

Endogenous Constitutional Democracy Capital and Economic Development

Wei Liang* Heng-fu Zou[†]

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Abstract

This paper develops a dynamic representative-citizens model that endogenizes the accumulation of both constitutional democracy capital and physical capital. Drawing from Douglass North's seminal works, our model integrates interactions between institutional capital and economic performance, focusing on how changes in institutional frameworks impact economic dynamics. By exploring the reciprocal relationships between democratic institutions and economic variables over time, the model elucidates how democratic structures facilitate economic growth and how economic conditions affect the vitality of democratic governance. It demonstrates that exogenous economic variables significantly influence both democracy and physical capital, while political institutional factors critically shape economic outcomes. This approach highlights the importance of a supportive democratic environment in enhancing capital accumulation and shows how threats to democracy can hinder both democratic integrity and economic development, providing a comprehensive framework to understand the interplay between political systems and economic performance.

Key Words: Democracy Capital, Constitutional Democracy, Liberty, Rule of Law, Property Rights, Institutions, Economic Growth, Development

JEL Classification Numbers: E20; E22; H56; O10; O40; P16, P48

1 Introduction

A constitutional democracy is a system of government where authority is derived from a constitution that delineates foundational laws and principles. This governance model is characterized by the separation of powers across various governmental branches, adherence to the rule of law, and the protection of fundamental human rights. The power of the government is both derived from and limited by the constitution, often ratified by representatives of the people or through direct vote, ensuring political equality where each citizen's voice is heard through free and fair elections, and officials are held accountable to the populace. This structured governance, where operations are clearly defined and regulated by constitutional law, is termed constitutional democracy capital.

Constitutional democracy functions as a democracy capital structure within any given institution, organization, or state, marked by collective power sharing among its members. This system serves as both institutional capital and a decision-making framework, continuously strengthened

*China Economics and Management Academy, Central University of Finance and Economics, Beijing, 100081, China. Email address: w3liang2@foxmail.com.

[†]The World Bank, Washington, D. C. 20433, USA. Email address: Hzhoucema@gmail.com.

by ongoing investments in democratic principles. Unlike earlier governance forms, modern democracies are noted for their robust institutional capital, empowering citizens to actively participate in societal governance and ensuring adherence to international laws. This distinction enhances modern democracies, solidifying them as resilient and effective governance platforms.

The foundational preferences and values of a constitutional democracy encompass the rule of law, ensuring that all individuals, institutions, and entities are accountable to laws that are publicly established, equally enforced, independently adjudicated, and in line with international human rights norms. The separation of powers divides governmental responsibilities among distinct branches to prevent any one branch from monopolizing core functions, thereby maintaining a balance that avoids power abuse and tyranny. The constitution often enshrines fundamental human rights, such as freedom of speech, religion, assembly, and the press, safeguarded by an independent judiciary. Governance in a constitutional democracy is transparent, with regular, free, and fair elections that hold power bearers accountable. Moreover, it upholds political equality, providing every citizen equal access to the political process and fostering broad participatory engagement in civic life, including public discussions and legislative activities. This participatory framework is vital for the continuous accumulation and enhancement of constitutional democracy capital through sustained investments in democratic processes and institutions.

The roots of democracy, or more precisely, the democracy capital structure, can be traced back to ancient Greece, a period often regarded by 18th-century intellectuals as the cornerstone of Western civilization. These thinkers endeavored to reinterpret ancient democratic frameworks into new models for political organization post-monarchy. Following World War II, there was a significant resurgence of democratic ideals and an enhancement of institutional capital that dramatically altered the global political landscape. Modern representative democracies strive to merge Rousseau’s concept of the state of nature with Hobbes’s perspective of society as inherently authoritarian. This is achieved through the formulation of ‘social contracts’ that protect citizens’ rights, curtail state power, and enhance individual empowerment through voting rights. These developments underscore the transformation and reinforcement of democracy as both a capital structure and vital institutional capital in the contemporary world.

Investments in modern constitutional democracy and the accumulation of constitutional democracy capital are crucial drivers of economic growth, as evidenced by the research of Persson and Tabellini (2009); and Acemoglu et al. (2019). In this paper, we develop a simple dynamic model that simultaneously endogenizes the accumulation of both constitutional democracy capital and physical capital. This endeavor is part of a broader initiative to synthesize the interplay among institutional capital, changes in institutional frameworks, and economic performance, drawing on the foundational theories proposed by North (1981, 1990, 2005). Our model aims to provide a deeper understanding of how democratic institutions impact and are impacted by economic variables over

time.

Section 2 of this paper offers a historical overview of investments in constitutional democracy and the gradual build-up of constitutional democracy capital. This section aims to contextualize the evolution of democratic institutions, tracing the historical trajectory from early democratic forms to contemporary democratic practices. Sections 3 and 4 introduce and explore our proposed model. Here, we detail the mechanisms through which constitutional democracy capital accumulation interacts with physical capital accumulation, emphasizing how enhancements in democratic governance can lead to significant economic outcomes. We analyze the implications of this interaction, illustrating how sustained investments in democracy can foster economic growth and stability. The model provides a theoretical framework for examining the interdependence between democratic depth and economic performance. Section 5 concludes the paper by summarizing our findings and discussing the broader implications of our model.

Our study is timely because recently, significant regressions in global political rights and civil liberties were marked by numerous events that effectively acted as negative investments in constitutional democracy and led to the degradation of constitutional democracy capital. These events, arising from both internal and external sources, frequently involved violent or coercive measures that directly threatened the stability and integrity of democratically elected governments:

Foreign Military Interventions: A stark example of these negative democracy investments and the destruction of constitutional democracy capital was observed in foreign military actions, such as Russia’s invasion of Ukraine. This military offensive not only sought territorial expansion but was fundamentally aimed at undermining a sovereign, democratically elected government. The intention was to replace it with a regime more aligned with the aggressor’s geopolitical goals. This conflict resulted in extensive casualties, massive civilian displacement, and significant destruction, severely destabilizing Ukraine’s democratic institutions and impairing the functionality of its political system.

Coups d’État: Another significant force destructively impacting constitutional democracy capital recently has been the occurrence of coups d’état. These sudden and illegal overthrows of governments dismantled established democratic processes and often installed military or autocratic regimes. For example, Burkina Faso experienced two coups during this period, drastically reversing democratic progress and eroding political freedoms and civil liberties under military rule.

Power Grabs by Incumbent Officials: Recent actions by incumbent officials in various countries have posed significant threats to constitutional democracy capital by exploiting their positions to dismantle democracy investments and consolidate undemocratic control. These actions often involved manipulating legal frameworks, purging opposition, suppressing dissent, and violating both the rule of law and electoral integrity. In Peru, President Pedro Castillo attempted to dissolve parliament and govern by decree, leading to widespread protests and political instability.

Similarly, in Tunisia and Hungary, leaders systematically weakened checks and balances, restricted press freedom, and undermined judicial independence. In Hungary, Prime Minister Viktor Orbán's Fidesz party benefited from legislative changes aimed at reducing political competition, further entrenching its power following the 2022 elections. In El Salvador, President Nayib Bukele used a parliamentary supermajority to pass measures that suppressed democratic controls under the guise of addressing gang violence. This led to the indefinite detention of thousands without due process and the suspension of anti-corruption measures. Bukele also announced his candidacy for a second term after manipulating the Constitutional Court to overturn a ban on consecutive presidential terms. These examples underscore a global pattern where political leaders undermine democratic institutions and processes to extend their rule, highlighting the ongoing challenges to maintaining and strengthening constitutional democracy capital worldwide. ¹

These direct assaults on democratic institutional capital have led to profound and far-reaching consequences worldwide. Such actions have not only eroded public trust in political systems and leaders—fundamental for democracy's effective functioning—but have also incited widespread civil unrest as citizens protest the erosion of their democratic rights. Additionally, these assaults have precipitated severe human rights violations, including arbitrary detentions and torture, while stifling the voices of opposition and criticism. The economic impact has been equally detrimental, significantly damaging physical capital investment, production, and overall economic development. In response, the international community has often imposed isolation, economic sanctions, and reduced foreign investments, exacerbating the economic crises in these nations. Together, these developments underscore the vulnerability of democratic systems and emphasize the critical need for continued investments in democracy and the protection of constitutional democracy capital against authoritarian threats. This situation highlights an ongoing global need for vigilance and resilience in safeguarding democratic institutions.

2 The investments on constitutional democracy and the accumulation of constitutional democracy capital: A very brief history

The development of democratic principles and the accumulation of democracy capital can be traced to the earliest human societies, where anthropologists have identified rudimentary forms of democratic decision-making among small hunter-gatherer bands. These early groups, usually numbering between 50 to 100 individuals linked by kinship, typically made decisions collectively, through consensus or majority rule, without formal leadership structures.

This early form of governance, sometimes referred to as tribalism or primitive democracy, was prevalent in small communities where direct communication was possible and effective. As these

¹For much more details, please read Freedom House, 2023. Freedom in the World: The Global Expansion of Authoritarian Rule.

communities expanded, the simplicity of direct democratic practices became impractical, giving rise to more complex forms of governance such as monarchies, tyrannies, aristocracies, and oligarchies in burgeoning urban centers..

The exploration of proto-democratic practices before classical Athenian democracy has captivated scholars, extending to regions like Phoenicia, Mesopotamia, and the Indian subcontinent. In Phoenicia, there are indications of communal decision-making in city assemblies, although details about their structure and influence are sparse.

In Mesopotamia, the notion of a "primitive democracy" has been suggested by researchers such as Thorkild Jacobsen, who pointed to early Sumerian city-states where free male citizens wielded political authority. This theory, however, is debated due to the vague and often interpretative nature of the evidence.

The Indian subcontinent featured republics known as *gaṇas* and *saṅghas*, characterized by significant assembly-based governance. The democratic nature of these assemblies is a subject of academic debate, with some interpretations highlighting democratic elements and others emphasizing the dominance of upper classes and ingrained social hierarchies.

Interpreting these early democratic forms is challenging due to the opaque nature of historical records on popular participation, the impact of caste systems, and the overall ambiguity surrounding the equality of citizens within these ancient systems. Nonetheless, studying these early instances provides invaluable insights into the genesis of collective governance and democratic processes throughout human history.

The democratic practices of ancient Germanic tribes, such as the Suebi, Cherusci, and Franks, during the pre-Roman and early medieval periods also reflect early democratic principles. These societies exhibited decentralized political structures with significant community involvement in decision-making:

Tribal Assemblies: Known as Thing or Moot, these gatherings were crucial for collective decision-making, offering a forum for public discussion and consensus on communal matters.

Leadership by Merit and Consensus: Leadership in these tribes was often based on merit and respect rather than heredity. Chiefs and elders, while influential, governed with the consent of the tribe, reflecting a rudimentary form of democratic governance.

Consultative Processes: Important decisions typically involved broad consultation with the assembly or a council of elders, emphasizing a participatory approach that valued diverse viewpoints.

Customary Laws: Governance was also guided by unwritten codes and traditions, which regulated social conduct and dispute resolution based on communal norms and values.

Although these early forms of democracy differ significantly from modern democratic institutions in complexity and structure, they represent foundational examples of participatory governance. These practices laid the groundwork for the political evolution that would shape later

democratic developments in Europe and beyond.

2.1 Investments in constitutional democracy and the accumulation of constitutional democracy capital in Jerusalem

Jerusalem's role in fostering constitutional democracy capital can be seen through its extensive historical and religious context. Central to the three major monotheistic religions—Judaism, Christianity, and Islam—Jerusalem has a long history of integrating ethical and legal principles from sacred texts into its governance. For instance, the Hebrew Bible, which is deeply ingrained in the city's legal and moral foundations, emphasizes themes of justice, fairness, and leadership accountability. These principles have shaped the governance of Jerusalem over millennia, contributing to its complex layering of democratic values amid its political and religious significance. This integration of religious doctrine and governance highlights a unique form of constitutional democracy development, where religious laws have historically intersected with civic administration, influencing both local governance structures and broader ideological movements toward democratic values.

2.2 Investments in constitutional democracy and the accumulation of constitutional democracy capital in Sparta

Investments in constitutional democracy and the accumulation of constitutional democracy capital in Sparta can be observed through its distinctive system of governance. Unlike many other ancient Greek city-states, Sparta developed a mixed governance system that integrated elements of both oligarchy and democracy. This system was characterized by a division of power among several key institutions: the dual kingship, the Gerousia (Council of Elders), the Ephors (representatives of the citizens), and the Ecclesia (assembly of Spartans).

Investments in democracy were evident in the structure of the dual kingship, where two kings ruled concurrently, sharing judicial responsibilities with the Gerousia. This council, composed of elders elected for life, held legislative powers, reflecting the oligarchic aspect of Spartan governance. In contrast, the accumulation of constitutional democracy capital is seen in the Apella, an assembly where Spartan citizens elected the members of the Gerousia and the Ephors, injecting a democratic element into the system. The Ephors, limited to one-year terms to prevent power accumulation, checked the kings' actions and managed foreign affairs, emphasizing democratic oversight.

Lycurgus, the renowned Spartan lawgiver, is credited with initiating significant reforms in response to a helot revolt, focusing on military preparedness, austere living, and equality among Spartiate citizens. His reforms, encapsulated in the Great Rhetra, represent early investments in constitutional democracy, often considered the world's first example of a written constitution. Spartan society, known for its emphasis on equality as seen in its education system and some rights

for women, however, displayed considerable restrictions on personal freedoms.

Despite the democratic features within Sparta's political system, it encountered criticism for its oligarchic tendencies, particularly the dominant role of the Gerousia. The stability of Sparta's political framework, which resulted in minimal constitutional changes over time, reflects a significant accumulation of constitutional democracy capital. Nevertheless, military conflicts, such as those with Athens and Thebes, including the Peloponnesian War and the defeat at Leuctra, progressively eroded Sparta's dominance. Eventually, Sparta's defeat by Philip II of Macedon marked a decline in its power and influence.

This historical analysis demonstrates how Sparta's unique amalgamation of oligarchic and democratic elements not only contributed to its strengths but also to the inherent tensions within its governance system, illustrating a complex evolution of constitutional democracy within the context of ancient Greek polis politics.

