

# The Spirit of Capitalism, Endogenous Preferences, Prosperity, and Depression

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## Abstract

When time preferences are a decreasing function of wealth, there will be prosperity; but when time preferences become an increasing function of wealth, there will be depression and stagnation.

## 1 Introduction

Max Weber's concept of the spirit of capitalism suggests that wealth accumulation can lead to a shift in individual attitudes towards time preference. According to this view, as people amass greater wealth, they tend to exhibit increased patience and place greater importance on future welfare and well-being. In other words, their subjective time discount rate, which reflects how they value present versus future outcomes, decreases as their wealth or capital accumulation grows. This decreasing function of wealth accumulation implies that individuals become more forward-thinking and prioritize long-term benefits over immediate gratification as they accumulate more resources. The shift towards increased patience and future orientation as a result of wealth accumulation can contribute to economic prosperity in several ways:

**Increased Investment:** Individuals with lower time discount rates are more likely to invest in long-term projects, such as infrastructure development, research and development, and education. These investments can lead to productivity gains and economic growth over time.

**Savings and Capital Formation:** Lower time discount rates encourage higher levels of savings and capital accumulation. This capital can be channeled into productive investments, fueling economic expansion and providing funds for entrepreneurship and innovation.

**Stability and Resilience:** Economies characterized by a population with lower time discount rates may be more stable and resilient to economic shocks. Long-term planning and investment can help mitigate the impact of short-term fluctuations and contribute to sustained growth.

**Human Capital Development:** With a focus on the future, individuals and societies are more likely to invest in human capital development, including education, skills training, and healthcare. A well-educated and healthy workforce can drive productivity and innovation, enhancing overall economic performance.

From this perspective, the alignment of individual time preferences with long-term economic goals can foster sustainable development and prosperity for society as a whole.

On the other hand, Weber suggests that with increasing wealth comes a rise in individuals' subjective time discount rate: People become more inclined to prioritize immediate gratification over long-run investment. As their financial security grows, they may feel less compelled to save and invest for the future, leading to a decline in the values of frugality and self-discipline. As wealth increases, individuals can experience a decline in the values and virtues that originally contributed to their economic success. The pursuit of material wealth can lead to a sense of complacency, entitlement, and hedonism, which may erode the ethic of hard work, thrift, and deferred gratification. Weber (1904-05) cites John Wesley's words to further this idea: "I fear, wherever riches have increased, the essence of religion has decreased in the same proportion. . . For religion must necessarily produce both industry and frugality, and these cannot but produce riches. But as riches increase, so will

pride, anger, and love of the world in all its branches... For the Methodists in every place grow diligent and frugal; consequently they increase in goods. Hence they proportionately increase in pride, in anger, in the desire of the flesh, the desire of the 4 eyes, and the pride of life. So, although the form of religion remains, the spirit is swiftly vanishing away..." (John Wesley in Weber, p.175).

Weber's analysis highlights the paradox of wealth accumulation and the Protestant ethic. While diligence and thrift often drive economic prosperity, amassing wealth can weaken the cultural and moral foundations supporting these values. As wealth grows, the essence of capitalism may diminish, challenging the principles of hard work and thrift. The values of frugality, industry, and self-discipline, crucial for economic growth, may lose significance as immediate consumption takes precedence over long-term investments. This shift poses a risk of economic decline, diverting focus from wealth accumulation to short-term gratification. Consequently, the economy becomes stagnant and depressive as people's subjective time discount rates rise with wealth accumulation.

