Mean Field Games of Capital Accumulation and Militarization among Nations

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Abstract

This paper develops a dynamic mean field game model of global arms races and capital accumulation among heterogeneous nations. Each country allocates its output among consumption, capital investment, and military spending, optimizing against an evolving global militarization level. Strategic interdependence is modeled through forward–backward stochastic differential equations, combining individual HJB equations, population-level Fokker–Planck dynamics, and a mean field consistency condition. Closed-form solutions are derived under a linear–quadratic benchmark, and steady-state distributions are simulated for major powers including the U.S., China, Russia, and the EU. Results reveal how cross-country differences in productivity, military efficiency, and volatility lead to divergent long-run equilibria and global militarization traps. The model captures strategic path dependence and hysteresis, showing how shocks or preemptive arming can entrench high-arms outcomes. Policy implications include the importance of disarmament mechanisms, transparency, and coordinated institutions to escape inefficient militarized equilibria. The framework offers a robust tool for analyzing geopolitical dynamics under uncertainty.

Key Words: arms race, capital accumulation, mean field games, global militarization, Hamilton–Jacobi–Bellman equation, Fokker–Planck equation, strategic interdependence, geopolitical equilibrium

1 Introduction

The global arms race is one of the most persistent and complex dynamics in modern international relations. Despite rising economic interdependence, military expenditures continue to escalate in both absolute and relative terms across regions. Countries invest in sophisticated military technologies, expand conventional and nuclear capabilities, and react defensively to the observed or anticipated buildups of their rivals. Yet the global consequences of these behaviors are deeply intertwined: one nation's military gain often translates into another's sense of insecurity, triggering cascading reactions that fuel systemic militarization.

This paper develops a dynamic and stochastic model of capital accumulation and arms races among multiple countries, using the tools of mean field games to capture the strategic interdependence that defines modern geopolitics. Each country is modeled as an optimizing agent that allocates its output across consumption, productive capital investment, and military buildup, facing not only internal economic constraints but also the evolving mean field — the global average level of militarization $\bar{m}(t)$. This global variable affects both output (via negative spillovers such as insecurity, inefficiency, or sanctions) and utility (via psychological stress, consumption anxiety, or social unrest). Crucially, each country takes $\bar{m}(t)$ as given when making its optimal decision, but in equilibrium, this average emerges endogenously from the collective behavior of all countries. The result is a system of forward–backward stochastic differential equations, governed by the Hamilton–Jacobi–Bellman (HJB) equations for optimal control, Fokker–Planck equations for population distribution dynamics, and a self-consistency condition that ties individual expectations to aggregate reality.

At the core of this model is a fundamental tradeoff between investment in prosperity and investment in deterrence. Capital accumulation builds long-run output and welfare, but military buildup serves as insurance, deterrence, or geopolitical leverage in a dangerous world. When the global environment becomes more militarized, countries face a distorted incentive structure: the marginal value of peace diminishes, and the marginal value of arms increases — even if everyone is made worse off in equilibrium. This arms–capital tradeoff, under strategic feedback and stochastic shocks, creates dynamics that are inherently path-dependent, self-reinforcing, and sometimes irreversible.

To render the system analytically tractable, we begin with a linear-quadratic benchmark that allows for closed-form solutions for optimal consumption, capital and military investment, and shadow value functions. We simulate these policies across eight major global actors — the United States, China, Russia, India, Iran, Turkey, Japan, and the European Union — assigning realistic parameter values for productivity, volatility, military efficiency, and geopolitical sensitivity. Our simulations reveal strong cross-country heterogeneity: peaceful democracies with high productivity and institutional disincentives to militarize converge to capital-rich equilibria, while volatile or authoritarian regimes with high military productivity persistently accumulate arms and raise global militarization levels. The endogenous feedback through $\bar{m}(t)$ traps even reluctant countries in a high-arms world, reducing global output and welfare.

We further analyze transition dynamics, showing how shocks — whether from strategic shifts, technological breakthroughs, or political instability — can trigger rapid rearmament, strategic overreaction, and long-term shifts in equilibrium. The model exhibits hysteresis: once militarization rises, returning to peace is difficult, even when the initiating threat fades. Comparative statics confirm that parameters like output productivity, militarization sensitivity, military efficiency, and stochastic volatility determine whether the world converges to peaceful cooperation or entrenched arms races.

The contribution of this paper is both theoretical and policy-relevant. Theoretically, we provide a general equilibrium framework that endogenizes the arms race as a decentralized game under uncertainty, with closed-form and simulation-based characterization of steady states and transitions. Policy-wise, we demonstrate that unilateral rationality can lead to global irrationality — unless strong institutional mechanisms, transparency, and disarmament incentives are in place. Our model suggests that treaties, norms, and coordinated expectations are not merely diplomatic niceties, but structural necessities to escape high-arms traps.

This paper proceeds in eleven sections. Section 2 sets up the model with country-specific optimization problems and stochastic dynamics. Section 3 derives the HJB equations and Section 4 the Fokker–Planck equations. Section 5 presents the mean field consistency condition. Section 6 provides closed-form solutions under a linear–quadratic benchmark, and Section 7 derives steady-state distributions. Section 8 conducts global simulations using real-world stylized facts. Section 9 analyzes comparative statics and policy scenarios. Section 10 explores transition dynamics and strategic path dependence. Section 11 concludes with broader implications and directions for future research. Together, these components form a flexible and powerful structure for understanding how nations compete, respond, and sometimes stumble into costly equilibria in the global strategic arena.

2 Model Setup

We consider a world composed of many strategic nations, indexed by $i = 1, 2, \dots, N$, each of which engages in continuous capital accumulation and military investment under uncertainty. Time is continuous and infinite in horizon. Each country simultaneously manages two dynamic state variables: the stock of productive capital $k_i(t)$, and the stock of military assets $m_i(t)$. Both evolve stochastically over time, subject to investment decisions and exogenous shocks. At each instant, a country allocates output between three competing uses: consumption c_i(t), capital investment $i_{k,i}(t)$, and military investment $g_i(t)$. These decisions are influenced by domestic considerations and the external geopolitical environment, particularly the aggregate level of global militarization.

Each country's output at time t is determined by a linear-exponential production function of the form:

$\mathcal{Y}_i(t) = A_i\left(k_i(t) + \epsilon_i m_i(t)\right) e^{-\gamma_i \bar{m}(t)}$.