2.3 Investments in constitutional democracy and the accumulation of constitutional democracy capital in Athen

The historical evolution of Athenian democracy provides a rich framework to examine the investments in constitutional democracy and the processes of accumulating constitutional democracy capital. These concepts can be explored through the sequential reforms enacted in Athens, each contributing to a more inclusive and participatory governance system.

Constitutional Democracy Investments

1. Legal Codification by Draco (621 BC): Draco's legal reforms represent a critical initial investment in Athenian constitutional democracy by transitioning from oral to written laws. This development not only ensured that laws were clear and accessible to citizens but also laid a foundational legal structure that would support more complex democratic governance.

2. Economic and Constitutional Reforms by Solon (594 BC): Solon's reforms were significant in reducing the power disparities within Athenian society. By redefining citizenship and property classes, Solon broadened political participation beyond the aristocracy, which facilitated a more equitable distribution of political power. His economic reforms, such as debt cancellation and the prohibition of debt slavery, alleviated major social tensions, paving the way for more stable democratic governance.

3. Cleisthenes' Reforms (508-507 BC): Cleisthenes' reorganization of the Athenian political structure into ten tribes based on geographic locations rather than familial ties further democratized the political process and integrated a wider segment of the population into governance. This restructuring reduced the influence of aristocratic families and enhanced the representational nature of the Athenian government.

4. Reforms by Ephialtes (462 BC): The reforms instigated by Ephialtes, particularly the dimin-

ishment of the Areopagus' powers, marked a significant shift towards increasing the democratic functioning of the Athenian state. This not only expanded political participation but also ensured that more citizens could hold the government accountable.

Constitutional Democracy Capital Accumulation

1. Institutional Development: The gradual enhancements to democratic governance, as seen through the reforms of Draco, Solon, Cleisthenes, and Ephialtes, contributed to the development of robust institutions that are essential for the accumulation of democratic capital. These institutions began to embody democratic norms and practices, making them part of the civic culture.

2. Civic Participation and Cultural Integration: As reforms progressively included more citizens in the governance process, democratic values became deeply ingrained in Athenian society. The establishment of the Ecclesia, where all male citizens could participate, was crucial in fostering a participative democratic culture. This cultural shift towards valuing civic engagement and collective decision-making exemplified the deepening of democratic capital.

3. Resilience and Adaptability: Despite several setbacks, such as the temporary overthrow of democratic governance by the oligarchy during the Peloponnesian War and the rule of the Thirty Tyrants, Athens demonstrated a significant resilience in reverting to democratic norms. This resilience is indicative of a society with a substantial accumulation of democratic capital, where the citizenry values and strives to return to democratic governance despite challenges.

4. Influence and Legacy: The Athenian model of democracy, despite its eventual decline, left a lasting legacy that influenced subsequent democratic thought and governance structures around the world. The historical significance of Athenian democracy in developing political theory and practice illustrates the profound impact of accumulated democratic capital over centuries.

Therefore, history of Athenian democracy, through its various reforms and the subsequent development of its institutions and civic culture, showcases a dynamic example of how constitutional democracy investments can lead to the accumulation of significant democratic capital. This capital not only facilitated the functioning of democracy within Athens but also contributed to the broader conceptual and practical frameworks of democratic governance in later civilizations.

Athenian democracy was direct and inclusive, involving citizens in decision-making through the Ecclesia, Boule, and courts. Although the modern sense of individual rights was not fully secured, Athenians enjoyed liberties within their city, free from external rule. The Athenian democracy, marked by its unique institutions and principles, represented a groundbreaking experiment in self-governance.

The Athenian democracy experienced periods of decline and revival throughout its two centuries of existence. Twice, during the crisis at the end of the Peloponnesian War, Athens voted against its democratic constitution, resulting in the establishment of oligarchic regimes. However, democracy was restored within a year both times. Subsequent reforms shifted law-making authority from the

Assembly to randomly selected juries known as "nomothetai." Despite restoration efforts, Athens faced political overshadowing by Hellenistic empires and eventual restriction to local administration under Roman conquest.

Internal criticisms from influential figures like Plato and Aristotle contributed to the decline of Athenian democracy. Their works praised the stability of Sparta's political system while portraying Athenian democracy as rule by the less well-born or the mob. It wasn't until George Grote's "A History of Greece" in the 19th century that modern thinkers began to view Periclean democracy positively. In the late 20th century, scholars re-evaluated Athenian democracy as a model for empowering citizens and as a post-modern example for communities and organizations.

2.4 Investments in constitutional democracy and the accumulation of constitutional democracy capital in Rome

The evolution of governance in Rome from a kingdom to a republic, and ultimately to an empire, underscores significant developments in constitutional democracy investments and the accumulation of constitutional democracy capital. This progression also highlights the dynamic nature of political systems and their impact on the concept of democracy.

Roman Kingdom to Republic

Initially governed by a monarchy, Rome's political landscape underwent a dramatic shift with the overthrow of its last king in 510 BCE. This pivotal moment led to the establishment of the Roman Republic, characterized by substantial investments in constitutional democracy. The creation of a new constitution by a group of aristocrats laid the foundational structure for more democratic governance. The conflict between the patricians (aristocrats) and plebeians (commoners) fueled demands for more inclusive rights, culminating in the drafting of the Twelve Tables in 451 BCE. These laws not only codified legal rights for all citizens but also gradually allowed plebeians to hold significant offices, such as the consulship, marking a crucial accumulation of constitutional democracy capital.

Structure and Stability of the Roman Republic

The Roman Republic featured a complex constitution that balanced powers among various political entities: two consuls who held supreme civil and military authority, a Senate that advised on foreign policy and civil administration, and assemblies that represented the populace with authority over elections, laws, and war declarations. This system, designed with checks and balances, fostered stability and prevented any single branch from dominating, reflecting Rome's core values of order, hierarchy, discipline, and obedience. These elements were instrumental in regulating both public and private lives, especially among the upper classes, further embedding democratic principles within the societal framework.

Challenges and Transformation into an Empire

As Rome expanded, it faced growing internal conflicts between the rich and the poor, exacerbated by land displacements and social unrest. The traditional republican constitution struggled to meet the needs of an expanding empire, leading to the rise of powerful military generals who eventually challenged and bypassed the Senate. The assassination of Julius Caesar in 44 BCE, following his temporary consolidation of power, triggered a succession of power struggles. These events underscored the limitations of the existing democratic structures to adapt to changing political dynamics.

Octavian, later known as Augustus, emerged victorious from these conflicts and, after his definitive triumph at the Battle of Actium in 31 BCE, transitioned Rome from a republic to an empire. While Republican institutions nominally remained, Octavian centralized power effectively, marking a significant shift from the accumulation of constitutional democracy capital to an imperial system centered around his personal authority. This period, though marked by the decline of the Republic's democratic elements, still retained vestiges of its earlier constitutional framework, which continued to influence governance.

Throughout its history, Rome made critical investments in constitutional democracy that facilitated the development and accumulation of democracy capital. From the establishment of the Republic through laws like the Twelve Tables to the complex interplay of powers within the republican system, these foundational elements contributed to the broader narrative of democracy. However, the transition to an empire illustrates the challenges faced by democratic systems in adapting to expansive and diverse political demands. The Roman experience with democracy, particularly through its Republic phase, remains a profound chapter in the history of constitutional democracy, offering enduring lessons for the evolution of democratic governance.

2.5 Investments in constitutional democracy and the accumulation of constitutional democracy capital in the Medieval Europe

During the Medieval period in Europe, spanning from approximately 500 CE to 1500 CE, the foundations of constitutional democracy as we understand it today were largely absent in the formal sense. However, there were significant developments and investments in practices and institutions that can be seen as precursors to modern constitutional democracy. These investments laid the groundwork for the evolution of democratic principles and the accumulation of what could be termed as constitutional democracy capital.

Feudalism and Early Democratic Practices

Feudalism dominated much of Medieval Europe, characterized by a hierarchical system where land was exchanged for service, often military. While this system is not democratic in the modern sense, it did foster a form of mutual obligation and limited power sharing between lords and vassals. This relationship necessitated negotiations and agreements, which sowed early seeds for the rule

of law and representative practices.

The Magna Carta

One of the most significant investments in the accumulation of constitutional democracy capital during this period was the signing of the Magna Carta in 1215. This document, forced upon King John of England by his barons, was crucial in the development of the modern legal and constitutional principles, notably the idea that the king's authority was not absolute but could be checked by other powers, an early form of the rule of law.

Rise of the Parliaments

The development of parliaments across Medieval Europe also marked a significant evolution in governance. For instance, the Simon de Montfort's Parliament in England in 1265 is considered one of the earliest forms of a more representative government, although initially limited to the nobility and clergy. These assemblies gradually included broader representation and were instrumental in shaping the governance structure that emphasized consultation and consent over unilateral rule by the sovereign.

Urban Communes and Merchant Guilds

In cities across Europe, especially within the burgeoning urban centers in Italy and the Hanseatic League in the north, merchant guilds and urban communes were making notable strides towards self-governance. These groups often drafted charters that allowed them some degree of autonomy from local lords or distant monarchs, fostering a civic culture that valued collective decision-making and economic freedom.

The Church's Role

The medieval Church also played a complex role in this development. On one hand, it often reinforced the divine right of kings, yet on the other, it sometimes acted as a counterbalance to the power of monarchies. The Church's canonical laws and courts introduced a form of legal consistency and intellectual debate that questioned and occasionally curbed secular authority.

Educational and Intellectual Revival

The revival of learning, particularly with the re-discovery and study of Roman and Greek classics during the late medieval period, spurred the intellectual underpinnings that questioned existing political and philosophical norms. Universities became centers for debate and discussion, which propagated ideas that were fundamental to democratic thought in later centuries.

These medieval developments represent early investments in what would become constitutional democracy capital. They contributed to a slow accumulation of democratic practices and principles that would later be critical in the development of modern democratic systems during the Enlightenment and beyond. The legacy of these institutions, ideas, and documents formed the bedrock upon which later more formalized constitutional democracies were built.

2.6 Investments in constitutional democracy and the accumulation of constitutional democracy capital during the Renaissance and Reformation:

During the Renaissance and Reformation from the 14th to the 17th centuries, there was significant advancement in the principles underlying constitutional democracy. This era, marked by a profound intellectual and cultural revival, played a crucial role in challenging entrenched authoritarian regimes and laying the groundwork for modern democratic governance. The period was characterized by a flourishing of humanistic values and a new emphasis on individual rights, which were pivotal in questioning and reshaping political structures.

The Italian city-states and the Dutch Republic were at the forefront of these changes, serving as experimental grounds for emerging republican ideas and enhanced civic engagement. These regions witnessed the development of institutions that fostered greater public participation and accountability, establishing early forms of the rule of law. This environment encouraged a reevaluation of the individual's role within the state, promoting ideas of self-governance and collective decision-making that were critical to the evolution of constitutional democracy.

These movements not only influenced political thought but also laid a foundational framework for the subsequent development of constitutional systems that prioritize the rule of law, individual rights, and democratic governance. The intellectual resurgence of this period, with its emphasis on revisiting classical antiquity, further enriched the dialogue around governance and personal freedom, setting the stage for the later institutionalization of these concepts into constitutional democracies.

2.7 Investments in constitutional democracy and the accumulation of constitutional democracy capital during the Glorious Revolution in the late 17th century and the Enlightenment:

During the periods of the Glorious Revolution in the late 17th century and the Enlightenment that followed, significant investments were made in constitutional democracy, contributing substantially to the accumulation of constitutional democracy capital. These investments were pivotal in shaping modern governance systems and broadening the scope of democratic principles.

The Glorious Revolution of 1688 in England was particularly instrumental in this process. It marked a profound shift in the balance of power between the monarchy and Parliament, leading to the establishment of a constitutional monarchy under William and Mary. This transition was underpinned by the Bill of Rights in 1689, which laid down limits on the powers of the crown and affirmed certain rights of the people, including regular parliaments and free elections. This period demonstrated a successful challenge to absolute monarchy and reinforced the importance of the rule of law and the rights of citizens within a constitutional framework, establishing precedents for democratic governance and civil liberties.

The Enlightenment further propelled the development of constitutional democracy by spreading and deepening the philosophical foundations necessary for democratic governance. Philosophers like John Locke, Montesquieu, Voltaire, and Jean-Jacques Rousseau debated and disseminated ideas concerning human rights, individual liberty, separation of powers, and the social contract. Their writings inspired a rethinking of traditional governance models and emphasized the role of reason and empirical evidence in shaping political systems. Locke's theories of government by consent and the right of rebellion against tyrants, in particular, became fundamental to Western political thought, influencing both the American and French Revolutions.

The Enlightenment also fostered an environment where ideas could be freely exchanged and critiqued, contributing to the development of a public sphere and civil society that are critical components of democratic systems. The period saw the emergence of salons, coffeehouses, and print culture, which facilitated widespread discussion of political, philosophical, and scientific ideas, thereby educating the populace and encouraging political participation.

These developments during the Glorious Revolution and the Enlightenment were critical in accumulating constitutional democracy capital by establishing legal and cultural norms that prioritized human rights, separation of powers, and the rule of law. These advances set the stage for the later democratic revolutions of the 18th and 19th centuries, providing a durable framework for the expansion of democratic institutions globally.

The Enlightenment was a watershed era for democratic theory, fueled by the philosophies of thinkers like John Locke, Montesquieu, and Jean-Jacques Rousseau. Their advocacy for popular sovereignty, the separation of powers, and natural rights laid the intellectual foundations that would inspire seminal events such as the American and French Revolutions, setting the stage for the emergence of modern constitutional democracies.

2.8 Investments in constitutional democracy and the accumulation of constitutional democracy capital during the American Revolution

During the American Revolution, significant strides were made in the investments in constitutional democracy and the accumulation of constitutional democracy capital. This pivotal period in history not only challenged the existing colonial governance imposed by the British monarchy but also laid the groundwork for the establishment of a system grounded in democratic governance and constitutional law, which would profoundly influence the development of democracies worldwide.

The American Revolution was driven by principles that would later define constitutional democracy, such as representation, rights, and the rule of law. Central to this was the rejection of tyranny under British rule, encapsulated in the slogan "no taxation without representation," which protested the lack of American seats in the British Parliament. This resistance to British policies catalyzed the philosophical and practical need to form a government that was accountable to its

citizens and protective of their liberties.

