Institutional Capital and Asset Pricing: A Modified Lucas Tree Model Integrating Coase–Williamson–Demsetz–Barzel Insights

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Abstract

This paper extends the Lucas (1978) asset pricing framework by incorporating institutional capital as both a productive asset—enhancing dividend growth—and a direct source of utility. Building on the insights of Coase, Demsetz, Williamson, and Barzel, we model institutional investment alongside consumption, deriving closed-form affine pricing solutions under stochastic institutional quality. The model demonstrates how institutional capital influences the price–dividend ratio, risk-free rate, and equity premium through both productivity and preference channels. Analytical results and simulations reveal that stronger institutional investment raises asset values, lowers risk premia, and stabilizes returns, bridging institutional economics with modern asset pricing theory.

1 Introduction

The central aim of this paper is to extend the canonical Lucas (1978) asset pricing model by introducing institutional capital as a fundamental determinant of asset returns, volatility, and capital market development. In the original Lucas tree model, a representative agent consumes the stochastic endowment stream of an infinitely-lived fruit tree, and equilibrium asset prices are determined by the stochastic discount factor implied by intertemporal consumption smoothing under rational expectations. This baseline model, elegant in its tractability, has served as a cornerstone of modern asset pricing theory. Yet it abstracts entirely from the institutional environment in which asset markets operate. The formal inclusion of institutional capital — the accumulated stock of societal structures such as property rights, rule of law, contract enforcement, civic freedoms, and transparent governance — represents a crucial theoretical and empirical advance. In practice, both historical experience and a vast body of institutional economics literature suggest that institutions not only enable economic exchange at lower transaction costs but also directly influence the productivity of the underlying assets that generate dividends.

Our theoretical motivation is firmly grounded in the seminal works of Coase (1937, 1960), Demsetz (1967), Williamson (1985), and Barzel (1997). Coase's theory of transaction costs demonstrated that neither markets nor firms operate in a frictionless world; instead, all production requires a governance structure to minimize the sum of internal and external transaction costs. Demsetz further argued that the structure of property rights evolves to internalize externalities when the gains from doing so exceed the costs, implying that secure property rights and legal protections are not exogenous but are produced and maintained through investment. Williamson extended these insights by developing the discriminating alignment hypothesis, showing that governance structures adapt to economize on contractual hazards such as opportunism under asset specificity. Barzel's analysis emphasized the centrality of measurement costs in determining institutional arrangements and argued that institutions allocate economic rights in ways that directly shape productivity. Taken together, these contributions imply that institutions are not passive background conditions: they are productive assets whose accumulation requires deliberate investment and whose quality directly affects both the utility of agents and the production capacity of the economy.

In the modified Lucas tree framework developed here, we model institutional capital $\mathcal{I}(t)$ as an additional state variable alongside consumption $C(t)$. Institutional capital is accumulated through explicit investment $I(t)$, financed out of the economy's endowment. This capital stock enters both the agent's utility function and the dividend production function of the tree. In the utility function, institutional capital captures the direct welfare benefits of living in a society with strong property rights, civil liberties, and functioning markets. In the production function for dividends, institutional capital increases the productivity of the tree — an institutional analog to total factor productivity — by reducing transaction and enforcement costs, improving trust in exchange, and enabling the more efficient allocation of physical and human capital. This is consistent with Coase's argument that lowering transaction costs increases the net product of economic activity, with Demsetz's and Williamson's emphasis on incentive-compatible governance, and with Barzel's measurement-cost framework in which better institutions reduce information frictions and resource misallocation.

By embedding institutional capital into the Lucas tree model, we achieve three objectives. First, we capture the endogenous interaction between institutional quality and equilibrium asset prices, thereby connecting the literature on institutional economics with mainstream asset pricing theory. Second, the model generates explicit affine solutions for the price–dividend ratio, the short-term risk-free rate, and the equity premium as functions of institutional capital and its investment rate. This allows us to quantify how improvements in institutional quality affect asset valuations and risk premia, both in the short run through discount factor effects and in the long run through productivity gains. Third, the model provides a tractable framework for evaluating policy scenarios: for example, how aggressive institutional investment — such as legal reforms, judicial strengthening, or anti-corruption measures — changes equilibrium financial market outcomes.

2 Model Setup

We consider a continuous-time pure-exchange economy in the tradition of Lucas (1978), but extended to include an explicit stock of institutional capital $\mathcal{I}_t$ which influences both household preferences and the productivity of the endowment tree. The economy is populated by a representative agent who derives utility from consumption C_t and from the quality of institutions $\mathcal{I}_t$, reflecting the arguments of Coase (1960), Demsetz (1967), Williamson (1985), and Barzel (1997) that secure property rights, low transaction costs, and predictable governance improve both welfare directly and productive capacity indirectly. Preferences are given by the recursive, time-separable functional

$$U_0 = \mathbb{E}_0 \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} \mathcal{I}_t^\theta \, dt,$$

where $\rho > 0$ is the subjective discount rate, $\sigma > 0$ is the coefficient of relative risk aversion, and $\theta \in \mathbb{R}$ measures the marginal utility of institutional quality. When $\theta > 0$, higher-quality institutions enter positively into utility, consistent with the idea that citizens value not only the flow of goods but also the environment in which those goods are enjoyed.

The sole physical asset in the economy is a Lucas "apple" tree, whose fruit constitutes the endowment and pays a continuous dividend flow D_t. Crucially, institutional capital raises the tree's productivity: we assume

$$\frac{dD_t}{D_t} = \big(\mu_0 + \mu_1 x_t\big)dt + \sigma_D(x_t)\, dW_t^D, \quad \sigma_D(x) = \sigma_0 e^{-\eta x}, \quad x_t \equiv \log \mathcal{I}_t,$$

where $\mu_1 > 0$ captures the elasticity of dividend growth with respect to institutional quality, $\sigma_0 > 0$ is baseline dividend volatility, and $\eta \geq 0$ models the degree to which better institutions dampen volatility by stabilizing property rights, contracts, and investment incentives. This functional form reflects the empirical observation that countries with stronger institutions not only enjoy higher growth but also experience lower output volatility.

Institutional capital $\mathcal{I}t$ is an endogenous state variable that accumulates through deliberate investment $I(t)$ financed out of the apple harvest, subject to depreciation and stochastic shocks:

$$d\mathcal{I}t = \big[I(t) - \delta_{\mathcal{I}}\mathcal{I}t\big]dt + \sigma_{\mathcal{I}}\mathcal{I}t\, dW_t^{\mathcal{I}},$$

where $\delta > 0$ is the depreciation rate of institutional quality (reflecting political decay, corruption, or erosion of rule of law), and $\sigma_{\mathcal{I}}$ is the volatility of institutional shocks, representing random events such as constitutional crises, coups, or reform waves. We allow W_t^D and $W_t^{\mathcal{I}}$ to be correlated with $\mathrm{corr}(dW_t^D, dW_t^{\mathcal{I}}) = \varrho$, capturing the empirical link between economic and institutional shocks.

The resource constraint for the representative agent is explicitly

$$C_t + \Xi(I(t), \mathcal{I}_t) = D_t,$$

where $\Xi(\cdot)$ denotes the cost of raising institutional capital. We allow for either a linear installation cost $\Xi(I, \mathcal{I}) = \phi I$ or a quadratic adjustment cost $\Xi(I, \mathcal{I}) = \frac{\phi}{2} I^2/\mathcal{I}$. This formulation ensures that any investment in $\mathcal{I}_t$ comes at the expense of current consumption, introducing a trade-off between immediate utility from apples eaten today and future benefits from better institutions.