Here, $A_i > 0$ captures the total productivity of country i; $\epsilon_i \in (0, 1)$ measures the relative productive contribution of military stock compared to capital stock; and $\gamma_i > 0$ quantifies how severely global militarization $\bar{m}(t)$ suppresses productive capacity. The exponential term captures the idea that geopolitical tension and international insecurity degrade trust, economic integration, and institutional effectiveness, thereby lowering output even for technologically capable countries.

A key innovation in this model is that the parameter ϵ_i is not assumed to be vanishingly small. While military hardware is often economically inefficient, it may still contribute meaningfully to welfare and productive stability in some contexts. For example, in nations with strong military–industrial complexes, defense may drive technology, demand, and infrastructure. Likewise, in high-risk environments, military deterrence indirectly supports long-run economic activity by preserving territorial integrity. Accordingly, we allow ϵ_i to vary across countries and be flexibly calibrated to empirical conditions — whether it be $\epsilon_i \approx 0.05$, $\epsilon_i \approx 0.2$, or higher, depending on the structural role of the defense sector in each nation.

Each country's budget constraint requires that output must fully cover the sum of consumption, capital investment, and arms investment:

$c_i(t) + i_{k,i}(t) + g_i(t) = \mathcal{Y}_i(t).$

The evolution of the state variables $k_i(t)$ and $m_i(t)$ follows stochastic differential equations with linear drift and independent Brownian shocks:

$dk_i(t) = \left(i_{k,i}(t) - \delta_i^k k_i(t)\right) dt + \sigma_{k,i}\, dW_{k,i}(t),$

$dm_i(t) = \left(g_i(t) - \delta_i^m m_i(t)\right) dt + \sigma_{m,i}\, dW_{m,i}(t),$

where $\delta_i^k,\ \delta_i^m > 0$ are the depreciation rates for capital and military stock respectively, $\sigma_{k,i},\ \sigma_{m,i} > 0$ capture volatility from investment shocks, political disturbances, or technology diffusion, and $W_{k,i}(t)$, $W_{m,i}(t)$ are independent standard Brownian motions. These stochastic terms ensure that even under identical policies, different countries may evolve differently due to idiosyncratic shocks.

Each country maximizes its expected discounted utility over an infinite horizon. The instantaneous utility function for country i is:

$\nu_i(c_i, m_i, \bar{m}) = a_i c_i - \frac{1}{2} b_i c_i^2 + \eta_{1,i} m_i + X_i c_i m_i - \eta_{2,i}\bar{m} - \chi_i c_i \bar{m}.$

The parameters $a_i,\ b_i > 0$ govern consumption preferences; $\eta_{1,i} > 0$ measures the direct utility from having military strength (e.g., domestic security or geopolitical status); $X_i > 0$ captures how arms spending enhances the utility of consumption (e.g., patriotic pride or national dignity); and $\eta_{2,i},\ \chi_i > 0$ quantify the psychological and material disutility of living in a militarized world. The interaction term $\chi_i c_i \bar{m}$ reflects the realistic behavioral effect that consumption becomes less satisfying when geopolitical tension is high — for example, due to uncertainty, anxiety, or instability.

Country i chooses control variables $c_i(t),\ i_{k,i}(t),\ g_i(t)$ to maximize the intertemporal objective:

$\max_{\{c_i, i_{k,i}, g_i\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\, \nu_i(c_i(t), m_i(t), \bar{m}(t))\, dt\right],$

subject to the stochastic dynamics of capital and military stock, and the per-period resource constraint. The constant $\rho > 0$ is the subjective time discount rate, assumed identical across countries for tractability.

The key feature that transforms this setup into a mean field game is the presence of the average global militarization level:

$\bar{m}(t) = \frac{1}{N} \sum_{j=1}^{N} m_j(t),$

which is treated by each country as exogenous when solving its control problem, but is in fact endogenously generated by the population-wide strategic choices. In equilibrium, each country's expectations about $\bar{m}(t)$ must match the average behavior that actually results from everyone's optimal actions. This fixed-point condition — known as the mean field consistency condition — will be rigorously formulated and solved in subsequent sections.

This model thus captures both the internal logic of resource allocation under uncertainty and the external dynamics of strategic interdependence across nations. It blends elements of capital theory, defense economics, and global game theory, making it a rich foundation for understanding arms races, military–economic tradeoffs, and the consequences of decentralized insecurity.

3 The Hamilton–Jacobi–Bellman Equation

To analyze how each country optimally allocates its resources across consumption, capital investment, and military buildup over time, we turn to the Hamilton–Jacobi–Bellman (HJB) framework. The HJB equation characterizes the dynamic programming problem faced by each country i, taking into account both the internal stochastic evolution of state variables and the global strategic environment reflected in the average militarization level $\bar{m}(t)$. The solution to this equation yields the optimal controls as feedback functions of capital $k_i(t)$, military stock $m_i(t)$, and the exogenous variable $\bar{m}(t)$, which is determined endogenously in equilibrium via the mean field.

Let us denote the value function of country i at time t as:

$$V_i(t, k_i, m_i) = \sup_{\{c_i, i_{k,i}, g_i\}} \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)} \nu_i(c_i(s), m_i(s), \bar{m}(s))\, ds \,\middle|\, k_i(t), m_i(t) \right].$$

This function represents the maximum expected discounted utility that country i can achieve, starting from current capital and military levels (k_i, m_i), while treating $\bar{m}(t)$ as a known function of time. The problem is solved subject to the capital and military dynamics:

$dk_i = \left(i_{k,i} - \delta_i^k k_i\right) dt + \sigma_{k,i}\, dW_{k,i}(t), \quad dm_i = (g_i - \delta_i^m m_i)\, dt + \sigma_{m,i}\, dW_{m,i}(t),$

and the resource constraint:

$c_i + i_{k,i} + g_i = A_i(k_i + \epsilon_i m_i) e^{-\gamma_i \bar{m}(t)}.$

To formulate the HJB equation, we apply Itô's Lemma to $V_i(t, k_i, m_i)$ and require that the expected discounted value satisfies the Bellman optimality condition. The result is the partial differential equation:

$\rho V_i(t, k_i, m_i) = \max_{c_i, i_{k,i}, g_i} \left\{ \nu_i(c_i, m_i, \bar{m}(t)) + \frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial k_i} \cdot \mu_{k,i} + \frac{\partial V_i}{\partial m_i} \cdot \mu_{m,i} + \frac{1}{2}\sigma_{k,i}^2 \frac{\partial^2 V_i}{\partial k_i^2} + \frac{1}{2}\sigma_{m,i}^2 \frac{\partial^2 V_i}{\partial m_i^2} \right\},$

where the drift terms for the state variables are given by:

$\mu_{k,i} = i_{k,i} - \delta_i^k k_i, \quad \mu_{m,i} = g_i - \delta_i^m m_i.$

Next, we substitute in the expression for instantaneous utility:

$\nu_i(c_i, m_i, \bar{m}) = a_i c_i - \frac{1}{2} b_i c_i^2 + \eta_{1,i} m_i + X_i c_i m_i - \eta_{2,i} \bar{m} - \chi_i c_i \bar{m},$

which contains both economic and strategic components. The quadratic form in consumption reflects diminishing marginal utility, while the interaction term $X_i c_i m_i$ models the political or psychological enhancement of consumption when military strength is high. The negative terms -$\eta_{2,i}\bar{m}$ and $-\chi_i c_i \bar{m}$ capture the direct and consumption-mediated anxiety from global militarization.

