we follow much of the literature (Blanchard and Gali, 2010;Nakamura and Steinsson, 2014; Christiano et al., 2014; and Gong et al., 2016) to set ϕ to 1. We set the elasticity of substitution between final goods ε to 6, implying that a gross steady state markup is 1.2. In line with much of the micro and macro evidence on price setting, we choose the probability of the final-goods firms not being able to adjust the prices at each period θ to be 0.75 such that the average duration of the nominal contracts lasts four quarters.

In accordance with Blanchard and Gali (2010), we set γ and α to 0.5 and 1 respectively. As for the degree of trade openness, we follow Wei and Xie (2020) and set ν to 0.4 which implies 60% domestic share of GDP. The productivity shock follows the AR(1) process with the persistence parameter and the standard deviation being 0.95 and 0.02, respectively.

Then we choose the parameter values describing the labor market. Consistent with Blanchard and Gali (2010) ,we set the unemployment rate u to 5 percent and the job finding rate x to 0.7, implying that the job separation rate $\delta = \frac{ux}{(1-u)(1-x)}$ is 0.12. Following Blanchard and Gali (2010) ,the fraction of the hiring cost in GDP $\delta B x^{\alpha}$ is 1 percent in the steady state, which means that $B = \frac{0.01}{\delta x^{\alpha}}$. Finally, from equation (25) ,we can solve for the parameter χ.

TABLE 1.

Parameter values in the benchmark case

Description	Parameter	Value
Discount factor	β	0.99
Frisch elasticity of labor supply	ϕ^{-1}	1
Degree of real wage rigidity	$1-\gamma$	0.5
Degree of trade openness	ν	0.4
Coefficient of hiring cost	α	1
Elasticity of substitution between final goods	ε	6
Duration of the nominal contracts	θ	0.75
Persistence of productivity shock	ρ_a	0.95
Standard deviation of productivity shock	σ_a	0.02

6.3. Quantitative analysis

Since the optimal monetary policy is not feasible in practice, we consider the macroeconomic implications of a simple Taylor-type monetary policy rule, which is given by

$$i_t = \rho + \rho_\pi \pi_{Ht} - \rho_u \hat{u}_t \tag{48}$$

where the coefficients ρ_π and ρ_u are chosen, for each calibration, to minimize the welfare losses. The choices of ρ_π and ρ_u are completed numerically by searching over a grid spanning the intervals $\rho_\pi \in [0,3]$ and $\rho_u \in [0,5]$.The optimal choices of ρ_π and ρ_u are 3 and 0.3536, respectively.

FIG. 1. Welfare losses under optimal monetary policy and optimized simple rule.

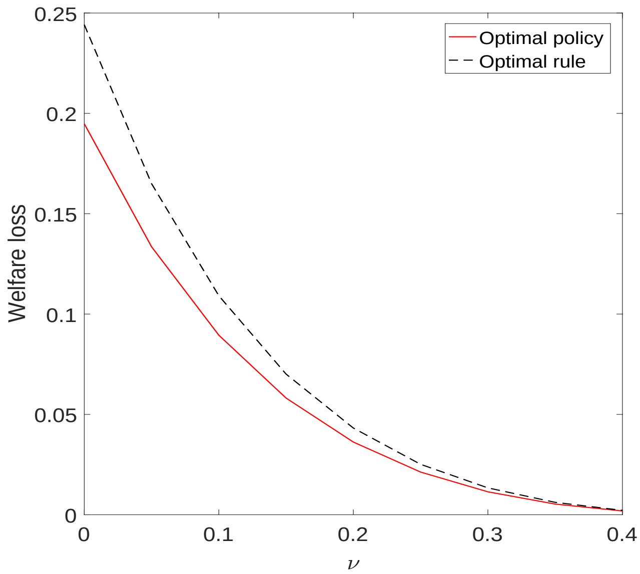
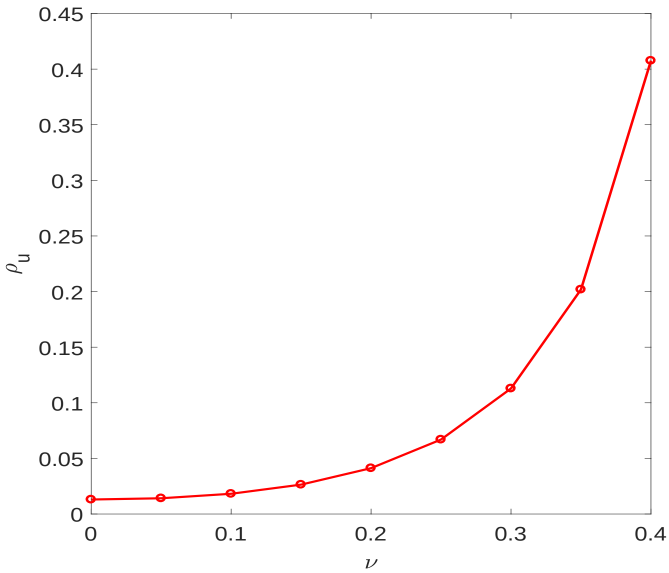


