The Optimal Timing of Institutional Reform in a Dynamic Optimization Framework*

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Abstract

We study the optimal timing of institutional reform in a growth model where reform is anticipated and causes a one-time disruption. Institutions are costly technologies that affect the economy's resource constraint and household saving. We solve for the optimal reform time by maximizing lifetime utility within a continuous-time optimal control framework with an endogenous switching time. For an analytically tractable case, we derive the necessary conditions for an optimal interior reform time. The core condition is the continuity of the optimized Hamiltonian at the point of reform, solved simultaneously with jump conditions for capital and its shadow price. This approach internalizes the fact that the pre-reform path is chosen in anticipation of the reform date. A calibration exercise illustrates three distinct regimes: immediate reform, delayed reform, and indefinite postponement, depending on the magnitude of the disruption and the quality of institutional improvement. Our analysis yields policy implications for sequencing and communicating reforms, highlighting conditions under which delay is optimal even when new institutions are unambiguously superior in the long run.

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1 Introduction

When should a country undertake major institutional reform? This question lies at the heart of economic development and public policy. While the long-run benefits of superior institutions—such as secure property rights, efficient contract enforcement, and low bureaucratic friction—are widely acknowledged (e.g., Acemoglu, Johnson, and Robinson (2005) and North (1990)), the transition itself is often fraught with disruption, cost, and uncertainty. A reform announced today but implemented tomorrow creates a period of anticipation, during which forward-looking agents adjust their behavior, altering the very economic conditions into which the reform is born. This paper addresses the timing question from a formal dynamic optimization perspective.

We build on a rich tradition that views institutions as mechanisms for economizing on transaction costs. Starting with the seminal work of Coase (1937, 1960), and elaborated by the theories of comparative governance of Williamson (1975, 1985, 2010) and the historical analysis of institutional performance by North (1990, 2005), this literature argues that the costs of defining, monitoring, and enforcing economic exchange are central to understanding economic outcomes. More recently, Zou (2025) integrated this insight directly into a neoclassical growth framework by modeling institutional frictions as real costs that enter the economy's resource constraint and influence household intertemporal saving decisions.

In this paper, we extend the governance-augmented Ramsey–Cass–Koopmans (RCK) framework of Zou (2025) to endogenize the timing of reform. We model an institutional reform as a discrete, anticipated switch from a high-cost institutional regime to a low-cost one. This switch promises a better long-run future but comes at a price: a one-time, multiplicative loss of physical capital at the moment of reform, following the approach of Blaquière, Gérard, and Leitmann (1969) and Seierstad and Sydsaeter (1986). This capital loss, denoted by a fraction κ, parsimoniously captures the disruptions inherent in systemic change, such as the scrapping of obsolete assets, legal and organizational restructuring, and the temporary breakdown of established economic relationships. The central planner, or a representative household, must choose the optimal reform date, τ, to maximize lifetime utility.

Our contribution is threefold. First, we provide a complete analytical characterization of the model's dynamics for the tractable case of constant relative risk aversion (CRRA) with a coefficient of $\sigma = 2$. We show that the system's state can be described by a set of solvable differential equations, with the consumption-capital ratio following an autonomous Riccati equation.

Second, we derive the exact necessary conditions for an interior optimal reform time, $\tau^* \in (0, \infty)$, using the principles of optimal control for problems with endogenous switching times. The solution requires satisfying simultaneous conditions for the jump in the state (capital) and costate (shadow price of capital) variables, along with the continuity of the optimized Hamiltonian at the switch. This rigorous approach accounts for the crucial fact that the entire pre-reform consumption and investment path is chosen optimally in anticipation of the reform date. We demonstrate that this leads to a complex but economically intuitive timing condition that cannot be reduced to a simple rule.

Third, we use a unified calibration to demonstrate how the model generates three distinct and economically intuitive policy regimes. For a small capital loss κ, reform is optimal immediately ($\tau^* = 0$). For a moderate loss, it is optimal to delay reform to a specific future date. For a large capital loss, the disruption is so severe that it is optimal to postpone reform indefinitely ($\tau^* = \infty$). These results provide a clear taxonomy for policy analysis and highlight the critical role of managing transition costs.

Our analysis connects the literature on optimal control and switching times (Seierstad and Sydsaeter (1986)) with modern macroeconomic models of institutional change (Besley and Persson (2011)). By embedding transaction and governance costs directly into the core equations of a growth model, we provide a microfounded basis for analyzing the trade-offs inherent in the timing of structural reforms. Relatedly, Zou (1994) analyzed the optimal timing of privatization in a partial-equilibrium setting without modeling household behavior, in contrast to our general-equilibrium treatment with forward-looking consumers.

