# Money, Growth, and Welfare in a Schumpeterian Model with Automation

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July 2024

**Abstract**

This paper explores the growth and welfare effects of monetary policy in a Schumpeterian vertical innovation model with automation. Money is introduced into the model via the cash-in-advance (CIA) constraints on consumption, production, automation and vertical innovation. We find that the relative strength of the cash constraints on automation and vertical innovations is crucial. If the CIA constraint is stronger (weaker) for automation, a higher nominal interest rate will lead to an increase (a decrease) in the amount of high-skilled labor allocated to vertical innovation. As a result, the automation level will decline (rise), but the vertical innovation and thereby aggregate economic growth will be faster (slower). We calibrate the model to the US economy and find a stronger cash constraint on automation. Our quantitative analysis shows that rising nominal interest rates are detrimental to automation but favorable to growth. In addition, higher nominal interest rates improve the welfare of different households and the aggregate welfare. As an empirical test, we find a significant, negative effect of the nominal interest rate on automation using cross-country panel data, consistent with our model prediction.

*JEL Classification*: O42 E42
*Keywords*: Monetary policy; Automation; Cash-in-advance; Schumpeterian model

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# 1 Introduction

The impact of monetary policy on economic growth and welfare is a classic topic in macroeconomics. One strand of literature has used the endogenous growth framework of Romer (1990) on expanding variety/horizontal innovation and Aghion and Howitt (1992) on quality improvement/vertical innovation (e.g., Marquis and Reffett, 1994; Funk and Kromen, 2010; Chu and Cozzi, 2014; He et al., 2023; Huang et al., 2023). Recently, researchers begin to highlight the role of automation in long-run growth (Acemoglu and Restrepo, 2018; Aghion et al., 2017; Berg et al., 2018; Prettner and Strulik, 2020; Hémous and Olsen, 2022; Chu et al., 2023; Jones and Liu, 2024). But how does monetary policy affect long-run growth and welfare when we also consider automation in an endogenous growth model? By answering this question, our study yields novel insights on the effects of monetary policy on long-run growth and welfare, as elaborated on below.

Both automation and innovation involve new technology: innovation (be it horizontal or vertical) does not change the capital-labor ratio, whereas automation enables machines to replace labor in production (Zeira, 1998; Acemoglu and Restrepo, 2018).[1] Hémous and Olsen (2022) incorporate automation (the replacement of low-skill workers with machines) into the Romer (1990) horizontal innovation (the creation of new products) model to analyze income inequality. Jones and Liu (2024) combine automation with vertical innovation (quality improvement) and find that automation determines the long-run growth. However, they assume that capital is fully depreciated, at which point it is no longer a stock variable. Chu et al. (2023) introduce automation into the Schumpeterian growth model (vertical innovation) of Aghion and Howitt (1992), studying the effects of R&D and automation subsidies. Our paper builds on Chu et al. (2023). We introduce money via the cash-in-advance (CIA) constraint on consumption (Clower, 1967; Lucas, 1980), production (Chu and Cozzi, 2014), vertical innovation (Chu and Cozzi, 2014; He et al., 2023) and automation to analyze the impact of monetary policy on economic growth and welfare.[2] We have three main findings.

First, we show that the relative strengths of the financing constraints on automation innovation and vertical innovation is critical (hereafter, automation and automation innovation are interchangeable). If the CIA constraint is stronger (weaker) for automation innovation, a higher nominal interest rate will lead to an increase (a decrease) in the amount of high-skilled labor allocated to vertical innovation, therefore, the automation level will decline (rise), the aggregate technology growth rate will increase (decrease) and economic growth will be faster (slower).

Second, we calibrate the model to the US economy and find a stronger financing constraint on automation innovation. Quantitative analysis shows that rising nominal interest rates are detrimental to automation but favorable to economic growth. In addition, we find that higher nominal interest rates always increase the welfare of capital owners and the aggregate welfare. When the CIA constraint of automation innovation is sufficiently strong, the welfare gains

[1] Automation is a special type of innovation. Therefore, there are three types of innovation: horizontal innovation (variety-expanding), vertical innovation (quality ladder/improvement) and automation/automation innovation. We use automation and automation innovation interchangeably in this paper.

[2] Although we use the Schumpeterian framework with automation as in Chu et al. (2023), we expect our insight that the different strengths of the CIA constraints on R&D and automation are crucial for monetary policy to impact growth and welfare to hold up in other endogenous models with automation.

from economic growth will dominate, and the welfare levels of high-skilled and low-skilled workers also rise.

Third, we empirically test the relationship between nominal interest rates and automation using cross-country panel data. The result shows a statistically significant negative correlation between the nominal interest rate and the growth rate of robot flows. The result holds up in instrumental variables regression that deals with the potential endogeneity of monetary policy. The robust evidence that a higher nominal interest rate is detrimental to automation is consistent with our model prediction. To the best of our knowledge, this is the first study to empirically test the correlation between nominal interest rates and automation.

Our paper adds to the existing literature on the effect of monetary policy on economic growth and welfare (Sidrauski, 1967; Dotsey and Sarte, 2000; Brunnermeier and Sannikov, 2016; Oikawa and Ueda, 2018; Moran and Queralto, 2018; Chu et al., 2019; He et al., 2023; Huang et al., 2023). Since Sidrauski (1967) proposed the superneutrality of money, many studies have challenged this conclusion. Chu and Cozzi (2014) show that an increase in the nominal interest rate will decrease economic growth in the Schumpeterian framework. He et al. (2023) find that nominal interest rates can promote economic growth when the spirit of capitalism is strong. Some researchers also find an inverted-U relation between the nominal interest rate and economic growth (Chu et al., 2019; Huang et al., 2023). Afonso and Forte (2023) incorporate automation (use of robots) into a directed technological change framework and find that an increasing in the nominal interest rate penalizes the economic growth. However, they neglect the innovation of automation technology. One of the contributions of our paper is that we show that the relationship between monetary policy and growth depends on the relative strengths of financing constraints on automation innovation and vertical innovation.

In addition, our paper is related to the literature on the relationship between monetary policy and automation. Fornaro and Wolf (2022) build a framework where monetary policy affects firms' automation decisions through firms' cost of capital relative to wages. They find that contractionary monetary policy may depress firms' use of automation technologies. However, they don't consider the firms' innovation activities. Our paper focuses on the endogenous growth model and finds that the impact of monetary policy on automation depends on the relative strengths of the financing constraints on automation innovation and vertical innovation. When automation innovation is subject to stronger financing constraints, nominal interest rates reduce the level of automation.

The rest of the paper proceeds as follows. Section 2 describes the monetary Schumpeterian model with automation. Section 3 presents theoretical results and intuitions. Section 4 conducts quantitative analysis. Section 5 provides supportive empirical evidence. Section 6 concludes.

# 2 A Monetary Schumpeterian Model with Automation

In this section, we introduce money into a Schumpeterian model with automation of Chu et al. (2023) to analyze the macroeconomic effect of monetary policy. We incorporate money demand via the CIA constraint on consumption (Clower, 1967; Lucas, 1980), production and vertical innovation (Chu and Cozzi, 2014). Moreover, we assume that automation innovation is also subject to the CIA constraint.

## 2.1 Household

There are three types of households in the economy: capital holders, high-skilled workers and low-skilled workers. Only capital owners make intertemporal consumption/saving choices, whereas workers consume all their income. Labor supply is inelastic for both types of workers and there is no population growth in the model.

