# Institutions and the Term Structure of Interest Rates: A Unified Lucas–Tree and Affine Pricing Framework

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**Abstract**

This paper develops a unified theoretical framework that integrates institutional quality into affine term structure models of interest rates. Building on a Lucas–tree economy with endogenous institutional capital, we derive the stochastic discount factor and show that zero–coupon bond prices admit an exponential–affine form. Using the Feynman–Kac representation, we obtain explicit Riccati equations and closed–form solutions under Vasicek- and CIR-type institutional dynamics. The analysis reveals how institutional persistence, mean reversion, and volatility affect the slope and curvature of the yield curve, generating distinct term premia across maturities. Comparative statics highlight that stronger and more stable institutions lower risk premia, while fragile or volatile institutions amplify borrowing costs. The framework bridges political economy and asset pricing, showing that institutional reforms not only promote long-run growth but also shape sovereign debt markets, financial development, and macroeconomic stability.

Keywords: Institutional capital; Affine term structure; Bond pricing; Lucas–tree model; Feynman–Kac formula; Vasicek model; CIR model; Sovereign yields; Political economy; Risk premia

## 1 Introduction

The central question of modern political economy and finance is how institutions shape economic performance and asset prices. A vast body of work, from North (1990), Acemoglu, Johnson, and Robinson (2001), and Acemoglu and Robinson (2012), has emphasized that persistent cross-country differences in prosperity can be traced to variations in property rights, the rule of law, and political governance. In parallel, the asset pricing literature, beginning with Lucas (1978), Breeden (1979), and Mehra and Prescott (1985), has established that equilibrium returns are determined by the dynamics of consumption, wealth, and risk. Yet these two literatures have remained largely disconnected: while institutions are treated as slow-moving background variables in macroeconomics, most asset pricing models abstract from institutional frictions altogether.

This paper develops a unified framework in which institutions are modeled as an endogenous form of institutional capital that co-evolves with economic shocks and directly affects asset prices. We embed an institutional state variable into a Lucas-tree economy, extend the representative agent's optimization problem to include investment in institutional quality, and derive the intertemporal Euler conditions and budget constraint that govern equilibrium allocations. The stochastic discount factor (SDF) inherits dependence on institutional capital, implying that both the risk-free rate and risk premia are functions of institutional quality and volatility.

To connect the model to observable bond markets, we adopt the affine term structure (ATS) framework of Vasicek (1977), Cox, Ingersoll, and Ross (1985), Duffie and Kan (1996), and Piazzesi (2010). Institutions and productivity jointly enter as affine state variables, yielding closed-form solutions for bond prices, yields, and forward rates through the Feynman–Kac representation. In this setting, institutional shocks generate distinct term structure implications: mean reversion in institutional quality determines the maturity profile of yield curve loadings, while institutional volatility enters convexity adjustments that create term premia. The model therefore links political economy fundamentals to bond market observables in a tractable and testable way.

The contributions of this paper are threefold. First, it provides a rigorous asset pricing model in which institutional capital is a state variable that influences the stochastic discount factor. Second, it integrates this structure with the affine term structure literature to obtain explicit solutions for yields and risk premia, both in Vasicek- and CIR-type formulations. Third, it offers comparative statics and closed-form characterizations of how institutional mean reversion, volatility, and persistence affect equilibrium prices of risk, the slope of the yield curve, and long-run forward rates. In doing so, the paper unifies insights from institutional economics and financial economics, and shows how political uncertainty and institutional fragility can be priced in asset markets.

The remainder of the paper is organized as follows. Section 2 introduces the model setup, combining the Lucas-tree framework with institutional capital. Section 3 develops the Feynman–Kac representation and derives affine bond pricing formulas. Section 4 conducts comparative statics from the Riccati system and interprets the role of institutional parameters. Section 5 presents closed-form solutions in Vasicek- and CIR-type cases and illustrates the yield curve implications. Section 6 concludes by discussing broader implications for political economy and financial market integration.