To make the problem analytically tractable while preserving its essential features, we conjecture a quadratic value function in total internal productive assets $z_i = k_i + m_i$:

$V_i(t, k_i, m_i) = \alpha_{0,i}(t) + \alpha_{1,i}(t)(k_i + m_i) + \alpha_{2,i}(t)(k_i + m_i)^2$.

This structure is motivated by the linear–quadratic form of utility and the production function. The marginal value of both capital and military stock is then given by:

$\frac{\partial V_i}{\partial k_i} = \frac{\partial V_i}{\partial m_i} = \alpha_{1,i}(t) + 2\alpha_{2,i}(t)(k_i + m_i) =: \lambda_i(t)$,

and the second derivatives are:

$\frac{\partial^2 V_i}{\partial k_i^2} = \frac{\partial^2 V_i}{\partial m_i^2} = 2\alpha_{2,i}(t), \quad \frac{\partial^2 V_i}{\partial k_i \partial m_i} = 2\alpha_{2,i}(t)$.

We define the current-period Lagrangian incorporating the instantaneous utility and the budget constraint:

$$\mathcal{L}i = a_i c_i - \frac{1}{2} b_i c_i^2 + \eta 1, i m_i + X_i c_i m_i - \eta_{2,i}\bar{m}(t) - \chi_i c_i \bar{m}(t)$$
$$+ \lambda_i(t) \left[A_i(k_i + \epsilon_i m_i) e^{-\gamma_i \bar{m}(t)} - c_i - i_{k,i} - g_i \right].$$

Taking first-order conditions:

• With respect to c_i:

$\frac{\partial \mathcal{L}_i}{\partial c_i} = a_i - b_i c_i + X_i m_i - \chi_i \bar{m}(t) - \lambda_i(t) = 0$,

yielding:

$$\boxed{c_i^*(t) = \frac{a_i + X_i m_i - \chi_i \bar{m}(t) - \lambda_i(t)}{b_i}}.$$

• With respect to $i_{k,i}$ and g_i:

$\frac{\partial \mathcal{L}i}{\partial i k,i} = -\lambda_i(t), \qquad \frac{\partial \mathcal{L}_i}{\partial g_i} = -\lambda_i(t)$,

which confirms that the marginal utility of resource use equals the shadow value of internal funds. These conditions allow for interior solutions where both capital and arms investment are chosen optimally based on the state variables and the value function.

The final step is to substitute $c_i(t)$ into the budget constraint:

$i_{k,i}(t) + g_i(t) = A_i(k_i + \epsilon_i m_i) e^{-\gamma_i \bar{m}(t)} - c_i(t)$,

which determines the total investable surplus, to be allocated between $i_{k,i}$ andg_i according to the country's own intertemporal trade-offs.

This derivation of the HJB equation, paired with an explicit guess for the value function, provides the structure necessary to solve for the dynamic policy paths and enables full characterization of the coupled system with the Fokker–Planck equations and the mean field consistency condition, which we develop next.

4 The Fokker–Planck Equation

To understand the macro-level implications of decentralized strategic behavior, we must track how the distribution of each country's state variables evolves over time. Let $p_i(t, k, m)$ denote the probability density function of country i's joint capital and military stock (k_i, m_i) at time t. This distribution evolves according to the Fokker–Planck equation (also called the Kolmogorov forward equation), which governs the dynamics of probability flow in state space induced by both optimal controls and stochastic perturbations.

Recall from Section 2 that the capital and military dynamics of country i are given by:

$$dk_i(t) = \left(i_{k,i}(t) - \delta_i^k k_i(t)\right) dt + \sigma_{k,i}\, dW_{k,i}(t),$$

$$dm_i(t) = (g_i(t) - \delta_i^m m_i(t))\, dt + \sigma_{m,i}\, dW_{m,i}(t),$$

where the optimal controls $i_{k,i}(t)$ and $g_i(t)$ are functions of current state (k_i, m_i), the global mean militarization $\bar{m}(t)$, and the endogenous shadow value $\lambda_i(t)$. These stochastic differential equations define the drift vector:

$$\mu_{k,i}(k, m; \bar{m}(t)) = i_{k,i}(k, m, \bar{m}(t)) - \delta_i^k k,$$

$$\mu_{m,i}(k, m; \bar{m}(t)) = g_i(k, m, \bar{m}(t)) - \delta_i^m m.$$

Given these drift terms and constant diffusion coefficients $\sigma_{k,i}$, $\sigma_{m,i}$, the Fokker–Planck equation describing the evolution of $p_i(t, k, m)$ is:

$$\begin{aligned}\frac{\partial p_i(t,k,m)}{\partial t} &= -\frac{\partial}{\partial k}\left(\mu_{k,i}(k,m;\bar{m}(t)) \cdot p_i(t,k,m)\right) - \frac{\partial}{\partial m}\left(\mu_{m,i}(k,m;\bar{m}(t)) \cdot p_i(t,k,m)\right) \\ &\quad + \frac{1}{2}\sigma_{k,i}^2 \frac{\partial^2 p_i(t,k,m)}{\partial k^2} + \frac{1}{2}\sigma_{m,i}^2 \frac{\partial^2 p_i(t,k,m)}{\partial m^2}.\end{aligned}$$

This partial differential equation (PDE) captures both the deterministic trends in capital and arms accumulation — as driven by national policy — and the random fluctuations arising from country-specific shocks. The equation is forward in time, in contrast to the backward HJB equation for the value function, and describes how the mass of probability "flows" across the state space (k, m).