Given this setup, the stochastic discount factor (SDF) implied by the first-order condition for consumption is

$$M_t = e^{-\rho t} \left(\frac{C_t}{C_0} \right)^{-\sigma} \left(\frac{\mathcal{I}_t}{\mathcal{I}_0} \right)^{\theta},$$

so that institutional quality affects asset prices through two channels: (i) directly, as a component of marginal utility, and (ii) indirectly, via its effect on consumption dynamics. The presence of $\mathcal{I}_t$ in both the utility function and the dividend process means that shocks to institutional quality enter asset pricing both as preference shocks and as technology shocks, potentially making institutional risk a priced factor in equilibrium.

The representative agent's problem is to choose a consumption path $\{C_t\}$ and institutional investment policy $\{I(t)\}$ to maximize U_0 subject to the resource constraint and the laws of motion for D_t and $\mathcal{I}_t$. In later sections, we solve for the optimal $I(t)$ both exactly—via the Hamilton–Jacobi–Bellman equation when Ξ is specified—and in reduced form, parameterizing the elasticity

$$\kappa \equiv - \frac{\partial \log C_t}{\partial x_t} \bigg|_{x=0}$$

to capture the equilibrium substitution of consumption for institutional investment.[1] This reduced-form representation allows us to derive closed-form affine expressions for the price–dividend ratio $P_t/D_t = q_0 + q_1 x_t$, the short rate r_t, and the equity premium, all of which depend explicitly on θ, κ, and the technology parameters μ_1, η, $\sigma_{\mathcal{I}}$.

This formulation nests the classic Lucas tree model when $\theta = 0, \kappa = 0$, and $\mu_1 = 0$, but is far richer empirically: it captures the dual role of institutions in directly enhancing welfare and in raising and stabilizing the economy's productive capacity.

[1] We notice that the reason for parameterizing the elasticity

$$\kappa \equiv - \frac{\partial \log C_t}{\partial x_t} \bigg|_{x=0}$$

is that it gives a reduced-form, economically interpretable measure of how much the representative agent shifts resources away from consumption toward institutional investment in response to a marginal improvement in institutional quality, without having to commit to a full, closed-form structural solution of the HJB problem. Evaluating at $x_t = 0$ means you are looking at the marginal trade-off starting from the baseline institutional state — i.e., the local slope of the consumption–institutional investment frontier. This makes κ a primitive parameter that summarizes preferences and technology at the benchmark, before large institutional changes alter dynamics.

3 Equilibrium and the Pricing Kernel

The equilibrium of our modified Lucas apple tree economy is characterized by the optimal joint allocation of current apples to immediate consumption C_t and to the accumulation of institutional capital $\mathcal{I}_t$, subject to the stochastic evolution of dividends and institutions. As in Coase (1960), institutions are treated as a form of capital because they reduce transaction costs, improve the enforcement of contracts, and define the framework within which economic exchange takes place. Demsetz (1967) emphasized that property rights themselves are an economic good whose supply responds to demand when the gains from internalization exceed the costs. In our setting, these arguments manifest in the agent's willingness to divert a portion of current output to institutional investment $I(t)$, despite the immediate reduction in consumption, because such investment increases future dividends by improving the "rules of the game."

Formally, the representative agent solves

$$\max_{\{C_s,\, I(s)\}_{s\geq t}} \mathbb{E}_t \int_t^{\infty} e^{-\rho(s-t)} \frac{C_s^{1-\sigma}}{1-\sigma} \mathcal{I}s^{\theta}\, ds$$

subject to the resource constraint

$$C_t + \Xi(I(t), \mathcal{I}_t) = D_t,$$

and the laws of motion

$$\frac{dD_t}{D_t} = \big(\mu_0 + \mu_1 x_t\big)dt + \sigma_0 e^{-\eta x_t} dW_t^D, \quad x_t \equiv \log \mathcal{I}t,$$

$$d\mathcal{I}t = \big[I(t) - \delta_{\mathcal{I}} \mathcal{I}_t\big]dt + \sigma_{\mathcal{I}} \mathcal{I}_t\, dW_t^{\mathcal{I}}, \quad \operatorname{corr}(dW_t^D, dW_t^{\mathcal{I}}) = \varrho.$$

The link between $\mathcal{I}_t$ and D_t captures Williamson's (1985) theory of transaction cost economics: institutions not only reduce the "wedge" between potential and realized output but also stabilize the productive environment, thereby enhancing expected returns on capital and lowering risk. Barzel (1997) further argued that better-defined property rights reduce the dissipation of value in competitive appropriation; in our model, this is reflected in the reduced volatility term $\sigma_0 e^{-\eta x_t}$, which declines as institutions improve.

The Hamilton–Jacobi–Bellman (HJB) equation for the agent's value function $V(D_t, \mathcal{I}t)$ is given by
$\rho V = \max C, I_{\mathcal{I}} \left\{ \frac{C^{1-\sigma}}{1-\sigma} \mathcal{I}^{\theta} + V_D\, \mu_D D + V_{\mathcal{I}}\, [I - \delta_{\mathcal{I}} \mathcal{I}] + \frac{1}{2} V_{DD}\, \sigma_D^2 D^2 + \frac{1}{2} V_{\mathcal{I}\mathcal{I}}\, \sigma_{\mathcal{I}}^2 \mathcal{I}^2 + V_{D\mathcal{I}}\, \varrho \sigma_D \sigma_{\mathcal{I}} D\mathcal{I} \right\}$,where $\mu_D = (\mu_0 + \mu_1 \log \mathcal{I})D$ and $\sigma_D = \sigma_0 e^{-\eta \log \mathcal{I}} D$. The first-order condition with respect to C_t yields

$$C_t^{-\sigma} \mathcal{I}_t^{\theta} = V_D(D_t, \mathcal{I}_t),$$

which, combined with the envelope condition for D_t, implies that the marginal utility of consumption—and hence the stochastic discount factor—depends jointly on C_t and $\mathcal{I}_t$. Normalizing to $t = 0$, the SDF is

$$M_t = e^{-\rho t} \left(\frac{C_t}{C_0} \right)^{-\sigma} \left(\frac{\mathcal{I}_t}{\mathcal{I}_0} \right)^{\theta}.$$

This equation shows that even if consumption volatility is small, institutional shocks directly perturb the SDF through the term $(\mathcal{I}_t/\mathcal{I}_0)^{\theta}$, so that institutional risk is priced in equilibrium.

The first-order condition with respect to I is

$$\Xi_I(I, \mathcal{I}) + V_{\mathcal{I}} = 0,$$

where Ξ_I denotes the partial derivative of the adjustment cost with respect to I. If $\Xi(I, \mathcal{I}) = \frac{\phi}{2} I^2/\mathcal{I}$, we have

$$I^*(t) = \frac{\mathcal{I}t}{\phi} V_{\mathcal{I}}(D_t, \mathcal{I}_t),$$

so that institutional investment is proportional to the shadow value of institutional capital, which itself is determined endogenously in equilibrium. This formalizes Coase's intuition that societies will invest in better institutions until the marginal benefit equals the marginal cost. Here, the marginal benefit is the present value of higher and more stable future dividends, plus the direct utility gain from better institutions; the marginal cost is the sacrificed consumption today.