# 2 Model Setup

We consider a representative agent who values both material consumption and the institutional environment that sustains exchange, contract enforcement, and property rights. Preferences are time-additive with subjective discount rate

$\delta > 0$ (notation chosen to avoid any clash with later short-rate loadings):

$$U = \mathbb{E}_0\left[\int_0^\infty e^{-\delta t}\left(\tfrac{C_t^{1-\gamma_C}}{1-\gamma_C} + \psi\,\tfrac{I_t^{1-\gamma_I}}{1-\gamma_I}\right)dt\right], \qquad \gamma_C, \gamma_I > 0,\ \psi > 0.$$

Here $C_t$ is consumption and $I_t$ is "institutional capital," a stock summarizing the quality and stability of legal/governance arrangements (rule of law, property rights, contract enforcement). The separability ensures that the intertemporal marginal rate of substitution—and thus the stochastic discount factor (SDF)—is driven by marginal utility of consumption:

$$U_C(C_t, I_t) = C_t^{-\gamma_C}, \qquad M_t = \exp(-\delta t)\,\frac{U_C(C_t, I_t)}{U_C(C_0, I_0)} = \exp(-\delta t)\Big(\frac{C_t}{C_0}\Big)^{-\gamma_C}.$$

Institutions therefore influence asset prices indirectly via their impact on the equilibrium consumption path and state dynamics, and directly through the optimality condition for institutional investment below.

Output is produced by a Lucas "tree" whose dividend flow is augmented by institutions:

$$D_t = \delta_y\, I_t^{\phi}\, e^{x_t}, \qquad \phi > 0,$$

where $x_t$ is a persistent productivity factor. Under the physical measure $\mathbb{P}$ we model $x_t$ as mean-reverting, i.e., Ornstein–Uhlenbeck or OU, process with volatility $\sigma_x$:

$$dx_t = -\kappa_x^{\mathbb{P}}(x_t - \bar{x}^{\mathbb{P}})\,dt + \sigma_x\, dW_t^{x,\mathbb{P}}, \qquad \kappa_x^{\mathbb{P}} > 0.$$

Institutional capital is a stock with purposeful investment $I_{\mathcal{I}}(t)$, depreciation $\delta_I$, and shocks $\sigma_{\mathcal{I}}$:

$$dI_t = \big[I_{\mathcal{I}}(t) - \delta_I I_t\big]\,dt + \sigma_{\mathcal{I}} I_t\, dW_t^{\mathcal{I},\mathbb{P}},$$

allowing correlation between $W^{x,\mathbb{P}}$ and $W^{\mathcal{I},\mathbb{P}}$. It is often convenient for pricing to work with $y_t \equiv \ln I_t$; in Section 3 we specify the risk-neutral dynamics of $y_t$ as OU to obtain closed-form affine bond prices.

Let the agent hold a money-market account $B_t$ and one risky tree asset $P_t$ (one share outstanding) that pays flow dividends $D_t\,dt$. With holdings $\theta_{0,t}$ and $\theta_t$ respectively,

$dB_t = r_t B_t\,dt, \qquad dP_t = P_t(\mu_t\,dt + \sigma_t\,dW_t^{S,\mathbb{P}}), \qquad$ dividend $D_t\,dt$ per share.

Total wealth $W_t = \theta_{0,t}B_t + \theta_t P_t$ evolves as

$$\boxed{dW_t = \theta_{0,t}\,dB_t + \theta_t\big(dP_t + D_t\,dt\big) - C_t\,dt - I_{\mathcal{I}}(t)\,dt.}$$

Equivalently, in "portfolio share" form with $\alpha_t \equiv \theta_t P_t/W_t$ and $c_t \equiv C_t/W_t,\ i_t \equiv I_{\mathcal{I}}(t)/W_t$,

$$\frac{dW_t}{W_t} = \Big[r_t + \alpha_t(\mu_t - r_t) + \frac{\theta_t D_t}{W_t} - c_t - i_t\Big]dt + \alpha_t \sigma_t \, dW_t^{S,\mathbb{P}}.$$

In the representative-agent Lucas endowment economy, the single tree is in unit net supply ($\theta_t \equiv 1$), the risk-free asset is in zero net supply ($\theta_{0,t} \equiv 0$), and wealth equals the ex-dividend tree price $W_t = P_t$. Market clearing in goods implies

$$\boxed{C_t + I_{\mathcal{I}}(t) = D_t}$$

so the consumption path is endogenously tied to institutional accumulation.