## 2 A Model of Endogenous Preferences

The representative individual chooses consumption  $c_t$  and capital  $k_t$  to maximize the lifetime utility subject to the resource constraint:

$$\begin{aligned} \max_{\{c_t, k_t\}} \int_0^\infty [u(c_t) + v(k_t)] e^{-\int_0^t \rho(k_s) ds} dt \\ \text{s.t. } \dot{k}_t = f(k_t) - c_t \end{aligned}$$

The instantaneous utility functions  $u(\cdot)$  and  $v(\cdot)$  are both increasing and concave. The instantaneous time preference is assumed to be a function of capital. Let

$$F(k_t) = e^{-\int_0^t \rho(k_s) ds} \left[ u(f(k_t) - \dot{k}_t) + v(k_t) \right].$$

Then the Euler equation is given by

$$F_k = \frac{dF_k}{dt},$$

where

$$F_k = e^{-\int_0^t \rho(k_s) ds} [u'(c_t) f'(k_t) + v'(k_t)],$$

and

$$F_{\dot{k}} = -e^{-\int_0^t \rho(k_s) ds} u'(c_t),$$

and

$$\frac{dF_k}{dt} = -e^{-\int_0^t \rho(k_s) ds} u''(c_t) \dot{c}_t + e^{-\int_0^t \rho(k_s) ds} u'(c_t) \rho(k_t).$$

As a result

$$[u'(c_t) f'(k_t) + v'(k_t)] = -u''(c_t) \dot{c}_t + u'(c_t) \rho(k_t)$$

In sum, the Euler equation becomes

$$-u''(c_t) \dot{c}_t = v'(k_t) + u'(c_t) [f'(k_t) - \rho(k_t)]. \quad (1)$$

The steady-state is characterized by the following two equations

$$\begin{aligned} v'(k^*) + u'(c^*) [f'(k^*) - \rho(k^*)] &= 0, \\ f(k^*) - c^* &= 0. \end{aligned} \quad (2)$$

As a result

$$f'(k^*) = \rho(k^*) - \frac{v'(k^*)}{u'(c^*)} < \rho(k^*).$$

Consider a benchmark scenario where there is no capital in utility,  $v \equiv 0$ . Let  $\tilde{k}$  denote the corresponding steady-state capital. Then

$$f'(\tilde{k}) = \rho(\tilde{k}).$$

So we have

$$\frac{f'(\tilde{k})}{\rho(\tilde{k})} = 1 > \frac{f'(k^*)}{\rho(k^*)}$$

For the special case where  $\rho(k) \equiv \bar{\rho}$  is a constant, the model becomes Zou (1994). Capital in utility leads to higher steady-state capital than the benchmark case without capital in utility:

$$k^* > \tilde{k}.$$

In general,  $\rho(\cdot)$  is a function of capital  $k$ . The optimal dynamic path is characterized by the following two equations:

$$\begin{aligned} \dot{c} &= \left[ \frac{v'(k)}{u'(c)} + f'(k) - \rho(k) \right] \sigma(c)^{-1}, \\ \dot{k} &= f(k) - c, \end{aligned} \quad (3)$$

where  $\sigma(c) \equiv -\frac{u''(c)}{u'(c)}$ .

We will consider an AK production function,  $f(k) = Ak$ , and log-utilities,  $u(c) = \log(c)$  and  $v(k) = \theta \log(k)$ , where  $\theta$  represents the intensity of the spirit of capitalism. In this case,  $\sigma(c) \equiv -\frac{u''(c)}{u'(c)} = \frac{1}{c}$ . The optimal dynamic path becomes

$$\begin{aligned} \dot{c} &= c \left[ \theta \frac{c}{k} + A - \rho(k) \right], \\ \dot{k} &= Ak - c. \end{aligned} \quad (4)$$

## 2.1 Properties of the Steady State

The steady state can be characterized by

$$\begin{aligned} c^* &= Ak^*, \\ \rho(k^*) &= A(\theta + 1), \end{aligned} \quad (5)$$

provided that the marginal impatience level  $A(\theta + 1)$  is attainable. Linearize the dynamic system around the steady-state yields

$$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = \begin{bmatrix} A\theta & -Ak^* \left[ \frac{A\theta}{k^*} + \rho'(k^*) \right] \\ -1 & A \end{bmatrix} \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix}. \quad (6)$$