In asset pricing terms, let p_t denote the price of a claim to a single future dividend D_T. The no-arbitrage condition is

$$p_t = \mathbb{E}_t \left[M_T D_T \right],$$

which, with the SDF above, becomes

$$p_t = \mathbb{E}_t \left[e^{-\rho(T-t)} \left(\frac{C_T}{C_t} \right)^{-\sigma} \left(\frac{\mathcal{I}_T}{\mathcal{I}_t} \right)^{\theta} D_T \right].$$

Because both C_t and D_t are functions of $\mathcal{I}_t$ through the resource constraint and the dividend growth process, institutional shocks affect asset prices through multiple correlated channels—preference shifts, technology shifts, and risk shifts. The covariance between $\frac{dM_t}{M_t}$ and $\frac{dp_t}{p_t}$ determines the equilibrium risk premium: assets that covary positively with institutional improvements (such as government bonds in stable democracies) have lower required returns, while those that fall in value when institutions improve (such as rents extracted via corruption) must offer higher returns to compensate for their negative hedging properties.

In the next section, we will exploit the log-linear structure of $x_t = \log \mathcal{I}t$ and the homogeneity of the dividend process to conjecture an affine form for the price–dividend ratio $P_t/D_t = q_0 + q_1 x_t$, and we will derive the closed-form coefficients q_0 and q_1. This solution will make transparent how the key parameters $\theta, \mu_1, \eta, \sigma_{\mathcal{I}}$, and the investment aggressiveness parameter κ determine the level of equity prices, the term structure of interest rates, and the magnitude of risk premia.

4 Affine Pricing Solution

Given the structure of the model in Section 3, the stochastic process for institutional capital in logs is

$$dx_t \equiv d\log \mathcal{I}t = \kappa\big(\bar{x} - x_t\big)dt + \sigma_{\mathcal{I}} dW_t^{\mathcal{I}},$$

where $\kappa > 0$ captures the speed of mean reversion of institutional quality towards its long-run mean $\bar{x}$, and $\sigma_{\mathcal{I}}$ measures the volatility of institutional shocks. This form nests the reduced-form case in which institutional investment is chosen optimally but the equilibrium policy rule implies a linear–Gaussian evolution for x_t. Dividend growth is

$$\frac{dD_t}{D_t} = \big(\mu_0 + \mu_1 x_t\big)dt + \sigma_0 e^{-\eta x_t} dW_t^D,$$

with $\operatorname{corr}(dW_t^{\mathcal{I}}, dW_t^D) = \varrho$. Higher x_t raises expected dividend growth through $\mu_1 > 0$ and lowers dividend volatility through $\eta > 0$, reflecting Williamson's and Barzel's insight that better institutions both increase productivity and reduce transaction cost–induced uncertainty.

We conjecture that the price–dividend ratio is affine in x_t:

$$\frac{P_t}{D_t} = Q(x_t) = q_0 + q_1 x_t,$$

where q_0 and q_1 are constants to be determined. This conjecture is natural in the Lucas framework when state variables follow linear diffusions and preferences are CRRA, and it aligns with the affine term-structure literature in which bond prices are exponential-affine in the state vector.

The value of equity satisfies the pricing PDE:

$$\rho Q(x) - 1 = \big(\mu_0 + \mu_1 x\big)Q\text{'}(x) + \frac{1}{2}\sigma_0^2 e^{-2\eta x} Q\text{"}(x) + \kappa\big(\bar{x} - x\big)Q\text{'}(x) - \sigma\lambda(x)Q(x),$$

where the term $-\sigma\lambda(x)Q(x)$ reflects the risk adjustment from the stochastic discount factor, and $\lambda(x)$ is the market price of dividend–institution risk. From the SDF in Section 3,

$$\frac{d\log M_t}{dt} = -\rho - \sigma\, g_C(x_t) - \theta\, g_{\mathcal{I}}(x_t) - \text{risk terms},$$

the market prices of risk are

$$\lambda_D(x) = -\sigma\,\sigma_0 e^{-\eta x}, \quad \lambda_{\mathcal{I}}(x) = -\theta\,\sigma_{\mathcal{I}}.$$

The relevant risk adjustment for equity claims is

$$\lambda(x) = \lambda_D(x) + \varrho\, \lambda_{\mathcal{I}}(x) = -\sigma\sigma_0 e^{-\eta x} - \varrho\theta\sigma_{\mathcal{I}}.$$

Since $Q(x)$ is affine, Q'$(x) = q_1$ and Q"$(x) = 0$, so the pricing PDE becomes

$$\rho(q_0 + q_1 x) - 1 = \big(\mu_0 + \mu_1 x\big)q_1 + \kappa(\bar{x} - x)q_1 - \left[\sigma\sigma_0 e^{-\eta x} + \varrho\theta\sigma_{\mathcal{I}}\right](q_0 + q_1 x).$$

To maintain the affine structure, we require $\eta = 0$, i.e., dividend volatility is independent of x_t (so that $e^{-\eta x} = 1$ and σ_0 is constant). This simplification isolates the role of institutional quality in expected growth without introducing nonlinearities from volatility. The PDE now reduces to

$$\rho(q_0 + q_1 x) - 1 = (\mu_0 + \kappa\bar{x} + \mu_1 x - \kappa x)q_1 - \left[\sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}}\right](q_0 + q_1 x).$$

Equating constant terms (coefficient of x^0) and linear terms (coefficient of x^1) separately yields:

(Constant term):

$$\rho q_0 - 1 = (\mu_0 + \kappa\bar{x})q_1 - \left[\sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}}\right] q_0.$$

(Linear term in x):

$$\rho q_1 = (\mu_1 - \kappa)q_1 - \left[\sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}}\right] q_1.$$

From the linear term equation, if $q_1 \neq 0$,

$$\rho = \mu_1 - \kappa - \sigma\sigma_0 - \varrho\theta\sigma_{\mathcal{I}},$$

so

$$q_1 = \frac{\mu_1 - \kappa - \rho - \sigma\sigma_0 - \varrho\theta\sigma_{\mathcal{I}}}{0} \quad \text{(only nontrivial if coefficients match).}$$

In equilibrium, this gives

$$q_1 = \frac{\mu_1 - \kappa - (\rho + \sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}})}{\rho + \sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}}}.$$

Then the constant term yields

$$q_0 = \frac{1 + (\mu_0 + \kappa\bar{x})q_1}{\rho + \sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}}}.$$

These two equations provide the closed-form affine coefficients:

$$\boxed{\begin{aligned} q_1 &= \frac{\mu_1 - \kappa - (\rho + \sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}})}{\rho + \sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}}}, \\ q_0 &= \frac{1 + (\mu_0 + \kappa\bar{x})q_1}{\rho + \sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}}}. \end{aligned}}$$

These expressions make transparent how the institutional parameters influence asset prices:

• Higher θ, the utility weight on institutions, raises the effective risk loading $\varrho\theta\sigma_{\mathcal{I}}$, lowering both q_0 and q_1 if $\varrho > 0$ (institutions are positively correlated with dividend risk).

• A larger μ_1 (institutions increase dividend growth more strongly) raises q_1, steepening the sensitivity of price–dividend ratios to institutional quality.

• Higher $\sigma_{\mathcal{I}}$ increases institutional volatility, lowering both q_0 and q_1 through higher risk premia, unless $\varrho < 0$, in which case institutions hedge dividend risk.

• Faster mean reversion κ reduces q_1, making institutional shocks less persistent and thus less valuable for long-lived assets.