The culmination of revolutionary thought and action was the drafting of foundational documents that became the bedrock of American democracy. The Declaration of Independence in 1776, penned by Thomas Jefferson, articulated the colonists' rights to freedom and self-governance and declared the colonies free from British rule. This document was profoundly influenced by Enlightenment ideals and underscored the universal rights to life, liberty, and the pursuit of happiness.

Following independence, the drafting of the Constitution in 1787 and its subsequent ratification laid a structural foundation for the new nation. The Constitution established a system of government featuring separation of powers among the executive, legislative, and judicial branches, checks and balances to prevent the abuse of power, and amendments that guaranteed individual rights. The first ten amendments, known as the Bill of Rights, were pivotal in ensuring the protection of fundamental rights such as freedom of speech, assembly, and religion, all critical components of constitutional democracy.

The American Revolution's investments in constitutional democracy did not only reshape governance in the United States but also had a wide-reaching impact, inspiring subsequent democratic movements and constitutional developments globally. The principles, policies, and institutions established during this era contributed significantly to the global accumulation of constitutional democracy capital, emphasizing the importance of government by the people and for the people, and laying a template for future democracies.

2.9 Investments in constitutional democracy and the accumulation of constitutional democracy capital during the French Revolution

The French Revolution, which erupted in 1789, was a monumental period for the advancement of constitutional democracy and the substantial accumulation of constitutional democracy capital. This epochal event radically transformed France's political landscape and had far-reaching implications that influenced the evolution of modern democracies around the world.

At the core of the French Revolution was the challenge to the ancient régime and its inequitable societal structure, characterized by the absolute rule of the monarchy, the aristocracy's privileges, and the systemic exclusion of the common people from political power. The revolution was ignited by profound dissatisfaction with severe economic hardships, social inequalities, and a desire for political reform. These catalysts propelled the populace to demand a system of governance that represented their interests and safeguarded their rights.

The Revolution's early phase saw the drafting of the Declaration of the Rights of Man and of the Citizen in August 1789, a foundational document that articulated fundamental rights and freedoms, including liberty, property, security, and resistance to oppression. This declaration was inspired by the Enlightenment ideals and mirrored the American Declaration of Independence,

emphasizing equality before the law, the sovereignty of the people, and the principles of national citizenship and inalienable rights.

As the Revolution progressed, France experimented with various forms of government, from constitutional monarchy to republic, and eventually to the radical phase led by the Jacobins. Each phase attempted to redefine the role of the state and its relationship with its citizens. The constitution of 1791 established a constitutional monarchy, which was soon followed by the first French Republic in 1792. These shifts were accompanied by significant institutional reforms aimed at democratizing the state apparatus and increasing public participation through newly established assemblies and a reformed legal system.

The tumultuous years of the Revolution also saw the implementation of wide-ranging policies to eradicate feudal privileges, reform the administrative system, and secularize society, all of which contributed to the dismantling of the old hierarchical structures and fostered a new political culture centered on republican values.

The enduring impact of the French Revolution on constitutional democracy was profound. It highlighted the importance of establishing a legal framework that guarantees equality and individual rights, influences that are embedded in modern democratic constitutions across the globe. The principles espoused and the reforms initiated during the French Revolution laid down critical components of what would become the modern notion of constitutional democracy, promoting ideas that would resonate in subsequent democratic movements and uprisings around the world.

In conclusion, the French Revolution was pivotal in the accumulation of constitutional democracy capital by profoundly challenging the old order and advocating for a political and social transformation that emphasized democratic governance, the rule of law, and civic rights, setting the stage for the modern democratic state.

2.10 Investments in constitutional democracy and the accumulation of constitutional democracy capital during the Industrial Revolution and Democratic Reform (18th - 19th centuries):

During the Industrial Revolution and Democratic Reform period of the 18th and 19th centuries, significant investments were made in constitutional democracy and the accumulation of constitutional democracy capital, driven by the profound societal and economic transformations of the era. The rapid industrialization brought about sweeping changes to the economic landscape and living conditions, which, in turn, sparked widespread demands for political reform and an expansion of democratic rights.

This period saw several pivotal developments that advanced the cause of democracy. Notably, the British Reform Acts were passed, progressively extending the electoral franchise to larger segments of the population. These Acts began with the Reform Act of 1832, which addressed gross

disparities in representation and reduced the property qualifications for voting, thereby enfranchising a broader portion of the middle class. Subsequent Acts continued this trend, gradually including more working-class men and eventually leading to near-universal male suffrage by the end of the century.

Simultaneously, the abolition of slavery across various nations redefined fundamental concepts of liberty and human rights. This monumental change not only freed millions from bondage but also laid a moral foundation that challenged other forms of social and political oppression, fostering movements that advocated for equality and justice within democratic frameworks.

Labor movements also gained momentum during this time, as workers sought to improve their often dire working conditions, secure fair wages, and gain the right to unionize. These movements not only pressed for economic rights but also championed broader democratic principles, such as the right to assemble, the right to representation, and the pursuit of policies beneficial to the working classes.

These changes were crucial in realigning constitutional frameworks with the needs of rapidly industrializing societies. They expanded democratic rights and representation, ensuring that new segments of the population could participate in the political process. This era of reform fundamentally transformed the political landscape by making governments more representative and responsive to their citizens' needs, thereby significantly enhancing the stock of constitutional democracy capital.

Overall, the Industrial Revolution and Democratic Reform period was marked by a dynamic interplay between economic development and political empowerment. The advancements in democratic governance that occurred during these centuries were instrumental in shaping modern democratic institutions, reflecting an evolving understanding of rights and responsibilities that continues to influence contemporary constitutional democracies.

2.11 Investments in constitutional democracy and the accumulation of constitutional democracy capital in the 20th century:

The 20th century witnessed significant global movements aimed at advocating for democracy and self-determination, marking substantial investments in constitutional democracy and the accumulation of constitutional democracy capital worldwide. This era was characterized by vigorous efforts toward decolonization, where numerous countries sought independence from colonial powers, aspiring to form sovereign nations governed by principles of self-rule and democracy. These movements often transitioned into robust nation-building exercises that embedded democratic frameworks into newly formed constitutions.

Additionally, the century was pivotal for the advancement of civil rights in the United States, where sustained campaigns against racial segregation and discrimination culminated in legislative

milestones like the Civil Rights Act and the Voting Rights Act. Globally, the push for women's suffrage gained momentum, resulting in women achieving the right to vote and participate in politics in numerous countries, dramatically transforming political landscapes by making them more inclusive.

The push for indigenous rights also became more pronounced, with indigenous populations advocating for recognition, autonomy, and rights to their ancestral lands, influencing national policies and contributing to the dialogue on human rights and equality.

Furthermore, the establishment of international frameworks and institutions such as the United Nations and the adoption of the Universal Declaration of Human Rights in 1948 significantly bolstered global governance models. These developments laid a foundation for international law and norms that supported democratic governance, human rights, and dignity, promoting a more inclusive and equitable global order. These efforts not only supported the formal structures of democracy but also emphasized the importance of universal human rights as integral components of democratic governance.

These movements and developments throughout the 20th century played crucial roles in reinforcing and spreading democratic ideals globally. They contributed to a broader understanding and implementation of democracy, influencing numerous societies to adopt democratic principles and integrate them into their political and legal systems, thereby enhancing global constitutional democracy capital. This period's legacy continues to impact contemporary global politics, underscoring the ongoing importance of democracy and human rights in international relations and domestic policies.

2.12 Investments in constitutional democracy and the accumulation of constitutional democracy capital in the 21st century:

The 21st century has been marked by significant technological advancements that have revolutionized democratic participation and the accumulation of constitutional democracy capital. The advent of the internet, social media, and mobile connectivity has dramatically transformed how citizens engage with each other and their governments, enhancing the public sphere by enabling faster communication, greater information dissemination, and more robust citizen engagement. These digital platforms have empowered individuals to organize, mobilize, and demand accountability and transparency from their governments, effectively broadening participation in democratic processes.

However, while technology has facilitated greater inclusivity and participation, it has also introduced complex challenges that threaten the integrity of democratic systems. Issues such as privacy concerns, the spread of misinformation, and the manipulation of public opinion through digital means have emerged as significant threats to democratic health. These challenges underscore the

necessity for continuous investments in regulatory frameworks that can adapt to the rapidly changing technological landscape. Additionally, there is a pressing need for educational programs that can enhance digital literacy and inform citizens about the critical evaluation of online information and the responsible use of technology in democratic engagement.

As we navigate these developments, the ongoing investments in constitutional democracy capital remain crucial. These efforts reflect a global and historical commitment to fostering justice, equality, and democratic governance. By continuously adapting and reinforcing the foundations of democratic systems to meet new challenges, societies can ensure that the advancements of the 21st century serve to enhance, rather than undermine, the democratic ideals that have shaped human progress throughout history. This enduring pursuit of political freedom, civil liberties, and human rights highlights the essential role of democracy.

3 The dynamic model

It has become routine in macroeconomic growth models to talk about representative agents. In the context of our following macro model on endogenous constitutional democracy capital and physical capital accumulation, the concept of "representative citizens" or agents embodies those individuals throughout history who have significantly contributed to the establishment and development of democratic systems. These representative citizens are not just historical figures like Solon, the Brutus brothers, William of Orange, Mary II, John Churchill, Francis Russell, Henry Compton, Washington, Jefferson, Franklin, Adams, Madison, and Hamilton, but also the common people whose collective actions and desires have driven democratic movements.

These individuals and groups played pivotal roles at various key moments in history, such as during Ancient Athens' democratic reforms, the Roman Republic's governance, the Glorious Revolution, and American Independence. Their actions, whether through leadership or mass participation, have been crucial in shaping the paths toward democracy. These representative agents in a model might encapsulate both the elite architects of political frameworks and the broader public whose engagement and support are essential for the sustainability and evolution of democratic systems. In the historical evolution of democratic systems, representative citizens encompass both influential figures and the general populace who have propelled democratic reforms. For example, in Athenian Democracy, Solon and Cleisthenes were crucial for setting reforms that enabled citizen engagement in governance, particularly among free males who actively participated in assemblies and voted on laws. In the Roman Republic, figures like Lucius Junius Brutus were pivotal in the transition from monarchy to a republic that featured a balance of power between the aristocratic Senate and the popular assemblies. The Glorious Revolution in England saw Parliament and figures like William of Orange play significant roles in shifting power dynamics towards a constitutional monarchy favoring parliamentary authority. Similarly, American Independence was shaped

by leaders like George Washington, Thomas Jefferson, John Adams, and James Madison, alongside a broad base of colonists who engaged in democratic processes, contributing to the establishment of a constitutionally framed government. These examples highlight the dual role of key figures and the broader populace in advancing and sustaining democratic reforms.

In particular, common citizens have historically played essential roles in shaping and supporting democratic movements across various epochs. In Athenian democracy, not just the notable figures like Solon and Cleisthenes were crucial, but also the general populace of Athens, particularly the free male citizens who actively participated in assemblies, voting on legislative and executive matters. Similarly, in the Roman Republic, commoners such as farmers, artisans, and soldiers engaged in assemblies that elected magistrates and passed laws, giving them a direct voice in governance. During the Glorious Revolution, the broader English public, though more passively, supported changes that restored Protestant and parliamentary governance, essential for the revolution's legitimacy and stability. In the American Revolution, common individuals like farmers, shopkeepers, and laborers played critical roles in supporting the war effort and ratifying the Constitution, participating actively in local assemblies and conventions. This widespread engagement and sometimes demand for accountability from common people were pivotal in initiating democratic reforms and ensuring the ongoing democratization process, providing the necessary pressure and legitimacy for the sustainability of democratic institutions. Abraham Lincoln's famous phrase "government of the people, by the people, for the people" from the Gettysburg Address encapsulates the ideal of democratic governance. This concept suggests that in a democracy, the citizens themselves are the ultimate source of governmental power and authority. The government is established to serve and represent the people's interests, deriving its legitimacy and direction from the collective will of its citizens. Thus, every citizen not only has a stake but also a role in the functioning of their government, emphasizing the responsibility and influence each individual holds in shaping public policy and governance. This notion reinforces the idea that democratic governance is participatory and inclusive, contrasting sharply with autocratic systems where power is held by a few. This is why in this paper we will treat democratic government itself just as the representative citizens. Keeping all this in mind, let us proceed to construct our model.

In a typical constitutional democracy, representative citizens or agents build and maintain constitutional democracy capital, denoted as P , through consistent democratic engagement and investments, represented by p . Concurrently, they also accumulate physical production capital, symbolized as k . The dynamics of this process are captured by the following equations, which describe the accumulation paths for both constitutional democracy capital and physical capital:

$$\frac{dP}{dt} = p - \delta_P P, \tag{1}$$

$$\frac{dk}{dt} = f(k, p, P) - c - \beta p - \delta_k k, \quad (2)$$

where δ_P is democratic erosion rate or democracy degradation rate;² δ_k is the depreciation rate of physical production capital; p is new investments in constitutional democracy capital, and β is their price; $f(k, p, P)$ is the production function; c is consumption; and the initial stocks of the constitutional democracy capital and physical production capital are give by $P(0) = P_0$ and $k(0) = k_0$, respectively.

It is important to emphasize here that investments in constitutional-democracy capital, denoted as p , encompass a variety of efforts, resources, and initiatives aimed at reinforcing democratic principles, institutions, and practices within a society. This involves establishing and maintaining robust legal and institutional frameworks, such as constitutions, judicial systems, and independent electoral commissions, which uphold fundamental rights, the separation of powers, and the rule of law. It also includes fostering the development of representative institutions like legislatures and courts that facilitate citizen participation and accountability. Additionally, significant investments are made in civic education and public awareness to enhance civic literacy and political engagement, alongside strengthening civil society organizations and grassroots activism that support democratic values and social justice. Moreover, ensuring media freedom, promoting access to diverse information, and safeguarding editorial independence are crucial for a vibrant media landscape that fosters transparency and government accountability. Strengthening judicial independence and the rule of law is also vital for protecting human rights and ensuring fair justice systems. Furthermore, engaging in international cooperation and diplomacy helps promote democratic governance globally, supporting democratic transitions and capacity-building in regions facing political challenges. These comprehensive investments require sustained commitment and political will from governments, civil society, international bodies, and citizens to effectively enhance democratic institutions, uphold legal standards, and advance fundamental democratic principles like freedom, equality, and justice for all.