2 A Governance-Augmented RCK Framework

We adopt the RCK framework augmented with institutional costs from Zou (2025). A representative household seeks to maximize lifetime utility from consumption, $c(t)$:

$$U = \int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t}\,\mathrm{d}t, \tag{1}$$

where $\rho > 0$ is the rate of time preference and $\sigma > 0$ is the coefficient of relative risk aversion.

The household's budget constraint is modified to include real costs associated with the quality of institutions. The law of motion for capital per capita, $k(t)$, is:

$$\dot{k} = Ak - (\delta + \Gamma)k - \frac{1}{2}\Theta k^2 - (1+\Phi)c. \tag{2}$$

The institutional cost parameters (Γ, Θ, Φ) represent:

- Γk: Flow costs proportional to the capital stock, such as the costs of protecting assets from expropriation or theft.
- $\frac{1}{2}\Theta k^2$: Costs that increase with economic scale, capturing congestion externalities or increasing opportunities for rent-seeking in a larger economy.
- Φc: Transaction costs on consumption, such as sales taxes used to fund an inefficient bureaucracy or the costs of verifying product quality.

The current-value Hamiltonian for this problem is:

$$\mathcal{H} = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda \left[Ak - (\delta + \Gamma)k - \frac{1}{2}\Theta k^2 - (1+\Phi)c\right]. \tag{3}$$

The first-order conditions with respect to consumption and capital yield the static optimality condition and the costate equation, respectively:

$$c^{-\sigma} = \lambda(1+\Phi), \tag{4}$$

$$\rho\lambda - \dot{\lambda} = \lambda\left[A - (\delta+\Gamma) - \Theta k\right]. \tag{5}$$

Combining these conditions and differentiating (4) with respect to time leads to the familiar Euler equation for consumption growth: $\frac{\dot{c}}{c} = \frac{1}{\sigma}\left[A - (\delta+\Gamma) - \Theta k - \rho\right]$. A reform improves institutions from (Γ, Θ, Φ) to (γ, θ, ϕ), where the new parameters represent lower costs.

2.1 Steady-State Analysis

The steady state of the economy is characterized by $\dot{k} = 0$ and $\dot{c} = 0$. Let the pre-reform steady-state values be (k^*, c^*) and post-reform be (k^{**}, c^{**}). From the Euler equation, $\dot{c} = 0$ implies that the net marginal product of capital equals the time preference rate:

$$A - (\delta+\Gamma) - \Theta k^* = \rho \implies k^* = \frac{A-\delta-\Gamma-\rho}{\Theta}. \tag{6}$$

Similarly, for the post-reform steady state: $k^{**} = \frac{A-\delta-\gamma-\rho}{\theta}$.

From the capital accumulation equation, $\dot{k} = 0$ implies:

$$(A-\delta-\Gamma)k^* - \frac{1}{2}\Theta(k^*)^2 = (1+\Phi)c^*. \tag{7}$$

Let $x = c/k$ be the consumption-capital ratio. In steady state, $x^* = c^*/k^*$. Substituting this and the expression for k^* into the $\dot{k} = 0$ condition and simplifying gives the steady-state consumption-capital ratio for the pre-reform economy:

$$x^* = \frac{A-\delta-\Gamma+\rho}{2(1+\Phi)}. \tag{8}$$

By analogy, the post-reform steady-state ratio is:

$$x^{**} = \frac{A - \delta - \gamma + \rho}{2(1 + \phi)}. \tag{9}$$

The transversality condition requires that after the reform, the economy converges to the post-reform steady state (k^{**}, c^{**}).

3 Optimal Reform Timing with Anticipation

Suppose the economy begins with institutions (Γ, Θ, Φ). A reform to (γ, θ, ϕ) is anticipated at time τ, causing a capital loss: $k_{\text{post}}(\tau) = (1 - \kappa)k_{\text{pre}}(\tau)$. The problem is to choose the consumption path $\{c(t)\}_{t=0}^{\infty}$ and the reform time $\tau \in [0, \infty]$ to maximize lifetime utility. We specialize to the case of $\sigma = 2$ for analytical tractability.

3.1 System Dynamics and Necessary Conditions

The problem is a multi-stage optimal control problem with an endogenous switching time. The necessary conditions for an interior optimum τ^* are given by Pontryagin's Maximum Principle, extended to handle such switches.