Figure 1 depicts the welfare losses under the optimal monetary policy and the optimized simple rule. Clearly, the welfare loss under the optimized simple rule is larger than that under the optimal monetary policy. However, the welfare gains from implementing the optimal monetary policy are small relative to the optimized simple rule. In particular, the welfare loss under the optimized simple rule is nearly identical to that under the optimal monetary policy in the benchmark case in which the degree of trade openness ν equals 0.4.

How does the coefficient ρ_u change when the degree of trade openness ν varies within a reasonable range? To answer this question, we keep the coefficient ρ_π at the optimal level 3 and search over the interval $[0,5]$ to find the optimal ρ_u. As shown in Figure 2, we find that the coefficient ρ_u increases with the degree of trade openness ν.

The reason that the coefficient ρ_u increases with the degree of trade openness ν can be found from the observation of the welfare loss function.

FIG. 2. The coefficient ρ_u increases with the degree of trade openness ν.



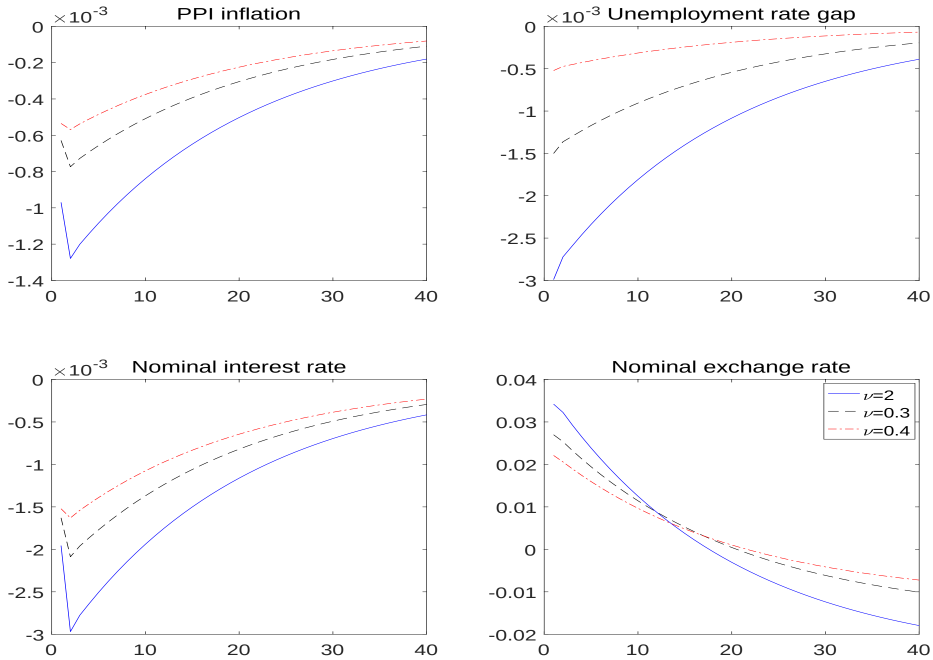
From equation (45) , we know that the welfare loss comes from the volatility in the PPI inflation and the unemployment rate gap. When the degree of trade openness ν goes up, the welfare loss from the volatility in the PPI inflation becomes smaller, which means that the welfare loss from the volatility in the unemployment rate gap increases relative to that from the volatility in the PPI inflation, accordingly, the monetary policymaker should raise the coefficient ρ_u to reflect the changed trade-off between the two components when implementing the optimized simple rule.