The cost of initiating new investments in constitutional democracy, represented by β , can be exceedingly high, particularly in authoritarian and totalitarian regimes where the stakes for such democratic endeavors are significantly elevated. In these contexts, the price of striving for democratic change often transcends financial and political challenges and may entail profound human sacrifices. The pursuit of establishing constitutional democracy under such regimes might necessitate large-scale social movements, enduring struggles against oppressive governmental forces, and, in extreme cases, revolution. This arduous process frequently involves risking lives and liveli-

²The term "democratic erosion rate" particularly highlights the gradual loss of democratic quality and the systemic weakening of institutions due to various detrimental actions and events. Similarly, the "democracy degradation rate" underscores the deterioration and decline in the overall health and effectiveness of a democracy. The term "democratic erosion rate" particularly highlights the gradual loss of democratic quality and the systemic weakening of institutions due to various detrimental actions and events. Similarly, the "democracy degradation rate" underscores the deterioration and decline in the overall health and effectiveness of a democracy.

hoods as citizens and reformers confront entrenched power structures in their fight to dismantle dictatorships and institute democratic governance. The historical and ongoing examples globally underscore that the transition towards constitutional democracy in such restrictive environments is not only a matter of political and economic investment but also a profound commitment to the ideals of freedom and justice, often paid for with the ultimate sacrifices of those who dare to challenge autocracy.

In our model, the erosion and degradation rate of constitutional democracy capital, denoted as δ_P , encompasses all factors that undermine or erode existing democratic structures. This includes actions that actively sabotage democracy, such as the overthrow of democratic systems, reinstatement of authoritarian or totalitarian regimes through rigged elections, power grabs, or military coups. When δ_P is high, it indicates a rapid degradation or loss of democratic capital. Essentially, a high δ_P value signifies that the mechanisms and events detrimental to democracy are occurring at a rate that significantly diminishes the strength and stability of constitutional democratic governance. In such scenarios, constitutional democracy is at risk of collapsing swiftly as the foundational democratic principles and structures are quickly undermined or destroyed, paving the way for non-democratic regimes to take hold and potentially reverse democratic gains. This parameter is crucial in understanding the vulnerability of democratic institutions to internal and external threats and highlights the need for vigilant maintenance and reinforcement of democratic systems to counteract these depreciative forces.

Representative citizens or agents maximize their discounted utility at a subjective time discount rate, $\rho > 0$

$$\max_{\{c,p,P,k\}} \int_0^\infty u(c,p,P)e^{-\rho t} dt, \quad (3)$$

subject to

$$\frac{dP}{dt} = p - \delta_P P, \quad (4)$$

$$\frac{dk}{dt} = f(k,p,P) - c - \beta p - \delta_k k, \quad (5)$$

for initially given $P(0) = P_0$ and $k(0) = k_0$. The utility and production functions described integrate elements from neoclassical consumption-production theory and new institutional economics to provide a comprehensive framework for examining constitutional democracies through economic and institutional lenses. These models incorporate variables such as physical consumption, investments in new constitutional democracy, the accumulation of constitutional democracy capital, and physical capital, demonstrating how investments in democratic processes and capital significantly enhance both citizen welfare and overall production. This synthesis highlights the vital interaction

between economic factors and institutional structures in a democracy, showing how democratic governance boosts economic performance and individual well-being.

Furthermore, by emphasizing the role of democratic engagement in promoting economic and societal prosperity, these functions resonate with Hayek's (1960) concept of the "creative powers of a free civilization." According to Hayek, freedom is essential for both personal and communal advancement, suggesting that a constitutional democracy, which nurtures individual freedoms and democratic participation, is key to enhancing personal welfare and economic health. Overall, these revised utility and production functions encapsulate how constitutional democracies foster conditions conducive to creativity and economic vitality, aligning with Hayek's views on the importance of the rule of law, property rights, and liberties in an open, free society.

Finally, institutions of constitutional democracy play a critical role in creating efficient markets with low monitoring and transaction costs. These institutions, which include the rule of law, transparent and accountable governance structures, and protection of property rights, are foundational for reducing uncertainties and risks associated with economic transactions. Here's how they contribute to market efficiency:

1. **Rule of Law:** The rule of law ensures that all market participants are subject to the same legal standards, reducing the risk of arbitrary changes in business conditions and providing a stable environment for transactions. This stability lowers the costs of monitoring and enforcing contracts.

2. **Transparent and Accountable Governance:** Institutions that promote transparency and accountability help reduce corruption and misuse of resources. When government actions are predictable and officials are held accountable, businesses and consumers face fewer unexpected obstacles, thus reducing transaction costs associated with navigating opaque regulatory environments.

3. **Protection of Property Rights:** Secure property rights are essential for efficient markets. They give individuals and businesses the confidence to invest, knowing that their investments are safe from expropriation or theft. This security reduces the need for extensive monitoring to protect assets, thereby lowering transaction costs.

At the same time, institutions of constitutional democracy evolve and become more efficient over time through various mechanisms:

1. **Legal Reforms:** As markets evolve, legal systems may be reformed to better suit the new economic realities, such as updating copyright laws for digital content or modifying regulations to accommodate new business models. These reforms can reduce the cost of compliance and enforcement, making markets more efficient.

2. **Technological Advancements:** Technology can drastically reduce transaction and monitoring costs. For example, online platforms can make government services more accessible and transparent, reducing bureaucracy and corruption. Digital land registries can enhance the security of

property rights, making property transactions simpler and less costly.

3. Civic Engagement and Political Participation: As citizens become more engaged, they demand better performance from their governments. This pressure can lead to more efficient governance as political leaders reform institutions to meet the expectations of their constituents.

4. International Standards and Practices: Globalization has led many countries to adopt international standards and best practices in governance and legal frameworks. This harmonization can facilitate international trade and investment, reduce barriers to entry, and lower transaction costs by creating a more predictable and consistent business environment across borders.

5. Feedback and Adaptation: Constitutional democracies often have built-in mechanisms for feedback and adaptation, such as periodic elections, judicial reviews, and legislative debates. These mechanisms allow for the constant updating and refining of institutions to better serve their economic and social purposes.

Thus, the institutions of constitutional democracy are not only required to create efficient markets but also need to continuously adapt and improve to maintain low transaction and monitoring costs in a changing economic landscape.

From all these perspectives stated above, it is obvious to assume that $\frac{\partial f}{\partial p} > 0$, $\frac{\partial f}{\partial P} > 0$, $\frac{\partial f}{\partial k} > 0$, $\frac{\partial u}{\partial p} > 0$, $\frac{\partial u}{\partial P} > 0$, $\frac{\partial u}{\partial c} > 0$, $\frac{\partial^2 f}{\partial p^2} < 0$, $\frac{\partial^2 f}{\partial P^2} < 0$, $\frac{\partial^2 f}{\partial k^2} < 0$, $\frac{\partial^2 u}{\partial p^2} < 0$, $\frac{\partial^2 u}{\partial P^2} < 0$, $\frac{\partial^2 u}{\partial c^2} < 0$.

The current-value Hamiltonian is:

$$\mathcal{H} = u(c, p, P) + m_1(p - \delta_P P) + m_2(f(k, p, P) - c - \beta p - \delta_k k). \quad (6)$$

The optimality conditions with respect to control variables c and p are given as:

$$\frac{\partial \mathcal{H}}{\partial c} = u_c(c, p, P) - m_2 = 0, \quad (7)$$

$$\frac{\partial \mathcal{H}}{\partial p} = u_p(c, p, P) + m_1 + m_2(f_p(k, p, P) - \beta) = 0. \quad (8)$$

The Euler equations are:

$$\frac{\partial \mathcal{H}}{\partial P} = \rho m_1 - \dot{m}_1 = u_P(c, p, P) - m_1 \delta_P + m_2 f_P, \quad (9)$$

$$\frac{\partial \mathcal{H}}{\partial k} = \rho m_2 - \dot{m}_2 = m_2(f_k(k, p, P) - \delta_k), \quad (10)$$

and the state equations are:

$$\frac{dP}{dt} = p - \delta_P P, \quad (11)$$

$$\frac{dk}{dt} = f(k, p, P) - c - \beta p - \delta_k k. \quad (12)$$

Therefore,

$$m_2 = u_c(c, p, P), \quad (13)$$

$$m_1 = -u_p(c, p, P) - m_2(f_p(k, p, P) - \beta). \quad (14)$$

Differentiating with respect to time t yields:

$$\dot{m}_2 = u_{cc} \cdot \dot{c} + u_{cp} \cdot \dot{p} + u_{cP} \cdot \dot{P}, \quad (15)$$

$$\begin{aligned} \dot{m}_1 = & [-u_{pc} - u_{cc}(f_p - \beta)]\dot{c} + [-u_{pp} - u_{cp}(f_p - \beta) - u_c f_{pp}]\dot{p} \\ & + [-u_{pP} - u_{cP}(f_p - \beta) - u_c f_{pP}]\dot{P} - u_c f_{pk}\dot{k}. \end{aligned} \quad (16)$$

Substituting out the costate variables m_1 and m_2 in the Euler equations leads to:

$$\begin{aligned} u_P + [u_p + u_c(f_p - \beta)]\delta_P + u_c f_P = & -\rho[u_p + u_c(f_p - \beta)] + \left[[u_{pc} + u_{cc}(f_p - \beta)]\dot{c} \right. \\ & + [u_{pp} + u_{cp}(f_p - \beta) + u_c f_{pp}]\dot{p} \\ & \left. + [u_{pP} + u_{cP}(f_p - \beta) + u_c f_{pP}]\dot{P} + u_c f_{pk}\dot{k} \right], \end{aligned} \quad (17)$$

and

$$u_c(f_k - \delta_k) = \rho u_c - [u_{cc} \cdot \dot{c} + u_{cp} \cdot \dot{p} + u_{cP} \cdot \dot{P}]. \quad (18)$$

Thus, the equilibrium dynamic system can be summarized as (17), (18) and

$$\begin{aligned} \dot{P} &= p - \delta_P P, \\ \dot{k} &= f(k, p, P) - c - \beta p - \delta_k k. \end{aligned}$$

If we assume that the utility function is separable of the form:

$$u(c, p, P) = \mathcal{U}(c) + \mathcal{V}(p) + \mathcal{W}(P),$$

where $\mathcal{U}'(c) > 0$, $\mathcal{U}''(c) > 0$; $\mathcal{V}'(p) > 0$, $\mathcal{V}''(p) > 0$; $\mathcal{W}'(P) > 0$, and $\mathcal{W}''(P) > 0$, the cross derivatives of the utility function $u(c, p, P)$ will all equal zero, i.e., $u_{cp} = u_{pc} = 0$, $u_{cP} = u_{Pc} = 0$, $u_{pP} = u_{Pp} = 0$. Accordingly, the equilibrium dynamic system can be simplified as:

$$u_P + [u_p + u_c(f_p - \beta)]\delta_P + u_c f_P = -\rho[u_p + u_c(f_p - \beta)] + [u_{cc}(f_p - \beta)]\dot{c} + [u_{pp} + u_c f_{pp}]\dot{p} + u_c f_{pP}\dot{P} + u_c f_{pk}\dot{k}, \quad (19)$$

$$u_{cc} \cdot \dot{c} = \rho u_c - u_c(f_k - \delta_k), \quad (20)$$

$$\dot{P} = p - \delta_P P, \quad (21)$$

$$\dot{k} = f(k, p, P) - c - \beta p - \delta_k k. \quad (22)$$

Substituting equations (20) and (21) into equation (19) to replace $\dot{c}$, $\dot{P}$ and $\dot{k}$, we have:

$$\dot{p} = \frac{1}{u_{pp} + u_c f_{pp}} \left[u_c [(f_p - \beta)(f_k - \delta_k + \delta_P) + f_P - f_{pP}(p - \delta_P P) - f_{pk}(f(k, p, P) - c - \beta p - \delta_k k)] + u_P + u_p \delta_P \right], \quad (23)$$

$$\dot{c} = \frac{u_c}{u_{cc}} (\rho - (f_k - \delta_k)), \quad (24)$$

$$\dot{P} = p - \delta_P P, \quad (25)$$

$$\dot{k} = f(k, p, P) - c - \beta p - \delta_k k. \quad (26)$$

To examine the stability of the steady state, we linearize the system around the steady state $\{p^*, c^*, P^*, k^*\}$:

$$\begin{bmatrix} \dot{p} \\ \dot{c} \\ \dot{P} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} J_{11}^* & J_{12}^* & J_{13}^* & J_{14}^* \\ -\frac{u_c^*}{u_{cc}^*} f_{kp}^* & J_{22}^* & -\frac{u_c^*}{u_{cc}^*} f_{kP}^* & -\frac{u_c^*}{u_{cc}^*} f_{kk}^* \\ 1 & 0 & -\delta_P & 0 \\ f_p^* - \beta & -1 & f_P^* & f_k^* - \delta_k \end{bmatrix} \begin{bmatrix} p - p^* \\ c - c^* \\ P - P^* \\ k - k^* \end{bmatrix} \quad (27)$$

where

$$J_{11}^* = -\frac{u_{ppp}^* + u_c^* f_{ppp}^*}{u_{pp}^* + u_c^* f_{pp}^*} \dot{p} + \frac{1}{u_{pp}^* + u_c^* f_{pp}^*} \left[u_c^* [f_{pp}^* (f_k^* - \delta_k + \delta_P) + f_{kp}^* (f_p^* - \beta) + f_{Pp}^* - f_{pP}^* (p^* - \delta_P P^*) - f_{pP}^* - f_{pkp}^* (f^* - c^* - \beta p^* - \delta_k k^*) - f_{pk}^* (f_p^* - \beta)] + u_{pp}^* \delta_P \right], \quad (28)$$

$$J_{12}^* = -\frac{u_{cc}^* f_{pp}^*}{u_{pp}^* + u_c^* f_{pp}^*} \dot{p} + \frac{1}{u_{pp}^* + u_c^* f_{pp}^*} \left[u_{cc}^* [(f_p^* - \beta)(f_k^* - \delta_k + \delta_P) + f_P^* - f_{pP}^* (p^* - \delta_P P^*) - f_{pk}^* (f^* - c^* - \beta p^* - \delta_k k^*)] + u_c^* f_{pk}^* \right], \quad (29)$$

$$J_{13}^* = -\frac{u_c^* f_{ppP}^*}{u_{pp}^* + u_c^* f_{pp}^*} \dot{p} + \frac{1}{u_{pp}^* + u_c^* f_{pp}^*} \left[u_c^* [f_{pP}^* (f_k^* - \delta_k + \delta_P) + f_{kp}^* (f_p^* - \beta) + f_{PP}^* - f_{pPP}^* (p^* - \delta_P P^*) + f_{pP}^* \delta_P - f_{pkp}^* (f^* - c^* - \beta p^* - \delta_k k^*) - f_{pk}^* f_P^*] + u_{PP}^* \right], \quad (30)$$

$$J_{14}^* = -\frac{u_c^* f_{ppk}^*}{u_{pp}^* + u_c^* f_{pp}^*} \dot{p} + \frac{1}{u_{pp}^* + u_c^* f_{pp}^*} \left[u_c^* [f_{pk}^* (f_k^* - \delta_k + \delta_P) + f_{kk}^* (f_p^* - \beta) + f_{Pk}^* - f_{pPk}^* (p^* - \delta_P P^*) - f_{pkk}^* (f^* - c^* - \beta p^* - \delta_k k^*) - f_{pk}^* (f_k^* - \delta_k)] \right], \quad (31)$$

$$J_{22}^* = \frac{(u_{cc}^*)^2 - u_c^* \cdot u_{ccc}^*}{(u_{cc}^*)^2} [\rho - (f_k^* - \delta_k)]. \quad (32)$$

Since this dynamic system has two pre-determined state variables, the steady-state equilibrium is locally saddle-point stable if and only if the Jacobian matrix has both negative eigenvalues and positive eigenvalues.