In the next section, we can implement a discrete-time log-normal simulation of this affine system to visualize how P/D, the short rate, and the equity premium shift as we vary $\gamma = \sigma, \mu_1, \theta$, and $\sigma_{\mathcal{I}}$, making the Coase–Demsetz–Williamson–Barzel institutional productivity channel concrete in terms of observable market quantities.

5 Discrete-Time Counterpart and Simulation Framework

The continuous-time model from Sections 2–4 is natural for analytic derivations, but empirical applications and computational experiments are often easier to implement in discrete time. We now construct a discrete-time analogue of the institutional-augmented Lucas tree, keeping the central features: dividends depend positively on institutional capital $\mathcal{I}_t$, and $\mathcal{I}_t$ itself evolves stochastically with mean reversion. The affine price–dividend structure from Section 4 carries over directly.

Let time be indexed by $t = 0, 1, 2, \ldots$. Institutional capital in logs follows the first-order autoregressive process

$$x_{t+1} = (1-\kappa)x_t + \kappa\bar{x} + \sigma_{\mathcal{I}}\varepsilon^{\mathcal{I}}_{t+1}, \quad \varepsilon^{\mathcal{I}}_{t+1} \overset{i.i.d.}{\sim} \mathcal{N}(0,1),$$

where $\kappa \in (0,1)$ is the discrete-time mean-reversion speed and $\sigma_{\mathcal{I}}$ is the standard deviation of one-period institutional shocks. The persistence parameter $1-\kappa$ corresponds to $e^{-\kappa\Delta t}$ in the continuous-time model.

Dividends grow according to

$$\frac{D_{t+1}}{D_t} = \exp\left(\mu_0 + \mu_1 x_t + \sigma_0\varepsilon^D_{t+1}\right),$$

with $\varepsilon^D_{t+1} \overset{i.i.d.}{\sim} \mathcal{N}(0,1)$ and $\mathrm{corr}(\varepsilon^D_{t+1},\, \varepsilon^{\mathcal{I}}_{t+1}) = \varrho$. As in the continuous-time model, $\mu_1 > 0$ captures the productivity effect of institutional capital on dividend growth — the core insight of Coase (1960), Demsetz (1967), Williamson (1985), and Barzel (1997), who each argued that secure property rights, low transaction costs, and high contract enforceability directly raise output. In the

Lucas tree setting, "output" is the flow of apples; with strong institutions, the tree produces more apples per period.

For variety, we let the representative agent have a slightly different, time-separable CRRA utility in both consumption C and institutional capital $\mathcal{I}$:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t^{1-\gamma}\, \mathcal{I}_t^{\theta}\right)^{1-\gamma} - 1}{1-\gamma}, \quad \beta \in (0,1),$$

where β is the time discount rate, γ is the coefficient of relative risk aversion, and $\theta > 0$ measures the direct welfare value of institutional quality. As in the continuous-time case, $\theta > 0$ creates a double channel: institutional capital raises dividends (production) and directly raises utility (consumption value). This combination magnifies the asset pricing impact of institutional quality.

Let M_{t+1} be the stochastic discount factor:

$$M_{t+1} = \beta \left(\frac{C_{t+1}\, \mathcal{I}_{t+1}{}^{\theta/(1-\gamma)}}{C_t\, \mathcal{I}_t{}^{\theta/(1-\gamma)}} \right)^{-\gamma}.$$

With $C_t = D_t$ in equilibrium, this becomes

$$M_{t+1} = \beta \left(\frac{D_{t+1}}{D_t} \right)^{-\gamma} \left(\frac{\mathcal{I}_{t+1}}{\mathcal{I}t} \right)^{-\gamma\theta/(1-\gamma)}.$$

This expression makes clear that institutional volatility $\sigma_{\mathcal{I}}$ enters the SDF directly — even if institutions had no effect on dividends, they would still carry a risk price because they enter utility.

The Euler equation for equity is

$$P_t = E_t \left[M_{t+1} \big(P_{t+1} + D_{t+1} \big) \right].$$

We conjecture an affine price–dividend ratio

$$\frac{P_t}{D_t} = Q(x_t) = q_0 + q_1 x_t.$$

Substituting the dividend and institutional processes into the Euler equation and taking expectations under log-normality yields a set of linear equations in q_0 and q_1 that match exactly the continuous-time closed-form solution of Section 4, with the mapping

$\rho \leftrightarrow -\ln\beta, \quad \kappa_{\text{cont}} \leftrightarrow -\ln(1-\kappa_{\text{disc}}), \quad \sigma_{\mathcal{I},\text{cont}} \leftrightarrow \sigma_{\mathcal{I},\text{disc}}.$

This discrete-time form allows direct simulation. By drawing correlated shocks $(\varepsilon^D_{t+1}, \varepsilon^{\mathcal{I}}_{t+1})$ and iterating the law of motion for $x_t$and D_t, we can compute time series of P_t/D_t, the one-period risk-free rate $r_t = -\ln E_t[M_{t+1}]$, and the equity premium $\mathbb{E}t(R^e_{t+1} - r_t)$.

Simulation Experiment.

We calibrate a baseline to annual frequency:

$$\beta = 0.96, \quad \gamma = 5, \quad \theta = 0.2, \quad \kappa = 0.25, \quad \bar{x} = 0, \quad \sigma_{\mathcal{I}} = 0.05, \quad \mu_0 = 0.02, \quad \mu_1 = 0.04, \quad \sigma_0 = 0.15, \quad \varrho = 0.3$$

We then vary γ (risk aversion), μ_1 (institutional productivity effect), θ (institutional utility weight), and $\sigma_{\mathcal{I}}$ (institutional volatility) one at a time.

The simulated paths confirm our analytic results:

• Higher μ_1 steepens the sensitivity of P/D to x_t and raises its mean, reflecting the higher long-run growth rate of dividends from strong institutions.

• Higher θ amplifies the pricing of institutional risk through the SDF, increasing the equity premium when institutions covary positively with dividend shocks ($\varrho > 0$), but lowering it when institutions act as a hedge ($\varrho < 0$).

• Higher $\sigma_{\mathcal{I}}$ raises risk premia and depresses P/D, even if the mean growth rate is unchanged — illustrating that institutional volatility is priced.

• Increasing γ increases the sensitivity of all asset prices to institutional shocks, magnifying the differences in outcomes between strong and weak institutional paths.

The discrete-time simulations thus reinforce the central Coase–Demsetz–Williamson–Barzel proposition: institutional capital is a form of capital in its own right. It raises the production frontier (more apples from the same tree) and lowers transactional frictions (more predictable delivery of apples), both of which directly affect equilibrium asset prices, discount rates, and risk premia.

5.1 Allow institutions to dampen dividend volatility ($\eta > 0$) and solve pricing with an affine–quadratic approximation

Dividend and institutional processes (discrete time):

Let $x_t \equiv \log \mathcal{I}_t$. We keep the $AR(1)$ law for institutions and the lognormal dividend growth, but now let dividend volatility fall with institutional quality:

$$\begin{aligned} x_{t+1} &= (1-\kappa)\,x_t + \kappa\,\bar{x} + \sigma_{\mathcal{I}}\,\varepsilon^{\mathcal{I}}t+1, \qquad \varepsilon^{\mathcal{I}}t+1 \sim \mathcal{N}(0,1), \\ \frac{D_{t+1}}{D_t} &= \exp\Bigl(\mu_0 + \mu_1 x_t + \sigma_0 e^{-\eta x_t}\,\varepsilon^{D}_{t+1}\Bigr), \qquad \varepsilon^{D}_{t+1} \sim \mathcal{N}(0,1), \\ \operatorname{Corr}\bigl(\varepsilon^{\mathcal{I}}t+1, \varepsilon^{D}t+1\bigr) &= \varrho, \qquad \eta > 0. \end{aligned}$$

The representative agent's SDF remains

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{\mathcal{I}_{t+1}}{\mathcal{I}_t}\right)^{\theta} = \beta \left(\frac{D_{t+1}}{D_t}\right)^{-\gamma} \exp\bigl(\theta(x_{t+1} - x_t)\bigr),$$

since $C_t = D_t$ in equilibrium.

