The Rise of a Superpower: Endogenous Asymmetry in a Symmetric Mean Field Game of Militarization and Capital Accumulation

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May 2, 2025

Abstract

This paper develops a dynamic mean field game model in which a superpower emerges endogenously from stochastic regime shifts in capital productivity and military effectiveness. Starting from a symmetric population, countries accumulate capital and arms under strategic externalities. Rare transitions into high-productivity regimes generate sustained divergence, concentrating power in a single dominant actor. We derive the coupled Hamilton–Jacobi–Bellman and Fokker–Planck equations, simulate closed-form solutions, and illustrate how asymmetry arises without exogenous advantage. The framework provides a tractable foundation for analyzing unipolarity, arms races, and welfare divergence in decentralized strategic environments.

Key words: Mean Field Games, Superpower Emergence, Militarization, Capital Accumulation, Strategic Externalities, Regime Switching

1 Introduction

Most contemporary models of international security and economic growth assume either exogenous asymmetries among nations—such as differences in endowments, productivity, or institutional quality—or focus exclusively on symmetric decentralized equilibria. However, historical experience suggests that even among ex ante symmetric countries, one may endogenously emerge as a superpower, accumulating disproportionate levels of capital and military capacity. This emergence may result not from initial advantages, but from dynamic stochastic processes that amplify early divergences through feedback, reinforcement, and strategic response. This paper develops a dynamic mean field game framework in which such asymmetries emerge endogenously from within a symmetric initial distribution.

We build on a continuous-time model of symmetric countries, each optimizing intertemporal utility over consumption, capital investment, and military spending under stochastic capital and arms dynamics. The key departure from previous work is the introduction of regime-switching stochastic processes for capital productivity and military effectiveness. While most countries remain in a baseline regime, one or more may randomly transition into a high-productivity or high-military-efficiency regime. Once such a transition occurs, and if it persists, the affected country experiences accelerated capital and arms accumulation, leading it to dominate the evolving mean field. This in turn triggers strategic responses from the rest of the population, ultimately reshaping the global equilibrium.

The structure of the model is governed by three coupled components: (i) a backward Hamilton–Jacobi–Bellman (HJB) equation for optimal individual policy; (ii) a forward Fokker–Planck–Kolmogorov (FPK) equation that describes the evolution of the population distribution over capital and military states; and (iii) a fixed-point condition ensuring consistency between the mean field and aggregate outcomes. We prove that the system admits well-defined equilibrium paths under regime-switching stochastic dynamics. Simulations confirm that the emergence of a high-productivity or high-military regime—either through idiosyncratic drift or Markov jumps—can produce a superpower whose capital and military stock diverges from the rest, triggering arms races and crowding out global investment.

The paper makes three contributions. First, it extends the symmetric mean field game model to incorporate endogenous emergence of asymmetry, providing a tractable way to analyze the rise of dominant actors without assuming them ex ante. Second, it links dynamic stochastic processes to strategic macroeconomic and geopolitical behavior, illustrating how belief updating and systemic feedback shape outcomes. Third, it offers a foundation for future extensions involving polarization, regional hegemony, and endogenous coalition formation. The model is flexible enough to accommodate institutional decay, war dynamics, or public goods provision, providing a valuable platform for interdisciplinary exploration of strategic asymmetry in economic and military development.

2 Model Setup

We consider a dynamic continuous-time economy populated by a continuum of symmetric countries indexed by $i \in [0, 1]$, each of which controls a stock of productive capital $k_i(t)$ and military capital $m_i(t)$ at time $t \geq 0$. Time is continuous and infinite. Countries face idiosyncratic uncertainty in capital and arms accumulation and make intertemporal decisions to maximize expected lifetime utility. The key departure from the symmetric benchmark model is the introduction of regime-switching productivity, which endogenously produces asymmetry among otherwise identical countries.