4 A special case with a logarithmic utility function and a Cobb-Douglas production function

For simplicity, let the representative citizens have a separable logarithmic utility function:

$$u(c, p, P) = \log(c) + \chi_p \log(p) + \chi_P \log(P). \quad (33)$$

In addition, let the representative citizens' production function take the Cobb-Douglas form:

$$f(k, p, P) = A(k^{\alpha_k} p^{\alpha_p} P^{1-\alpha_k-\alpha_p})^\zeta, \quad (34)$$

where $\bar{A}$ represents the level of technological productivity. In the steady state, $\dot{c} = \dot{p} = \dot{P} = \dot{k} = 0$, namely:

$$0 = u_c[(f_p - \beta)(f_k - \delta_k + \delta_P) + f_P - f_{pP}(p - \delta_P P) - f_{pk}(f(k, p, P) - c - \beta p - \delta_k k)] + u_P + u_p \delta_P, \quad (35)$$

$$0 = \rho - (f_k - \delta_k), \quad (36)$$

$$0 = p - \delta_P P, \quad (37)$$

$$0 = f(k, p, P) - c - \beta p - \delta_k k. \quad (38)$$

Plugging into equation (33) and (34) yields a system for the determination of $\{c^*, p^*, P^*, k^*, y^*\}$,

$$0 = \frac{1}{c^*} \left[(\zeta \alpha_p \frac{y^*}{p^*} - \beta)(\zeta \alpha_k \frac{y^*}{k^*} - \delta_k + \delta_P) + \zeta(1 - \alpha_k - \alpha_p) \frac{y^*}{P^*} \right] + \frac{\chi_P}{P^*} + \frac{\chi_p}{p^*} \delta_P, \quad (39)$$

$$0 = \rho - \zeta \alpha_k \frac{y^*}{k^*} + \delta_k, \quad (40)$$

$$0 = p^* - \delta_P P^*, \quad (41)$$

$$0 = y^* - c^* - \beta p^* - \delta_k k^*, \quad (42)$$

$$y^* = A \left((k^*)^{\alpha_k} (p^*)^{\alpha_p} (P^*)^{1-\alpha_k-\alpha_p} \right)^\zeta. \quad (43)$$

Now, we solve $\{c^*, p^*, P^*, k^*, y^*\}$, as a function of parameters $\{\rho, \beta, \delta_P, \delta_k, \chi_p, \chi_P, \alpha_k, \alpha_p, \zeta, A\}$, respectively.

4.1 Solving for the long-run steady state

First of all, we aim to reduce the equation system into a single equation for P^* . Equation (40) implies:

$$y^* = \frac{\rho + \delta_k}{\zeta \alpha_k} k^*. \quad (44)$$

From equation (41),

$$p^* = \delta_P P^*. \quad (45)$$

Substituting equations (44) and (45) into equation (43) yields:

$$\frac{\rho + \delta_k}{\zeta \alpha_k} k^* = A \left((k^*)^{\alpha_k} (\delta_P P^*)^{\alpha_p} (P^*)^{1-\alpha_k-\alpha_p} \right)^\zeta, \quad (46)$$

that is,

$$\frac{\rho + \delta_k}{A \zeta \alpha_k} = (k^*)^{\alpha_k \zeta - 1} \delta_P^{\alpha_p \zeta} (P^*)^{(1-\alpha_k)\zeta}. \quad (47)$$

After further manipulation, we have

$$(k^*)^{1-\alpha_k\zeta} = \frac{A\zeta\alpha_k}{\rho + \delta_k} \delta_P^{\alpha_p\zeta} (P^*)^{(1-\alpha_k)\zeta}, \quad (48)$$

or equivalently,

$$k^* = \left(\frac{A\zeta\alpha_k}{\rho + \delta_k} \delta_P^{\alpha_p\zeta} \right)^{\frac{1}{1-\alpha_k\zeta}} (P^*)^{\frac{(1-\alpha_k)\zeta}{1-\alpha_k\zeta}} \equiv \kappa(P^*). \quad (49)$$

Plugging equations (44), (45) and (49) into (42) results in:

$$c^* = \left(\frac{\rho + \delta_k}{\zeta\alpha_k} - \delta_k \right) \kappa(P^*) - \beta\delta_P P^*. \quad (50)$$

Substituting equations (44), (45), (49) and (50) into (39) leads to:

$$\begin{aligned} \mathcal{F}(P^*) = & \left(\left(\frac{\rho + \delta_k}{\zeta\alpha_k} - \delta_k \right) \kappa(P^*) - \beta\delta_P P^* \right)^{-1} \left[(\alpha_p \frac{\rho + \delta_k}{\alpha_k} \frac{\kappa(P^*)}{\delta_P P^*} - \beta)(\rho + \delta_P) \right. \\ & \left. + (1 - \alpha_k - \alpha_p) \frac{\rho + \delta_k}{\alpha_k} \frac{\kappa(P^*)}{P^*} \right] + \frac{\chi_P + \chi_p}{P^*} = 0. \end{aligned} \quad (51)$$

Given the parameter values in Table 1, the steady state can be solved as follows:

$$c^* = 0.1713, \quad k^* = 0.9554, \quad P^* = 0.8839, \quad p^* = 0.0884, \quad y^* = 0.4436. \quad (52)$$

Figure 1: The value of $\mathcal{F}(P^*)$ with respect to changes in P

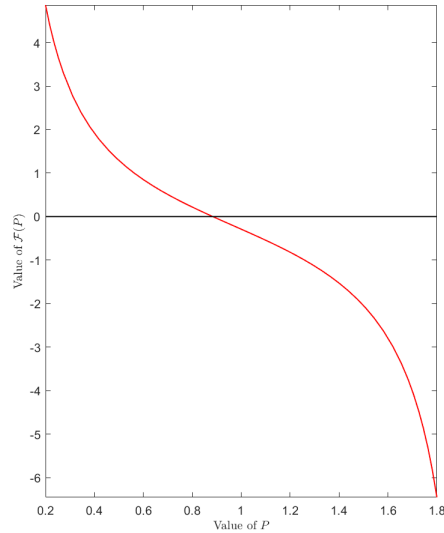


Table 1: Parameter values for the model

Parameter	Description	Value
ρ	Discount rate	0.03
β	Exogenous price of new investments in constitutional democracy capital	2
δ_P	Degradation rate of the constitutional democracy capital	0.1
δ_k	Depreciation rate of physical production capital	0.1
χ_p	Weight on new investments on democratic capital in the utility function	0.5
χ_P	Weight on the constitutional democracy capital in the utility function	0.5
α_k	Productivity of production capital in the production function	0.7
α_p	Productivity of new investments on democratic capital in the production function	0.1
ζ	Degree of decreasing return to scale in the production function	0.4
A	Exogenous level of production technology	0.5

To examine the stability of steady state, we rewrite (27) as

$$\begin{bmatrix} \dot{p} \\ \dot{c} \\ \dot{P} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} J_{11}^* & J_{12}^* & J_{13}^* & J_{14}^* \\ -\frac{u_c^*}{u_{cc}^*} f_{kp}^* & J_{22}^* & -\frac{u_c^*}{u_{cc}^*} f_{kP}^* & -\frac{u_c^*}{u_{cc}^*} f_{kk}^* \\ 1 & 0 & -\delta_P & 0 \\ f_p^* - \beta & -1 & f_P^* & f_k^* - \delta_k \end{bmatrix} \begin{bmatrix} p - p^* \\ c - c^* \\ P - P^* \\ k - k^* \end{bmatrix} \quad (53)$$

where

$$J_{11}^* = \frac{1}{u_{pp}^* + u_c^* f_{pp}^*} \left[u_c^* [f_{pp}^* (f_k^* - \delta_k + \delta_P) + f_{kp}^* (f_p^* - \beta) + f_{Pp}^* - f_{pP}^* - f_{pk}^* (f_p^* - \beta)] + u_{pp}^* \delta_P \right], \quad (54)$$

$$J_{12}^* = \frac{1}{u_{pp}^* + u_c^* f_{pp}^*} \left[u_{cc}^* [(f_p^* - \beta)(f_k^* - \delta_k + \delta_P) + f_P^*] + u_c^* f_{pk}^* \right], \quad (55)$$

$$J_{13}^* = \frac{1}{u_{pp}^* + u_c^* f_{pp}^*} \left[u_c^* [f_{pP}^* (f_k^* - \delta_k + \delta_P) + f_{kP}^* (f_p^* - \beta) + f_{PP}^* + f_{pP}^* \delta_P - f_{pk}^* f_P^*] + u_{PP}^* \right], \quad (56)$$

$$J_{14}^* = \frac{1}{u_{pp}^* + u_c^* f_{pp}^*} \left[u_c^* [f_{pk}^* (f_k^* - \delta_k + \delta_P) + f_{kk}^* (f_p^* - \beta) + f_{Pk}^* - f_{pk}^* (f_k^* - \delta_k)] \right], \quad (57)$$

$$J_{22}^* = \frac{(u_{cc}^*)^2 - u_c^* \cdot u_{ccc}^*}{(u_{cc}^*)^2} [\rho - (f_k^* - \delta_k)]. \quad (58)$$

The eigenvalues of the Jacobian matrix in (53) are

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix} = \begin{bmatrix} 0.0762 - 0.0974i \\ 0.0762 + 0.0974i \\ 0.0196 \\ -0.1406 \end{bmatrix}. \quad (59)$$

Since this dynamic system has two negative eigenvalues and two positive eigenvalues, the steady-state equilibrium is saddle-point stable.

4.2 The optimal path

Given the Jacobian matrix and eigenvalues in (53) and (59), the corresponding eigen-vectors are

$$\mathbf{V}_1 = \begin{bmatrix} -0.0438 + 0.0330i \\ 0.0237 + 0.0356i \\ -0.2698 + 0.0381i \\ 0.9596 \end{bmatrix}, \quad \mathbf{V}_2 = \begin{bmatrix} -0.0438 - 0.0330i \\ 0.0237 - 0.0356i \\ -0.2698 - 0.0381i \\ 0.9596 \end{bmatrix},$$

$$\mathbf{V}_3 = \begin{bmatrix} -0.0815 \\ 0.2367 \\ -0.6812 \\ 0.7164 \end{bmatrix}, \quad \mathbf{V}_4 = \begin{bmatrix} -0.0352 \\ 0.0132 \\ 0.8665 \\ -0.4977 \end{bmatrix}.$$

The optimal paths around the steady state are

$$\begin{bmatrix} p \\ c \\ P \\ k \end{bmatrix} = \begin{bmatrix} p^* \\ c^* \\ P^* \\ k^* \end{bmatrix} + C_1 \mathbf{V}_1 e^{\mu_1 t} + C_2 \mathbf{V}_2 e^{\mu_2 t} + C_3 \mathbf{V}_3 e^{\mu_3 t} + C_4 \mathbf{V}_4 e^{\mu_4 t}, \quad (60)$$

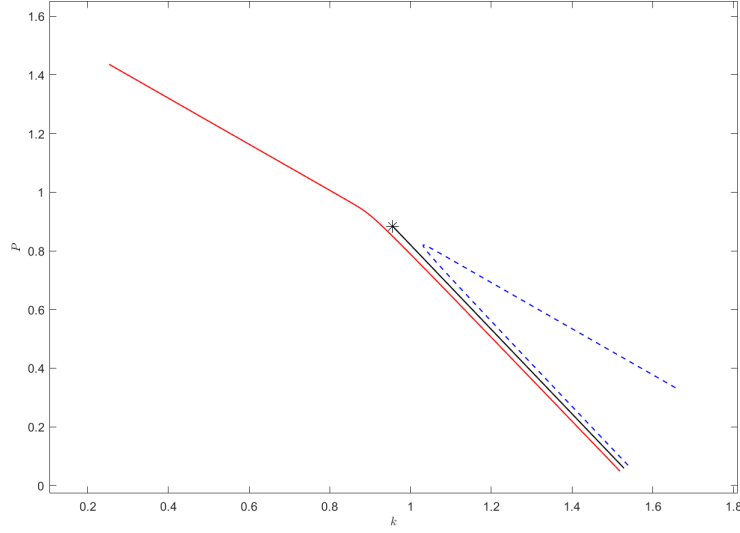
where C_1 , C_2 , C_3 , and C_4 are undetermined coefficients. It is challenging to draw a phase diagram for the four-dimension system. However, we can see the evolution of physical capital and constitutional-democracy capital over time in a two-dimension $P - k$ phase portrait. Therefore, the optimal paths in two-dimension $P - k$ phase around the steady state are

$$\begin{bmatrix} P \\ k \end{bmatrix} = \begin{bmatrix} P^* \\ k^* \end{bmatrix} + C_3 \mathbf{V}_3 e^{\mu_3 t} + C_4 \mathbf{V}_4 e^{\mu_4 t}, \quad (61)$$

the coefficients C_3 , and C_4 are determined by initial conditions, P_0 and k_0 . As illustrated in Figure 2, the economy starts from the initial state at the bottom right corner, then,

- (i) the solid black line depicts the trajectory wherein individuals optimize their physical capital and constitutional-democracy capital, proceeding along the saddle-path trajectory toward the long-term optimal steady state;
- (ii) the solid red line illustrates a trajectory where individuals allocate an excessive amount of resources to constitutional-democracy capital investment, resulting in an over-accumulation of constitutional-democracy capital and a deficiency in physical capital accumulation;
- (iii) the dashed blue line illustrates a scenario where individuals reduce their consumption and constitutional-democracy investments excessively, leading to an over-accumulation of physical capital and an insufficient accumulation of constitutional-democracy capital.