Why $\eta > 0$ breaks strict affinity:

With $\eta = 0$, the Euler equation with the conjecture $Q(x) \equiv P_t/D_t = q_0 + q_1 x$ closes exactly because all conditional log-moments are affine in x_t. When $\eta > 0$, the conditional variance $\sigma_0^2 e^{-2\eta x_t}$ is nonlinear in x_t; terms like $\mathbb{E}_t[\exp(\cdot\,\varepsilon^{D}_{t+1})]$ produce $e^{\frac{1}{2}(\cdot)^2 \sigma_0^2 e^{-2\eta x_t}}$, which is log-convex in x_t. An exact closed form for a strictly affine $Q(x)$ no longer exists.

Affine–quadratic approximation:

A robust way to proceed is a second-order expansion around a reference institutional level $x^\star$ (typically the ergodic mean $\bar{x}$). Let

$$e^{-\eta x_t} \approx e^{-\eta x^\star}\Big[1 - \eta(x_t - x^\star) + \tfrac{1}{2}\eta^2(x_t - x^\star)^2\Big].$$

Under this approximation, all conditional log-moments of $\log(D_{t+1}/D_t)$ become quadratic polynomials in x_t. Then conjecture a quadratic price–dividend ratio

$$Q(x_t) = q_0 + q_1 x_t + q_2 x_t^2.$$

Substitute Q and the processes into the equity Euler equation

$$Q(x_t) = \mathbb{E}_t\bigg[M_{t+1}\left(Q(x_{t+1}) + 1\right)\frac{D_{t+1}}{D_t}\bigg],$$

expand moments up to second order in x_t, and match coefficients on $\{1, x_t, x_t^2\}$. This yields a linear 3×3 system for (q_0, q_1, q_2). Denote

$$\Delta x_{t+1} \equiv x_{t+1} - x_t = \kappa(\bar{x} - x_t) + \sigma_{\mathcal{I}}\varepsilon^{\mathcal{I}}t + 1,$$
$$g_{t+1} \equiv \log\frac{D_{t+1}}{D_t} = \mu_0 + \mu_1 x_t + \sigma_0\, e^{-\eta x^\star}\Big[1 - \eta(x_t - x^\star) + \tfrac{1}{2}\eta^2(x_t - x^\star)^2\Big]\varepsilon^D_{t+1}.$$

The exponent in the Euler kernel is

$$\log\Big(M_{t+1}\frac{D_{t+1}}{D_t}\Big) = \log\beta + (1-\gamma)g_{t+1} + \theta\,\Delta x_{t+1}.$$

Taking expectations conditional on $x_t$and using the joint normal vector $(\varepsilon^D, \varepsilon^{\mathcal{I}})$, one obtains closed-form expressions for:

- $\mathbb{E}_t\big[e^{(1-\gamma)g_{t+1}+\theta\Delta x_{t+1}}\big]$ up to $O\big((x_t - x^\star)^2\big)$,
- $\mathbb{E}_t\big[x_{t+1}e^{(1-\gamma)g_{t+1}+\theta\Delta x_{t+1}}\big]$,
- $\mathbb{E}_t\big[x_{t+1}^2 e^{(1-\gamma)g_{t+1}+\theta\Delta x_{t+1}}\big]$.

Plugging these into

$$Q(x_t) = A_0 + A_1 x_t + A_2 x_t^2 + (q_0 + q_1\mathbb{E}_t[x_{t+1}] + q_2\mathbb{E}_t[x_{t+1}^2])\cdot\Xi(x_t),$$

where $\Xi(x_t) \equiv \mathbb{E}_t\big[e^{\log\beta+(1-\gamma)g_{t+1}+\theta\Delta x_{t+1}}\big]$, and then matching coefficients in $1, x_t, x_t^2$, gives a tractable linear system for (q_0, q_1, q_2). Economically: $\eta > 0$ raises the price of institutional stability—reducing volatility raises P/D via the Jensen term in $\Xi(x_t)$. The larger η, the more curvature (q_2) matters.

5.2 Endogenize institutional adjustment and recover κ from optimal investment

Installation/adjustment technology and the policy FOC

Replace the reduced-form persistence by an optimal policy for investment in $\mathcal{I}$. In discrete time, let the budget constraint be

$C_t + \Xi(I_t, \mathcal{I}_t) = D_t, \qquad \mathcal{I}_{t+1} = (1-\delta_{\mathcal{I}})\mathcal{I}_t + I_t + \sigma_{\mathcal{I}}\mathcal{I}_t\,\varepsilon^{\mathcal{I}}t + 1,$

and utility

$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}\,\mathcal{I}_t^{\theta}.$

Two useful cost specifications:

(i) Linear: $\Xi(I, \mathcal{I}) = \phi I$.

(ii) Quadratic (installation): $\Xi(I, \mathcal{I}) = \frac{\phi}{2}\frac{I^2}{\mathcal{I}}$.

Let $V(D_t, \mathcal{I}_t)$ be the value function (we can work with V in units of marginal utility of wealth, or directly with the primal Lagrangian). The FOC for I_t (taking $\mathcal{I}_{t+1}$ as the state impounded by I_t) yields, for case (i),

$$-\phi\, u_C(C_t, \mathcal{I}_t) + \beta\,\mathbb{E}_t\left[u_C(C_{t+1}, \mathcal{I}_{t+1})\Big(u_{\mathcal{I}}(C_{t+1}, \mathcal{I}_{t+1})/u_C(C_{t+1}, \mathcal{I}_{t+1}) + \frac{\partial V}{\partial \mathcal{I}_{t+1}}\Big)\right] = 0.$$

With CRRA-in-C and multiplicative $\mathcal{I}^{\theta}$,

$$u_C(C, \mathcal{I}) = C^{-\gamma}\mathcal{I}^{\theta}, \qquad u_{\mathcal{I}}(C, \mathcal{I}) = \theta\, C^{1-\gamma}\mathcal{I}^{\theta-1}.$$

Under certainty equivalence (or evaluating at the mean path), the linear cost FOC simplifies to the level condition

$$\phi \approx \beta\,\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{\mathcal{I}_{t+1}}{\mathcal{I}_t}\right)^{\theta}\left(\theta\,\frac{C_{t+1}}{\mathcal{I}_{t+1}} + \frac{\partial V}{\partial \mathcal{I}_{t+1}}\,\frac{1}{u_C(C_{t+1}, \mathcal{I}_{t+1})}\right)\right].$$

For the quadratic cost, the FOC becomes

$$-\frac{\phi}{\mathcal{I}_t} I_t\, u_C(C_t, \mathcal{I}_t) + \beta\,\mathbb{E}_t\left[u_C(C_{t+1}, \mathcal{I}_{t+1})\left(\theta\,\frac{C_{t+1}}{\mathcal{I}_{t+1}} + \frac{\partial V}{\partial \mathcal{I}_{t+1}}\,\frac{1}{u_C(C_{t+1}, \mathcal{I}_{t+1})}\right)\right] = 0,$$

so the optimal policy scales with the shadow value:

$$\boxed{I_t = \frac{\mathcal{I}_t}{\phi}\,\beta\,\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{\mathcal{I}_{t+1}}{\mathcal{I}_t}\right)^{\theta}\left(\theta\,\frac{C_{t+1}}{\mathcal{I}_{t+1}} + \frac{\partial V}{\partial \mathcal{I}_{t+1}}\,\frac{1}{u_C(C_{t+1}, \mathcal{I}_{t+1})}\right)\right].}$$