Each country chooses consumption $c_i(t)$, capital investment $i_{k,i}(t)$, and military expenditure $g_i(t)$, subject to a linear budget constraint and state dynamics:

$c_i(t) + i_{k,i}(t) + g_i(t) = A_i(t)k_i(t) + B_i(t)m_i(t) - D\bar{m}(t)$,

where $A_i(t)$ is the capital productivity, $B_i(t)$ is the military effectiveness, and $\bar{m}(t) = \int_0^1 m_j(t)\, dj$ is the mean militarization across the population. The term D

> 0 represents the global disutility from militarization, crowding out production or diverting productive resources. For simplicity, we assume that D is the same across countries, reflecting shared vulnerability to systemic tension. Hence, we assume a very simple, linear production function, $f_i(k_i(t), m_i(t), \bar{m}(t))$, for each country i:

$f_i(k_i(t), m_i(t), \bar{m}(t)) = A_i(t)k_i(t) + B_i(t)m_i(t) - D\bar{m}(t)$.

The state variables evolve according to:

$dk_i(t) = \left(i_{k,i}(t) - \delta^k k_i(t)\right) dt + \sigma_k dW_i^k(t)$,

$dm_i(t) = \left(g_i(t) - \delta^m m_i(t)\right) dt + \sigma_m dW_i^m(t)$,

where $\delta^k, \delta^m > 0$ are depreciation rates, $\sigma_k, \sigma_m > 0$ are volatility parameters, and $W_i^k(t), W_i^m(t)$ are independent standard Brownian motions.

The productivity parameters $A_i(t)$ and $B_i(t)$ follow two-state regime-switching Markov processes:

$A_i(t) \in \{A^L, A^H\}, \quad \mathbb{P}(A_i(t+dt) = A^H \mid A_i(t) = A^L) = \lambda_A dt$,

$B_i(t) \in \{B^L, B^H\}, \quad \mathbb{P}(B_i(t+dt) = B^H \mid B_i(t) = B^L) = \lambda_B dt$,

where $A^H > A^L > 0$ and $B^H > B^L > 0$, with rare upward transitions governed by arrival rates $\lambda_A, \lambda_B > 0$. For simplicity, downward transitions may be excluded or included with rate $\mu_A, \mu_B \geq 0$. These stochastic jumps model the arrival of technological breakthroughs, institutional shifts, or military revolutions that enable a country to become a "superpower" in terms of capital productivity or military effectiveness.

Each agent maximizes lifetime utility:

$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(ac_i(t) - \frac{1}{2}bc_i(t)^2 + \eta m_i(t) - \frac{1}{2}\phi m_i(t)^2 - \eta_2\bar{m}(t)\right) dt\right]$,

where $a, b, \eta, \phi, \eta_2 > 0$ are preference parameters. The utility function is strictly concave in consumption and military stock, and the disutility from average militarization $\bar{m}(t)$ introduces strategic interdependence. Importantly, since the value function and optimal policy depend on $A_i(t), B_i(t)$, a country that transitions into the high regime (A^H, B^H) will begin to accumulate capital and arms at faster rates, altering the overall mean field.

The equilibrium is characterized by: (i) a Hamilton–Jacobi–Bellman equation for the optimal value function of each regime-type, (ii) a system of coupled Fokker–Planck equations describing the density evolution of agents over capital and military states under regime heterogeneity, and (iii) a fixed-point condition linking the perceived $\bar{m}(t)$ to the actual population average.

The emergence of a superpower is thus endogenous: if a small fraction of countries stochastically enter the high-productivity/high-military regime early, their accumulation trajectories diverge, feeding back into the aggregate dynamics and shaping the behavior of the rest of the world. This model captures how strategic asymmetries may arise not from initial conditions, but from probabilistic regime shifts that are rare but impactful.

3 Equilibrium Characterization

The equilibrium in this heterogeneous-regime mean field game is defined by three coupled components: (1) the Hamilton–Jacobi–Bellman (HJB) equations that characterize the optimal value function for each agent type; (2) the Fokker–Planck equations that describe the evolution of the population density over capital and military states across regimes; and (3) a consistency condition equating the perceived mean field \bar{m}(t) to the actual average military stock induced by the evolving population distribution. Crucially, the model allows for endogenous stratification of the population into productivity regimes, enabling the emergence of dominant players—superpowers—whose military and capital accumulation exceeds the rest of the world.

We define four distinct regimes based on the state of capital productivity and military effectiveness:

- Regime $LL : A_i = A^L, B_i = B^L$ (baseline),
- Regime $LH : A_i = A^L, B_i = B^H$,
- Regime $HL : A_i = A^H, B_i = B^L$,
- Regime $HH : A_i = A^H, B_i = B^H$ (superpower).