Let the state be $(W_t, x_t, I_t)$ (or $(x_t, I_t)$ in the Lucas specialization $W_t = P_t$). The value function $V(W, x, I)$ solves the HJB

$$\delta V = \max_{C, I_{\mathcal{I}}, \theta} \Big\{ \underbrace{\tfrac{C^{1-\gamma_C}}{1-\gamma_C} + \psi \tfrac{I^{1-\gamma_I}}{1-\gamma_I}} + V_W \, [rW + \theta(\mu - r)W + \theta D - C - I_{\mathcal{I}}] + V_x \, \mu_x + V_I \, (I_{\mathcal{I}} - \delta_I I) + \tfrac{1}{2} V_{WW} (\theta \sigma W)^2 + \tfrac{1}{2} V_{xx} \sigma$$

First-order conditions (interior) are:
(consumption) $U_C(C, I) = V_W \implies C^{-\gamma_C} = V_W$,
(institutional investment) $- V_W + V_I = 0 \implies \boxed{U_C(C, I) = V_I}$,
(portfolio) $\theta^* = -\frac{\mu - r}{\sigma^2} \frac{V_W}{W V_{WW}}$ (Merton rule).
The key condition $U_C = V_I$ equates the marginal utility cost of diverting one unit from consumption to institutional investment to the marginal shadow value of one extra unit of $I$. This is the channel through which institutions affect consumption and, therefore, the SDF.

From $U_C = V_W$ and standard continuous-time asset-pricing theory, the SDF is

$$\boxed{M_t = \exp(-\delta t) \, C_t^{-\gamma_C} \Big/ C_0^{-\gamma_C}}.$$

For any gross return $R_{t,t+dt}$ on a traded asset,

$$1 = \mathbb{E}_t \big[M_{t+dt} \, R_{t,t+dt}\big],$$

and in diffusion form under $\mathbb{P}$,

$$\frac{dM_t}{M_t} = -r_t \, dt - \Lambda_t^\top dW_t^{\mathbb{P}},$$

with $\Lambda_t$ the vector of market prices of risk (a function of the states). Institutions show up in discounting via their effect on $C_t$ (through goods-market clearing $C + I_{\mathcal{I}} = D$) and in risk premia via their contribution to $\Lambda_t$when institutional shocks covary with asset payoffs.

This section establishes the primitives (preferences, technology, institutional accumulation) and the full, explicit budget constraint. It also pins down the SDF and the optimality condition $U_C = V_I$ that links consumption to institutional investment. In Section 3 we pass to the risk-neutral measure and impose affine (OU) dynamics for the state vector $X_t = (x_t, \ln I_t)$ to obtain closed-form, exponential-affine bond prices and yield curves consistent with this equilibrium foundation.

# 3 Feynman–Kac Representation and Affine Bond Pricing

We now build on the institutional Lucas-tree economy of Section 2 to derive closed-form bond prices and yields. Since the stochastic discount factor is affine in the underlying states $(x_t, \ln I_t)$, the zero-coupon bond market inherits an affine term structure. This section presents the mathematical derivation with care and explains the economic interpretation at each step.

The starting point is the definition of a zero-coupon bond with maturity $\tau > 0$. Its time-$t$ price is the risk-neutral expectation

$$P(t,\tau) = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\int_t^{t+\tau} r_s\, ds\right)\right],$$

where $r_t$ is the instantaneous short rate and the expectation is taken under the risk-neutral measure $\mathbb{Q}$. Consistent with the institutional economy, we adopt an affine specification of the short rate:

$$r_t = r_0 + \rho^\top X_t,$$

where $r_0$ is a constant and $X_t = (x_t, y_t)^\top$ is the vector of latent state variables, with $x_t$ representing productivity and $y_t = \ln I_t$ representing institutional capital. The vector $\rho = (\rho_x, \rho_I)^\top$ determines how each state factor contributes to the short rate.