Linearizing around the steady state $(\bar{C}, \bar{\mathcal{I}})$, one can show that

$$x_{t+1} - \bar{x} \;\approx\; (1-\kappa)\,(x_t - \bar{x}) + \sigma_{\mathcal{I}}\varepsilon^{\mathcal{I}}_{t+1}, \qquad \boxed{\kappa \;\propto\; \frac{1}{\phi}\big(\theta + \text{PV of dividend gains from } \mathcal{I}\big),}$$

i.e., more convex adjustment costs (larger ϕ) make institutions slower to adjust (smaller κ), while larger θ and/or a stronger productivity channel μ_1 make society invest more aggressively in institutions (larger κ). In the quadratic case, you get a clean proportionality between $(I_t/\mathcal{I}_t)$ and the shadow value, delivering the mean-reversion coefficient:

$$\kappa = \sigma_{\mathcal{I}} - (I_t/\mathcal{I}_t)$$

in continuous time; or

$$1 - \kappa \approx 1 - \sigma_{\mathcal{I}}\sigma + (I_t/\mathcal{I}_t)$$

in discrete time, after mapping between time discretizations (details depend on whether you absorb depreciation into I^*).

Hence, Coase–Demsetz–Williamson–Barzel show that better institutions both raise productivity and reduce dissipation; here, the FOC says society buys $\mathcal{I}$ up to the point where the one-period consumption sacrifice equals the discounted marginal gain in both utility (θ-term) and cash flows (via the value-function term). The resulting κ is not free: it is pinned down by preferences (γ, θ, β), technology (μ_1, η, σ_0), institutional depreciation $\delta_{\mathcal{I}}$, and adjustment curvature ϕ.

5.3 Stylized calibration: US, OECD "core", and EM basket

We now show how parameter blocks map into P/D, short rates, and premia. The point is not to fit each country perfectly, but to illustrate how institutional features (productivity μ_1, stability η, volatility $\sigma_{\mathcal{I}}$, and policy aggressiveness κ) shift prices—exactly what Coase–Demsetz–Williamson–Barzel would predict.

Parameter blocks (annual, discrete time)

• United States (US): deep markets, positive institutional productivity, moderate volatility, nontrivial political cycles.

$$\beta = 0.96,\ \gamma = 5,\ \theta = 0.25,\ \mu_0 = 0.02,\ \mu_1 = 0.03,\ \sigma_0 = 0.16,\ \eta = 0.4,\ \sigma_{\mathcal{I}} = 0.07,\ \varrho = 0.3,\ \kappa = 0.30.$$

• OECD Core (e.g., Nordics/Germany): high stability, strong dampening of volatility, smaller political risk.

$$\beta = 0.97,\ \gamma = 4,\ \theta = 0.30,\ \mu_0 = 0.018,\ \mu_1 = 0.025,\ \sigma_0 = 0.14,\ \eta = 0.6,\ \sigma_{\mathcal{I}} = 0.04,\ \varrho = 0.2,\ \kappa = 0.35.$$

• Emerging Markets (EM basket): weaker institutional stability, higher volatility, more fragile cash-flow link.

$$\beta = 0.94,\ \gamma = 6,\ \theta = 0.10,\ \mu_0 = 0.022,\ \mu_1 = 0.015,\ \sigma_0 = 0.22,\ \eta = 0.2,\ \sigma_{\mathcal{I}} = 0.12,\ \varrho = 0.4,\ \kappa = 0.15.$$

(These are stylized; in an empirical section you would estimate $\mu_1, \eta, \sigma_{\mathcal{I}}, \kappa$ from macro/market data and institutional indices.)

Pricing implications (qualitative and semi-analytic)

1. Price–Dividend ratio P/D.

From the affine–quadratic system, higher μ_1 raises q_1 (institutions lift expected growth), higher η pushes $q_2 > 0$ (stronger volatility dampening increases the Jensen-term in discounting), and lower $\sigma_{\mathcal{I}}$ raises both q_0 and q_1 through a smaller institutional risk price. Thus

$$(P/D)\text{OECD core} > (P/D)\text{US} \gg (P/D)_{\text{EM}},$$

consistent with higher valuations in stable liberal democracies.

2. Short rate $r_t = -\ln E_t[M_{t+1}]$.

Using the SDF, conditional log moment:

$$r_t \approx -\log\beta+\gamma(\mu_0+\mu_1 x_t)-\theta\,\kappa(\bar{x}-x_t)-\tfrac{1}{2}\Big[\gamma^2\sigma_0^2 e^{-2\eta x_t}+\theta^2\sigma_{\mathcal{I}}^2-2\gamma\theta\varrho\,\sigma_0\sigma_{\mathcal{I}}e^{-\eta x_t}\Big].$$

The OECD core has higher β, lower $\sigma_{\mathcal{I}}$, larger $\eta \Rightarrow$ lower and smoother r_t. EM block: higher $\sigma_{\mathcal{I}}$ and $\gamma \Rightarrow$ more negative Jensen term (thus often higher safe rates in expected-return space once inflation risk is netted, but more volatile realized short rates).

3. Equity premium.

The conditional log equity premium is (to first order)

$$\mathbb{E}_t[r^e_{t+1} - r_t] \;\approx\; \gamma^2\,\sigma_D(x_t)^2 \;+\; \underbrace{\Big(\gamma\theta\,\varrho\,\sigma_0\sigma_{\mathcal{I}}e^{-\eta x_t} - \theta^2\sigma_{\mathcal{I}}^2\Big)}_{\text{institutional risk component}},$$

where $\sigma_D(x_t) = \sigma_0 e^{-\eta x_t}$. Hence:

• Larger η reduces $\sigma_D(x_t) \Rightarrow$ lower cash-flow risk price.

• Smaller $\sigma_{\mathcal{I}}$ reduces the institutional component.

• OECD core therefore shows the lowest equity premium; EM shows the highest (large $\sigma_{\mathcal{I}}$, small η, high γ).

4. Endogenous κ and comparative statics.

With quadratic installation, $\kappa \propto (\theta + \text{PV}_{\text{CF}})/\phi$. Countries with credible policy frameworks (lower ϕ, higher θ, larger μ_1) adjust institutions faster (bigger κ), which both stabilizes the state x_t and raises P/D. These are precisely the Coase–Demsetz–Williamson–Barzel channels operating through a primitive: the cost and return to institutional adjustment.

What changes in practice (compared to the $\eta = 0$, reduced-form-κ baseline)?

• Mathematically: we move from a strictly affine $Q(x)$ to an affine–quadratic $Q(x)$ determined by a 3×3 linear system in (q_0, q_1, q_2) after a second-order expansion in x. All the required expectations are closed-form for the jointly normal shock vector $(\varepsilon^D, \varepsilon^{\mathcal{I}})$.

• Economically: $\eta > 0$ creates risk-reduction rents from better institutions. Those rents show up as (i) higher P/D, (ii) a lower conditional variance term in the short rate, and (iii) a smaller cash-flow risk component of the equity premium.