The lifetime utility function of the capital owner is

$$U = \int_0^\infty e^{-\rho t} \ln c_t^k dt, \tag{1}$$

where $c_t^k$ denotes the consumption of the capital owner at time $t$, and $\rho$ is the discount rate. The asset-accumulation equation of the capital owner is

$$\dot{a}_t + \dot{k}_t + \dot{m}_t = r_t a_t + (R_t - \delta) k_t - c_t^k - \pi_t m_t + i_t b_t + \tau_t, \tag{2}$$

where $a_t$ represents the shares of the monopolistic firms, and its return rate is the real interest rate $r_t$. The capital owner accumulates capital $k_t$ and earns a rate of return $R_t$. The depreciation rate of the capital is $\delta$. To introduce money, we assume that the capital owner faces a CIA constraint: $c_t^k + b_t \leqslant m_t$. This constraint requires the capital owner to hold the real money balance $m_t$ to meet consumption and borrowing needs. $b_t$ is the amount of money borrowed from each capital owner by firms, including the intermediate goods producers and R&D sectors, to finance for their costs. The return of $b_t$ is $i_t$ and the cost of holding money is the inflation rate $\pi_t$. Each capital owner receives a lump-sum transfer $\tau_t$ from the government.

Solving the optimization problem of the capital owner by Hamiltonian methods, we can derive the optimality condition for consumption:

$$\frac{1}{c_t^k} = (1 + i_t) \mu_t, \tag{3}$$

where $\mu_t$ is the Hamiltonian multiplier of (2). Thus, the Euler equation of the capital owner is

$$\frac{\dot{c}_t^k}{c_t^k} = r_t - \rho. \tag{4}$$

The no-arbitrage condition between $a_t$ and $b_t$ is

$$i_t = r_t + \pi_t, \tag{5}$$

which is the Fisher equation. And $i_t$ is the nominal interest rate.

The no-arbitrage condition between $a_t$ and $k_t$ is

$$r_t = R_t - \delta. \tag{6}$$

The form of the lifetime utility function for high-skilled and low-skilled workers is the same as that for capital owners. Each high-skilled worker inelastically supplies one unit of labor and consumes all of the wage income:

$$c_t^h = w_t^h, \tag{7}$$

where, $w_t^h$ is the wage rate of the high-skilled worker. A low-skilled worker inelastically supplies $l$ unit of labor, and the consumption of a low-skilled worker is

$$c_t^l = w_t^l l, \tag{8}$$

where, $w_t^l$ is the wage rate of the low-skilled worker.

## 2.2 Structure of the Economy

As is standard in the endogenous growth framework, the final goods sector is competitive and uses only intermediate goods to produce final goods. In the monopolistic intermediate goods sector, manufacturing (or the production of intermediate goods) needs either unskilled labor (in unautomated industries) or capital (in automated industries). And finally in the R&D sector, entrepreneurs hire solely high-skilled workers to innovate. There are two types of R&D: R&D in vertical innovation and R&D in automation innovation (we refer to as vertical innovation and automation, respectively).

In Jones and Liu (2024), automated industries will always remain automated (i.e., they employ capital to produce intermediate goods) and only experience quality improvement (i.e., vertical innovation) and unautomated industries (using labor to produce) are being automated by automation innovation. The sole input of R&D (R&D in vertical innovation and R&D in automation innovation) is final goods (i.e., a lab equipment model). In equilibrium, the long-run growth rate equals the rate of automation (the share of automated industries approaches 1 but it never reaches 1).

By contrast, we follow Chu et al. (2023) to assume that all industries (both unautomated and automated ones) face vertical innovation. When an automated industry experiences an vertical innovation, it becomes unautomated again (i.e., it uses labor to produce intermediate goods again). When an unautomated industry experiences a quality improvement (i.e., vertical innovation), it remains unautomated. Automation innovation can only target those unautomated industries as in Jones and Liu (2024). Therefore, in our framework, there are two opposing effects on the share of automated industries: automation innovation increases the share and vertical innovation decreases the share. In equilibrium, the share of automated industries $\theta$ is a constant and the long-run growth rate equals the growth rate of vertical innovation (or the level of technology).

## 2.3 Final Goods Production

Final goods producers use the intermediate goods to produce in a perfectly competitive market. The production function is

$$y_t = \exp\left(\int_0^1 \ln x_t(j)dj\right), \tag{9}$$

where $x_t(j)$ denotes the intermediate good $j$, $j \in [0,1]$. By solving the profit maximization problem, we can derive the conditional demand function of the intermediate good $j$

$$x_t(j) = \frac{y_t}{p_t(j)}, \tag{10}$$

where $p_t(j)$ is the price of $x_t(j)$.

## 2.4 Intermediate Goods Production

There is a unit continuum of industries producing differentiated intermediate goods. If an industry is not automated, it can only use labor for production. However, if an industry is automated, it can also use capital for production.

### 2.4.1 Unautomated industry

In the unautomated industry, only the leaders with the most advanced technology produce; however, they will be replaced when the next automation or vertical innovation comes. The production function of the leader in unautomated industry $j \in [\theta, 1]$, where $\theta$ is the share of automated industries, is

$$x_t(j) = z^{n_t(j)} l_t(j), \tag{11}$$

where $z > 1$ is the step size of each quality improvement, $n_t(j)$ is the total number of vertical innovations in industry $j$, and $l_t(j)$ is the amount of low-skilled labor employed in industry $j$.

Since the monopoly power exists, the industry leader can set the price of the intermediate good above its marginal cost. And we assume the markup is a constant $\mu > 1$. The leader needs to finance $\zeta$ fraction of manufacturing expenditure by borrowing from capital owners. Take together, we obtain

$$p_t(j) = \mu \frac{(1 + \zeta i_t)\, w_t^l}{z^{n_t(j)}}. \tag{12}$$

Combining (10) and (12), we can derive the profit of the unautomated industry $\pi_t^l(i)$:

$$\pi_t^l(j) = \frac{\mu - 1}{\mu} y_t. \tag{13}$$

And the low-skilled labor income of industry $j$ is

$$w_t^l l_t(j) = \frac{1}{\mu\,(1 + \zeta i_t)} y_t. \tag{14}$$

### 2.4.2 Automated industry

In the automated industry, the sole input of production is capital. The industry leaders will be replaced when the next vertical innovation arrives. In addition, the success of vertical innovation will lead to a comparative advantage in labor, so that the automated industry will become unautomated.

The production function of the current industry leader in the automated industry is

$$x_t(j) = \frac{A}{Z_t} z^{n_t(j)} k_t(j), \text{ where } j \in [0, \theta_t], \tag{15}$$

where the parameter $A$ measures the productivity difference between using labor and using capital, $Z_t$ is the aggregate technology, $k_t(j)$ is the capital input in industry $j$, and $\theta_t$ is the fraction of industries that are automated. A rise in $Z_t$ has a negative effect on the marginal output of capital, which reduces the adaptability of existing physical capital. If the capital-based production has a lower cost than the labor-based production, then all of the automated industries will use capital to produce intermediate goods. To ensure this result, we impose a technical condition that will be specified in Section 2.4.3.

We assume that the leader in automated industry also needs to finance $\beta$ fraction of the wage payments with cash borrowed from capital owners. Now the price of the intermediate good $j$ is

$$p_t(j) = \mu \frac{(1+\beta i_t) Z_t R_t}{A z^{n_t(j)}}, \text{ where } j \in [0, \theta_t]. \tag{16}$$

Similarly, the profit of the automation industry $\pi_t^k(j)$ is

$$\pi_t^k(j) = \frac{\mu - 1}{\mu} y_t. \tag{17}$$

And the capital rental payment of industry $j$ is

$$R_t k_t(j) = \frac{1}{\mu(1+\beta i_t)} y_t. \tag{18}$$

### 2.4.3 Technology Adoption

As discussed, there are two kinds of technological advances (i.e., two types of R&D/innovation). One is automation innovation, which enables capital to replace labor in production; the other is vertical innovation, which leads to quality improvement.