Figure 2: The paths with different initial conditions and choices



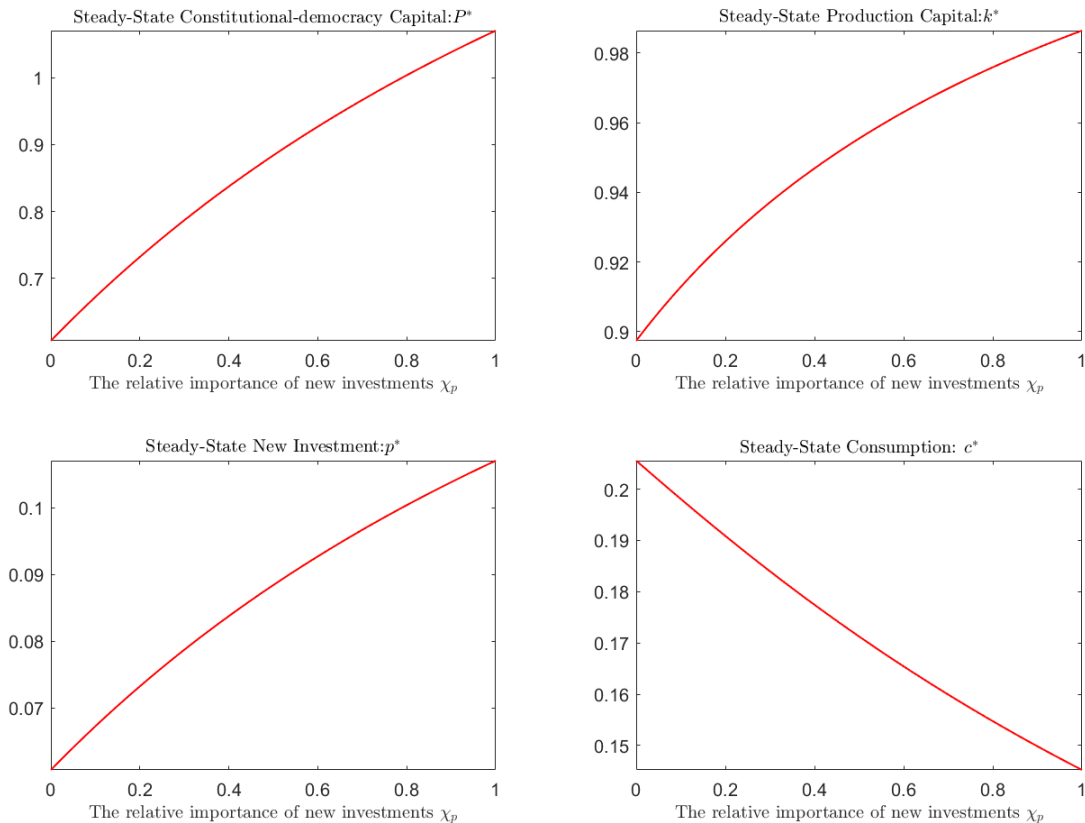
4.3 Dynamic implications

4.3.1 The relative importance of new investments on the constitutional-democracy capital in preferences

Proposition 1. An increase in χ_p leads to a decrease in consumption c^* and a rise in the constitutional-democracy investments p^* , physical capital k^* , and constitutional-democracy capital P^* .

Preferences for the constitutional-democracy investments lead to higher levels of investments on constitutional democracy, thereby stimulating accumulation of the constitutional-democracy capital and contributing to increased aggregate output in the economy. However, since citizens derive greater utility from the constitutional-democracy investments, it results in a corresponding decrease in consumption. Figure 3 shows that, as χ_p increases, new constitutional-democracy investments, the constitutional-democracy capital, and physical capital all increase, while consumption decreases.

Figure 3: The impact of the relative importance of new investments on the constitutional-democracy capital in preferences on long-run steady state

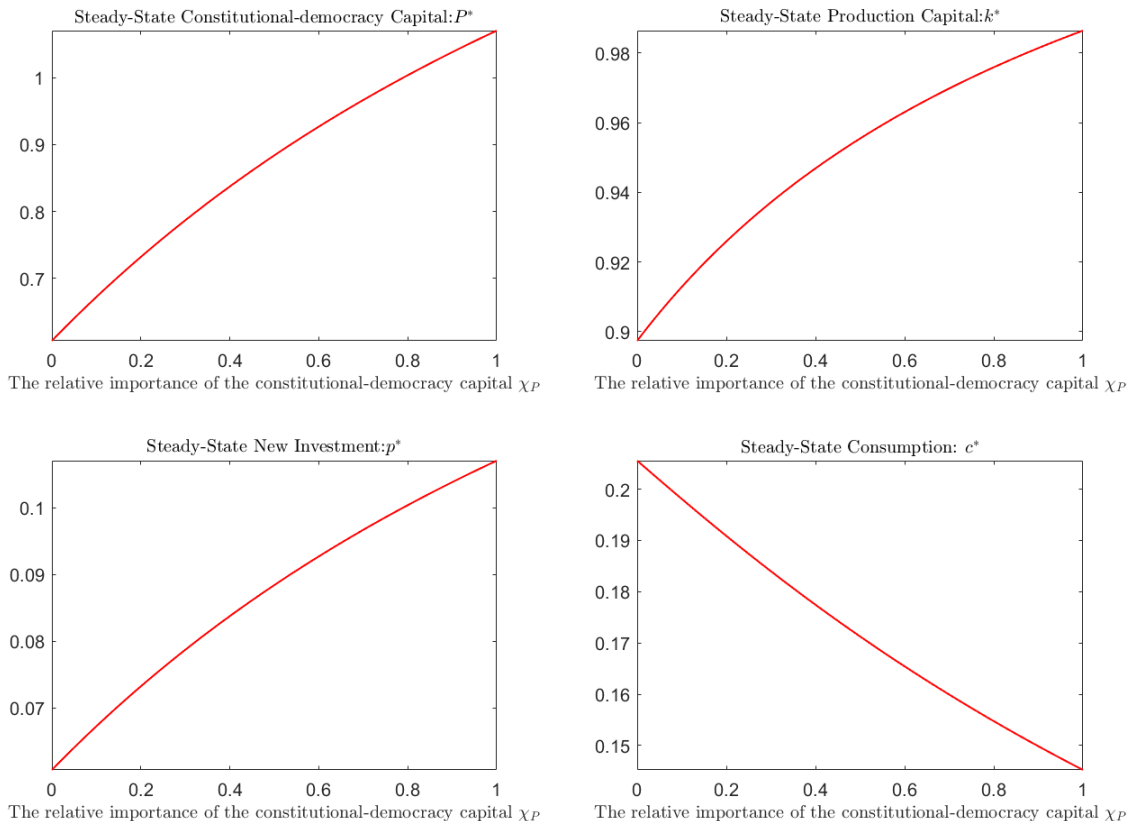


4.3.2 The relative importance of the constitutional-democracy capital in preferences

Proposition 2. An increase in χ_P leads to a decrease in consumption c^* , but results in higher levels of the constitutional-democracy investment p^* , capital k^* , and the constitutional-democracy capital P^* .

Preferences for the constitutional-democracy capital lead to higher levels of the constitutional-democracy capital accumulation, thereby contributing to increased aggregate output in society. However, since citizens derive greater utility from the constitutional-democracy capital, it results in a corresponding decrease in consumption. Figure 4 depicts that as χ_P increases, new constitutional-democracy investments, the constitutional-democracy capital, and physical capital all increase, while consumption decreases.

Figure 4: The impact of the relative importance of the constitutional-democracy capital in preferences on long-run steady state



Propositions 1 and 2 provide significant insights into the evolution of liberties, property rights, and constitutional democracy across different cultures and time periods. These propositions underscore the strong preferences for democratic investments and the accumulation of constitutional democracy capital, indicating a profound desire for liberties, equality, independence, and self-governance through democratic structures. This framework helps explain the rarity of democracy and liberties in historically despotic societies, the pioneering of freedom and democracy in ancient Greece and Rome, and the widespread acceptance of voluntary servitude throughout much of European history, as highlighted by Étienne de La Boétie in his seminal "Discourse on Voluntary Servitude."

Written over 450 years ago during his university years and published clandestinely in 1577, La Boétie's essay deeply explores the complex dynamics of power and obedience within political systems. Remarkable for its depth at a time when La Boétie was only about 18 years old, this work critically examines societal preferences for the security of subordination over the uncertainties of liberty, profoundly impacting the institutional structures of governance. His central thesis posits that tyrants maintain power solely through the consent of the governed, often given not by

active agreement but through passive acceptance and non-resistance. This insight challenges the legitimacy of oppressive governance and suggests that true freedom is forsaken when individuals, habituated to servitude, unknowingly uphold the psychological chains of their own subjugation.

La Boétie's discourse delves into the psychological and societal underpinnings of political submission and critically addresses the populace's complicity in their own servitude, advocating for reclaiming freedom through non-cooperation and civil disobedience. His call to action resonates with modern movements emphasizing non-violent resistance and the power of collective agency. La Boétie's work remains a cornerstone in political theory, offering timeless insights into the nature of power and the crucial role of individual and collective agency in overcoming authoritarianism. His revolutionary ideas advocate for the dismantling of authoritarian capital structures and the establishment of free institutional capital structures founded on the principles of liberty and democratic governance, underscoring the enduring relevance of his thoughts in the continuous global struggle for freedom and justice.

Propositions 1 and 2, in conjunction with Étienne de La Boétie's "Discourse on Voluntary Servitude," hold profound relevance in today's global context, where authoritarian and totalitarian regimes are resurging, and the fabric of democracy appears increasingly fragile. These theoretical frameworks offer critical insights into the essential nature of democratic resilience and the mechanisms through which societies may either resist or succumb to authoritarianism.

La Boétie's discourse, with its exploration of the dynamics between power and obedience, provides a vital lens through which to understand the current challenges facing democracies worldwide. His argument that tyrants maintain power solely through the consent of the governed—whether explicit or implicit—underscores the importance of public awareness and active engagement in safeguarding democratic institutions. Today, as authoritarian leaders gain traction, the need for a populace that is educated about and committed to democratic principles becomes ever more critical.

The call for individuals around the globe to actively invest in democracy capital is not merely philosophical but a practical strategy to strengthen the foundations of constitutional democracy. People who exhibit strong preferences for democratic investments and the accumulation of constitutional democracy capital play a pivotal role in this process. By fostering democratic values and participating in democratic processes, these individuals help to build and reinforce the structures necessary for resilient democratic governance.

In practical terms, investing in democracy capital means supporting free press, ensuring fair and transparent electoral processes, promoting the rule of law, and encouraging civic education. It involves not only defending existing democratic institutions but also innovating new ways to enhance political participation and accountability. Such efforts are crucial to consolidate the accumulation of constitutional democracy capital and build robust democratic nations capable of

withstanding the pressures of authoritarianism.

Thus, in an era marked by the rise of autocratic forces and the fragility of democratic systems, the teachings of Propositions 1 and 2, alongside La Boétie's reflections, serve as a clarion call. They remind us of the power of collective agency and the critical need for continuous, active engagement in democratic processes. This engagement is essential not only for the maintenance of democracy but also for its evolution and strengthening in response to the complex challenges of the 21st century. The defense and enhancement of democratic nations rely fundamentally on our commitment to these principles, ensuring that democracy, once secured, is never regarded as a permanent given but rather as a perpetually renewing project requiring constant vigilance and participation.