Proposition 1 (Necessary Conditions for Optimal Reform)**.** *Let an interior reform time $\tau^* \in (0, \infty)$ be optimal. Let the pre-reform regime be denoted by subscript 1 and the post-reform regime by subscript 2. The optimal path must satisfy:*

1. *The state and costate dynamics in each regime.*

2. *A state variable jump condition at τ^*:*

$$k_2(\tau^*) = (1 - \kappa)k_1(\tau^*). \tag{10}$$

3. *A costate variable jump condition at τ^*, derived from value function continuity (see Appendix A):*

$$\lambda_1(\tau^*) = (1 - \kappa)\lambda_2(\tau^*). \tag{11}$$

4. A Hamiltonian continuity condition at τ^*:

$$\mathcal{H}_1(k_1, c_1, \lambda_1)\Big|_{\tau^*} = \mathcal{H}_2(k_2, c_2, \lambda_2)\Big|_{\tau^*}. \tag{12}$$

Proof. Conditions 1 and 2 are direct consequences of the problem's setup. Condition 3, the costate jump, ensures that there is no arbitrage opportunity in the value of holding capital across the reform boundary. Condition 4 is the first-order condition for the optimal choice of τ^*; it states that at the optimum, the marginal value of delaying the switch is zero, which implies the maximized Hamiltonian flows must be equal just before and just after the switch (Seierstad and Sydsaeter (1986)). □

3.2 Analytical Solution for $\sigma = 2$

For $\sigma = 2$, the dynamics can be conveniently expressed in terms of the consumption-capital ratio, $x(t) = c(t)/k(t)$. Its time derivative is $\dot{x}/x = \dot{c}/c - \dot{k}/k$. Substituting the expressions for $\dot{c}/c$ and $\dot{k}/k$ yields:

$$\frac{\dot{x}}{x} = \frac{1}{\sigma}\left[A - (\delta + \Gamma) - \Theta k - \rho\right] - \left[A - (\delta + \Gamma) - \frac{1}{2}\Theta k - (1 + \Phi)x\right] \tag{13}$$

$$\text{For } \sigma = 2: \quad \dot{x} = x\left[-\frac{1}{2}(A - \delta - \Gamma + \rho) + (1 + \Phi)x\right]. \tag{14}$$

This simplification occurs because the terms involving the capital stock k in the expressions for $\dot{c}/c$ and $\dot{k}/k$ fortuitously cancel out when $\sigma = 2$. The result is an autonomous Riccati equation for $x_1(t)$ in the pre-reform regime. Let $C_1 = \frac{1}{2}(A - \delta - \Gamma + \rho)$ and $B_1 = 1 + \Phi$. The equation $\dot{x}_1 = B_1 x_1^2 - C_1 x_1$ has the solution:

$$x_1(t) = \frac{x^*}{1 - (1 - x^*/x_1(0))\, e^{C_1 t}}, \tag{15}$$

where $x^* = C_1/B_1$ is the pre-reform steady-state ratio. The laws of motion for capital stock follows a logistic equation with a time-varying linear term:

$$\dot{k_1} = k_1\left[A - (\delta + \Gamma) - (1 + \Phi) * x_1\right] - \frac{1}{2}\Theta k_1^2. \tag{16}$$

This ODE is Bernoulli and admits a closed form via the standard $y = 1/k_1$ transformation. Because $x_1(t)$ is explicit, computing the trajectory of $k_1(t)$ numerically is straightforward.

After the reform at time τ, the transversality condition requires the economy to be on the saddle path to the new steady state (k^{**}, c^{**}). For the $\sigma = 2$ case, this implies that the consumption-capital ratio must immediately jump to its new steady-state value and remain there:

$$x_2(t) = x^{**} \quad \text{for all } t \geq \tau. \tag{17}$$

This simplifies the analysis considerably, as the entire post-reform path is known once $k_2(\tau)$ is determined.