To characterize bond pricing, we must specify the dynamics of the state under the risk-neutral measure. We assume each component of $X_t$ follows an Ornstein–Uhlenbeck or OU process:

$$dX_t = \big(\kappa_0^{\mathbb{Q}} + K^{\mathbb{Q}} X_t\big)dt + \Sigma\, dW_t^{\mathbb{Q}},$$

with mean-reversion matrix $K^{\mathbb{Q}} = \mathrm{diag}(-\kappa_x^{\mathbb{Q}}, -\kappa_I^{\mathbb{Q}})$, long-run drifts collected in $\kappa_0^{\mathbb{Q}} = (\kappa_x^{\mathbb{Q}}\theta_x^{\mathbb{Q}}, \kappa_I^{\mathbb{Q}}\theta_I^{\mathbb{Q}})^\top$, volatility matrix $\Sigma = \mathrm{diag}(\sigma_x, \sigma_I)$, and covariance $\Omega = \Sigma\Sigma^\top$. Thus, both productivity and institutional quality revert to long-run levels $\theta_x^{\mathbb{Q}}, \theta_I^{\mathbb{Q}}$ at speeds $\kappa_x^{\mathbb{Q}}, \kappa_I^{\mathbb{Q}}$, and are hit by independent shocks. The use of the risk-neutral measure ensures consistency with no-arbitrage pricing.

Bond pricing can now be formulated as a partial differential equation via the Feynman–Kac theorem. Let $P(\tau, x)$ denote the price of a zero-coupon bond with maturity $\tau$, conditional on current state $X_t = x$. Then $P$ solves:

$$-\frac{\partial P}{\partial \tau} + (\kappa_0^{\mathbb{Q}} + K^{\mathbb{Q}} x)^\top \nabla_x P + \tfrac{1}{2}\,\mathrm{Tr}\big(\Omega \nabla_x^2 P\big) - (r_0 + \rho^\top x)P = 0, \qquad P(0,x) = 1.$$

The first term represents time-to-maturity decay, the second term the effect of mean-reverting drifts of productivity and institutions, the third term diffusion risk, and the last term the effect of discounting at the short rate.

To solve this PDE, we conjecture an exponential-affine form for bond prices:

$$P(\tau, x) = \exp\big(A(\tau) + B(\tau)^\top x\big), \qquad A(0) = 0, \quad B(0) = 0.$$

This ansatz is natural: if the short rate is affine in state variables, bond prices inherit an exponential-affine structure. Differentiating yields

$$\frac{\partial P}{\partial \tau} = (A\text{'}(\tau) + x^\top B\text{'}(\tau))P, \qquad \nabla_x P = B(\tau)P, \qquad \nabla_x^2 P = \big(B(\tau)B(\tau)^\top\big)P.$$

Substituting into the PDE and dividing through by $P$ gives

$$-(A\text{'}(\tau) + x^\top B\text{'}(\tau)) + (\kappa_0^{\mathbb{Q}} + K^{\mathbb{Q}} x)^\top B(\tau) + \tfrac{1}{2} B(\tau)^\top \Omega B(\tau) - (r_0 + \rho^\top x) = 0.$$

Equating coefficients in powers of x, we obtain the system of Riccati ordinary differential equations:

$$B\text{'}(\tau) = K^{\mathbb{Q}\top} B(\tau) - \rho, \qquad B(0) = 0,$$

$$A\text{'}(\tau) = \kappa_0^{\mathbb{Q}\top} B(\tau) + \tfrac{1}{2} B(\tau)^\top \Omega B(\tau) - r_0, \qquad A(0) = 0.$$

These equations determine the loadings $B(\tau)$ and intercept $A(\tau)$ that generate the affine bond price.

In the institutional two-factor model with independent OU dynamics, these Riccati equations admit closed forms. For the productivity factor,

$$B_x(\tau) = -\frac{\rho_x}{\kappa_x^{\mathbb{Q}}}\big(1 - e^{-\kappa_x^{\mathbb{Q}}\tau}\big),$$

and for the institutional factor,

$$B_I(\tau) = -\frac{\rho_I}{\kappa_I^{\mathbb{Q}}}\big(1 - e^{-\kappa_I^{\mathbb{Q}}\tau}\big).$$

Both loadings are negative, reflecting that a higher factor value increases the short rate and therefore lowers bond prices. Each loading decays exponentially with maturity at the respective mean-reversion rate, so shocks to productivity and institutions affect the short end of the yield curve most strongly and fade at longer horizons.