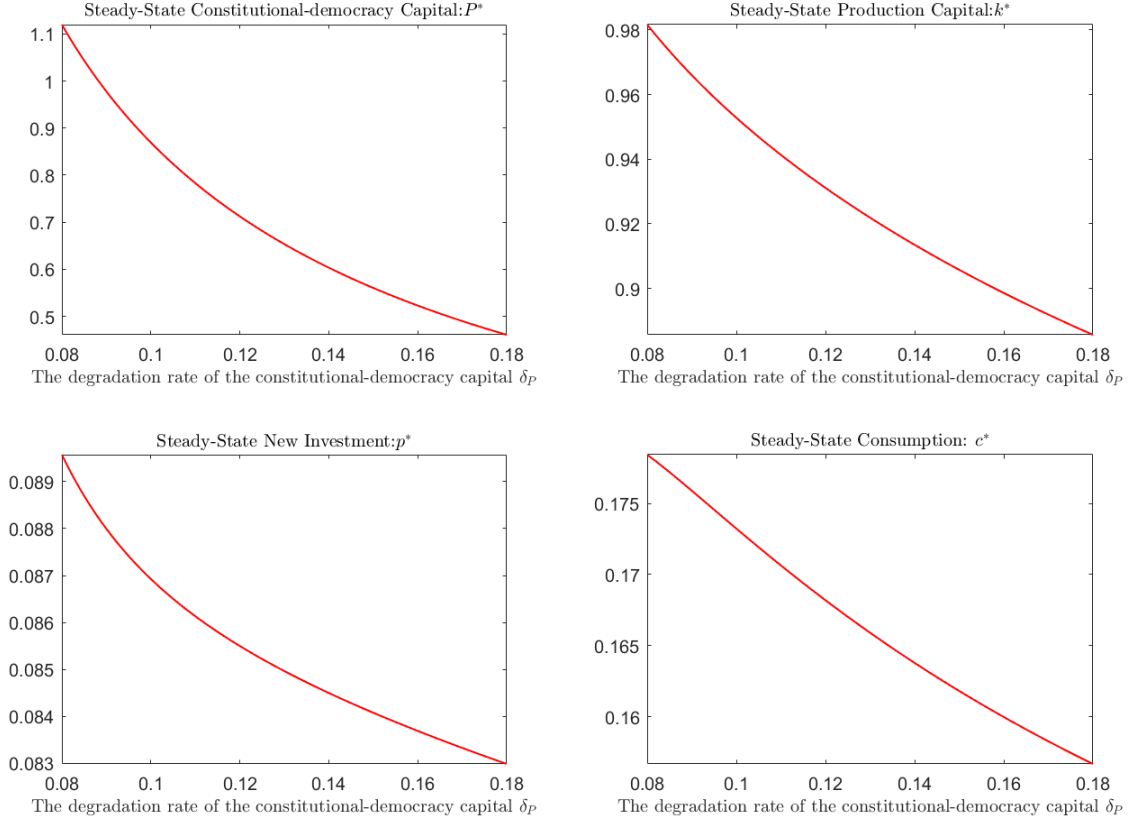
Propositions 1 and 2 also suggest that a robust and passionate commitment to investments in constitutional democracy and the accumulation of democracy capital can sometimes result in reduced physical consumption in the short term. This observation is supported by historical evidence showing that individuals and societies often endure significant sacrifices in their pursuit of liberties, equality, and democratic governance. Throughout history, people have been willing to endure hardships, engage in conflict, and even risk their lives for the cause of democracy, epitomized by the rallying cry: "Give me liberty or give me death!" This expression and the actions it inspires highlight the deep-seated values that drive societal shifts towards democratic systems, underscoring the complex trade-offs between immediate economic consumption and long-term democratic investments. These trade-offs reflect the profound impact that the pursuit of democratic ideals can have on a society's economic behavior and consumption patterns in the short run, as resources are often redirected towards achieving and safeguarding democratic freedoms.

4.3.3 The degradation rate of the constitutional-democracy capital

Proposition 3. An increase in the degradation rate of the constitutional-democracy capital, δ_P , leads to a decrease in consumption c^* , physical capital k^* , constitutional-democracy investment p^* , and the constitutional-democracy capital P^* .

In an environment with a higher degradation rate of constitutional-democracy capital, the accumulation of constitutional-democracy capital becomes more difficult, thereby reducing the level of output and individual income in the economy. Consumption and constitutional-democracy investments are also restricted. Figure 5 indicates that an increase in δ_P leads to a general deterioration in consumption c^* , capital k^* , constitutional-democracy investments p^* , and the constitutional-democracy capital P^* .

Figure 5: The impact of the depreciation rate of the constitutional-democracy capital on long-run steady state



The degradation and erosion of constitutional-democracy capital represent a critical threat to the foundations of democratic societies globally. This concept captures the gradual decline in democratic principles and institutions, a process that often occurs subtly and insidiously. It is crucial for citizens in every nation to remain vigilant and proactive in monitoring these changes because, as the adage goes, "democracy dies in silence." When democratic erosion is ignored or goes unchecked, it can lead to the silent death of freedoms and rights that many often take for granted. Let us recall: Alexis de Tocqueville (1835, 1840, 2022), in his work *Democracy in America*, already expressed concern over the "tyranny of the majority," a situation where the majority's interests prevail to the detriment of minority rights, potentially leading to oppression. He argued that this could undermine individual freedom and intellectual diversity. Tocqueville also described "soft despotism" as a subtle form of control in democratic societies, where the government gradually increases its influence over its citizens' decisions and lives in a paternalistic manner. This kind of government, while non-oppressive on the surface, could stifle individual initiative and freedom through excessive regulation and dependency on public services, leading to a decline in civic engagement and personal responsibility.

Today, the erosion and degradation can manifest in various forms, including the weakening of the rule of law, the curtailment of freedom of the press, the undermining of electoral integrity, and the gradual dismantling of checks and balances within government systems. Other indicators include the consolidation of power in the hands of a few, the restriction of public access to information, and the repression of dissent and political opposition. These trends are particularly dangerous because they can be incremental and made to seem legal or justified under the guise of other motives, such as national security concerns or emergency measures.

The responsibility to safeguard constitutional-democracy capital does not rest solely with politicians or the judiciary but extends to all citizens. Active citizenship requires awareness and engagement with political processes, critical scrutiny of government actions and policies, and the readiness to speak out against injustices. Civil society, educational institutions, and the media play pivotal roles in fostering this active engagement. They help to educate the public, provide platforms for debate and dissent, and hold leaders accountable.

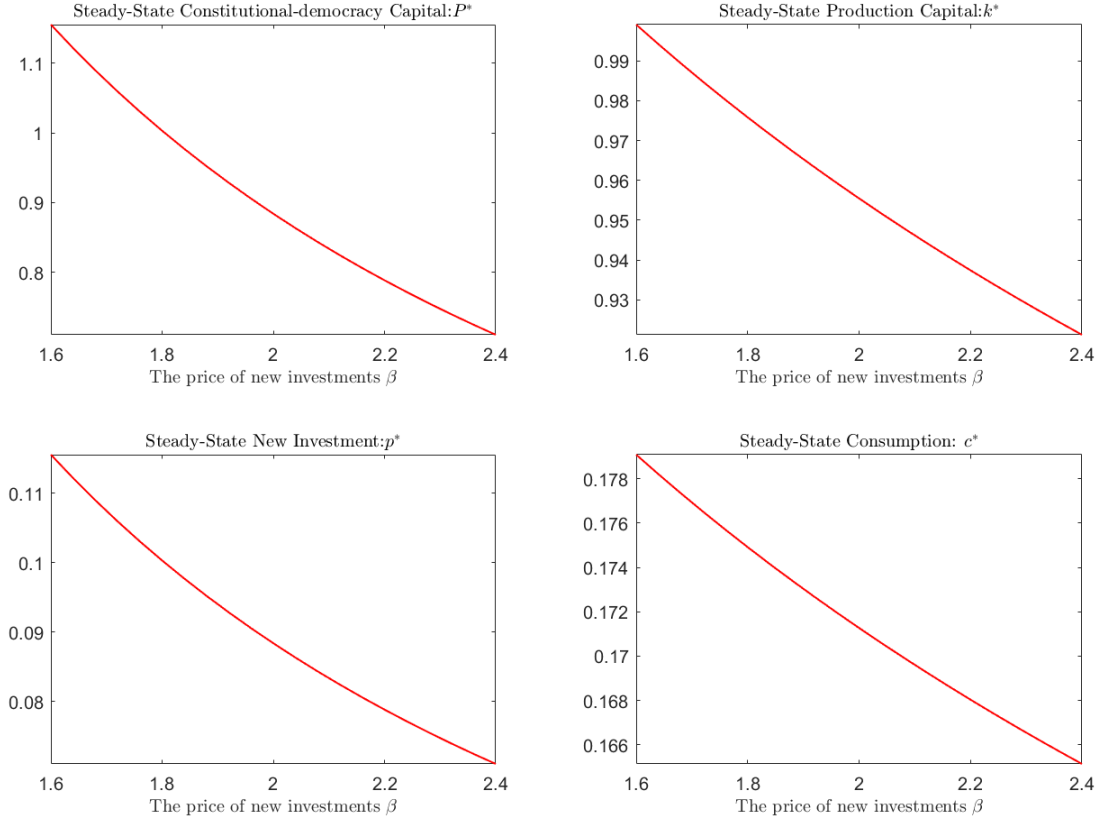
Therefore, the erosion of constitutional-democracy capital is a pernicious process that can lead to the dissolution of democratic norms and institutions if not actively contested. The vigilance and participation of citizens are indispensable in preserving the integrity and vitality of democracy. Recognizing the signs of democratic decay and responding with informed, concerted action are essential to ensuring that democracy does not die in silence but thrives openly and robustly in societies around the world.

4.3.4 The price of new investments on the constitutional-democracy capital

Proposition 4. An increase in β leads to a decrease in constitutional-democracy investments p^* , capital k^* , consumption c^* , and constitutional-democracy capital P^* .

If the costs of constitutional-democracy investments rise, citizens will reduce their corresponding investments in constitutional democracy, leading to less constitutional-democracy capital and thereby adversely affecting production. What's more, individuals may look for alternative investment channels, which may lead them to shift from constitutional-democracy investments to physical capital investment. Figure 6 demonstrates that as β rises, new constitutional-democracy investment p^* , constitutional-democracy capital P^* , consumption c^* , and production capital k^* all decline.

Figure 6: The impact of the price of new investments on the constitutional-democracy capital on long-run steady state



In the contemporary global landscape, the cost of investing in constitutional-democracy capital has been escalating for countries across the democratic and authoritarian spectrum. Over the past decade and a half, autocrats have successfully engineered a more advantageous international environment for themselves, bolstered by their growing political and economic clout and a noticeable decline in democratic pressures. This shift has led to a less favorable climate for fostering and sustaining constitutional democracy.

The emerging international order championed by these autocrats is not underpinned by a shared ideology or personal bonds among leaders. Rather, it is strategically designed to serve the interests of those in power, focusing on reducing oversight on their actions and perpetuating their control. This order does not prioritize the welfare of the global population nor does it facilitate opportunities for individuals to enhance their own lives. Instead, it fosters a governance framework where autocratic leaders are free to exploit their positions with minimal resistance.

Such a world order, dominated by unchecked autocratic rule, inherently breeds disorder characterized by armed conflicts, rampant lawlessness, pervasive corruption, and economic instability. The consequences of this disorder are profound, not only undermining global stability but also

exacting a severe human toll through violence and socioeconomic upheaval. The resultant global instability directly impacts the human cost, making it increasingly challenging and expensive to maintain and establish the foundations of constitutional democracy.

This scenario underscores the rising costs associated with investments in constitutional-democracy capital. As autocratic tendencies gain ground, the international community faces heightened challenges in upholding democratic norms and instituting checks on power abuses. The effort to sustain or establish constitutional democracy becomes not only more complex and demanding but also more crucial in countering the trends that threaten global peace and the rule of law. Thus, reinforcing democratic institutions and principles worldwide is not just a moral imperative but a practical necessity to ensure a stable, just, and orderly global society.

Propositions 1 through 4 in our model illustrate how variations in political institutional factors can significantly impact the accumulation of physical capital, treating both democratic capital and physical economic capital as endogenous variables. This approach integrates institutional and economic variables in both the utility function and the production function, a methodological advancement that aligns with the principles of new institutional economics, which has been advocating for this integrated approach since the 1960s.

Specifically, these propositions reveal the interdependent relationship between democratic preferences and capital accumulation. When there is a strong preference for democracy within a society, it not only fosters the growth of democracy capital but also stimulates the accumulation of physical capital. This dual enhancement arises because a robust democratic framework often encourages investment by ensuring a stable and predictable environment for economic activities, promoting transparency, and safeguarding property rights.

Propositions 1 through 4 under discussion reveal a dynamic and continually evolving process in the maintenance and evolution of constitutional democracy, challenging the notion of an "end of history" where democratic systems persist unchallenged. These insights emphasize that constitutional democracy is not static but fluctuates due to a variety of internal and external pressures. A key issue is the increasing degradation rate of democracy capital, which refers to the decline in the quality and effectiveness of democratic institutions and norms. This degradation can stem from corruption, political polarization, erosion of public trust, or external influences like foreign interference, making it increasingly difficult to maintain robust democratic governance and making systems more vulnerable to authoritarian tendencies. This weakening of democratic norms not only impacts the political landscape but also has significant economic repercussions. Concurrently, the weakening of democratic institutions typically results in reduced accumulation of both democracy and physical capital—assets crucial for both democratic governance and economic development—thereby stifling economic and democratic resilience. Additionally, rising costs associated with maintaining and strengthening democratic structures—due to factors like increased

conflict, heightened security needs, or greater expenditures to combat misinformation—lead to a noticeable decline in investments in democracy. Such high costs can deter domestic and international support for democratic initiatives, reducing investments critical for maintaining a functioning democracy. Furthermore, the combination of increasing rates of democracy capital degradation and rising investment costs fosters conditions conducive for authoritarian and totalitarian regimes to take root or re-emerge. As democratic systems become less effective and more costly to uphold, and public dissatisfaction increases, there may be greater receptivity to alternative, often more authoritarian forms of governance that promise stability or simpler governance models, despite potential sacrifices in freedom and democratic norms. These dynamics underscore the importance of vigilance, sustained investment, and adaptability to prevent democratic systems from regressing into authoritarian rule, highlighting the critical interplay between economic and political factors and illustrating how economic challenges can hasten the decline of democratic governance, whereas strong, responsive democratic institutions can enhance economic performance.

These four propositions underscore the critical role that political institutional factors play in shaping economic outcomes. They highlight how a supportive democratic environment can bolster economic prosperity by enhancing capital accumulation, while challenges to democracy can impede both democratic and economic development. This dual focus on institutions and economic variables provides a comprehensive framework for understanding the complex dynamics between political systems and economic performance.