3.3 The Optimal Timing Condition

We can now simplify the Hamiltonian continuity condition. The maximized Hamiltonian, $\mathcal{H}^*$, is found by substituting the optimal control rule back into the Hamiltonian. For $\sigma = 2$ and a utility function $u(c) = -1/c$, this simplifies to $\mathcal{H}^* = -2/c + \lambda f(k)$, where $f(k)$ is the part of $\dot{k}$ independent of consumption. The condition $\mathcal{H}_1^*(\tau^*) = \mathcal{H}_2^*(\tau^*)$ thus becomes:

$$-\frac{2}{c_1(\tau^*)} + \lambda_1(\tau^*) f_1(k_1(\tau^*)) = -\frac{2}{c_2(\tau^*)} + \lambda_2(\tau^*) f_2(k_2(\tau^*)), \tag{18}$$

where $f_1(k) = (A - \delta - \Gamma)k - \frac{1}{2}\Theta k^2$ and $f_2(k) = (A - \delta - \gamma)k - \frac{1}{2}\theta k^2$. This equation, combined with the jump conditions and the laws of motion, implicitly defines the optimal reform time τ^*.

Relationship between consumption and capital ratios. The costate jump condition provides a powerful link between the consumption paths. From $\lambda_1 = (1 - \kappa)\lambda_2$ and the first-order condition $c_i^{-2} = \lambda_i(1 + \Phi_i)$ (for $\sigma = 2$), we have:

$$\frac{c_1(\tau^*)^{-2}}{1 + \Phi} = (1 - \kappa)\frac{c_2(\tau^*)^{-2}}{1 + \phi} \implies c_1(\tau^*) = c_2(\tau^*)\sqrt{\frac{1 + \phi}{(1 - \kappa)(1 + \Phi)}}. \tag{19}$$

Since the post-reform path is the saddle path, $c_2(\tau^*) = x^{**}k_2(\tau^*) = x^{**}(1 - \kappa)k_1(\tau^*)$. Substituting this and dividing by $k_1(\tau^*)$ gives the relationship between the pre- and post-reform consumption-capital ratios at the moment of the switch:

$$x_1(\tau^*) = x^{**}\sqrt{\frac{(1-\kappa)(1+\phi)}{1+\Phi}}. \tag{20}$$

This elegant result is a necessary condition for an optimal switch. It defines the target consumption-capital ratio, $x_1(\tau^*)$, that the economy must reach at the moment of reform. This transforms the problem of finding the optimal reform time τ^* into a search for the correct initial consumption choice, $c_1(0)$, that steers the economy to the optimal switching point where the Hamiltonian continuity condition (18) holds.

4 Numerical Illustration and Policy Regimes

To illustrate the model's predictions, we solve the system numerically for a specific calibration. An interior solution $\tau^* \in (0, \infty)$ requires that it is better to wait at $t = 0$ (i.e., $\mathcal{H}_1^*(k_0) > \mathcal{H}_2^*(k_0)$), and that at time τ^*, the Hamiltonian flows are equalized.

Primitives and Derived Parameters:

- Technology & Preferences: $A = 0.33$, $\delta = 0.05$, $\rho = 0.03$, $\sigma = 2$.
- Pre-Reform Institutions: $(\Gamma, \Theta, \Phi) = (0.06, 0.20, 0.10)$.
- Post-Reform Institutions: $(\gamma, \theta, \phi) = (0.05, 0.18, 0.08)$.
- Initial State: $k_0 = 0.2$.
- Derived Steady-State Values:
  - Pre-reform: $k^* = 0.95$, $x^* \approx 0.1136$.
  - Post-reform: $k^{**} \approx 1.111$, $x^{**} \approx 0.1204$.

The solution method involves a shooting algorithm: we guess an initial consumption-capital ratio $x_1(0)$, evolve the system forward, and find the time τ at which $x_1(\tau)$ hits the target value from (20). We then check if the Hamiltonian continuity condition is met at that point. If not, we adjust the initial guess $x_1(0)$ and repeat until the condition holds.

Case 1: Immediate Reform (Small Loss, e.g., $\kappa = 0.05$). With a very small capital loss, the benefits of the new institutions are immediately accessible and outweigh the minor disruption. At the initial state $k_0 = 0.2$, a calculation of the maximized Hamiltonians shows that $\mathcal{H}_2^*(k_0) > \mathcal{H}_1^*(k_0)$. The marginal value of switching is strictly positive. It is therefore optimal to reform immediately. The solution is a corner solution at $\tau^* = 0$.