The intercept term $A(\tau)$ captures the accumulated drift and convexity adjustments. Explicitly,

$$
\begin{aligned}
A(\tau) = & -r_0\tau - \rho_x\theta_x^{\mathbb{Q}}\Big[\tau - \tfrac{1-e^{-\kappa_x^{\mathbb{Q}}\tau}}{\kappa_x^{\mathbb{Q}}}\Big] - \rho_I\theta_I^{\mathbb{Q}}\Big[\tau - \tfrac{1-e^{-\kappa_I^{\mathbb{Q}}\tau}}{\kappa_I^{\mathbb{Q}}}\Big] \\
& + \frac{\sigma_x^2\rho_x^2}{2(\kappa_x^{\mathbb{Q}})^2}\Big[\tau - \tfrac{2(1-e^{-\kappa_x^{\mathbb{Q}}\tau})}{\kappa_x^{\mathbb{Q}}} + \tfrac{1-e^{-2\kappa_x^{\mathbb{Q}}\tau}}{2\kappa_x^{\mathbb{Q}}}\Big] \\
& + \frac{\sigma_I^2\rho_I^2}{2(\kappa_I^{\mathbb{Q}})^2}\Big[\tau - \tfrac{2(1-e^{-\kappa_I^{\mathbb{Q}}\tau})}{\kappa_I^{\mathbb{Q}}} + \tfrac{1-e^{-2\kappa_I^{\mathbb{Q}}\tau}}{2\kappa_I^{\mathbb{Q}}}\Big].
\end{aligned}
$$

The first line contains deterministic discounting and mean-reversion adjustments; the second and third lines contain convexity corrections due to volatility of productivity and institutional shocks. These convexity terms are strictly positive, which implies upward term premia in the presence of uncertainty.

The yield curve and forward rates follow directly. The yield on a $\tau$-maturity bond is

$$y(t,\tau) = -\frac{1}{\tau}\big(A(\tau) + B(\tau)^\top X_t\big),$$

and the instantaneous forward rate at maturity $\tau$ is

$$f(t,\tau) = -A'(\tau) - B'(\tau)^\top X_t.$$

The economic interpretation is clear. The institutional factor enters the short rate linearly through $\rho_I y_t$ and influences the slope of the yield curve through its loading $B_I(\tau)$. Because $B_I(\tau)$ decays exponentially, institutional shocks primarily affect short maturities, while long-term yields are determined by the long-run institutional mean $\theta_I^{\mathbb{Q}}$. Higher institutional volatility $\sigma_I$ raises convexity terms in $A(\tau)$, thereby generating larger positive term premia and a steeper yield curve. In this way, institutional quality and stability become key determinants of the entire term structure of interest rates.

# 4 Comparative Statics from the Affine Coefficients

Having established the affine bond pricing structure, we now turn to comparative statics. The objective is to trace, directly from the Riccati solutions, how institutional and productivity parameters shape bond yields, forward rates, and term premia. Rather than appeal to intuition alone, we derive the effects systematically from the coefficients $A(\tau)$ and $B(\tau)$. This ensures the results follow rigorously from the mathematical structure of the affine model and can be mapped precisely to economic reasoning.

The central equations are the solutions for the loadings:

$$B_x(\tau) = -\frac{\rho_x}{\kappa_x^{\mathbb{Q}}}\big(1 - e^{-\kappa_x^{\mathbb{Q}}\tau}\big), \qquad B_I(\tau) = -\frac{\rho_I}{\kappa_I^{\mathbb{Q}}}\big(1 - e^{-\kappa_I^{\mathbb{Q}}\tau}\big),$$

and for the intercept,

$$
\begin{aligned}
A(\tau) = & -r_0\tau - \rho_x\theta_x^{\mathbb{Q}}\Big[\tau - \tfrac{1-e^{-\kappa_x^{\mathbb{Q}}\tau}}{\kappa_x^{\mathbb{Q}}}\Big] - \rho_I\theta_I^{\mathbb{Q}}\Big[\tau - \tfrac{1-e^{-\kappa_I^{\mathbb{Q}}\tau}}{\kappa_I^{\mathbb{Q}}}\Big] \\
& + \frac{\sigma_x^2\rho_x^2}{2(\kappa_x^{\mathbb{Q}})^2}\Big[\tau - \tfrac{2(1-e^{-\kappa_x^{\mathbb{Q}}\tau})}{\kappa_x^{\mathbb{Q}}} + \tfrac{1-e^{-2\kappa_x^{\mathbb{Q}}\tau}}{2\kappa_x^{\mathbb{Q}}}\Big] \\
& + \frac{\sigma_I^2\rho_I^2}{2(\kappa_I^{\mathbb{Q}})^2}\Big[\tau - \tfrac{2(1-e^{-\kappa_I^{\mathbb{Q}}\tau})}{\kappa_I^{\mathbb{Q}}} + \tfrac{1-e^{-2\kappa_I^{\mathbb{Q}}\tau}}{2\kappa_I^{\mathbb{Q}}}\Big].
\end{aligned}
$$

The yield curve is

$$
y(t,\tau) = -\frac{1}{\tau}\Big(A(\tau) + B_x(\tau)x_t + B_I(\tau)y_t\Big).
$$

Each parameter affects the yield through either its impact on the state loadings $B(\tau)$ or on the convexity term in $A(\tau)$.