Crucially, the drift functions $\mu_{k,i}$ and $\mu_{m,i}$ depend not only on country i's internal state, but also on the global militarization average$\bar{m}(t)$, which is endogenously determined by the entire distribution:

$\bar{m}(t) = \frac{1}{N} \sum_{j=1}^{N} \int_0^\infty \int_0^\infty m \cdot p_j(t, k, m)\, dk\, dm.$

This creates a circular interdependence: the policy functions used in the drift terms require knowledge of \bar{m}(t), which itself is computed from the evolving distributions $\{p_j(t, k, m)\}$. This interdependence is resolved through the mean field consistency condition, which we explore in Section 5.

The Fokker–Planck equation allows us to analyze the population-level behavior of capital and military stock accumulation under stochastic investment and interdependent optimization. It enables the computation of time-evolving probability densities, long-run steady-state distributions, and responses to shocks and policy changes. In particular, the solution $p_i(t, k, m)$ reflects the joint impact of national preferences, initial conditions, volatility, and systemic exposure to global militarization.

In Section 6 and beyond, we use this framework to simulate how different countries settle into different statistical equilibria, and how global militarization $\bar{m}(t)$ feeds back into and amplifies these outcomes. Before doing so, however, we must formally define and solve the mean field consistency condition that closes the full system of forward–backward equations. This is the task of the next section.

5 The Mean Field Consistency Condition

In a world of strategically interdependent nations, each country optimizes its behavior by taking the global militarization level \bar{m}(t) as given. However, in equilibrium, this global variable is not exogenous — it emerges endogenously from the collective choices of all countries. The mathematical condition that ensures coherence between these individual decisions and the aggregate environment is known as the mean field consistency condition.

Recall that for each country i, the optimal controls — consumption $c_i(t)$, capital investment $i_{k,i}(t)$, and military investment $g_i(t)$ — are functions of its own state variables $k_i(t)$, $m_i(t)$, and the global militarization level $\bar{m}(t)$. These controls determine the drift of the stochastic differential equations governing capital and military dynamics. In turn, these dynamics determine the evolution of the probability distribution $p_i(t, k, m)$ via the Fokker–Planck equation described in Section 4.

The key point is this: in equilibrium, the average militarization level $\bar{m}(t)$ — the strategic environment perceived by each country — must equal the actual population average, computed across all countries and weighted by their evolving state distributions. That is, the realized average must match the anticipated average, forming a self-fulfilling expectation. Mathematically, this requires that:

$$\boxed{\bar{m}(t) = \frac{1}{N} \sum_{j=1}^{N} \int_0^\infty \int_0^\infty m \cdot p_j(t, k, m) \, dk \, dm}$$

for all $t \geq 0$.

This condition closes the forward–backward system of our mean field game. The backward HJB equation determines the optimal controls given a path $\bar{m}(t)$; the forward Fokker–Planck equation determines the evolution of the state distributions under those controls; and the mean field consistency condition ensures that the resulting average $\bar{m}(t)$ matches what countries assumed in their optimization.

This mutual dependence is both a conceptual and computational challenge. Solving the model requires finding a fixed point: a path \bar{m}(t) such that, when every country optimizes with respect to it, the induced evolution of state distributions yields back the same $\bar{m}(t)$. In practice, this often requires iterative solution methods, where an initial guess for $\bar{m}(t)$ is refined through repeated forward–backward passes until consistency is achieved.

In steady state, this condition simplifies. If each country's distribution $p_i(t, k, m)$ converges to a time-invariant density $p_i^(k, m)$, then the steady-state global militarization level must satisfy:

$$\boxed{\bar{m} = \frac{1}{N} \sum_{j=1}^{N} \int_0^\infty \int_0^\infty m \cdot p_j(k, m) \, dk \, dm.}$$

This equation implicitly defines $\bar{m}$ as a fixed point of a mapping from militarization to equilibrium state distributions. It reflects the core structure of a decentralized equilibrium: every agent behaves rationally given their beliefs, and the world turns out exactly as they expected — because everyone's beliefs were mutually consistent.

The mean field consistency condition plays a central role in all subsequent analysis. It links micro-level incentives with macro-level outcomes and captures the emergent properties of strategic competition.

6 Closed-Form Solution — Linear-Quadratic Benchmark

To understand the strategic dynamics of capital and arms accumulation in a tractable and transparent way, we now analyze a linear–quadratic benchmark model. This specification permits explicit solutions for the value function and all optimal controls. Each country retains its own structure and parameters, including productivity, volatility, military effectiveness, disutility from global militarization, and initial capital and arms stocks. Closed-form expressions allow us to investigate how individual optimization generates interdependent global behavior.

We assume that each country i's utility function takes the quadratic form:

$\nu_i(c_i, m_i, \bar{m}) = a_i c_i - \frac{1}{2} b_i c_i^2 + \eta_{1,i} m_i + X_i c_i m_i - \eta_{2,i} \bar{m} - \chi_i c_i \bar{m}$,

where a_i, $b_i > 0$ define the shape of preferences over consumption, $\eta_{1,i}$ captures the utility from national military power, X_i reflects complementarities between consumption and national security, and $\eta_{2,i}$, χ_i represent the disutility from global militarization in both direct and consumption-affected forms.

Each country's production function is given by:

$\mathcal{Y}_i(t) = A_i(k_i(t) + \epsilon_i m_i(t)) \cdot e^{-\gamma_i \bar{m}(t)}$,

where $A_i > 0$ is total factor productivity and $\epsilon_i \in (0, 1)$ indicates the relative efficiency of arms in producing output. The exponential term $e^{-\gamma_i \bar{m}(t)}$, with $\gamma_i > 0$, captures the idea that excessive global militarization undermines productivity by generating strategic instability or international insecurity.

Each country's budget constraint requires that:

$c_i(t) + i_{k,i}(t) + g_i(t) = \mathcal{Y}i(t)$,

with capital and military dynamics given by:

$dk_i(t) = \left(ik, i(t) - \delta_i^k k_i(t)\right) dt + \sigma_{k,i} dW_{k,i}(t)$,

$dm_i(t) = \left(g_i(t) - \delta_i^m m_i(t)\right) dt + \sigma_{m,i} dW_{m,i}(t)$,

where δ_i^k, $\delta_i^m > 0$ are depreciation rates, and $\sigma_{k,i}$, $\sigma_{m,i}$ capture stochastic volatility in capital and arms, respectively.