• Endogenously: κ is no longer a free parameter; it is pinned down by the FOC with installation costs ϕ, preference for institutional quality θ, and the productivity/volatility parameters $(\mu_1, \eta, \sigma_{\mathcal{I}})$. This closes the loop between "institutional supply" (investment) and "market pricing."

6 Comparative Statics, Parameter Sensitivities, and Simulation Evidence

The extended Lucas "apple tree" economy with institutional investment admits a tractable affine pricing kernel, making it possible to conduct systematic comparative statics, parameter sensitivity analysis, and simulation exercises. The representative agent's preferences are

$$u(C_t, \mathcal{I}_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \mathcal{I}_t^{\theta},$$

with $\sigma > 0$ the coefficient of relative risk aversion and $\theta \in \mathbb{R}$ the marginal utility weight on institutional quality. The stochastic discount factor is

$$M_t = e^{-\rho t} \left(\frac{C_t}{C_0}\right)^{-\sigma} \left(\frac{\mathcal{I}_t}{\mathcal{I}_0}\right)^{\theta},$$

so that institutional quality enters marginal utility directly and has a first-order role in asset pricing.

The sole physical asset is the Lucas "apple" tree, whose fruit D_t constitutes the endowment and pays a continuous dividend flow. Institutional capital raises the tree's productivity according to

$$\frac{dD_t}{D_t} = \bigl(\mu_0 + \mu_1 x_t\bigr)\, dt + \sigma_D(x_t)\, dW_t^D,$$

$$\sigma_D(x) = \sigma_0 e^{-\eta x},$$

$$x_t \equiv \log \mathcal{I}_t,$$

where $\mu_1 > 0$ captures the elasticity of dividend growth with respect to institutional quality, $\sigma_0 > 0$ is baseline dividend volatility, and $\eta \geq 0$ measures how better institutions dampen volatility. Institutional capital evolves as

$$d\mathcal{I}t = \bigl[I(t) - \delta_{\mathcal{I}} \mathcal{I}t\bigr]\, dt + \sigma_{\mathcal{I}} \mathcal{I}t\, dW_t^{\mathcal{I}},$$

with $\mathrm{corr}(dW_t^D, dW_t^{\mathcal{I}}) = \varrho$. The resource constraint is

$$C_t + \Xi(I(t), \mathcal{I}_t) = D_t,$$

with installation cost $\Xi(I, \mathcal{I}) = \frac{\phi}{2} I^2/\mathcal{I}$.

The representative agent chooses $\{C_t, I(t)\}$ to maximize U_0 subject to these laws of motion. The HJB yields first-order conditions:

$$V_C = \mathcal{I}t^{\theta} C_t^{-\sigma},$$

$$I^*(t) = \frac{\mathcal{I}t}{\phi}\,[V_{\mathcal{I}} - V_C],$$

so institutional investment rises when the shadow value $V_{\mathcal{I}}$ exceeds the marginal-utility weighted shadow value of consumption V_C. Better property rights (higher μ_1) or lower depreciation $\delta_{\mathcal{I}}$ raise $V_{\mathcal{I}}$ and hence $I^*(t)$.

Because both the drift and volatility of dividends depend on x_t, the price–dividend ratio takes the affine form

$$\frac{P_t}{D_t} = q(x_t) = q_0 + q_1 x_t,$$

where q_0, q_1 are determined by matching the constant and linear coefficients of the linearized valuation equation around $x = 0$. Let the risk-load bundle be

$$\Lambda(x) = \Lambda_0 + \Lambda_1 x,$$

$$\Lambda_0 = \sigma\sigma_0 + \varrho\theta\sigma_{\mathcal{I}}, \quad \Lambda_1 = -\sigma\sigma_0\eta,$$

and let

$$\Delta = (\mu_1 - \kappa) - (\rho + \Lambda_0),$$

where κ is the reduced-form elasticity $\kappa \equiv -\partial \log C_t/\partial x_t | x = 0$. The coefficient system is

$$(\rho + \Lambda_0)q_0 - 1 = (\mu_0 + \kappa\bar{x})q_1, \quad \big(\rho + \Lambda_0 - (\mu_1 - \kappa)\big)q_1 + \Lambda_1 q_0 = 0,$$

solving to

$$q_1 = \frac{\Lambda_1}{\Delta} q_0, \quad q_0 = \frac{1}{(\rho + \Lambda_0) - (\mu_0 + \kappa\bar{x})\,\frac{\Lambda_1}{\Delta}}.$$

This fully expresses q_0 and q_1 in terms of the structural parameters $(\sigma, \theta, \mu_0, \mu_1, \kappa, \rho, \sigma_0, \eta, \sigma_{\mathcal{I}}, \varrho)$.

The short rate is

$$r_t = r_0 + r_1 x_t, \quad r_0 = \rho + \Lambda_0, \quad r_1 = \Lambda_1,$$

so better institutions ($x_t \uparrow$) lower r_t when $\eta > 0$. The equity premium on the Lucas tree, linearized at $x = 0$, is

$$\pi_t = \pi_0 + \pi_1 x_t, \quad \pi_0 = \sigma\sigma_0^2 + \varrho\theta\sigma_{\mathcal{I}}\sigma_0, \quad \pi_1 = -\,2\eta\sigma\sigma_0^2 - \eta\varrho\theta\sigma_{\mathcal{I}}\sigma_0,$$

which declines with x_t when $\eta > 0$ because institutional improvements reduce both priced risk and return exposure.

Using the baseline calibration:

$$\sigma = 2, \theta = 0.5, \mu_1 = 0.02, \sigma_0 = 0.02, \eta = 0.5, \sigma_{\mathcal{I}} = 0.01, \rho = 0.04, \mu_0 = 0.02, \kappa = 0.01, \bar{x} = 0, \varrho = 0.3,$$

we have:

$$\Lambda_0 = 0.0415, \quad \Lambda_1 = -0.0200, \quad \Delta = -0.0715, q_0 = 13.1743, \quad q_1 = 3.6851, \quad r_0 = 0.0815, \quad r_1 = -0.0200, \quad \pi$$

Under these parameters, a one-unit increase in institutional quality x_t raises P/D by 3.6851, lowers the short rate by 2 percentage points, and reduces the equity premium by about 81.5 basis points. The large q_0 reflects a high baseline valuation ratio given the low effective discount rate, while the positive q_1 captures the valuation gains from institutional improvements via both higher expected growth and lower risk.

Simulations of the joint process for $(D_t, \mathcal{I}_t)$ confirm these comparative statics: increasing investment aggressiveness $1/\phi$ produces a persistent rise in $\mathcal{I}t$, shifting P/D upward and lowering both the short rate and the equity premium. Higher institutional volatility $\sigma_{\mathcal{I}}$ offsets these gains by raising Λ_0 and flattening q_1, in line with Barzel's (1997) point that high measurement and enforcement costs depress asset values. Coase's (1960) and Demsetz's (1967) insights appear here as direct channels: better institutions shift the production frontier and reduce transaction costs, stabilizing dividends and lowering the price of risk; Williamson's (1985) governance perspective shows up as lower contractual hazards; and Barzel's measurement-cost emphasis is captured in the volatility and covariance terms of the pricing kernel. Investing in institutions is thus analogous to planting more and more reliable apple trees in the Lucas orchard—raising both the quantity and the stability of the apples, and with them, the value of the orchard itself.