Why does the welfare loss from the volatility in the PPI inflation become smaller when the degree of trade openness ν goes up? Due to the existence of price rigidity, the price dispersion causes inefficiency with the result that the PPI inflation is a source of the welfare loss. When the degree of trade openness ν goes up, the effect of domestic price dispersion on the consumption of the home representative household decreases, thus, the welfare loss from the volatility in the PPI inflation becomes smaller.

6.4. Impulse responses

To understand how the degree of trade openness affects the transmission mechanism of monetary policy when the monetary policymaker implements the optimized simple rule. We compare the impulse responses of the relevant variables of the model to a positive productivity shock hitting the home intermediate-goods production sector. Figure 3 depicts the impulse responses of the relevant variables of the model for three cases: $\nu = 0.2; \nu = 0.3; \nu = 0.4$.

FIG. 3. Impulse responses to a positive productivity shock hitting the home intermediate-goods production sector.



In open economies, the monetary policy in one country generally produces expenditure level and expenditure switching effects, with the former affecting aggregate demand and the latter affecting the relative demand for one country's goods. When one country is more open in the sense that the expenditure share on imported goods becomes larger, the expenditure switching effect is more powerful. Facing a positive productivity shock hitting the home intermediate-goods production sector, the home monetary policymaker needs to lower the nominal interest rate to boost the demands for domestic goods. When the home country is more open, the degree of decrease in the nominal interest rate is smaller with the aim of producing a desirable expenditure switching effect. Thus the degree of depreciation of the nominal exchange rate decreases with the degree of trade openness.

Since the degree of decrease in the nominal interest rate is smaller when the home country is more open, the demands for domestic goods are lower than the cases in which the home country is less open, thus the firms will produce less goods. It means that the unemployment rate increases with the degree of trade openness. Due to the fact that the unemployment rate in the flexible-price equilibrium is a constant, the unemployment rates shift downwards with the result that the unemployment rate gap in the case in which the home country is more open is smaller than those in which the home country is less open. A positive productivity shock tends to drive down the price of domestic goods, however, fewer supply in the case in which the home country is more open implies that the degree of decrease in the price of domestic goods is smaller than those in which the home country is less open.