In order for automation innovation to be adopted, we need a technical condition that makes it cheaper to produce with capital than with labor. Therefore, this first technical condition is $(1+\beta i)Z_t R_t/(A z^{n_t(j)}) < (1+\zeta i) w_t^l / z^{n_t(j)}$. Vertical innovation can increase the productivity level $z^{n_t(j)}$ in both automated and unautomated industries. However, if the new vertical innovation is adopted, the automated industry will become an unautomated one where labor has a comparative advantage. Therefore, we require the second technical condition $(1+\zeta i) w_t^l / z^{n_t(j)+1} < (1+\beta i) R_t / (A z^{n_t(j)})$ to ensure that vertical innovations are adopted. Thus, the combined technical condition is

$$(1+\zeta i) w_t^l > (1+\beta i) \frac{Z_t R_t}{A} > (1+\zeta i) \frac{w_t^l}{z}, \tag{19}$$

which supports a cycle of automation and vertical innovation.

**Lemma 1** *The steady-state equilibrium condition for the automation-vertical innovation cy-*

*cle is*

$$\frac{1}{z} < \left[\frac{\mu(1+\beta i)}{A}(g+\rho+\delta)\right]^{\frac{1}{1-\theta}} < 1. \tag{20}$$

**Proof.** See Appendix A. ■

## 2.5 Innovation Process

The R&D sectors innovate by employing high-skilled labor. As discussed, vertical innovations for quality improvement target all industries (both automated and unautomated ones), whereas automation innovations only target unautomated industries.

Denote $v_t^l(j)$ as the value of the monopolistic firm in the unautomated industry $j$. Since $\pi_t^l(j) = \pi_t^l$, we obtain $v_t^l(j) = v_t^l$. The no-arbitrage condition of $v_t^l$ is

$$r_t = \frac{\pi_t^l + \dot{v}_t^l - (\lambda_t + \alpha_t)\, v_t^l}{v_t^l}, \tag{21}$$

where $\lambda_t$ is the arrival rate of the vertical innovation, and $\alpha_t$ is the arrival rate of the next automation innovation. This equation means that the return from investing in $v_t^l$ is equal to the sum of monopolistic profit, capital gain and expected capital loss due to creative destruction and automation. The innovation in the unautomated industry is at risk of being replaced by both automation and vertical innovations at the same time.

Similarly, denote $v_t^k$ as the value of the monopolistic firm in the automated industry $j$. The no-arbitrage condition of $v_t^k$ is

$$r_t = \frac{\pi_t^k + \dot{v}_t^k - \lambda_t v_t^k}{v_t^k}. \tag{22}$$

Different from $v_t^l$, only the new vertical innovation can replace the monopolistic leader in the automation industry.

### 2.5.1 Vertical innovation

The arrival rate of the vertical invention in industry $j$ is:

$$\lambda_t(j) = \varphi_t h_{r,t}(j), \text{where } \varphi_t = \varphi h_{r,t}^{\epsilon-1}, \tag{23}$$

where $\varphi_t$ is the innovation efficiency parameter, $h_{r,t}(j)$ is the high-skilled labor hired and $h_{r,t}$ is the aggregate high-skilled labor undertaking vertical innovation. Since $h_{r,t}(j) = h_{r,t}$, we obtain

$$\lambda_t = \int_0^1 \lambda_t(j)dj = \varphi h_{r,t}^{\epsilon}. \tag{24}$$

The parameter $\epsilon \in (0, 1)$ captures congestion externalities as in Jones and Williams (2000), which we refer to as the stepping on toes effect.

We assume that vertical innovation also needs to finance part of the R&D cost with cash.

Thus, the free-entry condition of vertical innovation is

$$\lambda_t v_t^l = (1 + \gamma i_t)\, w_t^h h_{r,t}, \tag{25}$$

where $\gamma$ is the share of wage payment paid by cash borrowed from capital owners. The left-hand-side (LHS) of (25) is the expected return of vertical innovation, and the right-hand-side (RHS) is the total cost. Substituting (24) into (25) yields

$$\varphi v_t^l = (1 + \gamma i_t)\, w_t^h h_{r,t}^{1-\epsilon}. \tag{26}$$

### 2.5.2 Automation innovation

The arrival rate of the automation innovation follows a similar structure as in (23):

$$\alpha_t(j) = \phi_t h_{at}(j), \text{and } \phi_t = \phi\,(1 - \theta_t)\, h_{at}^{\epsilon-1}, \tag{27}$$

where $\phi$ is the automation innovation efficiency parameter, $h_{at}(j)$ is the high-skilled labor hired for developing automation technology in unautomated industry $j \in [\theta_t, 1]$ and $h_{at}$ is the aggregate value of $h_{at}(j)$. As we can see, when the fraction of automated industries rises, there are fewer and fewer industries that are not automated, and the arrival rate of the automation technology decreases. This setting reflects the reality that one automates the easy things first, so the industries which are automated last are often the most difficult to automate.

Since $h_{at}(j) = h_{at}/(1 - \theta_t)$, we derive

$$\alpha_t = \int_{\theta_t}^{1} \alpha_t(j)dj = \phi h_{at}^{\epsilon}. \tag{28}$$

The free entry condition of the R&D sector developing automation innovation is

$$\alpha_t v_t^k = (1 + \kappa i_t)\, w_t^h \frac{h_{a,t}}{1 - \theta_t}, \tag{29}$$

where $\kappa$ is the share of the wage payment that needs to be covered by cash borrowed. Combining (28) and (29) yields

$$\phi\,(1 - \theta_t)\, v_t^k = (1 + \kappa i_t)\, w_t^h h_{at}^{1-\epsilon}. \tag{30}$$

## 2.6 Monetary authority

The monetary authority determines the supply of nominal money $M_t$. By the definition of the real money balance $m_t = M_t/P_t$, the growth rate of the nominal money is

$$\frac{\dot{M}_t}{M_t} = \frac{\dot{m}_t}{m_t} + \pi_t. \tag{31}$$

The government transfers the seigniorage revenue to currency holders in the form of a lump-sum transfer. Using (31) yields

$$\tau_t = \frac{\dot{M}_t}{p_t} = \dot{m}_t + \pi_t m_t. \tag{32}$$

On the balanced growth path (BGP), combining the CIA constraint and the Euler equation (4) yields

$$\frac{\dot{m}_t}{m_t} = \frac{\dot{c}_t^k}{c_t^k} = r_t - \rho. \tag{33}$$

By Fisher equation (5), (31) and (33), we can derive the relationship between the nominal interest rate and the growth rate of money supply

$$i_t = \frac{\dot{M}_t}{M_t} + \rho. \tag{34}$$

Thus, the monetary authority can choose either the nominal interest rate or the growth rate of money supply as the policy instrument. When one is determined, the other is also pinned down by (34). Since firms' behavior is mainly affected by the nominal interest rate on their financing costs, we choose the nominal interest rate as the policy instrument.