Let  $\mu_1$  and  $\mu_2$  be the characteristic roots of the linearized system, then

$$\begin{aligned} \mu_1 \mu_2 &= A^2 \theta - Ak^* \left[ \frac{A\theta}{k^*} + \rho'(k^*) \right] = -Ak^* \rho'(k^*), \\ \mu_1 + \mu_2 &= A(\theta + 1) > 0. \end{aligned}$$

- When  $\rho'(k) < 0$ , individual's patience increases with capital. Both  $\mu_1 \mu_2$  and  $\mu_1 + \mu_2$  are positive. The steady state is unstable. When both  $\mu_1$  and  $\mu_2$  are real and positive. The steady state is a source. The optimal consumption and capital will follow the direction of the disturbance and move away from the steady state. When both characteristic roots are complex and have positive real parts. The steady state is a spiral source. The optimal consumption and capital will keep expanding along the trajectory of a cycle. In sum, when patience increases with capital, there is neither a balanced growth path nor stationary steady state equilibrium.

- When  $\rho'(k) > 0$ , individual's patience decreases with capital. We have  $\mu_1\mu_2 < 0$ . The steady state is a saddle.
- When  $\rho(k)$  is not monotonic, the dynamic system might have multiple equilibria. The transition dynamics at each equilibrium might be either saddlepoint stable or unstable. According to Weber, it is reasonable to assume that  $\rho(k)$  is initially decreasing with  $k$  until capital reaches the threshold  $K$  and  $\rho$  reaches its minimum  $\rho(K)$ , after which  $\rho(k)$  starts to increase with  $k$ . Let  $k_L^*$  and  $k_H^*$  be the two solutions to  $\rho(k) = A(\theta + 1)$ , then

$$k_L^* < K < k_H^*.$$

- At the steady state  $(c^*, k^*) = (Ak_L^*, k_L^*)$ ,  $\rho'(k^*) < 0$  and thus  $\mu_1\mu_2 > 0$ . In this case, the steady state is unstable.
- At the steady state  $(c^*, k^*) = (Ak_H^*, k_H^*)$ ,  $\rho'(k^*) > 0$  and thus  $\mu_1\mu_2 < 0$ . In this case, the steady state is saddlepoint stable.

## 2.2 The Balanced Growth Path

When the marginal impatience level  $A(\theta + 1)$  is not attainable, the steady state does not exist. This can happen if the instantaneous preference is bounded from above such that  $\rho(k) \leq \bar{\rho} < A(\theta + 1)$  for all  $k$ . Assume that there exists a threshold  $K$ , such that  $\rho(k) = \bar{\rho}$  for all  $k \geq K$ , then we may show that there exists a balanced growth path. On the balanced growth path,

$$\frac{\dot{c}}{c} = \theta \frac{c}{k} + A - \bar{\rho} \equiv \gamma.$$

Then

$$\frac{c}{k} = \frac{1}{\theta} (\gamma + \bar{\rho} - A)$$

Also

$$\frac{\dot{k}}{k} = A - \frac{c}{k} = \gamma,$$

which implies

$$\gamma = A - \frac{\bar{\rho}}{1 + \theta} > 0.$$

The long-run growth rate for consumption and capital,  $\gamma$ , is higher, when the technological coefficient  $A$  is higher, or the intensity of the spirit of capitalism  $\theta$  is higher, or the upper bound of individual's impatience  $\bar{\rho}$  is lower.

## 2.3 Limit Cycles

Denote (4) as

$$\begin{aligned} \dot{c} &= f(c, k) = c \left[ \theta \frac{c}{k} + A - \rho(k) \right], \\ \dot{k} &= g(c, k) = Ak - c. \end{aligned}$$

Consider

$$\begin{aligned} f_c + g_k &= \frac{2\theta c}{k} + A - \rho(k) + A \\ &= 2 \left( A + \frac{\theta c}{k} \right) - \rho(k). \end{aligned}$$

If the time preference  $\rho(k) \leq 2A$  for all  $k$ , then  $f_c + g_k > 0$  for all  $(c, k)$  and the dynamic system has no closed trajectory. (Bendixson-Dulac Theorem)