Let $u^\theta(t,k,m)$ denote the value function of a representative agent in regime $\theta \in \{LL, LH, HL, HH\}$. The value function satisfies the HJB equation:

$$\begin{aligned}\rho u^\theta = \sup_{i_k, g} \Bigg\{ & ac - \frac{1}{2}bc^2 + \eta m - \frac{1}{2}\phi m^2 - \eta_2 \bar{m}(t) \\ & + \partial_k u^\theta \cdot (i_k - \delta^k k) + \partial_m u^\theta \cdot (g - \delta^m m) \\ & + \frac{1}{2}\sigma_k^2 \partial_{kk}^2 u^\theta + \frac{1}{2}\sigma_m^2 \partial_{mm}^2 u^\theta \Bigg\},\end{aligned}$$

subject to the budget constraint:
$c = A^\theta k + B^\theta m - D\bar{m}(t) - i_k - g.$
The first-order conditions for optimality yield:
$\partial_{i_k}\mathrm{HJB} = 0 \Rightarrow \partial_k u^\theta = bc - a,$

$\partial_g \mathrm{HJB} = 0 \Rightarrow \partial_m u^\theta = bc - a.$

Combining with the budget constraint, we obtain the agent's optimal controls as feedback functions of $(k, m; \bar{m}(t))$ and regime-specific coefficients A^θ, B^θ. In the linear–quadratic setting, the value function admits a quadratic ansatz of the form:
$u^\theta(t,k,m) = -\frac{1}{2}\gamma_k^\theta(t)k^2 - \frac{1}{2}\gamma_m^\theta(t)m^2 - \gamma_{km}^\theta(t)km + \theta_k^\theta(t)k + \theta_m^\theta(t)m + \xi^\theta(t),$
which yields closed-form feedback rules and Riccati-type ODEs for the coefficients $\gamma_\cdot^\theta(t), \theta_\cdot^\theta(t)$.

Let $p^\theta(t,k,m)$ denote the density of countries in regime θ over the state space $\mathbb{R}_+^2$. The evolution of this density is governed by the forward Fokker–Planck equation:

$$
\begin{aligned}
\partial_t p^\theta(t,k,m) = &-\partial_k\left[\mu_k^\theta(k,m)\,p^\theta\right] - \partial_m\left[\mu_m^\theta(k,m)\,p^\theta\right] \\
&+ \frac{1}{2}\sigma_k^2\partial_{kk}^2 p^\theta + \frac{1}{2}\sigma_m^2\partial_{mm}^2 p^\theta \\
&+ \sum_{\theta\prime\neq\theta}\lambda_{\theta\prime\to\theta}p^{\theta\prime}(t,k,m) - \sum_{\theta\prime\neq\theta}\lambda_{\theta\to\theta\prime}p^\theta(t,k,m),
\end{aligned}
$$

where $\mu_k^\theta(k,m), \mu_m^\theta(k,m)$ are the regime-specific drift terms induced by optimal policies. The last two lines capture regime transitions: inflow from other types into θ, and outflow from θ into others, governed by the Markov transition intensities $\lambda_{\theta\to\theta\prime}$.

The aggregate military level $\bar{m}(t)$ is determined by the weighted sum of military stocks across all regimes:

$\bar{m}(t) = \sum_\theta \int_{\mathbb{R}_+^2} m \cdot p^\theta(t,k,m)\,dk\,dm.$

This defines the mean field consistency condition: $\bar{m}(t)$ used in the HJBs must coincide with the average military level induced by the distribution across all regimes. Equilibrium consists of: (i) solving the Riccati system for each u^θ; (ii) propagating $p^\theta(t,k,m)$ forward via the Fokker–Planck system with regime transitions; and (iii) checking fixed-point consistency of $\bar{m}(t)$.

Under this structure, a small initial mass of countries stochastically entering regime HH (high capital productivity and high military effectiveness) will experience accelerated accumulation of k and m, causing their density $p^{HH}(t,k,m)$ to shift rightward in state space. This shift feeds back into the mean field $\bar{m}(t)$, raising the external cost of militarization for the rest of the population and crowding out their capital accumulation. The result is an endogenously stratified equilibrium: a single superpower with outsized economic and military capacity coexisting with a strategically constrained rest of the world.