## 2.7 Aggregation

The aggregate technology $Z_t$ is defined as

$$Z_t \equiv \exp\left(\int_0^1 n_t(j)dj \ln z\right) = \exp\left(\int_0^t \lambda_s ds \ln z\right), \tag{35}$$

where the last equality uses the law of large numbers. Differentiating both sides of (35) with respect to $t$ yields the growth rate of the aggregate technology

$$g_{Zt} = \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z. \tag{36}$$

Substituting (11) and (15) into the final goods production function (9), and using the definition of $Z_t$, we can derive the aggregate production function:

$$y_t = \left(\frac{Ak_t}{\theta_t}\right)^{\theta_t} \left(\frac{Z_t l}{1-\theta_t}\right)^{1-\theta_t}, \tag{37}$$

which is a Cobb-Douglas form. The evolution of the automation level $\theta_t$ is

$$\dot{\theta}_t = \alpha_t (1 - \theta_t) - \lambda_t \theta_t. \tag{38}$$

Successful automation innovation makes $\theta_t$ go up, however, the next vertical innovation makes the automated industry turn back into an unautomated one, decreasing $\theta_t$.

The law of motion of capital is

$$\dot{k}_t = y_t - c_t - \delta k_t, \tag{39}$$

where $c_t \equiv c_t^k + c_t^h + c_t^t$. And the capital income share and low-skilled labor income share are

$$\frac{R_t k_t}{y_t} = \frac{\theta_t}{\mu}\frac{1}{1+\beta i_t}, \tag{40}$$

$$\frac{w_t^l l}{y_t} = \frac{1-\theta_t}{\mu}\frac{1}{1+\zeta i_t}. \tag{41}$$

## 2.8 Decentralized Equilibrium

*Given a nominal interest $i_t$ and an initial conditions of $Z_0$, the equilibrium is a time path of prices $\left\{p_t(j), r_t, R_t, i_t, w_t^l, w_t^h, v_t^l, v_t^k\right\}$ and allocations $\left\{c_t^k, c_t^l, c_t^h, a_t, k_t, m_t, b_t, y_t, x_t(j), k_t(j), l_t(j), h_{rt}(j), h_{at}(j)\right\}$ such that:*

- capital owners maximize utility taking prices $\{i_t, r_t, R_t\}$ as given;
- final-goods producers maximize profit taking $\{p_t(j)\}$ as given;
- each monopolistic intermediate-goods sector chooses $\{k_t(j), l_t(j), p_t(j)\}$ to maximize profit taking prices $\left\{w_t^l, R_t\right\}$ as given;
- R&D firms choose $\{h_{rt}(j), h_{at}(j)\}$ to maximize expected profit taking prices $\left\{w_t^h, v_t^l, v_t^k\right\}$ as given;
- capital market clears such that $\int_0^{\theta_t} k_t(j)\,dj = k_t$;
- low-skilled labor market clears such that $\int_{\theta_t}^1 l_t(j)\,dj = l$;
- high-skilled labor market clears such that $\int_0^1 h_{rt}(j)dj + \int_{\theta_t}^1 h_{at}(j)dj = 1$;
- final goods market clears, i.e., $y_t = c_t^k + c_t^l + c_t^h + \dot{k}_t + \delta k_t$;
- the value of monopolistic firms adds up to the value of households' assets such that $\int_0^{\theta_t} v_t^k(j)dj + \int_{\theta_t}^1 v_t^l(j)dj = a_t$;
- the amount of money borrowed by intermediate-goods firms and R&D sectors is $\zeta w_t^l l + \beta R_t k_t + \gamma w_t^h h_{rt} + \kappa w_t^h h_{at} = b_t$.

# 3 Main Theoretical Results and Intuitions

On the BGP, both arrival rates are constants. From (29) and (41), the growth rate of $v_t^l$ is equal to the growth rate of the output $g$. The final goods market clearing condition means the growth rate of output equals the growth rate of $c_t^k$. Thus, using the Euler equation (4) and (21), we can derive

$$v_t^l = \frac{\pi_t^l}{\rho + \lambda_t + \alpha_t}. \tag{42}$$

Similarly, the value of $v_t^k$ satisfies

$$v_t^k = \frac{\pi_t^k}{\rho + \lambda_t}. \tag{43}$$

Substituting (42) and (43) into the corresponding free entry condition (26) and (30) yields

$$\frac{\varphi \pi_t^l}{\rho + \lambda_t + \alpha_t} = (1 + \gamma i_t)\, w_t^h h_{r,t}^{1-\epsilon}, \tag{44}$$

$$\frac{\phi (1 - \theta_t)\, \pi_t^k}{\rho + \lambda_t} = (1 + \kappa i_t)\, w_t^h h_{at}^{1-\epsilon}. \tag{45}$$

Combining (44) and (45), we can derive

$$\frac{\varphi(\rho + \lambda_t)}{\phi (1 - \theta_t)(\rho + \lambda_t + \alpha_t)} = \frac{1 + \gamma i_t}{1 + \kappa i_t} \left(\frac{h_{rt}}{h_{at}}\right)^{1-\epsilon}. \tag{46}$$

On the BGP, the fraction of automation is a constant. Using (38) and $\dot{\theta}_t = 0$, we can obtain

$$\theta = \frac{\alpha}{\lambda + \alpha}. \tag{47}$$

Substituting (47), the arrival rate (24) and (28) into (46), we can derive

$$\frac{1 + \kappa i}{1 + \gamma i}\left[\frac{\varphi}{\phi} + \left(\frac{1 - hr}{hr}\right)^{\epsilon}\right] = \left(\frac{h_r}{1 - h_r}\right)^{1-\epsilon} + \left(\frac{h_r}{1 - h_r}\right)^{1-2\epsilon} \frac{\phi}{\varphi + \rho / h_r^{\epsilon}}. \tag{48}$$

This equation pins down the value of aggregate high-skilled labor conducting vertical innovation.

**Proposition 1** *The balanced growth path exists and is unique if* $\epsilon \leq \frac{1}{2}$ .

**Proof.** The LHS of (48) is a decreasing function of $h_r$ because

$$\frac{\partial LHS}{\partial h_r} = -\frac{1 + \kappa i}{1 + \gamma i}\left(\frac{1 - hr}{hr}\right)^{\epsilon - 1} \frac{\epsilon}{h_r^2} < 0. \tag{49}$$

Derivation of the RHS of (48) with respect to $h_r$ yields

$$\frac{\partial RHS}{\partial h_r} = \frac{h_r^{-\epsilon}}{(1 - h_r)^{2-\epsilon}}\left[1 - \epsilon + \phi h_r^2 \left(\frac{hr}{1 - hr}\right)^{-\epsilon} \frac{(1 - 2\epsilon)(\varphi h_r^{\epsilon} + \rho) + \rho\epsilon(1 - h_r)}{(\varphi h_r^{\epsilon} + \rho)^2}\right]. \tag{50}$$

From (50), $\epsilon \leq 1/2$ is a sufficient condition for the RHS of (48) to rise monotonically in $h_r$.

As $h_r \to \infty$, the LHS of the equation tends to infinity, while the RHS of the equation starts at the origin. Thus, the two functions corresponding to the two sides of (48) have only one intersection at some positive value of $h_r$. ■

Then, we analyze the impact of monetary policy on automation and economic growth. From the aggregate production function (37), we can derive the growth rate of the economy on balanced growth path

$$g = g_z = \varphi h_r^{\epsilon} \ln z. \tag{51}$$

(51) shows that in the long run, the economic growth depends on the aggregate technology growth. A higher growth rate of the aggregate technology increases productivity in all intermediate goods industries through quality improvements, which leads to a higher growth rate of the economy. On the BGP, automation does not affect the growth rate of output, but it does affect the level of output.