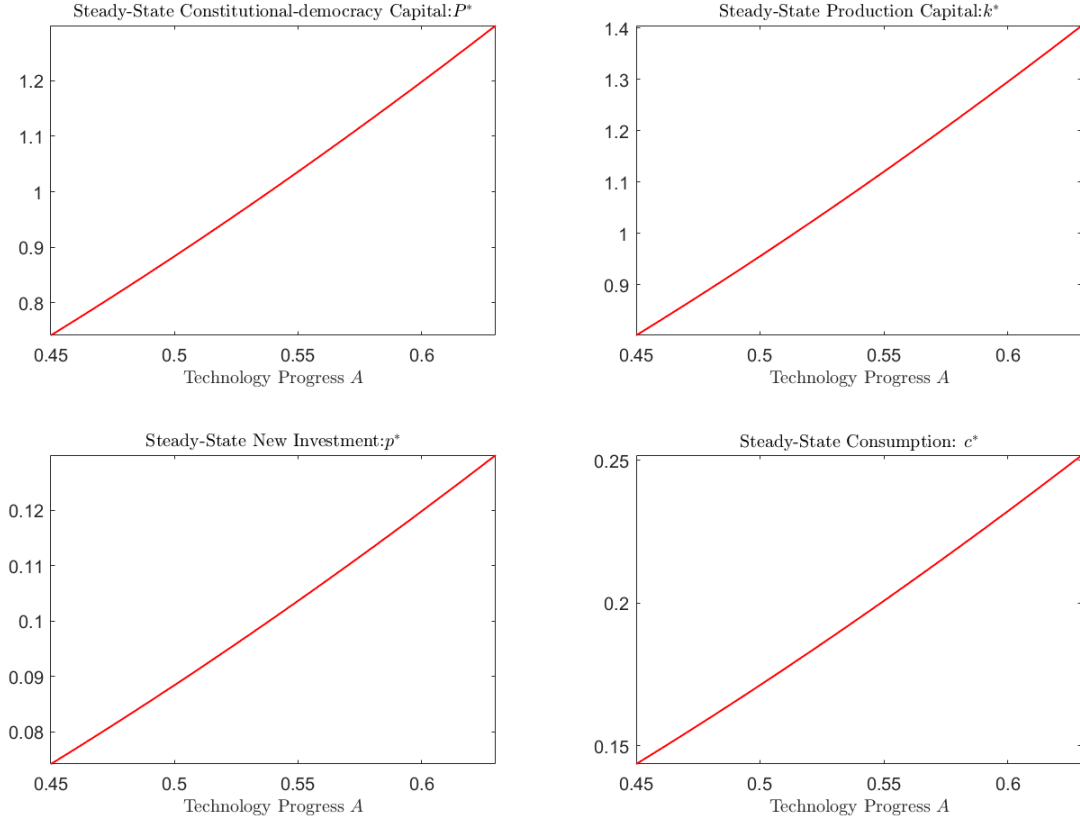
At the same time, in our model, which focuses on the endogenous determination of both physical capital and democracy capital, it becomes clear how changes in exogenous economic variables can significantly influence both types of capital. This interaction is highlighted through the following two propositions.

4.3.5 The effects of technology progress

Proposition 5. An increase in the exogenous level of technology A leads to an increase in constitutional-democracy investments p^* , physical capital k^* , constitutional-democracy capital P^* , and consumption c^* .

The implication of this proposition is rather straightforward: as technological advancement increases, so does production and individual income. Consequently, citizens have the means to invest more in constitutional democracy. Investments in the constitutional democracy contribute to the accumulation of constitutional-democracy capital, which subsequently leads to higher production incomes. Figure 7 illustrates that an increase in A leads to a rise in capital k^* , constitutional-democracy investments p^* , and the constitutional democracy capital P^* , and consumption c^* .

Figure 7: The impact of technology productivity on long-run steady state



Technological advancements have historically played a crucial role in enhancing the accumulation of democracy capital by broadening access to information and facilitating civic engagement. From the 16th-century printing press to modern internet and mobile technologies, these innovations have democratized knowledge, enabling wider participation in political processes and enhancing government transparency. Furthermore, technologies like the telegraph, radio, and social media have been pivotal in shaping public opinion and organizing movements such as the Civil Rights Movement and the Arab Spring. Additionally, the implementation of e-governance and digital platforms has revolutionized the delivery of government services, making them more accessible and efficient, thereby increasing public participation in governance and improving the responsiveness of governments to citizen needs.

Of course, technological progress has significantly accelerated the accumulation of physical capital and consumption through several key advancements. The Industrial Revolution introduced innovations like the steam engine and electrical power, dramatically boosting industrial productivity. Similarly, the Green Revolution enhanced agricultural output, supporting larger populations and economic growth. Technological advancements have also spurred economic expansion and diversification, creating new markets and industries such as the automotive and information

technology sectors, which have diversified economic bases and increased resilience. Additionally, advancements in financial technologies, logistics, and communications have reduced the costs of economic transactions, facilitating faster and more efficient global trade and economic integration.

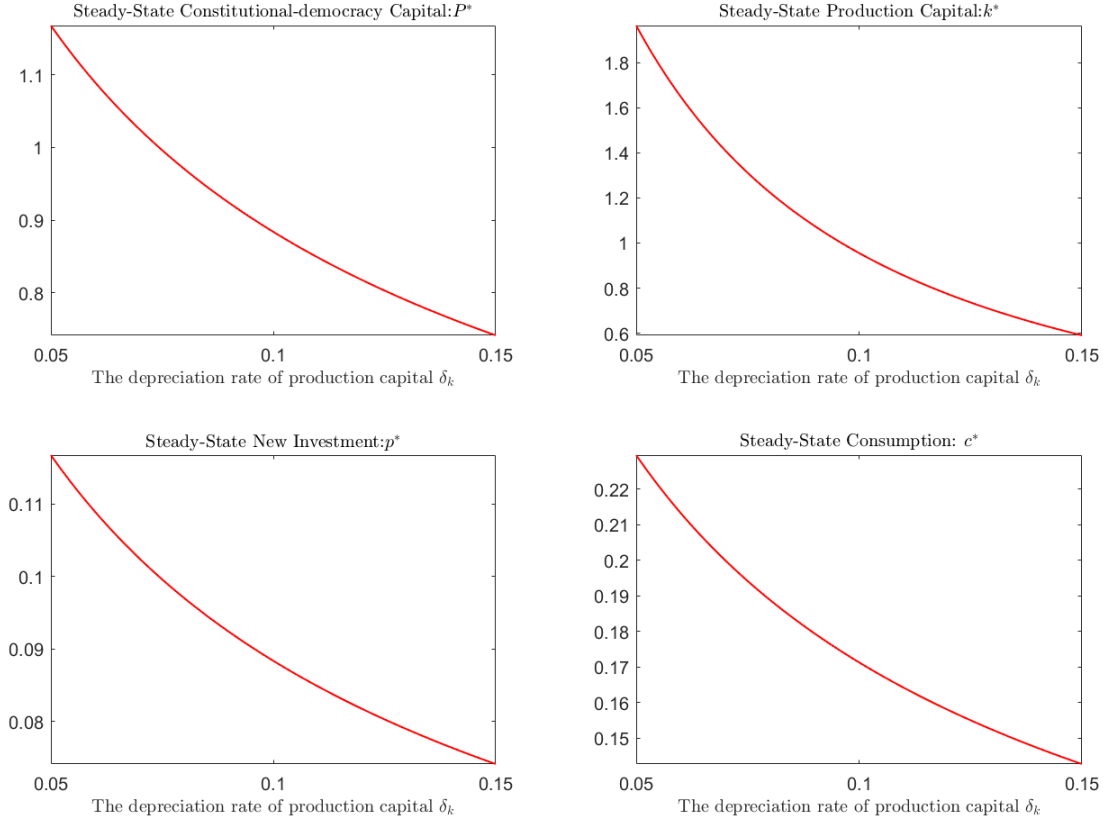
Therefore, the interplay between technology, democracy, and economic growth is deeply intertwined and cyclical. Technological advancements not only bolster democratic mechanisms but also thrive under democratic systems that promote freedom, protect intellectual property, and prioritize education—all vital for innovation. This relationship supports the flattening of social hierarchies and disperses power, aligning closely with democratic values and practices. While earlier scholars like North and Acemoglu & Robinson may not have focused on technology's direct impact on institutions, its crucial role in shaping democratic and economic frameworks is undeniable and essential for fostering both democracy and economic growth.

4.3.6 The depreciation rate of production capital

Proposition 6. The higher the depreciation rate of physical capital, δ_k , the lower the long-run consumption c^* , constitutional-democracy investments p^* , capital k^* , and the constitutional-democracy capital P^* .

In an environment with a higher depreciation rate of physical capital, the accumulation of physical capital becomes more difficult, agents have less motivation to produce, which implies a lower income for citizens. This leads to a reduction in consumption c^* and constitutional-democracy investments p^* , and less formation of the constitutional-democracy capital P^* . Figure 8 indicates that an increase in δ_k leads to a general deterioration in consumption c^* , capital k^* , constitutional-democracy investments p^* , and the constitutional-democracy capital P^* .

Figure 8: The impact of the depreciation rate of production capital on long-run steady state



A higher depreciation rate of physical capital can exert significant economic strain, initiating a chain reaction that impacts various facets of a nation's economic and political landscape. When physical assets depreciate more rapidly, citizens face an increased financial burden as more resources are needed for the replacement or repair of these assets. This necessity diverts funds from other potential areas of investment, including those crucial for supporting and developing democratic institutions and processes. As financial resources become constrained, there may be a reduction in investments towards democratic infrastructure like legal systems, education, and public services, which, over time, could lead to weaker democratic institutions and a decrease in democracy capital. Simultaneously, the continuous allocation of resources towards maintaining existing capital rather than its expansion or modernization may slow physical capital accumulation, further impeding economic growth and development. This stagnation in economic progress and the diversion of resources away from democratic investments may cause the quality and robustness of democratic institutions to deteriorate, eroding democracy capital and leading to diminished political stability and governance effectiveness. In the long run, a higher depreciation rate presents considerable challenges to both economic stability and democratic development, highlighting the essential interconnection between economic policies, capital maintenance, democratic investment,

and overall societal well-being.

A lower depreciation rate of physical capital significantly benefits a nation's economy and democratic framework. When physical assets depreciate more slowly, there is a reduced financial burden on citizens due to lower maintenance and replacement costs, freeing up resources for other uses. This financial saving increases disposable income, enhancing consumer spending and boosting overall economic activity. Additionally, the savings from lower depreciation rates allow for increased investments in new physical capital, which improves productive capacity and supports economic expansion. With more financial resources available, there is also greater scope for investing in democratic institutions. This includes funding for legal frameworks, civic education, and public services, all crucial for strengthening democratic governance. Such investments lead to the accumulation of democracy capital, which fosters a more robust and stable democratic environment, thereby enhancing the overall health and sustainability of the nation's democratic systems.

The six propositions presented in this section are essential for dissecting the patterns of economic booms and busts as analyzed by Pritchett et al. (2018) in "Deals and Development: The Political Dynamics of Growth Episodes." These propositions incorporate exogenous political and economic variables that are reflected in the events observed across the ten countries studied in their work. These variables serve as external factors that influence the economic and political dynamics within these nations, providing a framework to understand the complex interactions that lead to periods of rapid growth or significant decline. This approach helps to clarify the underlying causes and mechanisms driving the economic and political fluctuations detailed in the book.

5 Concluding remarks

This paper develops a dynamic model to analyze the growth of constitutional democracy capital and physical capital, focusing on the role of representative citizens rather than government control. It investigates the interactions between institutional capital and economic performance through the actions of these representative agents, examining how modifications in institutional frameworks influence economic dynamics over time. The model reveals a symbiotic relationship between democratic institutions and economic factors, showing how robust democratic structures can propel economic growth and how economic conditions can affect the strength of democratic governance managed by representative citizens.

By incorporating both democracy capital and physical capital into the model, we reveal the substantial influence of exogenous economic variables on these capitals. Additionally, the model emphasizes the crucial role of political institutions in shaping economic outcomes, suggesting that a supportive democratic environment not only boosts capital accumulation but also safeguards democratic integrity. Conversely, the model highlights how threats to democratic structures can impede economic development. This dual focus provides a comprehensive framework for understanding the

nuanced interactions between political systems and economic performance, illustrating the critical need for maintaining healthy democratic institutions to foster sustainable economic growth.

Most existing studies such as Lipset (1959); Barro (1996, 1997, 1999); Acemoglu, et al. (2019); Przeworski and Limongi (1993); Rodrik and Wacziarg (2005); and Persson and Tabellini (2006, 2008), among many others, examining the relationship between economic development and democracy often face criticism for being one-sided because they tend to treat either economic development as a determinant of democracy or democracy as a driver of economic development. This approach can be limiting as it overlooks the possibility that both democracy and economic development are endogenous and interdependent. Such studies might fail to capture the complex dynamics where both democracy and economic development influence each other reciprocally. In reality, the relationship is likely bidirectional: economic development can provide the resources and middle class that demand and support democratic institutions, while established democratic institutions can create a stable environment that fosters economic growth. Ignoring this bidirectionality and the endogeneity of both variables can lead to incomplete or misleading conclusions about the nature of their relationship. Therefore, a more nuanced approach that considers the simultaneous interactions and feedback loops between democracy and economic development is essential for a more accurate understanding of how these important factors influence one another.

In ending the paper, we repeat that constitutional democracy capital refers to the foundational and operational elements critical to a functional democratic system. It stems from a constitution that defines laws and governmental structures, ensuring power distribution among various branches to prevent centralization, uphold the rule of law, and protect human rights. Such a system empowers the government through the constitution, often ratified by public representatives or direct vote, ensuring political equality and accountability. Constitutional democracy capital represents the accumulation of democratic principles and practices vital for maintaining and strengthening democratic governance, emphasizing the role of the people in sustaining these structures daily.

Data on constitutional democracy capital and its associated investments can be tracked through annual publications such as the Democracy Index 2023, Freedom in the World 2023, and the World Justice Project 2023. These reports provide structured metrics to assess the health of democratic institutions globally, offering insights into the dynamics of democratic governance. For instance, the Democracy Index 2023 documents a worrying decline in global democracy, with fewer than 8% of the world population living in full democracies and nearly 40% under authoritarian regimes. This decrease is exacerbated by rising global conflicts, highlighting the challenges democracies face in maintaining peace. Furthermore, in his 2018 study, Seva Gunitsky identifies thirteen waves of democratic expansion and consolidation from 1776 to 2012, each characterized by a rejection of absolute rule. These include movements such as the Enlightenment-driven Atlantic Wave, the Latin American wars of independence, and modern movements like post-Soviet wave and the Arab

Spring. Each wave is marked by the adoption of Enlightenment ideals, such as liberalism and republicanism, which challenged established hierarchies and fostered democratic governance and constitutional democracy capital accumulation across various regions and times. This historical analysis highlights the complex interplay of political, social, and economic factors that influence democratic transitions and underscore the cyclical nature of democratic resilience and fragility.

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