First, consider the role of the mean-reversion coefficients $\kappa_x^{\mathbb{Q}}, \kappa_I^{\mathbb{Q}}$. These parameters govern the exponential decay of the loadings. When $\kappa_I^{\mathbb{Q}}$ is high, institutional shocks die out quickly, and $B_I(\tau)$ approaches zero rapidly as $\tau$ grows. In that case, short-term yields are sensitive to current institutional conditions, but long-term yields depend almost exclusively on the long-run mean $\theta_I^{\mathbb{Q}}$. Conversely, when $\kappa_I^{\mathbb{Q}}$ is low, institutions are highly persistent, so $\mathrm{B}_I(\tau)$ decays slowly, allowing shocks in institutional quality to affect the entire yield curve. This mathematical property translates economically into the idea that weak institutional mean-reversion magnifies the long-run impact of institutional risk on asset pricing.

Second, examine the sensitivity to the volatility parameters $\sigma_x, \sigma_I$. These appear only in the convexity terms of $A(\tau)$. Higher institutional volatility $\sigma_I$ raises the last term of $A(\tau)$, increasing the value of $-A(\tau)/\tau$, which in turn raises yields. This effect is maturity-dependent: the convexity corrections grow approximately linearly in $\tau$ for long horizons, so institutional uncertainty generates a persistent upward shift in the yield curve. Economically, this convexity effect corresponds to term premia demanded by investors to bear uncertainty about institutional stability.

Third, consider the effects of the long-run means $\theta_x^{\mathbb{Q}}, \theta_I^{\mathbb{Q}}$. They enter $A(\tau)$ with coefficients proportional to $-\rho_x, -\rho_I$. If, for instance, higher institutional quality lowers the short rate (i.e. $\rho_I > 0$), then raising $\theta_I^{\mathbb{Q}}$ reduces yields across maturities, because the bond market anticipates more favorable long-run institutional conditions. If instead higher institutions raise the short rate (e.g. by facilitating growth and capital returns, so $\rho_I < 0$), then the opposite occurs. Thus, the sign and magnitude of $\rho_I$ encode the channel through which institutions affect macro-financial conditions.

Fourth, the loadings reveal how current state realizations influence the slope of the curve. Because $B_I(\tau) < 0$ when $\rho_I > 0$, a higher current institutional state $y_t$ lowers bond prices and raises yields, especially at short maturities. The exponential decay of $B_I(\tau)$ guarantees that this effect diminishes at long horizons. Hence, institutional shocks are primarily a short-term phenomenon unless persistence is strong.

Finally, we turn to forward rates. Since

$$f(t,\tau) = -A\text{'}(\tau) - B\text{'}(\tau)^{\top} X_t,$$

their comparative statics reflect the Riccati dynamics directly. A higher $\sigma_I$ steepens forward rates at long maturities through convexity. A higher $\kappa_I^{\mathbb{Q}}$ reduces the persistence of institutional influences, flattening the forward curve. These insights follow rigorously from the Riccati system and show how affine structure translates institutional dynamics into the geometry of the yield curve.

Taken together, these comparative statics reveal the core intuition: institutional quality and stability affect both the level and shape of the yield curve through two distinct channels. First, persistence parameters determine how institutional shocks propagate across maturities. Second, volatility generates convexity corrections that translate into term premia. This decomposition allows us to interpret observed yield-curve shifts as evidence of underlying institutional dynamics.

# 5 Analytical Implications for Yields and Returns

The affine framework developed above allows us to derive explicit solutions for the impact of institutional capital on the term structure of interest rates and asset returns. Since bond prices admit exponential–affine representations, the yield curve depends on the interaction between mean–reverting institutional dynamics, volatility of institutional shocks, and their loadings in the short rate specification. These closed–form solutions make it possible to analyze the comparative statics of institutional quality in a tractable and transparent way.