Substituting (24) and (28) into (47), we can rewrite the automation level on the BPG as

$$\theta = \frac{\phi h_a^{\epsilon}}{\phi h_a^{\epsilon} + \varphi h_r^{\epsilon}} = \left[1 + \frac{\varphi}{\phi}\left(\frac{h_r}{(1-h_r)}\right)^{\epsilon}\right]^{-1}. \tag{52}$$

It can be seen that both the automation level and the growth rate of the economy depend on $h_r$, so we start by analyzing how the nominal interest rate affects the high-skilled labor hired for vertical innovation ($h_r$).

**Proposition 2** *The impact of the nominal interest rate on the two R&D sectors' labor allocation depends on the relative strengths of the CIA constraints. If the CIA constraint is stronger for R&D in automation innovation, a higher nominal interest rate leads to an increase in the amount of high-skilled labor allocated to vertical innovation; if the CIA constraint is stronger for R&D in vertical innovation, a higher nominal interest rate leads to a decrease in the amount of high-skilled labor hired for vertical innovation.*

**Proof.** As we can see from (48), only the LHS of the equation is affected by the nominal interest rate. Derivation of the LHS of (48) with respect to $i$ yields

$$\frac{\partial \text{ LHS}}{\partial i} = \left[\frac{\varphi}{\phi} + \left(\frac{1-hr}{hr}\right)^{\epsilon}\right] \frac{\kappa - \gamma}{(1+\gamma i)^2} \begin{cases} \geqslant 0, & \kappa > \gamma \\ < 0, & \kappa < \gamma \end{cases}.$$

If $\kappa > \gamma$, the LHS of (48) is an increasing function of the nominal interest rate. As we can see in Figure 1, when the nominal interest rate rises, the $L$ curve (capturing the LHS of (48)) moves up to the position of the $L_1$ curve and intersects the $R$ curve (showing the RHS of (48)) at the new intersection point $F$. Compared with the old intersection $E$, the new point $F$ has higher value of $h_r$. If $\kappa < \gamma$, the LHS of (48) is decreasing with the nominal interest rate, so the $L$ curve shifts downward to the position of the $L_2$ curve, and the high-skilled labor hired for vertical innovation decreases on the new BGP.

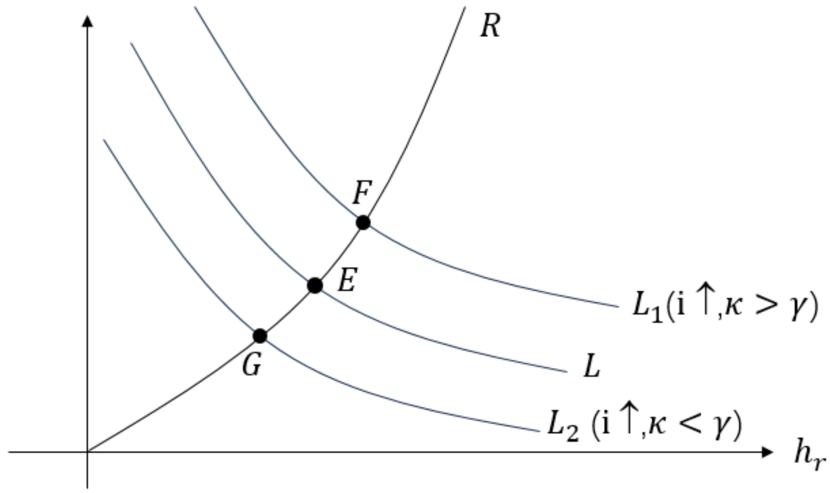


Figure 1. Effect of nominal interest rates on high-skilled labor allocated to vertical innovation

■

The intuition is as follows. A higher nominal interest rate raises costs in both R&D sectors, especially in the R&D sector with a stronger CIA constraint. As a result, high-skilled labor will shift away from the sector with a stronger financing constraint to the sector with a weaker financing constraint when the nominal interest rate rises.

**Proposition 3** *When the automation innovation facing a stronger (weaker) CIA constraint, the automation level will decline (rise), the aggregate technology growth rate will increase (decrease) and economic growth will be faster as the nominal interest rate rises (declines).*

**Proof.** According to (52), we can derive

$$\frac{\partial \theta}{\partial h_r} = -\frac{\varphi}{\phi}\left(\frac{h_r}{(1-h_r)}\right)^{\epsilon-1}\left[1+\frac{\varphi}{\phi}\left(\frac{h_r}{(1-h_r)}\right)^{\epsilon}\right]^{-2}\frac{\epsilon}{(1-h_r)^2} < 0, \tag{53}$$

$$\frac{\partial \theta}{\partial i} = \frac{\partial \theta}{\partial h_r}\cdot\frac{\partial h_r}{\partial i}. \tag{54}$$

According to (51), we can derive

$$\frac{\partial g}{\partial i} = \frac{\partial g_Z}{\partial i} = \varphi \epsilon h_r^{\epsilon-1}\frac{\partial h_r}{\partial i}\ln z. \tag{55}$$

Using the conclusion in Proposition 2, if $\kappa > \gamma$, we have $\partial h_r/\partial i > 0$, then $\partial \theta/\partial i < 0$, $\partial g_z/\partial i > 0$, and $\partial g/\partial i > 0$. Similarly, if $\kappa < \gamma$, we have $\partial h_r/\partial i < 0$, then $\partial \theta/\partial i > 0$, $\partial g_z/\partial i < 0$, and $\partial g/\partial i < 0$. ■

If the R&D sectors conducting automation innovation facing a stronger CIA constraint, a rise in the nominal interest rate will have a more adverse impact on such firms. So that high-skilled labor will shift to vertical innovation sectors with relatively weaker financing constraints, which will have two effects: on the one hand, the automation level will decrease; on the other hand, an increase in highly skilled labor hired for vertical innovation will accelerate aggregate technological progress, leading to faster economic growth.

# 4 Quantitative Analysis

In this section, we calibrate the model to match the U.S. economy. Then we simulate how changes in nominal interest rates affect the level of automation and economic growth. Finally, we quantitatively analyze the welfare impact of changes in nominal interest rates.

## 4.1 Calibration

The calibration is done in two steps. First, we pin down some parameters with reference to the existing literature and the US data. We set the discount rate $\rho = 0.04$ and the parameter of congestion externalities $\epsilon = 0.5$ as in Chu et al. (2023). According to Basu (1996) and Norrbin (1993), the step size of innovation is in the range of 1.05 to 1.4, and we set $z = 1.17$. We assume the value of markup is equal to the value of step size, so $\mu = 1.17$. We set the strength of CIA constraint on vertical innovation $\gamma = 0.05$ as in He et al. (2023). As we can see in Section 3, only the CIA constraints on R&D sectors affect the rate of economic growth, so for simplicity, we set the CIA constraints on intermediate goods producers $\zeta = \beta = 0.05$. We use (34) to calibrate the value of the nominal interest rate. The average annual growth rate of broad money in the US during 1961-2019 is 7.27%, which gives the nominal interest rate $i = 0.1127$ using (34).

Second, we jointly calibrate four parameters $\{\varphi, \phi, \kappa, \delta\}$ to match four moments in the US data: (1) the annual growth rate of real GDP per capita from 1961 to 2019, which is 2%; (2) the capital share, which is 35%; (3) the average labor share from 1961 to 2019, which is 61.6%; (4) the capital-output ratio, which is 4.[3]

We use equations (25), (25), (29) and (6) to jointly calibrate the four undetermined parameters. The results are reported in Table 1.