To solve this stochastic control problem, we conjecture that the value function takes the quadratic form:

$V_i(t, k_i, m_i) = \alpha_{0,i}(t) + \alpha_{1,i}(t) z_i + \alpha_{2,i}(t) z_i^2, \quad \text{where } z_i := k_i + m_i.$

This structure reflects the additivity of capital and arms in production and allows marginal values to be cleanly expressed as:

$\frac{\partial V_i}{\partial k_i} = \frac{\partial V_i}{\partial m_i} = \lambda_i(t) = \alpha_{1,i}(t) + 2\alpha_{2,i}(t)(k_i + m_i)$.

The Lagrangian of country i's optimization problem at each time t, incorporating the budget constraint, is:

$$\mathcal{L}i = a_i c_i - \frac{1}{2} b_i c_i^2 + \eta 1, i m_i + X_i c_i m_i - \eta_{2,i} \bar{m}(t) - \chi_i c_i \bar{m}(t)$$
$$+ \lambda_i(t) \left[A_i(k_i + \epsilon_i m_i) e^{-\gamma_i \bar{m}(t)} - c_i - i_{k,i} - g_i \right].$$

We now derive the first-order conditions:

• With respect to consumption c_i:

$\frac{\partial \mathcal{L}_i}{\partial c_i} = a_i - b_i c_i + X_i m_i - \chi_i \bar{m}(t) - \lambda_i(t) = 0,$

which yields the optimal consumption rule:

$$\boxed{c_i^*(t) = \frac{a_i + X_i m_i - \chi_i \bar{m}(t) - \lambda_i(t)}{b_i}}.$$

Substituting $c_i(t)$ into the budget constraint, we determine the residual available for investment:

$i_{k,i}(t) + g_i(t) = A_i(k_i + \epsilon_i m_i) e^{-\gamma_i \bar{m}(t)} - c_i(t).$

To preserve analytic tractability while keeping investment endogenous, we assign a constant investment share rule:

$$\boxed{\begin{aligned} i_{k,i}(t) &= \xi_i \left(A_i(k_i + \epsilon_i m_i) e^{-\gamma_i \bar{m}(t)} - c_i(t) \right), \\ g_i(t) &= (1 - \xi_i) \left(A_i(k_i + \epsilon_i m_i) e^{-\gamma_i \bar{m}(t)} - c_i(t) \right), \end{aligned}}$$

where $\xi_i \in (0,1)$ represents country i's investment orientation — more capital-focused when ξ_i is large, more militarized when ξ_i is small.

Together, the equations for $c_i(t)$, $i_{k,i}(t)$, $g_i(t)$, and the marginal value $\lambda_i(t)$ form the basis of country i's optimal policy behavior. These expressions now feed into the drift terms in the Fokker–Planck equation (Section 4) and determine the evolution of the population distribution $p_i(t,k,m)$. The global mean militarization $\bar{m}(t)$ is updated using the mean field consistency condition (Section 5), closing the system dynamically.

We are now in a position to study how these optimal policies generate long-run distributions of capital and military power and how the global militarization equilibrium emerges from the decentralized interaction of diverse national strategies. These issues are taken up in the next section.

7 Steady-State Distribution and Global Militarization

With optimal policies and interdependencies explicitly defined, we now turn to the steady-state behavior of the model. The core question is whether each country's joint distribution of capital k_i and military stock m_i converges to a stable statistical configuration over time — and, if so, what that implies for the average global militarization level \bar{m}^*. This steady state emerges from the dynamic interaction of investment policies, stochastic shocks, depreciation, and mutual strategic influence through the mean field.

Let $p_i^*(k,m)$ denote the steady-state joint probability density for country i's capital and military stock. In steady state, the probability density no longer evolves with time, so the time derivative in the Fokker–Planck equation vanishes. From Section 4, we then obtain the following steady-state Fokker–Planck equation for each country i:

$0 = -\frac{\partial}{\partial k}\left(\mu_{k,i}(k,m;\bar{m}(t)) \cdot p_i(t,k,m)\right) - \frac{\partial}{\partial m}\left(\mu_{m,i}(k,m;\bar{m}(t)) \cdot p_i(t,k,m)\right) + \frac{1}{2}\sigma_{k,i}^2 \frac{\partial^2 p_i(t,k,m)}{\partial k^2} + \frac{1}{2}\sigma_{m,i}^2 \frac{\partial^2 p_i(t,k,m)}{\partial m^2}$.

where the drift terms $\mu_{k,i}$, $\mu_{m,i}$ are computed using the optimal investment policies derived in Section 4:

$\mu_{k,i}(k,m;\bar{m}(t)) = i_{k,i}(k,m,\bar{m}(t)) - \delta_i^k k$,

$\mu_{m,i}(k,m;\bar{m}(t)) = g_i(k,m,\bar{m}(t)) - \delta_i^m m$.

The optimal consumption function $c_i(k,m,\bar{m})$ is given explicitly by:

$c_i(k,m,\bar{m}) = \frac{a_i + X_i m - \chi_i \bar{m}^* - \alpha_{1,i} - 2\alpha_{2,i}(k+m)}{b_i}$.

To complete the equilibrium, we impose the mean field consistency condition at steady state:

$$\boxed{\bar{m}^* = \frac{1}{N}\sum_{j=1}^{N}\int_0^\infty \int_0^\infty m \cdot p_j^(k,m)\, dk\, dm,}$$

ensuring that the assumed global militarization $\bar{m}$ in each country's policy calculation matches the actual global average emerging from the joint distributions $\{p_j^*\}_{j=1}^N$.

Interpretation of Steady-State Behavior

Each country's steady-state distribution $p_i^*(k,m)$ reflects the balance between deterministic investment choices and stochastic volatility. When volatility terms $\sigma_{k,i}$, $\sigma_{m,i}$ are small, the distribution is tightly concentrated around deterministic equilibrium values. With higher volatility, the distribution spreads, and the country faces greater uncertainty over future capital and arms levels — increasing the importance of precautionary motives.

Countries with high productivity A_i, low militarization sensitivity γ_i, and low psychological disutility$\eta_{2,i}$, χ_i tend to sustain higher military levels and contribute disproportionately to $\bar{m}^*$. In contrast, countries with high disutility from arms buildup or low military productivity ϵ_i tend to allocate more toward capital, resulting in more peaceful steady-state behavior.

However, because global militarization enters every country's output function through the exponential term $e^{-\gamma_i \bar{m}^*}$, even peaceful countries are harmed when others aggressively arm. This generates a negative externality that cannot be internalized in a decentralized system. The model thus illustrates how rational behavior at the national level can lead to collectively inefficient equilibria at the global level.

Depending on parameter values, the world may settle into one of two regimes:

• A peaceful equilibrium, where psychological and economic disincentives to militarize are strong enough to keep $\bar{m}^*$ low, even in the presence of uncertainty.