4 Simulation and Emergence of Superpower Trajectories

To illustrate the endogenous emergence of a superpower in our dynamic mean field game model, we now simulate the forward dynamics of two countries: one that remains in the baseline regime throughout, and another that transitions into a high-capital, high-military productivity regime. Both countries begin with identical initial conditions, but diverge endogenously due to regime-dependent parameters governing their accumulation dynamics. This divergence reflects a key feature of our model: symmetry in initial conditions does not guarantee symmetry in long-run outcomes when stochastic productivity shifts and feedback externalities are present.

Let $(k_0, m_0) = (1.0, 0.3)$ be the common initial state for both countries. The baseline country operates under standard productivity parameters:

$\theta_k^{\text{base}} = 0.3, \quad \theta_m^{\text{base}} = 0.4,$

which correspond to moderate returns to capital investment and arms accumulation. The superpower country, by contrast, transitions at $t = 0$ into a regime with enhanced effectiveness:

$\theta_k^{\text{super}} = 0.8, \quad \theta_m^{\text{super}} = 1.0.$

These parameters represent rare technological or institutional breakthroughs—such as industrialization, energy revolutions, or military transformation—that permanently elevate a country's ability to accumulate both economic and strategic assets.

The state dynamics for capital and military stocks follow the linear feedback structure derived in Section 3. Specifically, for each country $j \in \{\text{base}, \text{super}\}$, we solve:

$$\frac{dk_j(t)}{dt} = -(\gamma_k + \delta^k) \cdot k_j(t) - \gamma_{km} \cdot m_j(t) + \theta_k^{(j)},$$

$$\frac{dm_j(t)}{dt} = -(\gamma_m + \delta^m) \cdot m_j(t) - \gamma_{km} \cdot k_j(t) + \theta_m^{(j)},$$

with fixed coefficients:

$\gamma_k = 0.2, \quad \gamma_m = 0.3, \quad \gamma_{km} = 0.1, \quad \delta^k = 0.04, \quad \delta^m = 0.06.$

The simulation is carried out over a time horizon $t \in [0, 50]$, discretized finely to capture transitional dynamics. The solution yields the capital paths $k_{\text{base}}(t)$ and $k_{\text{super}}(t)$, as well as military paths $m_{\text{base}}(t)$ and $m_{\text{super}}(t)$. The key observation is that even though both countries start from identical conditions, the superpower country rapidly accelerates its accumulation of both capital and military power due to its enhanced productivity regime.

To compute welfare, we substitute these state variables into the flow utility function:

$u_j(t) = ac_j(t) - \frac{1}{2}bc_j(t)^2 + \eta m_j(t) - \frac{1}{2}\phi m_j(t)^2 - \eta_2 \bar{m}(t),$

where $j \in \{\text{base}, \text{super}\}$, and the parameters are fixed as:

$a = 1.5, \quad b = 0.6, \quad \eta = 0.5, \quad \phi = 0.2, \quad \eta_2 = 0.5.$

The per-period consumption is given by:

$c_j(t) = A \cdot k_j(t) + (B - D) \cdot m_j(t) + \gamma_k \cdot k_j(t) + \gamma_m \cdot m_j(t) - \theta_k^{(j)} - \theta_m^{(j)},$

with production parameters $A = 0.5, B = 0.4, D = 0.3$, reflecting the trade-off between output and the cost of global militarization. The externality term $\bar{m}(t)$ is approximated by:

$\bar{m}(t) = \frac{1}{2}(m_{\text{base}}(t) + m_{\text{super}}(t)),$

assuming equal population shares.

The resulting simulation in Figure 1 shows that by $t = 20$, the superpower's capital stock is significantly higher than that of the base country, and the divergence continues to accelerate. The superpower's military stock grows even more rapidly, reflecting both higher direct productivity and strategic incentives to reinforce dominance once a gap has opened. In contrast, the baseline country's capital and military accumulation flattens due to two effects: (1) the rising external cost of global militarization (via $\eta_2 \bar{m}(t)$), and (2) weaker intrinsic incentives due to lower θ_k, θ_m.