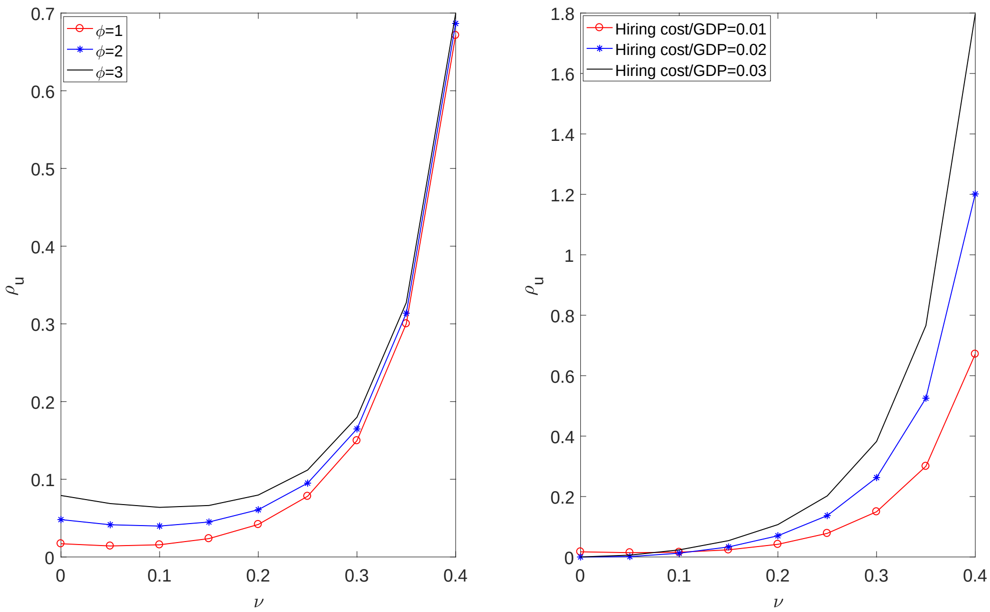
6.5. Sensitivity analysis

In this section, we perform the sensitivity analysis to show that our conclusion is robust to two key parameters, namely the Frisch elasticity of labor supply and the hiring cost. In the benchmark model, we set the Frisch elasticity of labor supply to 1. To check whether our conclusion depends on the calibrated value, we plot the relationship between the degree of trade openness and the coefficient ρ_u for three cases in the left panel of Figure 4: $\phi = 1, \phi = 2$,and $\phi = 3$. Moreover, we also perform the sensitivity analysis for the ratio of the hiring cost to GDP, which is reported in the right panel of Figure 4

As shown in the left panel of Figure 4, our conclusion that the coefficient ρ_u increases with the degree of trade openness ν is robust to the change in the Frisch elasticity of labor supply. In addition, we also find that, given the degree of trade openness, the coefficient ρ_u decreases with the Frisch elasticity of labor supply. After a casual observation of the expression of the welfare loss function, we can find that the welfare loss from the volatility in the unemployment rate gap increases when the Frisch elasticity of labor supply goes down. Thus, the monetary policymaker should pay more attention to the unemployment rate gap when the Frisch elasticity of labor supply goes down.

From the right panel of Figure 4, we know that the coefficient ρ_u increases with the degree of trade openness ν regardless of the ratio of the hiring cost to GDP. In addition, we know that, given the degree of trade openness, the coefficient ρ_u increases with the ratio of the hiring cost to

FIG. 4. Sensitivity analysis.



GDP. Since a higher ratio of the hiring cost to GDP implies that the welfare loss caused by labor market frictions is greater, it is evident that the monetary policymaker should respond more to the volatility of the unemployment.

7. CONCLUSION

This paper examines how the optimal monetary policy is conducted in a small open economy with labor market frictions in the spirit of Blanchard and Gali (2010) .As in Blanchard and Gali (2010) , when conducting the optimal monetary policy, the monetary policymaker should target the PPI inflation and the unemployment rate gap, but the degree of trade openness affects the trade-off between the two targets.

Due to the unfeasibility of the optimal monetary policy in practice, we consider the implications of the optimized simple Taylor-type monetary policy rule for the macroeconomic stabilization. We find that the optimized simple rule performs better in the sense that the welfare gains from implementing the optimal monetary policy are small relative to the optimized simple rule. Moreover, we find that, under the optimized simple rule, the monetary policymaker attaches a greater weight to the unemployment rate gap when the degree of trade openness becomes larger.

Since the prices are sticky, the fluctuation in PPI inflation causes the welfare loss. When the degree of trade openness goes up, the welfare loss from the fluctuation in PPI inflation becomes smaller, thus the monetary policymaker should pay more attention to the unemployment rate gap, when conducting the optimized simple rule. The sensitivity analysis shows that our conclusion is robust to the changes in the Frisch elasticity of labor supply and the hiring cost.

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