Let the short rate be defined as

$$r_t = r_0 + \rho_x x_t + \rho_I y_t,$$

where $x_t$ denotes the productivity factor and $y_t = \ln I_t$ is the log of institutional capital. The bond price with maturity $\tau$ is given by

$$P(t,\tau) = \exp\bigl(A(\tau) + B_x(\tau)x_t + B_I(\tau)y_t\bigr),$$

with the Riccati equations for $B(\tau)$ and $A(\tau)$ solved in closed form for both factors. The corresponding yield is

$$y(t,\tau) = -\frac{1}{\tau}\bigl(A(\tau) + B_x(\tau)x_t + B_I(\tau)y_t\bigr).$$

The comparative statics are immediate. A higher volatility of institutional shocks, $\sigma_I$, raises the convexity term in $A(\tau)$, thereby steepening the yield curve and increasing term premia, consistent with the notion that greater institutional uncertainty commands higher compensation for bearing duration risk. Faster mean reversion of institutions, $\kappa_I^{\mathbb{Q}}$, accelerates the decay of the loading $B_I(\tau)$, dampening the influence of current institutional conditions on long maturities.

Conversely, slower mean reversion allows institutional shocks to propagate further into the future, affecting yields across the maturity spectrum.

To provide further clarity, we compute explicit elasticities such as

$$\frac{\partial y(t,\tau)}{\partial \sigma_I} > 0, \qquad \frac{\partial y(t,\tau)}{\partial \kappa_I^{\mathbb{Q}}} < 0,$$

capturing, respectively, the upward pressure of volatility and the dampening role of faster institutional adjustment. These elasticities summarize the essential channels by which institutional stability or fragility enters the pricing kernel and the term structure.

A natural way to illustrate these effects is to contrast two benchmark affine specializations for the institutional factor: the Gaussian Vasicek specification and the square–root CIR specification.

In the Vasicek–type institutional model, the log of institutional capital follows an Ornstein–Uhlenbeck process under the risk–neutral measure:

$$dy_t = \kappa_I^{\mathbb{Q}}(\theta_I^{\mathbb{Q}} - y_t)dt + \sigma_I dW_t^{\mathbb{Q}}.$$

The corresponding bond price takes the exponential–affine form

$$P(t,\tau) = \exp\bigl(A^V(\tau) + B_I^V(\tau)y_t\bigr),$$

with

$$B_I^V(\tau) = -\frac{\rho_I}{\kappa_I^{\mathbb{Q}}}\Bigl(1 - e^{-\kappa_I^{\mathbb{Q}}\tau}\Bigr),$$

and

$$A^V(\tau) = -r_0\tau - \rho_I\theta_I^{\mathbb{Q}}\left(\tau - \frac{1 - e^{-\kappa_I^{\mathbb{Q}}\tau}}{\kappa_I^{\mathbb{Q}}}\right) + \frac{\sigma_I^2\rho_I^2}{2(\kappa_I^{\mathbb{Q}})^2}\left[\tau - \frac{2(1 - e^{-\kappa_I^{\mathbb{Q}}\tau})}{\kappa_I^{\mathbb{Q}}} + \frac{1 - e^{-2\kappa_I^{\mathbb{Q}}\tau}}{2\kappa_I^{\mathbb{Q}}}\right].$$

In the CIR–type institutional model, positivity of the institutional factor is ensured by specifying

$$dI_t = \kappa_I^{\mathbb{Q}}(\theta_I^{\mathbb{Q}} - I_t)dt + \sigma_I\sqrt{I_t}\,dW_t^{\mathbb{Q}}.$$

The bond price is then

$$P(t,\tau) = \exp\bigl(A^{CIR}(\tau) - B_I^{CIR}(\tau)I_t\bigr),$$

with

$$B_I^{CIR}(\tau) = \frac{2\bigl(e^{\gamma\tau} - 1\bigr)}{(\kappa_I^{\mathbb{Q}} + \gamma)\bigl(e^{\gamma\tau} - 1\bigr) + 2\gamma},$$

$$\gamma = \sqrt{(\kappa_I^{\mathbb{Q}})^2 + 2\sigma_I^2\rho_I},$$

and

$$A^{CIR}(\tau) = \frac{2\kappa_I^{\mathbb{Q}}\theta_I^{\mathbb{Q}}}{\sigma_I^2} \ln\left( \frac{2\gamma e^{(\kappa_I^{\mathbb{Q}}+\gamma)\tau/2}}{(\kappa_I^{\mathbb{Q}}+\gamma)(e^{\gamma\tau}-1)+2\gamma} \right).$$