• A militarized trap, where global fear and strategic insecurity drive countries to build up arms preemptively, sustaining a high $\bar{m}^*$ that depresses output and welfare globally.

In the next section, we numerically compute and simulate these steady-state distributions for selected countries using stylized parameter values reflective of their real-world military and economic characteristics. These simulations allow us to compare how heterogeneous strategic preferences and constraints shape global equilibrium outcomes.

8 Simulations of Global Militarization and Strategic Divergence

This section presents simulation-based evidence of how heterogeneity in national characteristics leads to divergent long-run patterns of capital accumulation, military investment, and global militarization. Using the closed-form policy rules derived in Section 6 and the steady-state structure developed in Sections 4 and 7, we solve for the equilibrium distributions of capital and arms across a selected group of major countries: the United States, China, Russia, India, Iran, Turkey, Japan, and the European Union. These countries represent a wide range of strategic orientations, technological capabilities, institutional constraints, and military priorities. By calibrating key parameters to reflect these differences, we uncover how individual decisions aggregate into global outcomes through the mechanism of mean field interaction.

For each country i, we assign a set of structural parameters reflecting its economic productivity A_i, the efficiency of military assets ϵ_i, the disutility from global militarization $(\eta_{2,i}, \chi_i)$, the internal value of national power $\eta_{1,i}$, and the degree of complementarity between consumption and arms X_i. We also calibrate capital and military depreciation rates(δ_i^k, δ_i^m) and volatility parameters $(\sigma_{k,i}, \sigma_{m,i})$, along with the strategic weight placed on capital investment via ξ_i. The production function used in simulation takes the form $Y_i(t) = A_i(k_i(t) + \epsilon_i m_i(t))e^{-\gamma_i \bar{m}(t)}$, which penalizes national output when global militarization rises. Each country's utility function follows the quadratic structure defined in earlier sections, with consumption, arms, and global tension jointly shaping welfare.

Solving the system involves iterating between three components: the country-level HJB-derived controls, the Fokker–Planck equation governing the steady-state distribution $p_i^(k, m)$, and the mean field consistency condition requiring that the global average militarization $\bar{m}$ equals the realized weighted average of military stocks across all countries. For each candidate value of $\bar{m}^*$, we solve the Fokker–Planck equation until the induced average $\int m \cdot p_i(k, m)\, dk\, dm$ aligns with $\bar{m}$. The result is a fixed point of the decentralized global system.

The simulation results reveal clear divergence in strategic equilibrium behavior. The United States and Japan, both characterized by high productivity, high consumption preference, and strong disutility from global militarization, settle into equilibria with high capital accumulation and low average military stock. Their distributions are concentrated around capital-intensive, investment-driven paths. Japan in particular has very low military efficiency and strong institutional constraints on arms buildup, which makes it highly sensitive to $\bar{m}^*$ and committed to peaceful equilibrium.

By contrast, China and Russia accumulate substantial military stock in steady state. Both countries are assigned high A_i and ϵ_i, reflecting the importance of state-led military industries and the strategic use of arms to expand global influence. In the case of China, relatively low values of χ_i and $\eta_{2,i}$ mean that global militarization has limited psychological or policy deterrent effect. Russia, with high volatility in both capital and arms accumulation, exhibits a broader distribution but sustains high arms investment under uncertainty, amplified by high $\eta_{1,i}$ and low consumption preference.

India and Turkey occupy intermediate positions. India combines moderate productivity with regional strategic pressure, leading to moderate arms accumulation and a mixed investment profile. Turkey has high volatility and moderate productivity, but also high internal value placed on military independence, which results in a relatively fat-tailed distribution of military investment. Iran exhibits the most militarized equilibrium after Russia, driven by very high $\eta_{1,i}$, low χ_i, and an arms-centric utility function. Despite economic fragility and high volatility, Iran's policy functions favor sustained defense investment as a substitute for capital accumulation.

The European Union achieves a capital-rich, low-militarization equilibrium due to high productivity, high preference for consumption, and strong disutility from militarization. EU countries have low volatility, strong internal institutional checks on military escalation, and no immediate border pressures equivalent to those faced by China or Russia. As a result, their optimal policies sustain capital investment while minimizing arms buildup, even when other countries arm more aggressively.

The global average militarization $\bar{m}^*$ is driven disproportionately by the high-armament countries — especially China, Russia, and Iran — whose strategic behavior exerts negative externalities on the rest of the system. Because militarization reduces output for all countries via the exponential term $e^{-\gamma_i \bar{m}^*}$, the actions of a few militarized powers suppress global output and force even peaceful countries to reduce consumption or adjust investment behavior. In this way, global militarization emerges not as a collective choice, but as an emergent result of decentralized optimization under fear, rivalry, and mutual distrust.

The simulations therefore confirm the model's theoretical prediction: when countries act independently under uncertainty and interdependence, the world can settle into a militarization trap, where excessive arms investment reduces output and welfare for all. Avoiding such equilibria would require coordination, trust-building, and institutionalized disarmament — issues we address further in the comparative statics of the next section.

9 Comparative Statics and Counterfactual Analysis

To better understand the strategic architecture of the model, we now perform comparative statics and counterfactual analysis by systematically varying key structural parameters across countries. The goal is to isolate the forces driving divergence in long-run capital and military distributions, identify thresholds where qualitative behavior changes, and assess the potential for policy interventions to shift global militarization toward more peaceful equilibria. We focus on four sets of parameters: military efficiency ϵ_i, output sensitivity to global militarization γ_i, volatility terms $\sigma_{k,i}\}$, $\sigma_{m,i}$, and disutility from global arms buildup χ_i, $\eta_{2,i}$.

We begin with military efficiency ϵ_i, which measures the contribution of military assets to domestic production. In our benchmark model, countries like China and Russia have relatively high ϵ_i, reflecting the dual-use role of military capital in manufacturing, defense-related R&D, and internal regime support. As ϵ_i increases, military investment becomes more productive, shifting the optimal allocation away from capital toward arms. In steady state, this leads to an increase in the marginal value of military stock $\frac{\partial V_i}{\partial m_i}$, a higher mean of the distribution$p_i(k, m)$ in the m-direction, and a greater contribution to global militarization $\bar{m}$. In contrast, countries with low ϵ_i, such as Japan or the European Union, have little incentive to substitute arms for capital, even when others do. A one-unit increase in ϵ_i for a large country like China can shift the global $\bar{m}^*$ upward by a measurable margin, thereby tightening the resource constraints for all countries and depressing consumption and output globally.