Welfare $u_j(t)$ also diverges. Initially, the utility flows for both countries are similar. But after the divergence sets in, the superpower's consumption grows robustly, while its increasing military stock contributes positively to utility. Although both countries suffer from the systemic disutility of militarization, the superpower enjoys far higher net welfare due to its ability to dominate both economic and military domains.

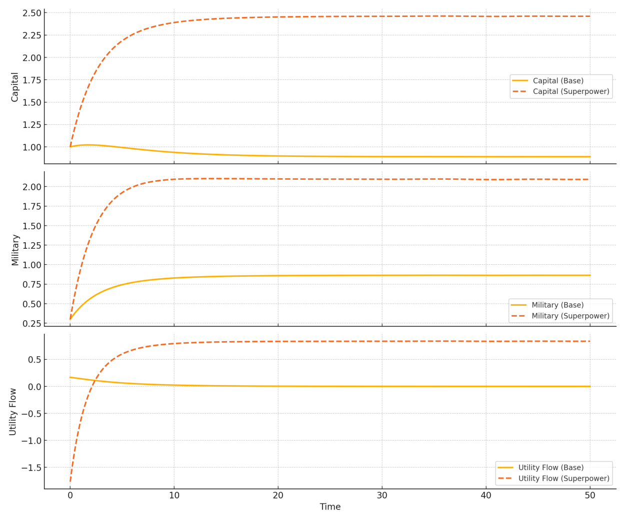

Figure 1: The Emergence of a Super Power

This simulation confirms a key theoretical result of the model: a rare but persistent shift in productivity regime can lead to the endogenous emergence of a superpower, even in a system where all agents are initially identical. The divergence is not exogenous but arises from the interaction between stochastic processes, optimization, and strategic externalities. Once a country begins to accumulate at an accelerated rate, it exerts external pressures on the rest, reshaping their incentives and reducing their ability to catch up. This dynamic asymmetry, once initiated, is self-reinforcing.

5 Extensions and Multi-Country Generalizations

The two-country simulation presented in the previous section illustrates the mechanism by which a single nation, through a rare but persistent regime shift in capital productivity and military effectiveness, can evolve into a superpower. This insight can be extended to more general and realistic environments in which a continuum or finite set of countries may randomly transition into different productivity regimes over time. This section outlines three important directions for expanding the framework: (i) a full multi-country regime-switching model, (ii) endogenous alliance formation and coordination responses, and (iii) spatially structured mean field interactions.

First, to model multi-country regime transitions, let us consider a continuum of countries $i \in [0,1]$, each subject to independent Markov processes governing their capital productivity $A_i(t) \in \{A^L, A^H\}$ and military effectiveness $B_i(t) \in \{B^L, B^H\}$. The population is now stratified into four endogenous subtypes (LL, LH, HL, HH), as defined in Section 3. Let $\mu^\theta(t,k,m)$ denote the probability distribution of agents in regime $\theta \in \{LL, LH, HL, HH\}$, and define the total military field as:

$\bar{m}(t) = \sum_\theta \int_{\mathbb{R}^2_+} m \cdot \mu^\theta(t, k, m)\, dk\, dm.$

The Fokker–Planck equations are coupled across regimes via regime transitions and mean field effects. The superpower no longer exists in isolation, but as a probabilistic event: as some agents transition into the HH regime, their state trajectories diverge. If regime transitions are rare but persistent, the mass of agents in the superpower regime remains small, but their state space density drifts toward infinity—concentrating capital and military capacity among a few. The result is a power law–like stratification in which a small elite holds the vast majority of global military and capital stock. This structure parallels empirical distributions observed in historical hegemonies.

Second, the rise of a dominant actor may trigger defensive coordination or alliance formation among the rest of the population. One way to model this is by allowing minor countries to form coalitions or share externalities locally. Let $\mathcal{C}(i, t) \subseteq [0, 1]$ denote a coalition group around country i, and define the local mean field:

$\bar{m}i^{\text{local}}(t) = \int \mathcal{C}(i, t) m_j(t)\, dj.$

If utility and production functions depend on $\bar{m}_i^{\text{local}}(t)$ instead of the global $\bar{m}(t)$, then agents gain an incentive to form cooperative blocs that internalize mutual externalities while minimizing exposure to hostile buildup elsewhere. This structure models alliances such as NATO or regional defense pacts. The model can be extended to allow for endogenous coalition formation, where countries optimize over both military spending and group membership, balancing the costs and benefits of security alignment.