Both Vasicek and CIR models belong to the affine class, but their economic interpretations differ. The Vasicek model allows symmetric Gaussian shocks to institutional quality, consistent with the idea that institutions fluctuate around a long-run mean due to alternating cycles of reform and erosion. The CIR specification, by contrast, enforces positivity and captures one–sided fragility, where institutions may deteriorate toward a floor but cannot become negative. This distinction has consequences for term premia: Gaussian fluctuations generate symmetric convexity effects, while square–root dynamics generate asymmetric, volatility–dependent adjustments, producing stronger persistence and risk compensation in economies with fragile institutional structures.

Therefore, by embedding institutional dynamics into both Vasicek and CIR affine frameworks, we demonstrate that the influence of institutions on the yield curve is robust to the choice of modeling environment. These results highlight that the persistence, volatility, and lower–bound features of institutions all have distinct, analytically tractable signatures in the term structure of interest rates, providing a powerful theoretical foundation for the empirical study of institutional asset pricing.

# 6 Conclusion

This paper has developed a unified theoretical framework that integrates institutional capital into the affine term structure of interest rates. Building on the Lucas–tree model of asset pricing, we have shown how institutional quality can be treated as a persistent state variable whose dynamics interact with productivity shocks to determine equilibrium consumption, portfolio choice, and the stochastic discount factor. By applying the Feynman–Kac representation, we derived exponential–affine bond pricing formulas in which institutional capital directly enters the Riccati equations for yield curve loadings. This approach allowed us to connect macro–institutional dynamics with the canonical affine term structure literature in a fully rigorous way.

The analysis established several core results. First, institutional persistence and volatility systematically shape the slope and curvature of the yield curve. When institutional capital is modeled as an Ornstein–Uhlenbeck process (Vasicek–type), shocks are Gaussian and symmetric, producing smooth adjustments to term premia. By contrast, when institutions are modeled as a square–root process (CIR–type), positivity and asymmetric risks are enforced, yielding stronger volatility–induced convexity effects. Second, comparative statics showed that higher institutional volatility raises bond yields and term premia, while faster institutional mean reversion dampens the influence of current institutional conditions on long maturities. Third, the framework demonstrated that institutional fragility and reform both leave analytically tractable signatures in yields, risk premia, and forward rates, bridging the gap between political economy and financial asset pricing.

Our findings provide a conceptual foundation for future empirical research. Rather than relying solely on statistical fits, the model generates closed–form yield curve implications that can be directly mapped to institutional quality indicators, such as rule of law or governance capital, observed across countries. Extensions could incorporate rare disasters, nonlinearities in political shocks, or global spillovers in institutional quality, allowing the framework to address open puzzles in asset pricing, including the equity premium and the term premium variation. The comparative analysis of Gaussian and square–root institutional specifications suggests that both developed and emerging economies may be captured within the same affine class, but with different institutional parameterizations reflecting their stability and fragility.

From a policy perspective, the results highlight that institutional reform is not only a determinant of long–run economic growth but also an immediate driver of sovereign borrowing costs and the pricing of public and private debt. Stronger rule of law, lower institutional volatility, and credible policy commitments reduce risk premia and flatten the yield curve, lowering the cost of capital and fostering financial deepening. Conversely, fragile institutions amplify volatility in interest rates, generate higher term premia, and exacerbate rollover risks in sovereign debt markets. These insights provide a rigorous asset pricing channel through which institutional investments and reforms directly influence macroeconomic stability, fiscal sustainability, and financial development.

In conclusion, this paper demonstrates that institutions are not merely background conditions for growth but are fundamental state variables shaping the intertemporal tradeoffs of investors, the pricing of risk, and the structure of interest rates. By embedding institutional dynamics into affine term structure models, we provide a unified theoretical channel through which institutional quality governs both macroeconomic outcomes and financial market valuations. This synthesis opens the way toward an institutionally grounded asset pricing theory that can be tested, calibrated, and extended in future research.

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