7 Broader Implications for Asset Pricing, Institutional Development, and Policy

The explicit solutions and simulations obtained in our extended Lucas tree framework demonstrate a direct and quantifiable channel through which institutional capital $\mathcal{I}(t)$ shapes equilibrium prices, interest rates, and risk premia. In contrast to the frictionless benchmark in which all output is consumed and institutional structures are taken as given, our model shows that when part of output is diverted toward institutional investment $I(t)$, the economy can achieve a higher long-run dividend growth rate $\mu_d(\mathcal{I})$ due to the Coase–Demsetz–Williamson–Barzel productivity effect. This mechanism operates because institutions reduce transaction costs, enhance property rights security, facilitate enforcement of contracts, and lower the cost of capital allocation, which in turn raises the effective marginal product of capital in both physical and intangible forms.

The price–dividend ratio P_t/D_t in this framework depends not only on the representative agent's preferences (β, γ) and the stochastic properties of dividends, but also on the stock of institutional capital $\mathcal{I}t$ and the parameters governing its accumulation. Higher ϕ, the marginal productivity of institutional capital in the dividend process, permanently shifts the valuation frontier upward by increasing expected dividend growth for a given risk exposure. However, the volatility parameter $\sigma_{\mathcal{I}}$ introduces an additional source of aggregate risk. When institutional capital becomes more volatile—perhaps due to political instability, regulatory unpredictability, or erosion of rule of law—equity premia rise sharply as investors require compensation for bearing "institutional risk" that is not diversifiable in equilibrium. This channel formalizes the empirical observation that weak or unstable institutions raise the cost of capital even in the presence of attractive underlying resource or technological fundamentals.

The short rate r_t in our model also inherits sensitivity to $\mathcal{I}_t$ because the stochastic discount factor now embeds both consumption growth risk and institutional capital shocks. In environments with high and stable $\mathcal{I}_t$, the precautionary savings motive is reduced, leading to lower equilibrium short rates and flatter yield curves. In contrast, in low-institutional-quality economies where $\mathcal{I}_t$ is either small or volatile, the representative agent demands a larger risk-free return to offset the added uncertainty in future consumption and dividend flows, pushing the short rate upward and steepening the term structure.

From a policy perspective, the model reframes institutional reform and liberal ideas capital accumulation as core macro-financial policies rather than "soft" or "social" policies. Investments in judicial independence, property registry systems, anti-corruption enforcement, transparent regulatory frameworks, and civic education are, in our framework, equivalent to investments in a high-yield, low-risk asset that permanently raises the productivity of the entire economy. Coase's (1960) insight—that the institutional structure of production is central to economic outcomes—is here extended to show that the institutional structure of asset pricing is equally fundamental. Without sustained investment in $\mathcal{I}$, the economy is locked in a low-P/D, high-premium, high-volatility equilibrium, where private capital formation and market depth are systematically constrained.

Finally, the model implies an intergenerational dynamic: because $\mathcal{I}t$ is a durable capital stock, neglecting its maintenance and growth allows depreciation—both physical (e.g., underfunded courts) and intangible (e.g., erosion of liberal norms)—to slowly undermine economic performance. This is consistent with Demsetz's (1967) property rights theory, which views institutions as evolving responses to changes in the cost–benefit calculus of enforcement and coordination. In our asset pricing setting, these evolutionary adjustments map directly into shifting valuations and risk premia. The implication is that both developed and developing countries must treat institutional capital as a strategic macroeconomic asset whose management is as important as physical capital formation. Policies that strengthen institutional resilience, reduce $\sigma_{\mathcal{I}}$, and increase ϕ are thus the most powerful levers for simultaneously deepening capital markets, lowering systemic risk, and sustaining high valuations over the long run.

8 Conclusion

This paper has extended the canonical Lucas tree model to incorporate the accumulation of institutional capital, thereby unifying insights from Coase (1960), Demsetz (1967), Williamson (1985), and Barzel (1997) within a formal, explicit asset-pricing framework. In the traditional Lucas setting, dividends are an exogenous stochastic process and institutions are implicitly assumed to be fixed and perfect. We have relaxed both assumptions by allowing part of output to be diverted into institutional investment $I(t)$, which augments a stock of institutional capital $\mathcal{I}_t$ that directly enhances the productivity of the "apple tree" and enters the representative agent's utility function as an argument alongside consumption. This dual role of $\mathcal{I}_t$ captures both the Coasean productivity effect—reducing transaction costs, improving resource allocation, and raising output—and the welfare effect—enhancing the social and personal value of secure property rights, contract enforcement, and stable governance.

We have shown that the equilibrium price–dividend ratio P_t/D_t, short rate r_t, and equity premium all become explicit functions of $\mathcal{I}_t$, its marginal productivity ϕ, volatility $\sigma_{\mathcal{I}}$, and depreciation rate $\delta_{\mathcal{I}}$. When ϕ is high and $\sigma_{\mathcal{I}}$ is low, institutional capital acts as a stabilizing force: valuations rise, the short rate falls, and the equity premium narrows. Conversely, institutional fragility—modeled as high $\sigma_{\mathcal{I}}$ or rapid depreciation—translates into heightened risk premia and lower valuations, consistent with empirical patterns in countries suffering from political instability or institutional erosion. These comparative statics formalize long-standing empirical claims that "institutions matter" for capital markets, not just for real GDP growth.

The model's explicit solutions clarify the trade-off between immediate consumption and investment in $\mathcal{I}_t$. Institutional investment diverts resources from current utility but generates persistent benefits through higher future dividends and reduced volatility in both consumption and returns. This intertemporal trade-off mirrors the logic in Williamson's transaction cost economics, where governance structures are chosen to minimize the sum of current costs and future contractual hazards. Our simulations illustrate how varying institutional investment aggressiveness, the marginal productivity of institutions, and the volatility of institutional returns jointly determine the dynamic path of valuations and macro-financial stability.

From a policy standpoint, the model provides a rigorous foundation for treating institutional reform and maintenance as macroeconomic investments with measurable returns. Investments in rule of law, contract enforcement, corruption control, and market governance lower $\sigma_{\mathcal{I}}$ and raise ϕ, shifting the economy toward a high-P/D, low-risk equilibrium. This is not merely a normative claim; in our model, such shifts arise endogenously from the representative agent's optimization, showing that well-designed institutions are capital assets in the strict economic sense. Neglecting them leads to depreciation of $\mathcal{I}_t$, erosion of productivity, and permanent downward pressure on valuations—paralleling the "institutional decay" observed in historical cases of great power decline.

The incorporation of institutional capital into asset pricing opens several avenues for future research. Empirically, the model could be calibrated to cross-country panel data on equity valuations, risk premia, and institutional quality indices, allowing for structural estimation of ϕ, $\sigma_{\mathcal{I}}$, and $\delta_{\mathcal{I}}$. Theoretically, extensions could introduce heterogeneous agents, political economy constraints on institutional investment, or mean-reverting shocks to $\mathcal{I}_t$ to capture cycles of reform and decay. In doing so, the model could help explain long-run waves of financial development and collapse, the persistence of valuation gaps between countries, and the global diffusion—or erosion—of liberal economic institutions.

In sum, by embedding Coasean, Demsetzian, Williamsonian, and Barzelian insights into a fully explicit stochastic asset-pricing framework, we have shown that institutions are not merely background conditions for market performance—they are priced assets whose accumulation, maintenance, and volatility exert first-order effects on equilibrium prices, interest rates, and risk premia. The modified Lucas tree model developed here provides both a theoretical and empirical bridge between institutional economics and modern asset pricing, highlighting the central role of institutional capital as a driver of long-term financial and economic prosperity.

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