**Remark:** when  $\rho(k) > 2A$  for some  $k$ , we may study a few special functional forms of  $\rho(k)$ ...(to be added)

### 3 Three Examples

#### 3.1 Example 1: Patience Increases with Capital

In this case, individual's instantaneous time preference  $\rho$  is a decreasing function in capital,  $\rho' < 0$ . The relationship between  $k^*$  and  $\tilde{k}$  is uncertain because the sign of the first derivative for  $f'/\rho$  is unknown. Consider a special functional form

$$\rho(k) = \bar{\rho} + B/k, \quad (7)$$

where the constant  $\bar{\rho} > 0$  and  $B > 0$ . Then

$$\rho'(k) = -B/k^2 < 0.$$

This special functional form captures the idea that the instantaneous time preference becomes a constant when capital grows to infinity. The optimal dynamic path becomes

$$\begin{aligned} \dot{c} &= c \left[ \theta \frac{c}{k} + A - \bar{\rho} - B/k \right], \\ \dot{k} &= Ak - c. \end{aligned}$$

The steady-state is characterized by

$$\begin{aligned} c^* &= Ak^*, \\ k^* &= \frac{B}{(\theta + 1)A - \bar{\rho}}, \end{aligned}$$

provided that

$$\bar{\rho} < (\theta + 1)A.$$

Clearly

$$k^* < \tilde{k} = \frac{B}{A - \bar{\rho}}.$$

In this case, capital in utility leads to lower steady-state capital than the benchmark case without capital in utility.

Linearize the dynamic system around the steady-state yields

$$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = \begin{bmatrix} A\theta & A(A - \bar{\rho}) \\ -1 & A \end{bmatrix} \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix},$$

whose characteristic roots  $\mu_1$  and  $\mu_2$  are given by

$$\mu_1 + \mu_2 = A(\theta + 1) > 0, \quad \mu_1\mu_2 = A^2\theta + A(A - \bar{\rho}) > 0.$$

Solve for the characteristic roots we have

$$\begin{aligned} \mu_1 &= \frac{A(\theta + 1) + \sqrt{A^2(\theta + 1)(\theta - 3) + 4A\bar{\rho}}}{2}, \\ \mu_2 &= \frac{A(\theta + 1) - \sqrt{A^2(\theta + 1)(\theta - 3) + 4A\bar{\rho}}}{2}. \end{aligned}$$

When

$$A(\theta + 1)(\theta - 3) + 4\bar{\rho} \geq 0,$$

the two characteristic roots are real and positive. The steady state is a source and unstable. The optimal consumption and capital will follow the direction of the disturbance and move away from the steady state. When

$$A(\theta + 1)(\theta - 3) + 4\bar{\rho} < 0,$$

Both characteristic roots are complex and have positive real parts. The steady state is a spiral source and unstable. The optimal consumption and capital will oscillate away from the steady state.

In sum, when patience increases with capital, there is neither a balanced growth path nor stationary steady state equilibrium.

### 3.2 Example 2: Patience Decreases with Capital

When an individual's instantaneous time preference  $\rho$  is an increasing function in capital,  $\rho' > 0$ ,  $\frac{f'(\cdot)}{\rho(\cdot)}$  is decreasing in  $k$ , and as a result  $k^* > \tilde{k}$ . In this case, capital in utility leads to higher steady-state capital than the benchmark case without capital in utility

The steady-state can be characterized by

$$\begin{aligned} c^* &= Ak^*, \\ \rho(k^*) &= A(\theta + 1), \end{aligned}$$

provided that the marginal impatience level  $A(\theta + 1)$  is attainable. Linearize the dynamic system around the steady-state yields

$$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = \begin{bmatrix} A\theta & -Ak^* \left[ \frac{A\theta}{k^*} + \rho'(k^*) \right] \\ -1 & A \end{bmatrix} \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix}.$$

Let  $\mu_1$  and  $\mu_2$  be the characteristic roots of the linearized system, then

$$\mu_1 \mu_2 = A^2 \theta - Ak^* \left[ \frac{A\theta}{k^*} + \rho'(k^*) \right] = -Ak^* \rho'(k^*) < 0.$$

This implies that the steady-state is saddle-point stable. Capital accumulation through the spirit of capitalism will stop at some point in the future, given that individual's marginal impatience can grow to a sufficiently high level.