Table 1: Parameters and moments in the joint calibration

| Parameter | Value | Joint Targets | Data | Model |
|---|---|---|---|---|
| Vertical innovation efficiency $\varphi$ | 0.1620 | Growth rate | 0.02 | 0.02 |
| Automation efficiency $\phi$ | 0.1443 | Capital share | 0.35 | 0.35 |
| CIA constrain on automation $\kappa$ | 0.2368 | Labor share | 0.616 | 0.616 |
| Deprecation rate $\delta$ | 0.0275 | Capital-output ratio | 4 | 4 |

The calibration result $\kappa > \gamma$ shows that the CIA constraint on automation innovation is stronger than that on vertical innovation, which is also consistent with the findings of the empirical evidence in Section 5. To ensure that Lemma 1 holds, the productivity difference parameter $A$ should take a value between 0.1030 and 0.1129, and we set $A = 0.11$.

[3] We use the data from Federal Reserve Economic Data (FRED) and Penn World Table 10.01 (PWT10.01) to obtain the moments.

## 4.2 Growth Implications

The effect of nominal interest rates on highly skilled labor for vertical innovation is shown in Figure 2. Based on the calibration ($\kappa = 0.2368$), we find that the R&D sectors conducting automation innovation are subject to stronger CIA constraints, so that higher nominal interest rates cause the R&D costs of those sectors to rise more rapidly. Therefore, high-skilled labor shifts to the vertical innovation sector. When the CIA constraint on automation innovation becomes weaker, e.g. $\kappa = 0.1$, the reallocation effect of high-skilled labor diminishes, so that the increase in high-skilled labor hired for vertical innovation is smaller when the nominal interest rate increases. If $\kappa = \gamma = 0.05$, nominal interest rates have no effect on the allocation of highly skilled labor in the R&D sectors.

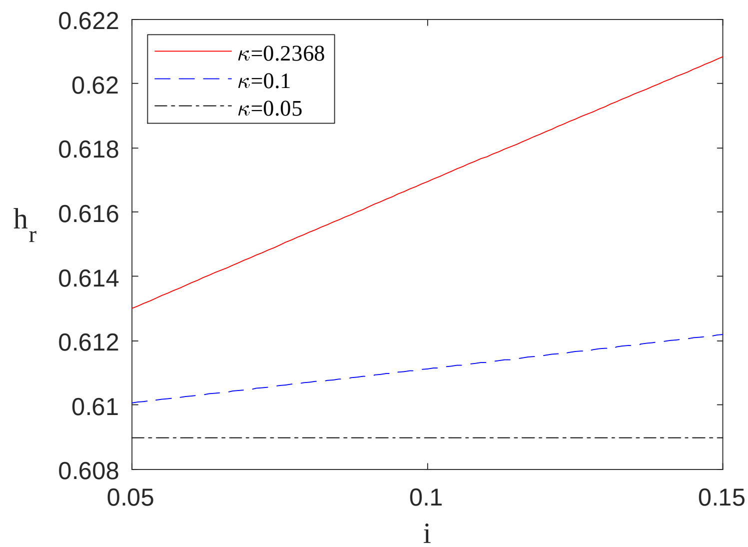


Figure 2. Effect of nominal interest rates on high-skilled labor allocated to vertical innovation

Then we analyze how monetary policy affects automation level and economic growth. Figure 3 presents the results. According to Figure 3, a rise in nominal interest rates leads to a decrease in the arrival rate of automation innovation and an increase in the arrival rate of vertical innovation through the reallocation of high-skilled labour. Therefore, higher nominal interest rates decrease the level of automation, and increase the growth rate of aggregate technology as well as that of the economy.

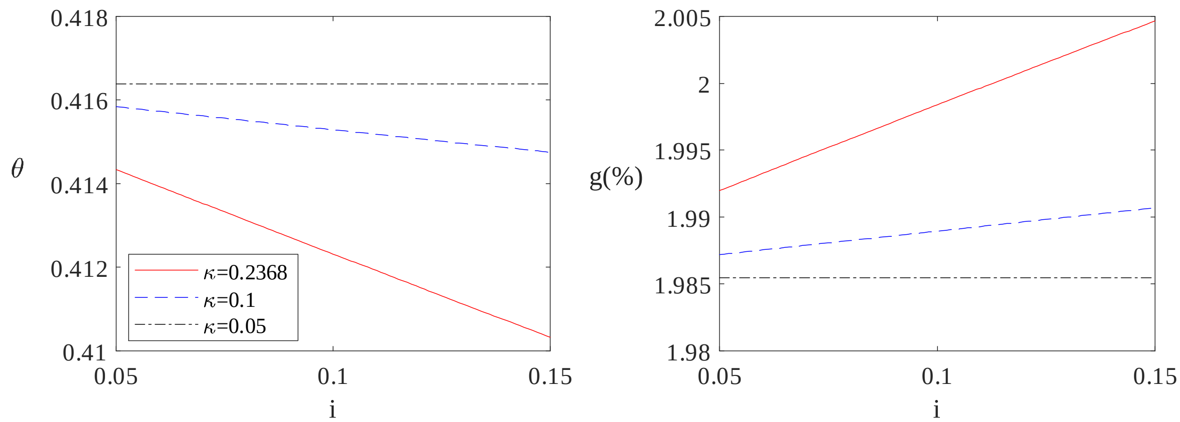


Figure 3. Effect of nominal interest rates on automation and aggregate technology growth

When $\kappa$ is smaller (but still larger than $\gamma$), i.e., the strength of the CIA constraint on automation innovation is slightly stronger than that on vertical innovation, the impact of nominal interest rates on automation and economic growth weakens. When $\kappa = \gamma$, nominal interest rates have no effect on the automation level and long-run growth.

## 4.3 Welfare Analysis

In this section, we analyze the impact of monetary policy on individual and aggregate welfare levels.

On the BGP, rewriting the lifetime utility function of household $f$ yields the welfare function:

$$\begin{aligned} U^f &= \int_0^\infty e^{-\rho t}(\ln c_0^f + gt)dt \\ &= \frac{1}{\rho}\ln c_0^f + \frac{g}{\rho^2}, \quad f \in \{k, h, l\} \end{aligned} \tag{56}$$

where the second equation requires the use of integration by parts. We define the aggregate welfare as the sum of the welfare of the three types of households:

$$U = U^k + U^h + U^l$$

Figure 4 illustrates the quantitative impact of nominal interest rates on welfare. Using the calibrated parameters we find that higher nominal interest rates increase the welfare of each individual and the aggregate. However, this result varies with different values of $\kappa$. When $\kappa = 0.1$, welfare of high-skilled workers will fall with nominal interest rates, and when $\kappa = 0.05$, higher nominal interest rates lead to lower welfare of both high-skilled and low-skilled workers.