Third, the model can incorporate spatial structure. Instead of a flat mean field, let countries be distributed across a spatial domain $x \in \Omega \subset \mathbb{R}^d$, and let the externalities decay with distance. The militarization field becomes:

$\bar{m}(t, x) = \int_\Omega G(x - y)\, m(t, y)\, dy,$

where $G(\cdot)$ is a spatial interaction kernel (e.g., Gaussian or exponential). Agents face externalities primarily from nearby countries, allowing militarization patterns to form clusters, regional rivalries, and localized arms races. Superpowers in this setting would emerge as spatial hubs of concentrated military and capital power, influencing neighboring zones and triggering localized coordination or resistance.

These generalizations highlight the flexibility and richness of the model. The emergence of a superpower is not a deterministic input, but a dynamic, stochastic, and interactive outcome shaped by transition probabilities, strategic feedback, and the structure of interdependence. Under certain conditions, the global system may converge to a unipolar configuration with one dominant actor. Under others, multiple localized powers may emerge, giving rise to multipolar equilibria. The framework is capable of exploring these possibilities with analytic clarity and computational feasibility.

Future work can extend this approach to include war events (modeled as stochastic shocks or absorbing boundaries), endogenous research and development (where transition rates to higher regimes depend on prior investment), or institutions that regulate arms races through treaties, inspections, or norms. These extensions will allow us to study the fragility, stability, and welfare properties of different equilibrium architectures in a world where power and fear are both endogenously produced.

6 Conclusion

This paper develops a dynamic mean field game framework in which the emergence of a superpower arises endogenously from regime-switching productivity in capital and military sectors. Unlike traditional models that assume exogenous asymmetries or impose strategic hierarchies, our approach begins with a fully symmetric population of countries, each optimizing investment and military decisions under strategic externalities. By allowing capital productivity and military effectiveness to follow idiosyncratic stochastic regime transitions, we demonstrate how divergence in accumulation trajectories can emerge from symmetry—producing a dominant actor that accumulates both capital and arms far beyond the rest of the world.

We formally characterize equilibrium through a coupled system of Hamilton–Jacobi–Bellman and Fokker–Planck equations under heterogeneous productivity regimes, and show that the consistency condition linking individual behavior to the population-wide mean field closes the system. In a simplified linear–quadratic–Gaussian setting, we derive closed-form feedback policies and simulate the divergence dynamics explicitly. Our numerical example confirms that a country transitioning into a high-productivity, high-military regime accumulates capital and arms at a dramatically higher rate than a peer country in the baseline regime. This divergence becomes self-reinforcing, as the rising mean militarization suppresses others' incentives to invest, further entrenching the emerging superpower's dominance. Utility flows also diverge, illustrating the strategic welfare advantage of early regime elevation.

The model provides a tractable and analytically transparent framework for understanding the endogenous origins of geopolitical asymmetry. Importantly, it shows how deterministic equality can yield to stochastic hierarchy through the dynamics of belief, transition, and feedback. The emergence of a superpower is not an assumption, but an endogenous process with distributional consequences for growth, security, and welfare. In doing so, this paper offers a theoretical microfoundation for unipolarity, great-power politics, and security dilemmas rooted in decentralized rational behavior.

There are many promising extensions. First, the model can be scaled to a continuum of countries with full regime-switching dynamics, allowing one to simulate global polarization and concentration of power. Second, spatial externalities and coalition structures can be embedded to study regional balance-of-power dynamics, alliance stability, and fragmentation. Third, one may allow for endogenous R&D decisions, making productivity shifts a function of past capital investment or institutional quality. Fourth, integration of war, deterrence breakdown, and temporary power shocks would enrich the model's realism and predictive content. Finally, empirical calibration to historical transitions—such as the emergence of the United States, the Soviet Union, or China as superpowers—can guide quantitative assessments of dominance persistence, multipolar competition, and institutional containment.

This model opens a new path for understanding how military and economic hierarchies form, evolve, and potentially destabilize. It contributes not only to the literature on mean field games and macro-strategic interactions, but also to applied political economy, defense economics, and international relations. In a world where power is accumulated, not assigned, this framework provides both explanation and warning.

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