If the instantaneous time preference is bounded from above such that

$$\rho(k) \leq \bar{\rho} < A(\theta + 1),$$

then the economy will exhibit endogenous growth. Further assume that there exists a threshold  $K$ , such that  $\rho(k) = \bar{\rho}$  for all  $k \geq K$ . On the balanced growth path,

$$\frac{\dot{c}}{c} = \theta \frac{c}{k} + A - \bar{\rho} \equiv \gamma.$$

Then

$$\frac{c}{k} = \frac{1}{\theta} (\gamma + \bar{\rho} - A)$$

Also

$$\frac{\dot{k}}{k} = A - \frac{c}{k} = \gamma,$$

which implies

$$\gamma = A - \frac{\bar{\rho}}{1 + \theta} > 0.$$

The long-run growth rate for consumption and capital,  $\gamma$ , is positively determined by the technological coefficient  $A$  and the intensity of the spirit of capitalism  $\theta$ .

### 3.3 Example 3: Patience as a Nonmonotone Function of Capital

Assume that the preference is characterized by

$$\rho(k) = \bar{\rho} - \frac{1}{(M - k)^2 + N}, \quad M > 0, \quad N > 0.$$

Then

$$\rho'(k) = 2 \frac{k - M}{[(M - k)^2 + N]^2} \begin{cases} < 0, & \text{if } k < M, \\ = 0, & \text{if } k = M, \\ > 0, & \text{if } k > M. \end{cases}$$

During the process of initial capital accumulation, the patience increases with capital. At the later stage of capital accumulation such that  $k > M$ , the patience becomes a decreasing function of capital. We maintain the AK technology. The optimal dynamic path is

$$\begin{aligned}\dot{c} &= c \left[ \theta \frac{c}{k} + A - \bar{\rho} + \frac{1}{(M - k)^2 + N} \right], \\ \dot{k} &= Ak - c.\end{aligned}$$

When

$$\bar{\rho} > A(\theta + 1),$$

the steady-state is characterized by

$$\begin{aligned}c^* &= Ak^*, \\ (k^* - M)^2 &= \frac{1}{\bar{\rho} - A(\theta + 1)} - N.\end{aligned}$$

Existence of a steady-state further requires  $\frac{1}{\bar{\rho} - A(\theta + 1)} > N$ , and then we have

$$A(\theta + 1) < \bar{\rho} < A(\theta + 1) + \frac{1}{N}.$$

In this case, there are two equilibria

$$k^* = M \pm \sqrt{\frac{1}{\bar{\rho} - A(\theta + 1)} - N}.$$

Linearize the optimal path around the steady-state to obtain

$$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = H \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix},$$

where

$$H = \begin{pmatrix} A\theta & \frac{2Ak^*(M - k^*)}{[(M - k^*)^2 + N]^2} - A^2\theta \\ -1 & A \end{pmatrix}.$$

The trace and the determinant of  $H$  are given by

$$\begin{aligned}tr(H) &= A(\theta + 1) > 0, \\ det(H) &= \frac{2Ak^*(M - k^*)}{[(M - k^*)^2 + N]^2}.\end{aligned}$$

- When  $k^* = M - \sqrt{\frac{1}{\bar{\rho} - A(\theta + 1)} - N} < M$ , the economy is in the regime of increasing patience and  $det(H) > 0$ . The steady state is unstable.
- When  $k^* = M + \sqrt{\frac{1}{\bar{\rho} - A(\theta + 1)} - N} > M$ , the economy is in the regime of decreasing patience and  $det(H) < 0$ . The steady state is a saddlepoint.