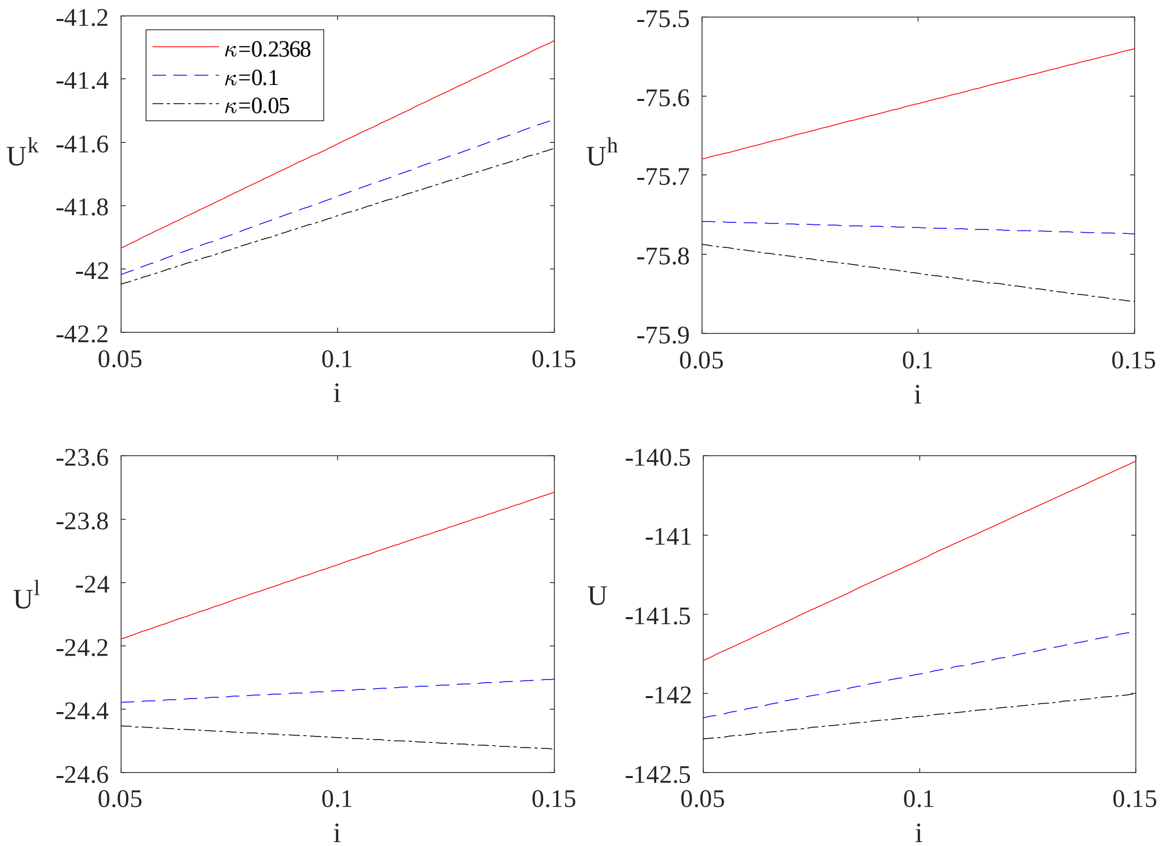


Figure 4. Effect of nominal interest rates on economic growth

The key intuitions of this finding are as follows. Nominal interest rates have three effects on welfare. First, nominal interest rates improve welfare by increasing interest income from borrowing. Second, nominal interest rates improve welfare by promoting economic growth. Since financing constraints for R&D on automation innovation are stronger, rising nominal interest rates lead to an increase in the amount of high-skilled labor devoted to vertical innovation. More input in vertical innovation will accelerate economic growth by promoting technological progress, which improves the welfare of each individual. Third, nominal interest rates have a negative impact on welfare by pulling down initial wages. Rising nominal interest rates make production of intermediate goods and R&D activity more costly, thereby decreasing the incentives to produce and innovate. Since labor supply is inelastic, the wage of high-skilled and low-skilled workers will fall, leading to a lower level of initial consumption and welfare.

In particular, when $\kappa = \gamma$, an increase in the nominal interest rate has no effect on economic growth, so the second effect disappears. The first effect increases the welfare of capital owners, and the third effect makes the welfare of high-skilled and low-skilled workers fall. Overall, the first positive effect dominates and, therefore, aggregate welfare rises.

The second positive effect increases gradually as $\kappa$ rises. When $\kappa = 0.1$, the welfare of low-skilled workers rises with nominal interest rates since the second positive effect dominates the third negative effect. However, the welfare of high-skilled workers is still declining because the CIA constraints are stronger in the R&D sectors than in the intermediate goods sector, so the decline in wages for high-skilled worker is greater and the third negative effect remains stronger than the second positive effect. When $\kappa$ is sufficiently large, the second positive effect dominates, i.e., the welfare gains from economic growth exceed the welfare losses from the initial wage decline. Therefore, the welfare of all households rises, so does the aggregate welfare.

# 5 Empirical Evidence

Our model has many testable predictions. In this section, we use the cross-country panel data to test the relationship between the monetary policy and automation.

Proposition 3 predicts that when the CIA constraint on automation innovation is stronger, a rise in the nominal interest rate is detrimental to automation, while it favours automation when the CIA constraint on vertical innovation is stronger. To test how nominal interest rates affect automation, we run the following panel data regression:

$$\ln Robot_{m,t} = \eta_0 + \eta_1 i_{m,t} + \eta_{other} Controls_{m,t} + \sigma_t + \chi_m + \varepsilon_{m,t} \tag{57}$$

where $Robot_{m,t}$ represents the robot flow per unit of labor of country $m$ in year $t$, $i_{m,t}$ is the nominal interest rate and $\eta_1$ is the coefficient of our interest. Control variables $Controls_{m,t}$ include the natural logarithm of the real GDP per capita (*gdp*), the natural logarithm of total population (*pop*), the share of net exports in GDP (*trade*), the share of investment in GDP (*invest*) and the share of government consumption in GDP (*gov*). $\sigma_t$ and $\chi_m$ stand for year and country fixed effects, respectively. $\varepsilon_{m,t}$ is the error term.

There are three sources of data: (1) the data on robots are from the International Federation of Robotics (IFR); (2) nominal interest rates are calculated using the sum of the money growth rate and the discount rate as in (34); we use the annual broad money growth data from the World Bank; (3) the data on the control variables are from the Penn World Table 10.01 (PWT 10.01). To avoid the influence from outliers, we drop all observations with nominal interest rates greater than 50% and observations with missing values. In sum, we obtain 610 observations for 39 countries from 1993 to 2019. Table 2 presents the summary statistics of our data.

Table 3 reports the baseline regression results for the impact of nominal interest rates on robot flows. All the regressions show a significantly negative relationship between the nominal interest rate and the robot flow. Column (1) of Table 3 reports the estimates of equation (57) using all observations with nominal interest rates below 50%. The result shows that a 1% rise in the nominal interest rate will decrease the growth of the amount of the robot flow by 1.776%, which is statistically significant at the 5% level. When we use samples with lower nominal interest rates, the coefficient of the nominal interest rate is robustly negative, as shown in columns (2)-(4). This finding is consistent with the analysis of the calibrated

Table 2: Decriptive statistics

| Variables | Obs. | Mean | Std. dev. | Min | Max |
|---|---|---|---|---|---|
| $\ln Robot$ | 610 | 4.011 | 2.259 | -2.743 | 8.465 |
| $i$ | 610 | 0.147 | 0.089 | -0.164 | 0.491 |
| $\ln gdp$ | 610 | 10.020 | 0.785 | 7.622 | 11.560 |
| $\ln pop$ | 610 | 3.538 | 1.640 | -1.234 | 7.268 |
| $trade$ | 610 | 0.003 | 0.107 | -0.626 | 0.479 |
| $invest$ | 610 | 0.251 | 0.071 | 0.090 | 0.666 |
| $gov$ | 610 | 0.173 | 0.056 | 0.045 | 0.323 |

theoretical model: a rise in the nominal interest rate is detrimental to accelerating the use of robots, which leads to a decrease in the level of automation.