Case 2: Interior Reform (Moderate Loss, e.g., $\kappa = 0.20$). With a moderate capital loss, the disruption is significant enough to make immediate reform suboptimal. At the initial capital stock $k_0 = 0.2$, the Hamiltonian flow from waiting is greater than from switching: $\mathcal{H}_1^*(k_0) > \mathcal{H}_2^*(k_0)$. It is better to wait and accumulate capital. The optimal path involves choosing an initial consumption ratio $x_1(0) \approx 0.112651$, which is slightly lower than the pre-reform steady-state ratio ($x^* \approx 0.1136$). This leads to a waiting period during which the capital stock grows and the consumption-capital ratio $x_1(t)$ decreases towards its target.

$$\boxed{\text{For } \kappa = 0.20 \text{ and } k_0 = 0.2\text{, the numerical solution yields } \tau^* \approx 16 \text{ years.}}$$

At $t \approx 16$ years, the economy reaches a capital stock of $k_1(\tau^*) \approx 0.5466$ and the consumption ratio hits its target $x_1(\tau^*) \approx 0.1067$. At this precise point, the Hamiltonian continuity condition is satisfied, making it the optimal moment to enact the reform.

Case 3: Never Reform (Large Loss, e.g., $\kappa = 0.30$). If the capital loss is too large, the disruption is so costly that it is never optimal to switch. The condition $\mathcal{H}_1^*(k) > \mathcal{H}_2^*(k)$ holds for all economically relevant capital stocks. The marginal value of switching is always negative. The optimal policy is to postpone reform indefinitely. The solution is $\tau^* = \infty$, and the economy remains in the institutional trap, converging to its pre-reform steady state (k^*, c^*).

5 Discussion and Conclusion

We have developed a tractable dynamic model to analyze the optimal timing of an anticipated institutional reform. By applying the proper methods from optimal control theory, we characterized the optimal timing decision with a set of necessary conditions that account for the forward-looking nature of economic agents. Our central finding is that delay can be optimal. The presence of transition costs creates a powerful incentive to postpone reform, allowing the economy to reach a more favorable state from which to absorb the shock. The optimal waiting period is not arbitrary; it is the precise amount of time needed for the economy to evolve to a point where the complex trade-offs embodied in the Hamiltonian continuity condition are perfectly balanced.

To achieve this clarity, our model deliberately employs several simplifying assumptions, most notably the deterministic setting, the representative-agent structure, and the specialization to CRRA utility with $\sigma = 2$. Relaxing these assumptions provides a rich agenda for future research. For instance, if $\sigma \neq 2$, the post-reform consumption-capital ratio no longer jumps to its steady-state value but follows a dynamic saddle path. While this removes some analytical convenience and requires a fully numerical solution, the core principles of our approach—satisfying the Hamiltonian continuity and the state/costate jump conditions simultaneously—remain the correct methodology. We conjecture that our main qualitative result, the existence of three distinct timing regimes (immediate, delayed, or never), would survive this generalization. Introducing uncertainty, for example, regarding the magnitude of the disruption cost κ, would create an "option value of waiting" that would likely reinforce the incentive to delay reform.

Our benevolent planner framework also abstracts from important political economy considerations. In reality, reforms create winners and losers. A model with heterogeneous agents could show how distributional conflicts lead to a politically determined reform time, τ^P, that differs from the socially optimal time, τ^*. Similarly, if the government's promise to reform is not fully credible, the problem shifts from one of optimal control to one of dynamic game theory, where one would solve for a time-consistent or Markov perfect equilibrium.

The framework is flexible and can be extended to explore alternative modeling choices. One could model gradual reform with convex adjustment costs, or enrich the model by allowing institutional quality to enter the utility function directly, reflecting the idea that political freedom and rule of law provide direct well-being (Frey and Stutzer (2000) and Sen (1999)). Furthermore, the nature of the transition cost itself can be varied. As shown in the appendices, an additive capital loss implies a continuous shadow price of capital, unlike the multiplicative loss analyzed in the main text. Finally, future work could apply the model empirically by calibrating the institutional cost parameters using governance indicators and estimating transition costs from historical reform episodes.

In conclusion, despite these avenues for extension, our work offers a rigorous and transparent benchmark for analyzing the fundamental trade-offs in reform timing. It establishes the core dynamic principles that must underpin any richer, more complex analysis. The main policy implications are clear and robust: managing transition costs is paramount, and credibility is key. Policies that reduce the disruption from reform can dramatically alter the optimal timing from "never" or "later" to "now." For reforms with unavoidable, significant transition costs, our analysis provides a rationale for a well-communicated, deliberate delay, which can be superior for long-run welfare to a rushed and disruptive implementation.

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A Derivation of the Costate Jump Condition for Multiplicative Loss

The costate jump condition is a fundamental result in optimal control with state jumps, arising from the principle of value function continuity. Let $V_1(k,t)$ be the value function in the pre-reform regime and $V_2(k,t)$ be the value function in the post-reform regime.