Next, we examine the role of output sensitivity to global militarization, parameterized by γ_i. This exponential penalty appears in every country's production function as $Y_i = A_i(k_i + \epsilon_i m_i)e^{-\gamma_i \bar{m}(t)}$. A higher γ_i implies that a given level of global militarization sharply reduces effective output. Countries with high γ_i, such as the EU or Japan, experience large losses in production when $\bar{m}$ rises, which incentivizes restraint in arms accumulation. These countries also become more reactive to others' militarization, as they suffer more from the resulting suppression of economic capacity. Conversely, countries with low γ_i, such as Russia or Iran, can endure higher $\bar{m}$ without major output loss, and thus maintain or escalate arms buildup even in unstable environments. As a result, increasing γ_i for a peaceful country typically has little effect on $\bar{m}^*$, but decreasing γ_i for a militarized state can push the entire system toward a more aggressive equilibrium.

We then explore the impact of volatility parameters $\sigma_{k,i}$ and $\sigma_{m,i}$. These parameters govern the diffusion terms in the Fokker–Planck equation and control how concentrated or dispersed the steady-state distributions $p_i(k, m)$ are. When volatility is high, especially for military assets, countries face greater uncertainty over the effectiveness of arms accumulation. In such environments, precautionary motives may justify larger arms reserves, even for risk-averse nations. At the same time, higher volatility flattens the population density, increasing the variance and reducing the predictability of aggregate behavior. This unpredictability further elevates $\bar{m}$, as all countries must respond to both mean and higher-order moments of the military distribution. In simulations, doubling $\sigma_{m,i}$ for Russia or Iran led to visible increases in their expected military stock and triggered mild militarization responses even from distant countries like Japan and the EU.

Finally, we vary the psychological disutility from global arms buildup, captured by χ_i and $\eta_{2,i}$. These parameters reflect how strongly citizens, institutions, and governments internalize the social and political costs of living in a militarized world. A high χ_i means that consumption becomes less enjoyable in tense global environments, while a high $\eta_{2,i}$ captures direct disutility from observing global arms accumulation. These terms directly influence the first-order condition for optimal consumption, and indirectly affect investment policies through the shadow value of resources. Increases in χ_i or $\eta_{2,i}$ for any single country strongly dampen its arms buildup, but have limited effect on $\bar{m}^*$ unless adopted by multiple countries simultaneously. Collective increases in disutility — for instance, via global norms, media coverage, or public education — can generate a sharp reduction in arms accumulation across the board, indicating the existence of nonlinear coordination thresholds.

The counterfactual experiments underscore several key insights. First, militarized equilibria are not solely the result of individual aggressiveness, but also of global strategic spillovers and asymmetries in sensitivity. Second, small shifts in parameters for large or influential countries (e.g., China, the U.S.) have disproportionately large effects on the global mean field. Third, psychological and institutional disincentives to militarize play a critical role in stabilizing peaceful equilibria, especially when backed by credible policy frameworks. Fourth, uncertainty amplifies arms accumulation, not just by raising direct risk but by inducing more defensive postures from others.

Together, these comparative statics reinforce the model's policy implications: global militarization is an emergent property of decentralized, rational, and heterogeneous agents operating under uncertainty. Coordinated adjustments to strategic sensitivities, military effectiveness, and disutility perceptions offer the most promising path toward reducing $\bar{m}^*$ and avoiding long-run economic and political instability. In the next section, we explore transition dynamics to show how shocks, crises, or sudden escalations can alter the trajectory of militarization over time — and whether peaceful equilibria, once disturbed, can be restored.

10 Transition Dynamics and Strategic Hysteresis

While the steady-state analysis in previous sections provides insight into the long-run configuration of global militarization and capital distributions, real-world strategic environments evolve through time under persistent uncertainty, asymmetric shocks, and episodic geopolitical events. Understanding how countries adjust dynamically toward equilibrium — or diverge away from it — is essential for analyzing the persistence and reversibility of militarization patterns. This section investigates the transition dynamics of the model, with a focus on hysteresis: whether militarization shocks have long-term effects even after their immediate causes dissipate.

In our framework, each country i's joint distribution of capital $k_i(t)$ and military stock $m_i(t)$ evolves over time according to the Fokker–Planck equation derived in Section 4. The forward PDE for the probability density $p_i(t,k,m)$ is driven by endogenous drift terms — determined by the optimal policy functions $c_i(t), i_{k,i}(t), g_i(t)$ — and stochastic diffusion governed by volatility parameters. The time evolution of this distribution is shaped not only by country i's own policies, but by the global mean field \bar{m}(t), which in turn evolves endogenously as:

$\bar{m}(t) = \frac{1}{N}\sum_{j=1}^{N}\int_0^{\infty}\int_0^{\infty} m \cdot p_j(t,k,m)\,dk\,dm.$

To simulate transition dynamics, we initialize each country with empirically plausible values for capital and military stock and track how the state distributions $p_i(t,k,m)$ evolve under optimal policy functions, which are continuously updated as the global $\bar{m}(t)$ changes. The system evolves through a coupled forward–backward structure: the value function is solved backward via the HJB equation, the distribution evolves forward via the Fokker–Planck equation, and the equilibrium path of $\bar{m}(t)$ is determined consistently at each time step.

In a baseline case without major shocks, the system gradually converges to the steady state described in Section 7. Capital- and consumption-oriented countries slowly accumulate productive assets while maintaining low military stocks, and militarily focused states gradually converge toward higher m_i and lower k_i, with their distributions stabilizing in shape and location. The global militarization trajectory $\bar{m}(t)$ initially increases due to fear-driven adjustments, then flattens and settles at $\bar{m}^*$. This trajectory reflects endogenous strategic adjustment and mutual learning across countries.

We then introduce an exogenous militarization shock: a sudden increase in perceived external threat or internal instability in a large country (e.g., China or Russia), modeled as a temporary exogenous jump in arms investment $g_i(t)$ and/or military utility weight $\eta_{1,i}$. This perturbation raises the drift term in the military accumulation equation and causes a spike in $m_i(t)$, which quickly propagates through the mean field $\bar{m}(t)$. In response, other countries adjust their controls defensively, increasing arms spending and reducing capital and consumption. The global average $\bar{m}(t)$ rises rapidly and flattens at a new, elevated level. Even after the initial shock is removed and $\eta_{1,i}$ reverts to its pre-shock value, the global system does not return to the original equilibrium. Instead, it converges to a new steady state with higher militarization and lower capital across the entire population.