Table 3: Baseline regression results

| Dependent Variable: $\ln Robot$ | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | $i < 0.5$ | $i < 0.4$ | $i < 0.3$ | $i < 0.2$ |
| $i$ | -1.776** | -1.519** | -1.594** | -2.251** |
| | (0.685) | (0.722) | (0.770) | (0.986) |
| $\ln gdp$ | 2.431*** | 2.374*** | 2.397*** | 2.665*** |
| | (0.549) | (0.557) | (0.543) | (0.679) |
| $\ln pop$ | -2.503 | -2.594* | -3.102* | -3.072* |
| | (1.510) | (1.507) | (1.786) | (1.737) |
| $trade$ | -1.761 | -1.571 | -1.900 | -2.131 |
| | (1.300) | (1.315) | (1.443) | (1.548) |
| $invest$ | 2.247 | 2.401 | 2.371 | 1.855 |
| | (2.161) | (2.181) | (2.227) | (2.469) |
| $gov$ | -7.237** | -7.511** | -10.176** | -10.479** |
| | (3.349) | (3.428) | (3.773) | (4.352) |
| Observations | 610 | 597 | 573 | 480 |
| R-squared | 0.711 | 0.705 | 0.711 | 0.710 |
| Number of country | 39 | 39 | 39 | 37 |

Notes: All estimates include country and year fixed effects. Robust standard errors are reported in parentheses. *** stands for the 1% significance level,** stands for the 5% significance level,and * stands for the 10% significance level.

In order to mitigate the potential endogeneity problem caused by reverse causality and omitted variables, we introduce the group mean $\bar{i}_{m,t}$, the lagged term of the difference between $\bar{i}_{m,t}$ and $\bar{i}_{m,t-1}$, denoted as *Lagged diff* $(\bar{i}_{m,t})$, as well as the lagged term of the difference between the nominal interest rate and its lagged term *Lagged diff* $(i_{m,t})$ as instrumental variables (IVs) for the nominal interest rate and use the two-stage least squares (2SLS) method. $\bar{i}_{m,t}$ denotes the average of nominal interest rates in year $t$ for countries other than country $m$.

In Table 4, columns 2 to 3 report the results obtained using $\bar{i}_{m,t}$ and *Lagged diff* $(\bar{i}_{m,t})$

as instrumental variables, while columns 4 to 5 show the results from the instrumental variable regression using 3 IVs. The $p$ values of the Sargan over-identification test are 0.329 and 0.126, respectively, which means that the instrumental variables are uncorrelated with the error term (i.e., the instruments are valid). The large and statistically significant $F$-statistics for the excluded instruments show that the instruments are strong (i.e., no weak instruments). The results of the second-stage regression show that there is a statistically significant negative correlation between the nominal interest rate and the growth rate of robot flows. The IV regression results in Table 4 are similar to the OLS (ordinary least squares) regression results reported in Table 3. The estimated coefficients on the nominal interest rate are negative and significant and have similar magnitudes in both OLS and IV regressions. Therefore, we find robust evidence that the rise in nominal interest rates is detrimental to automation.

Table 4: Results of IV regression

| | 2 IVs | | 3 IVs | |
|---|---|---|---|---|
| Dependent Variable: | $i$ | $\ln Robot$ | $i$ | $\ln Robot$ |
| Independent Variables | First stage | Second stage | First stage | Second stage |
| $i$ | | -1.429** | | -1.488** |
| | | (0.616) | | (0.617) |
| $\ln gdp$ | 0.043*** | 2.707*** | 0.043*** | 2.706*** |
| | (0.016) | (0.311) | (0.016) | (0.312) |
| $\ln pop$ | 0.014 | -4.446*** | 0.012 | -4.436*** |
| | (0.037) | (0.802) | (0.037) | (0.802) |
| $trade$ | -0.067* | -2.458*** | -0.068* | -2.450*** |
| | (0.039) | (0.773) | (0.039) | (0.773) |
| $invest$ | 0.035 | 1.049 | 0.035 | 1.061 |
| | (0.052) | (1.024) | (0.052) | (1.023) |
| $gov$ | 0.031 | -9.288*** | 0.029 | -9.264*** |
| | (0.086) | (1.918) | (0.086) | (1.920) |
| $\bar{i}$ | -15.894*** | | -15.865*** | |
| | (0.933) | | (0.927) | |
| Lagged diff($\bar{i}$) | 0.053 | | 1.494* | |
| | (0.298) | | (0.825) | |
| Lagged diff($i$) | | | 0.086* | |
| | | | (0.046) | |
| Observations | 518 | 518 | 518 | 518 |
| R-squared | 0.890 | 0.924 | 0.891 | 0.924 |
| $p$ value of Sargan test | 0.329 | | 0.126 | |
| $F$ test of excluded instruments | 788.000 | | 530.440 | |

Notes: All estimates include country and year fixed effects. Robust standard errors are reported in parentheses. *** stands for the 1% significance level,** stands for the 5% significance level,and * stands for the 10% significance level.

# 6 Conclusion

This paper explores the macroeconomic effects of monetary policy in a Schumpeterian model with automation. We show that the relative strengths of the financing constraints on automation innovation and vertical innovation is critical. If the CIA constraint is stronger (weaker) for R&D in automation innovation, a higher nominal interest rate will lead to an increase (a decrease) in the amount of high-skilled labor allocated to vertical innovation, therefore, the automation level will decline (rise), the aggregate technology growth rate will increase (decrease) and economic growth will be faster (slower).

We calibrate the model to the US economy and find a stronger financing constraint on automation innovation. Quantitative analysis shows that rising nominal interest rates are detrimental to automation but favorable to economic growth. In addition, we find that higher nominal interest rates always increase the welfare of capital owners and the aggregate welfare. When $\kappa$ is sufficiently large, the welfare gains from economic growth will dominate, and the welfare of high-skilled and low-skilled workers also rises.

Finally, we empirically examine the relationship between nominal interest rates and automation using cross-country panel data. The result shows a statistically significant negative correlation between the nominal interest rate and the growth rate of robot flows. The result holds up in the instrumental variables regression. Our empirical evidence provides support to theoretical model. Nevertheless, it may be useful to reexamine the growth and welfare effects of monetary policy using the Jones and Liu (2024) framework where automation is the source of long-run growth. We leave this to future research.

# 7 Appendix

## Appendix A: Proof of Lemma 1

(19) can be rewritten as:

$$\frac{1+\zeta i}{z} < \frac{Z_t R_t (1+\beta i)}{A w_t^l} < 1+\zeta i. \tag{A.1}$$

According to (4) and (6), we have

$$R_t = g + \rho + \delta. \tag{A.2}$$

Using the equation of low-skilled labor income share (41), we can derive

$$w_t = \frac{1-\theta}{\mu} \frac{1}{1+\zeta i} \frac{y_t}{l}, \tag{A.3}$$

where $l$ can be written as

$$l = \frac{1-\theta}{Z_t} \left[ y_t \left( \frac{A k_t}{\theta} \right)^{-\theta} \right]^{\frac{1}{1-\theta}}, \tag{A.4}$$

using the aggregate production function (37).

Substituting (A.2), (A.3) and (A.4) into (A.1) yields

$$\frac{1}{z} < \frac{\mu(1+\beta i)(g+\rho+\delta)}{A^{\frac{1}{1-\theta}}} \left(\frac{\theta y_t}{k_t}\right)^{\frac{\theta}{1-\theta}} < 1. \tag{A.5}$$

Using the equation of capital share (40), we have

$$\frac{\theta y_t}{k_t} = \mu R(1+\beta i). \tag{A.6}$$

Combining (A.6) with (A.5) yields the steady-state equilibrium condition for the automation-vertical innovation cycle (20).

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