At the moment of the switch τ, an agent holding capital $k_1(\tau)$ transitions to a state with capital $k_2(\tau) = (1-\kappa)k_1(\tau)$. For the switch to be seamless in value, the utility value just before the switch must equal the utility value just after. Since the problem is autonomous, the value functions do not depend explicitly on time, so we write $V_1(k)$ and $V_2(k)$.

$$V_1(k_1(\tau)) = V_2(k_2(\tau)) = V_2((1-\kappa)k_1(\tau)). \tag{21}$$

This equality must hold for any level of capital $k_1(\tau)$ at which a switch might occur. We can therefore treat it as an identity in k_1 and differentiate both sides with respect to $k_1(\tau)$.

The costate variable (shadow price of capital) is defined as the derivative of the value function with respect to the state variable:

$$\lambda_1(\tau) = \left.\frac{\mathrm{d}V_1(k_1)}{\mathrm{d}k_1}\right|_{k_1=k_1(\tau)} \quad \text{and} \quad \lambda_2(\tau) = \left.\frac{\mathrm{d}V_2(k_2)}{\mathrm{d}k_2}\right|_{k_2=k_2(\tau)}. \tag{22}$$

Differentiating the value continuity equation with respect to $k_1(\tau)$ yields:

$$\frac{\mathrm{d}V_1(k_1(\tau))}{\mathrm{d}k_1(\tau)} = \frac{\mathrm{d}}{\mathrm{d}k_1(\tau)}\left[V_2((1-\kappa)k_1(\tau))\right]. \tag{23}$$

Applying the chain rule to the right-hand side:

$$\frac{\mathrm{d}V_1}{\mathrm{d}k_1} = \frac{\mathrm{d}V_2}{\mathrm{d}k_2} \cdot \frac{\mathrm{d}k_2}{\mathrm{d}k_1}. \tag{24}$$

Substituting the definitions of the costates and the state jump rule $\frac{\mathrm{d}k_2}{\mathrm{d}k_1} = 1-\kappa$, we get:

$$\lambda_1(\tau) = \lambda_2(\tau) \cdot (1-\kappa). \tag{25}$$

This is the required jump condition for the costate variable. It shows that if the reform destroys a fraction κ of the capital stock, the marginal value of a unit of capital just before the reform must be smaller by a factor of $(1-\kappa)$ than the marginal value of a unit of capital just after. This is because one pre-reform unit of capital becomes only $(1-\kappa)$ post-reform units, so its value must be discounted accordingly at the margin.

B Derivation of the Costate Jump Condition for Additive Loss

Consider an alternative scenario where the reform incurs an additive capital loss, $k_{\text{post}}(\tau) = k_{\text{pre}}(\tau) - \xi$, where $\xi > 0$ is a fixed amount of capital. The principle of value function continuity at the switch time τ still holds:

$$V_1(k_1(\tau)) = V_2(k_2(\tau)) = V_2(k_1(\tau) - \xi). \tag{26}$$

As before, we treat this as an identity in $k_1(\tau)$ and differentiate with respect to $k_1(\tau)$ to find the relationship between the shadow prices.

$$\frac{\mathrm{d}V_1(k_1(\tau))}{\mathrm{d}k_1(\tau)} = \frac{\mathrm{d}}{\mathrm{d}k_1(\tau)} \left[V_2(k_1(\tau) - \xi) \right]. \tag{27}$$

Applying the chain rule to the right-hand side gives:

$$\frac{\mathrm{d}V_1}{\mathrm{d}k_1} = \frac{\mathrm{d}V_2}{\mathrm{d}k_2} \cdot \frac{\mathrm{d}k_2}{\mathrm{d}k_1} = \frac{\mathrm{d}V_2}{\mathrm{d}k_2} \cdot \frac{\mathrm{d}(k_1 - \xi)}{\mathrm{d}k_1}. \tag{28}$$

Since the derivative of $(k_1 - \xi)$ with respect to k_1 is simply 1, the condition simplifies to:

$$\lambda_1(\tau) = \lambda_2(\tau). \tag{29}$$

In the case of an additive loss, the costate variable is continuous across the reform. The intuition is that a marginal unit of capital is not lost in the transition; it is fully transferred from the pre-reform to the post-reform regime, even though the total stock of capital is reduced by a fixed amount ξ. Therefore, its marginal value must be the same immediately before and after the switch.