This irreversibility illustrates strategic hysteresis: once militarization has escalated, the cost of disarming becomes politically and economically too high for many countries. The long-term value functions $V_i(t,k,m)$ become skewed toward sustaining arms to preserve deterrence and respond to others' persistent buildup. Consumption and capital investment remain suppressed due to tighter budgets and elevated shadow prices. Unlike standard economic models with smooth convergence, here the transition dynamics reveal path dependence and locking-in effects.

We also examine transition paths under positive coordination shocks — for instance, a globally enforced disarmament agreement or institutional constraint on arms investment. In this case, we introduce a structural decrease in ϵ_i and $\eta_{1,i}$ for all countries simultaneously. The global militarization level $\bar{m}(t)$ drops gradually, and the drift in capital investment increases. However, the rate of convergence back to the low-arms equilibrium is significantly slower than the rate of escalation in the earlier scenario. In the absence of perfect coordination or credible enforcement, some countries remain skeptical and maintain higher arms as insurance, preventing full convergence to the peaceful steady state.

These transition experiments emphasize a critical insight of the model: even when steady states are formally symmetric and stable, the dynamics of strategic interaction make some equilibria far more accessible than others. The high-arms equilibrium is "easier" to reach and harder to escape due to asymmetric risk, informational frictions, and the one-sided nature of deterrence. Once $\bar{m}(t)$ rises, it increases the marginal benefit of arming and reduces the marginal utility of consumption, leading to systemic inertia.

Hence, the transition dynamics confirm that global militarization is not merely a long-run consequence of structural heterogeneity — it is shaped continuously by shocks, expectations, and strategic reactions. The path toward global peace or conflict is not only determined by steady-state parameters but also by the sequence of actions and reactions along the way. In the final section, we now summarize the paper's findings and highlight directions for further theoretical, empirical, and policy-oriented extensions.

11 Conclusion and Directions for Further Research

This paper has developed and solved a dynamic stochastic mean field game model of capital accumulation and military competition among heterogeneous nations. In doing so, we have constructed a framework in which each country optimally allocates its economic resources between consumption, capital investment, and military spending, while accounting for the strategic consequences of global militarization. Countries differ in productivity, volatility, psychological disutility from arms races, and military efficiency, and each makes forward-looking decisions in response to both domestic circumstances and the evolving global environment. Their interactions are mediated through a mean field term — the global average militarization level — which captures the externalities generated by the arms decisions of all nations.

We established a rigorous analytical structure that combines the Hamilton–Jacobi–Bellman (HJB) equation for optimal decision-making with the Fokker–Planck equation for the evolution of state distributions. The closed-form solutions under a linear–quadratic benchmark revealed that national behavior depends critically on shadow prices of internal resources, which themselves are shaped by military tensions and volatility. We also showed that in equilibrium, each country's beliefs about the global arms environment must match the aggregate outcome emerging from the decentralized system — a consistency condition that closes the mean field game and determines the global militarization level endogenously.

Through steady-state analysis, we demonstrated how countries converge to heterogeneous distributions of capital and military power. Militarized equilibria emerge not only from offensive ambition but also from defensive anticipation, particularly when global militarization suppresses productivity and elevates strategic anxiety. Simulations across major countries — including the United States, China, Russia, India, Iran, Turkey, Japan, and the European Union — illustrated how structural differences produce divergent investment patterns and joint contributions to a shared militarized environment. Comparative statics further showed how key parameters — military efficiency, volatility, sensitivity to global militarization — shape both national and global outcomes.

Crucially, our analysis of transition dynamics uncovered hysteresis effects: once a militarized equilibrium is reached, it is difficult to escape. Escalatory shocks have persistent consequences because optimal behavior adjusts defensively, locking countries into high-arms, low-capital equilibria even after the original threat has faded. The global average militarization level $\bar{m}(t)$ is thus path-dependent, shaped as much by history and expectations as by structural fundamentals. This finding underscores the importance of credible coordination and trust-building to prevent self-reinforcing arms races that reduce global welfare.

Several promising avenues remain for future research. First, the model can be extended to include endogenous regime change, political instability, or ideological preferences, allowing domestic institutions to interact with international dynamics. Second, empirical estimation of country-specific parameters would allow for more precise calibration and forecasting of real-world arms race trajectories. Third, game-theoretic extensions involving coalitions, alliances, or reputation dynamics could capture the layered complexity of international relations beyond the mean field average. Finally, one may explore optimal international policy mechanisms — such as arms control treaties, defense subsidies, or transparency commitments — that shift equilibria toward peaceful paths by internalizing the externalities that strategic militarization imposes on the global system.

In sum, this paper contributes a rigorous and flexible framework for understanding how decentralized strategic behavior shapes global militarization, investment, and welfare. It bridges dynamic optimization, stochastic game theory, and geopolitical analysis in a way that highlights both the structural causes of militarized equilibria and the strategic logic required to overcome them.

REFERENCES

Carmona, R., & Delarue, F. (2018). Probabilistic Theory of Mean Field Games with Applications I: Mean Field FBSDEs, Control, and Games. Springer.

Carmona, R., & Delarue, F. (2018). Probabilistic Theory of Mean Field Games with Applications II: Mean Field Games with Common Noise and Master Equations. Springer.

Fearon, J. D. (1995). Rationalist explanations for war. International Organization, 49(3), 379–414.

Gong, L., & Zou, H.-F. (2003). Military spending and stochastic growth. Journal of Economic Dynamics and Control, 28(1), 153–170.

Hobbes, T. (1651). Leviathan. Oxford University Press.

Huang, M., Malhamé, R. P., & Caines, P. E. (2006). Large population stochastic dynamic games: Closed-loop McKean–Vlasov systems and the Nash certainty equivalence principle. Communications in Information and Systems, 6(3), 221–252.

Lasry, J.-M., & Lions, P.-L. (2007). Mean field games. Japanese Journal of Mathematics, 2(1), 229–260.

Nozick, R. (1974). Anarchy, State, and Utopia. Basic Books.

Powell, R. (2006). War as a commitment problem. International Organization, 60(1), 169–203.

Tilly, C. (1990). Coercion, Capital, and European States, AD 990–1992. Blackwell.

Zou, H.-F. (1995). A dynamic model of capital and arms accumulation. Journal of Economic Dynamics and Control, 19(1–2), 371–393.

Zou, H.-F. (1997). Dynamic analysis in the Viner model of mercantilism. Journal of International Money and Finance, 16(4), 637–651.